Strange form factors in ν scattering

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- Looking for strangeness with neutrino scattering
- Interesting observables
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Looking for strangeness with ν scattering

The measurement of NC neutrino cross sections

$$\nu_\mu(\overline{\nu}_\mu) + N \longrightarrow \nu_\mu(\overline{\nu}_\mu) + N \tag{1}$$

is very important tool for the determination of the matrix elements of the strange current:

$$\langle p, s|\bar{S}\gamma^\alpha\gamma^5 S|p, s\rangle = 2Ms^\alpha g_A^s$$

$S, \bar{S}$ strange quark fields

$|p, s\rangle$ proton (momentum, spin) state vector.

CC processes also considered:

$$\begin{aligned} \nu_\mu + n &\longrightarrow \mu^- + p\,, \\ \overline{\nu}_\mu + p &\longrightarrow \mu^+ + n\,. \end{aligned} \tag{2}$$

Nucleon currents involved:

$$J_\alpha^Z = V_\alpha^3 + A_\alpha^3 - 2\sin^2\theta_W J_\alpha^{em} - \frac{1}{2}V_\alpha^s - \frac{1}{2}A_\alpha^s$$

$$J_\alpha^W = V_{ud}\langle N|\bar{U}\gamma_\alpha(1+\gamma_5)D|N\rangle$$

Weak neutral current and $s\bar{s}$ content

One nucleon matrix element of axial quark current:

$$\langle p, s|\bar{q}\gamma^\alpha\gamma^5 q|p, s\rangle = 2Ms^\alpha g_A^q$$

constants g_A^u, g_A^d, g_A^s determined from:

- QCD sum rule (polarized structure function)

$$\Gamma_1^p = \int_0^1 dx g_1^p(x) = \frac{1}{2}\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right)$$

- relation $g_A = g_A^u - g_A^d$
  with $g_A = 1.2573 \pm 0.0028$ from neutron decay

- relation $3F - D = g_A^u + g_A^d - 2g_A^s$
  F, D from semileptonic decay of hyperons.

Determination of various g_A^q subject to several assumptions (small x extrapolation, QCD corrections, SU(3) invariance, etc.)

Notations

One nucleon matrix elements of the vector and axial NC:

$$
{}_{p(n)}\langle p'|V_\alpha^{NC}|p\rangle_{p(n)} = \overline{u}(p')\left[\gamma_\alpha F_1^{NC;p(n)}(Q^2) + \right.
$$

$$
\left. + \frac{i}{2M}\sigma_{\alpha\beta}q^\beta F_2^{NC;p(n)}(Q^2)\right] u(p)
$$

$$
{}_{p(n)}\langle p'|A_\alpha^{NC}|p\rangle_{p(n)} = \overline{u}(p')\gamma_\alpha\gamma_5 G_A^{NC;p(n)} u(p)
$$

where the NC form factors are given by

$$
F_{1,2}^{NC;p(n)}(Q^2) = \pm\frac{1}{2}\left\{F_{1,2}^p(Q^2) - F_{1,2}^n(Q^2)\right\} -
$$

$$
-2\sin^2\theta_W F_{1,2}^{p(n)}(Q^2) - \frac{1}{2}F_{1,2}^s(Q^2)
$$

$$
G_A^{NC;p(n)}(Q^2) = \pm\frac{1}{2}G_A(Q^2) - \frac{1}{2}G_A^s(Q^2)
$$

Equivalently, NC Sachs form factors are used:

$$
G_E{}^{NC;p(n)}(Q^2) = \pm\frac{1}{2}\left\{G_E^p(Q^2) - G_E^n(Q^2)\right\} -
$$

$$
-2\sin^2\theta_W G_E^{p(n)}(Q^2) - \frac{1}{2}G_E^s(Q^2)
$$

$$
G_M{}^{NC;p(n)}(Q^2) = \pm\frac{1}{2}\left\{G_M^p(Q^2) - G_M^n(Q^2)\right\} -
$$

$$
-2\sin^2\theta_W G_M^{p(n)}(Q^2) - \frac{1}{2}G_M^s(Q^2)
$$

Interesting Observables

Consider ν–proton elastic cross sections or ν–nucleus elastic and inelastic cross sections

NC over CC ratio:

$$R_{NC/CC}(Q^2) = \frac{\left(d\sigma/dQ^2\right)_{\nu}^{NC}}{\left(d\sigma/dQ^2\right)_{\nu}^{CC}} \tag{3}$$

Asymmetry:

$$\mathcal{A}(Q^2) = \frac{\left(\frac{d\sigma}{dQ^2}\right)_{\nu}^{NC} - \left(\frac{d\sigma}{dQ^2}\right)_{\overline{\nu}}^{NC}}{\left(\frac{d\sigma}{dQ^2}\right)_{\nu}^{CC} - \left(\frac{d\sigma}{dQ^2}\right)_{\overline{\nu}}^{CC}} \tag{4}$$

Ratio p/n

$$R^{\nu}_{p/n}(Q^2) = \frac{\left(\frac{d\sigma}{dQ^2}\right)_{(\nu,p)}^{NC}}{\left(\frac{d\sigma}{dQ^2}\right)_{(\nu,n)}^{NC}} \tag{5}$$

Elastic NC ν–nucleon scattering

Differential cross sections:

$$\left(\frac{d\sigma}{dQ^2}\right)^{NC}_{\nu(\overline{\nu})} = \frac{G_F^2}{2\pi}\left[\frac{1}{2}y^2(G_M^{NC})^2 + \right.$$

$$+\left(1-y-\frac{M}{2E}y\right)\frac{(G_E^{NC})^2+\frac{E}{2M}y(G_M^{NC})^2}{1+\frac{E}{2M}y}$$

$$+\left(\frac{1}{2}y^2+1-y+\frac{M}{2E}y\right)(G_A^{NC})^2$$

$$\left.\pm 2y\left(1-\frac{1}{2}y\right)G_M^{NC}G_A^{NC}\right].$$

with

$$y=\frac{p\cdot q}{p\cdot k}=\frac{Q^2}{2p\cdot k}$$

E is the energy of neutrino (antineutrino) in the laboratory system.

Note

To obtain information on the strange form factors it is compulsory to know the axial CC form factos G_A (via charge-exchange quasi–elastic processes).

The $\nu - \bar{\nu}$ asymmetry

The neutrino-antineutrino asymmetry in $\nu(\bar{\nu})$-nucleon elastic scattering reads:

$$\mathcal{A}_{p(n)} = \frac{1}{4}\left(\pm 1 - \frac{G_A^s}{G_A}\right) \times$$
$$\times \left(\pm 1 - 2\sin^2\theta_W \frac{{G_M}^{p(n)}}{G_M^3} - \frac{1}{2}\frac{G_M^s}{G_M^3}\right).$$

Thus, in the asymmetry $\mathcal{A}$ the strange axial and vector form factors enter in the form of ratios, G_A^s/G_A and G_M^s/G_M^3.

Taking into account only terms which linearly depend on the strange form factors:

$$\mathcal{A}_{p(n)} = \mathcal{A}^0_{p(n)} \mp \frac{1}{8}\frac{G_M^s}{G_M^3} \mp \frac{G_A^s}{G_A}\mathcal{A}^0_{p(n)}$$

with

$$\mathcal{A}^0_{p(n)} = \frac{1}{4}\left(1 \mp 2\sin^2\theta_W \frac{{G_M}^{p(n)}}{G_M^3}\right)$$

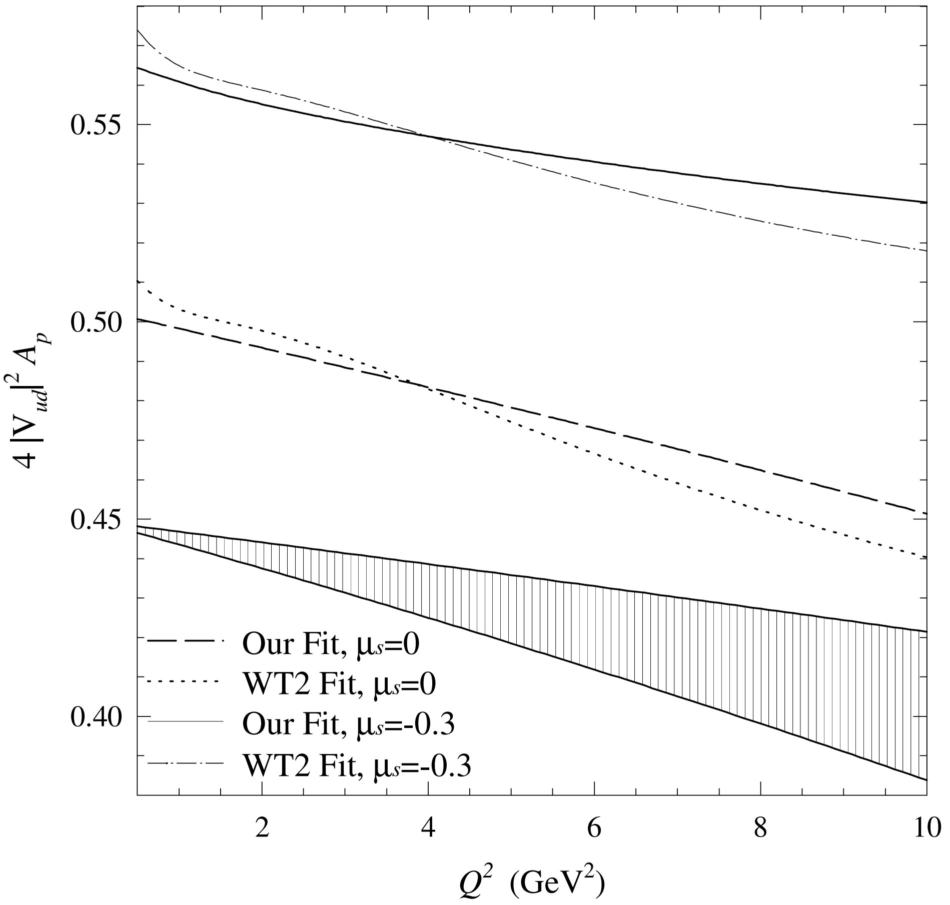


Fig. 1 – Plot of $4|V_{ud}|^2\mathcal{A}_p$ as a function of Q^2. The shadowed area corresponds to the present uncertainty in the magnetic form factors. The other curves were obtained using a dipole form for $F_A^s(Q^2)$ with $g_A^s = -0.15$. The dashed (dotted) curve was obtained with $G_M^S(Q^2) = 0$ utilizing our fit (respectively, the WT2 fit) for the magnetic FF of the nucleon. T he solid (dot–dashed) line was analogously obtained, setting $\mu_s = -0.3$.

The BNL - 734 experiment

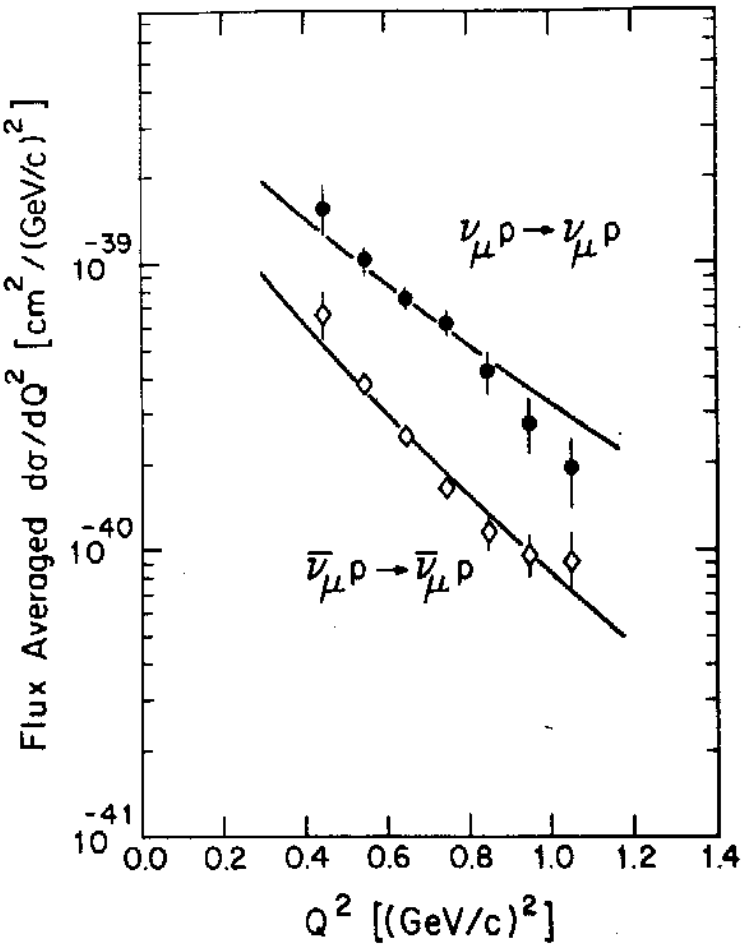


Fig. 2 – Flux averaged differential cross sections measured by Ahrens et al., PRD35 (1987). Solid curves are the best fit to the combined data using $M_A = 1.06$ GeV and $\sin^2_W = 0.220$. The same data are compatible (at 90% CL) with $-0.25 \leq G_A^s(0) \leq 0$ if the axial cutoff is constrained to $M_A = 1.032 \pm 0.036$ GeV.

Information about the above mentioned ratios has been obtained by the BNL - 734 experiments.
They measured:

$$R_\nu = \frac{\langle\sigma\rangle_{(\nu p\to\nu p)}}{\langle\sigma\rangle_{(\nu n\to\mu^- p)}} = 0.153 \pm 0.007 \pm 0.017$$

$$R_{\overline{\nu}} = \frac{\langle\sigma\rangle_{(\overline{\nu} p\to\overline{\nu} p)}}{\langle\sigma\rangle_{(\overline{\nu} p\to\mu^+ n)}} = 0.218 \pm 0.012 \pm 0.023$$

$$R = \frac{\langle\sigma\rangle_{(\overline{\nu} p\to\overline{\nu} p)}}{\langle\sigma\rangle_{(\nu p\to\nu p)}} = 0.302 \pm 0.019 \pm 0.037 \ ,$$

where $\langle\sigma\rangle_{\nu(\bar\nu)}$ is a total cross section integrated over the incident neutrino (antineutrino) energy and weighted by the $\nu(\bar\nu)$ flux. The first error is statistical and the second is the systematic one.
In terms of these ratios, the "integrated" asymmetry reads:

$$\langle\mathcal{A}_p\rangle = \frac{R_\nu(1-R)}{1-RR_\nu/R_{\overline{\nu}}}$$

and from the experimental data we found

$$\langle\mathcal{A}_p\rangle = 0.136 \pm 0.008(\text{stat}) \pm 0.019(\text{syst})$$

Fig. 3 – The ratios R_ν and $R_{\bar{\nu}}$, R and $\langle \mathcal{A}_p \rangle$ versus μ_s. Both for $g_A^s = 0$ and $g_A^s = -0.15$ three choices of ρ_s are shown: $\rho_s = 0$ (solid), $\rho_s = -2$ (dot–dashed) and $\rho_s = +2$ (dashed). The shadowed regions correspond to the experimental band.

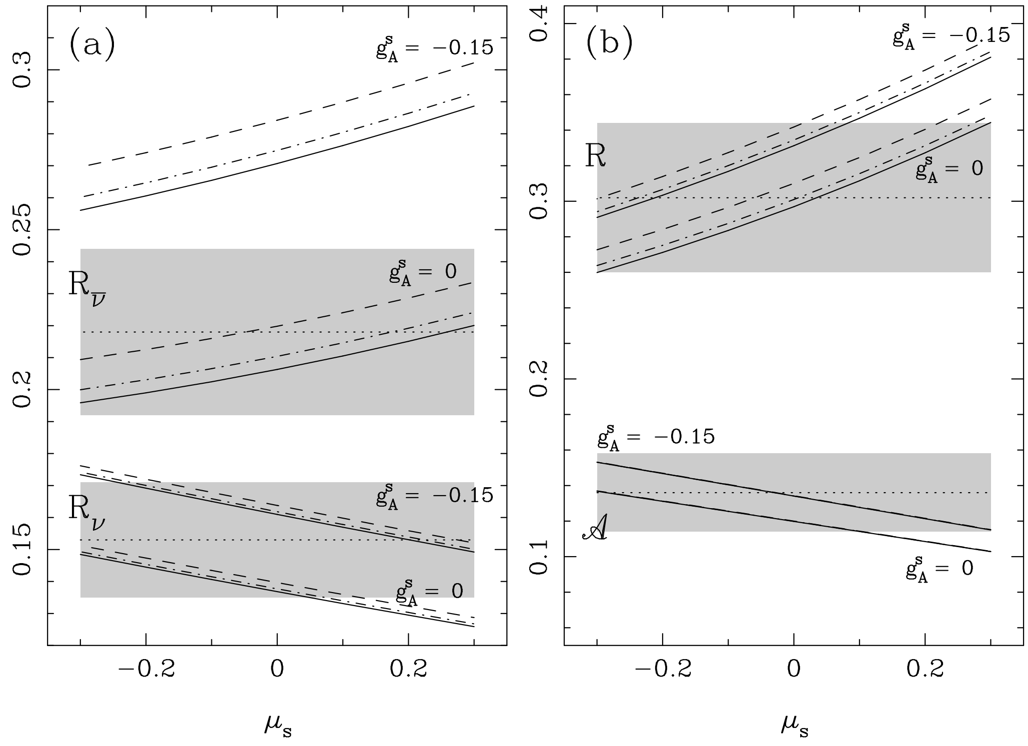
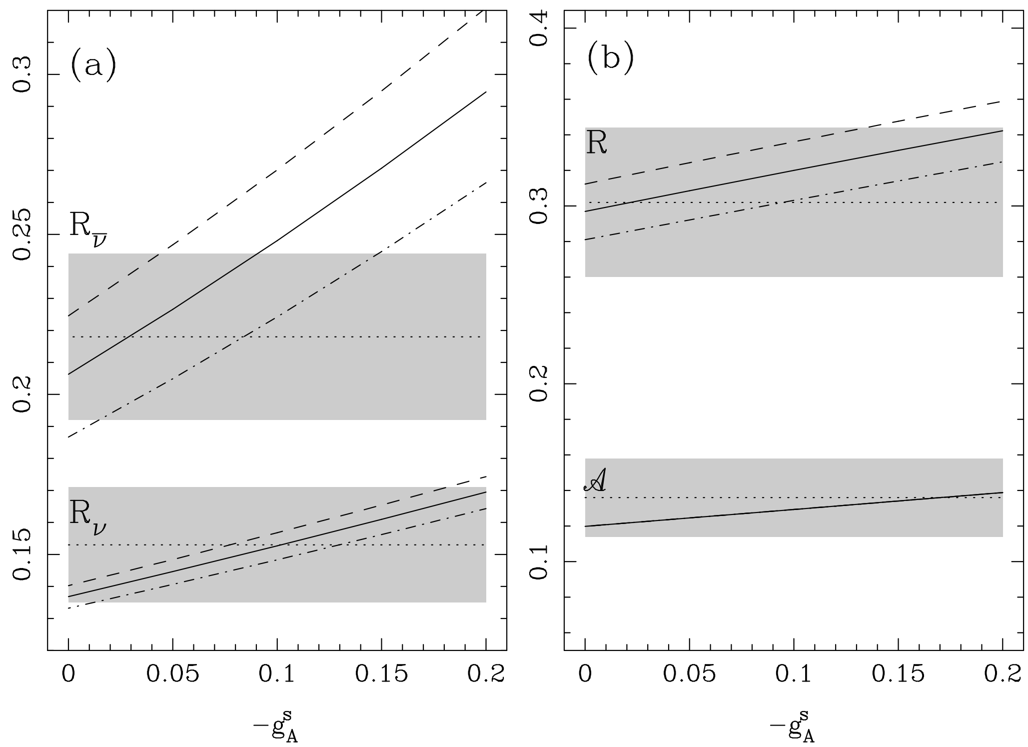


Fig. 4 – The ratios R_ν, $R_{\bar{\nu}}$, R and $\langle \mathcal{A}_p \rangle$ versus g_A^s: the sensitivity to M_A is shown: $M_A = 1.032$ GeV (solid), $M_A = 1.068$ GeV (dashed) and $M_A = 0.996$ GeV (dot–dashed). Here $\mu_s = \rho_s = 0$.



The ratio of proton to neutron yield

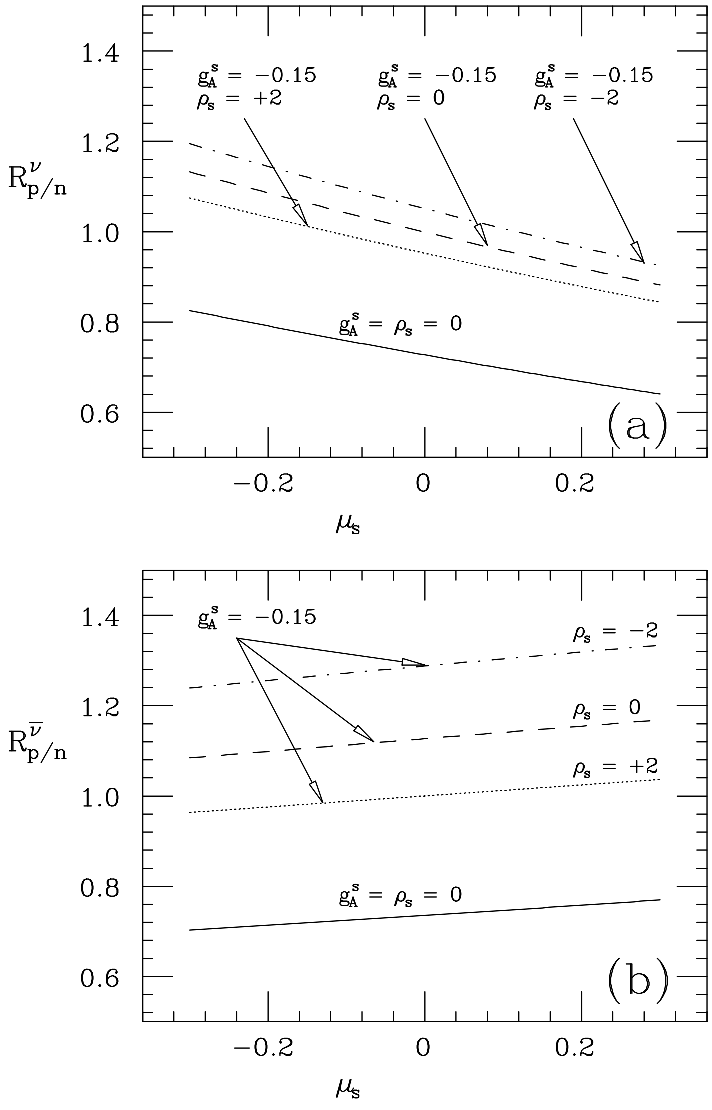


Fig. 5 – Ratio of the integrated inelastic ν ($\bar{\nu}$)– nucleus scattering cross sections, with emission (and detection) of a proton or neutron, calculated in RFG.
$E_{\nu(\bar{\nu})} = 1$ GeV.

Perspectives at NuMi: sensitivity to strange FF

We have considered, for the NuMi low–energy neutrino flux:

- The ratio of NC and CC elastic νp scattering

  1. It is sensitive to g_A^s, but not much affected by the cutoff mass of the axial form factors, assumed in the dipole form:

$$G_A(Q^2) = \frac{1.26}{\left(1 + Q^2/M_A^2\right)}, \quad G_A^s(Q^2) = \frac{g_A^s}{\left(1 + Q^2/M_A^2\right)}$$

  2. Different parameterizations of the e.m. form factors do not sensibly affect the ratio
  3. The interference between axial and vector strange form factors (in particular: magnetic strange ff) can hinder the effetc of g_A^s alone.
  4. The sensitivity to the flux is negligible, because of ratio
  5. Nuclear effects (again negligible, because of ratio)

- The ratio of NC and CC elastic $\bar{\nu} p$ scattering (if possible, gives great complementary information)

- The ratio of NC and CC elastic νp integrated cross sections

  (integration over Q^2 between 0.15 and 0.64 GeV^2.)

  The ratio $\sigma^{NC}_{\nu p}/\sigma^{CC}_{\nu}$ as a function of g^s_A, shows the correlation with other effects, e.g.:

  1. Different values of M_A
  2. Different values of μ_s
  3. Different cutoff in strange ff

- The ratio of proton to neutron yield in quasi-elastic neutrino scattering on ^{12}C

  1. Sensitivity to g^s_A
  2. Interference between axial and vector strange ff
  3. Elastic versus relativistic Fermi gas

Fig. 6 – Ratio of elastic NC/CC ν-proton scattering: effects of the axial strange form factor

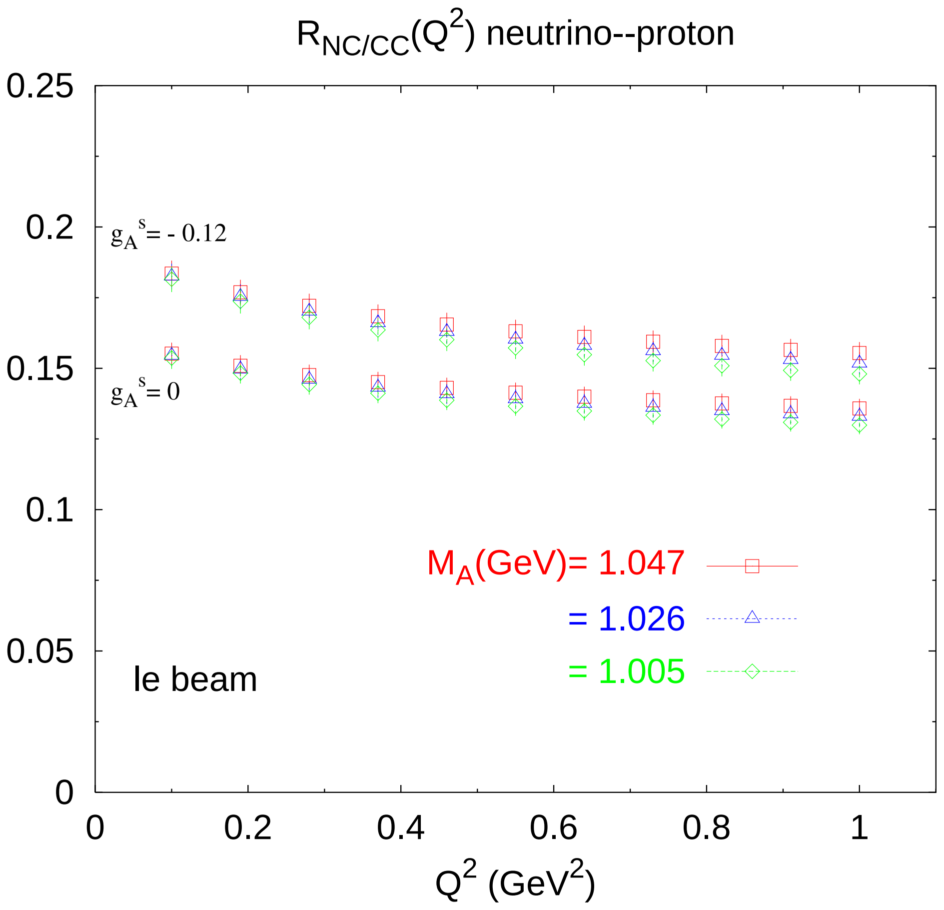


Fig. 7 – Sensitivity to the parameterization of electromagnetic form factors.

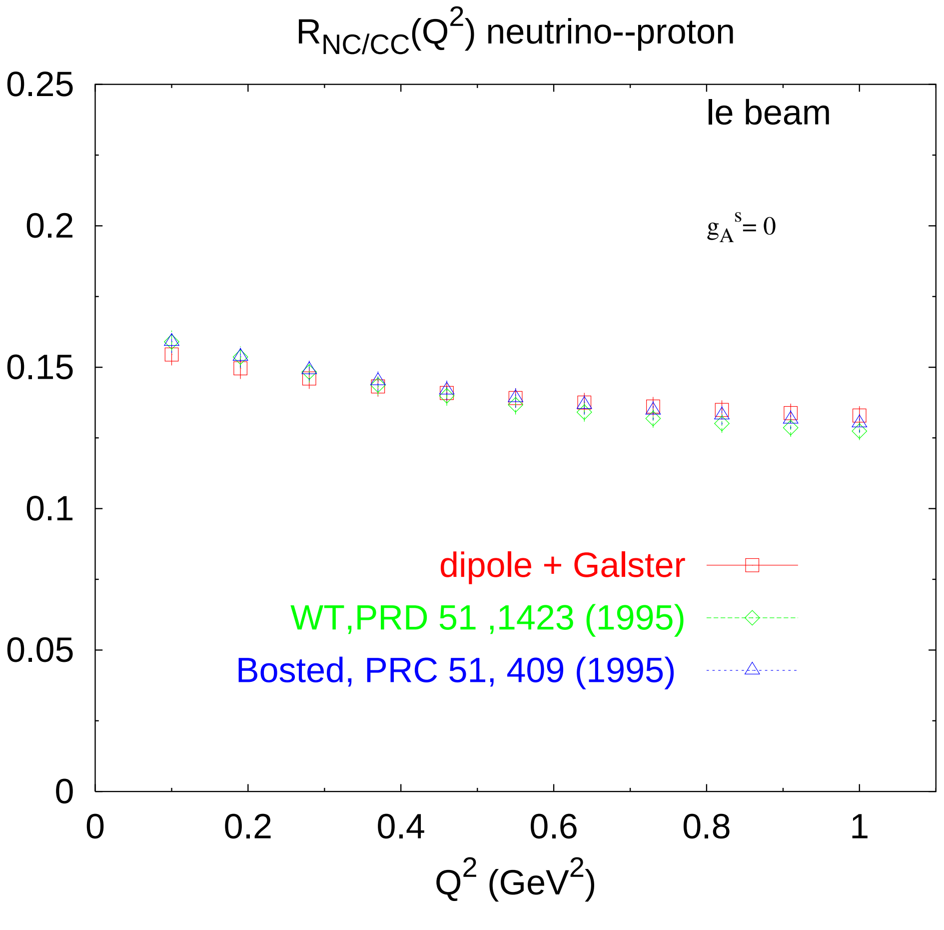


Fig. 8 – Sensitivity to the vector strange form factors

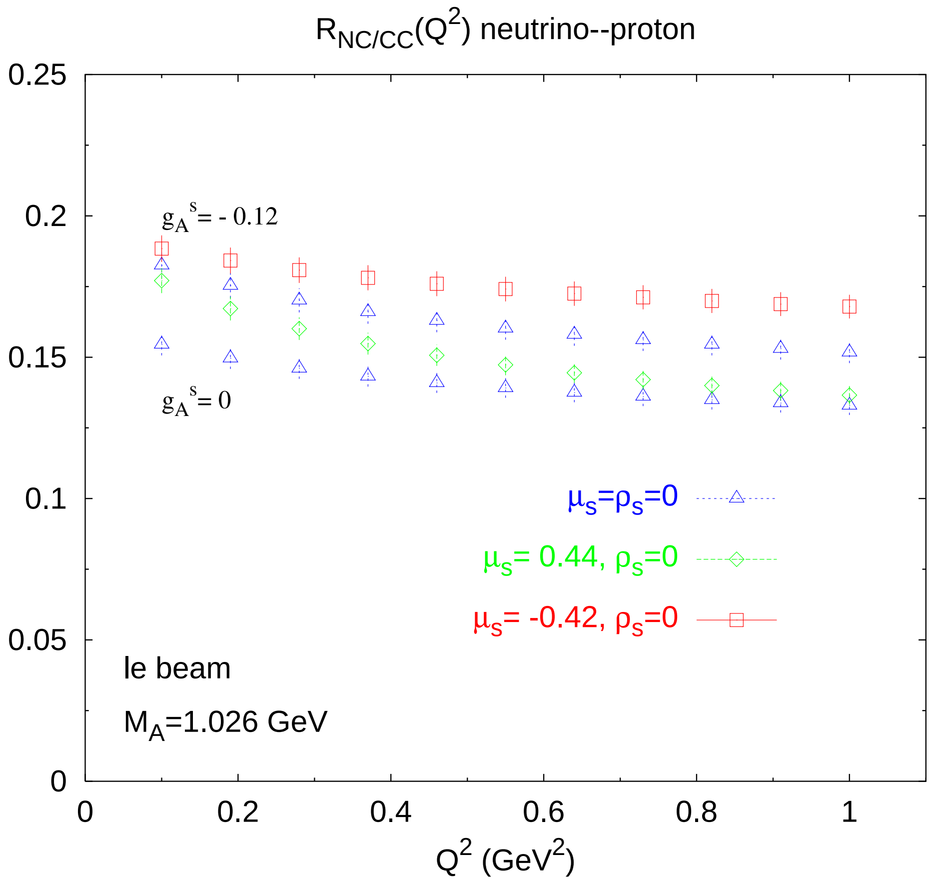


Fig. 9 – Example of the ratio of $\bar{\nu}$–proton cross sections.

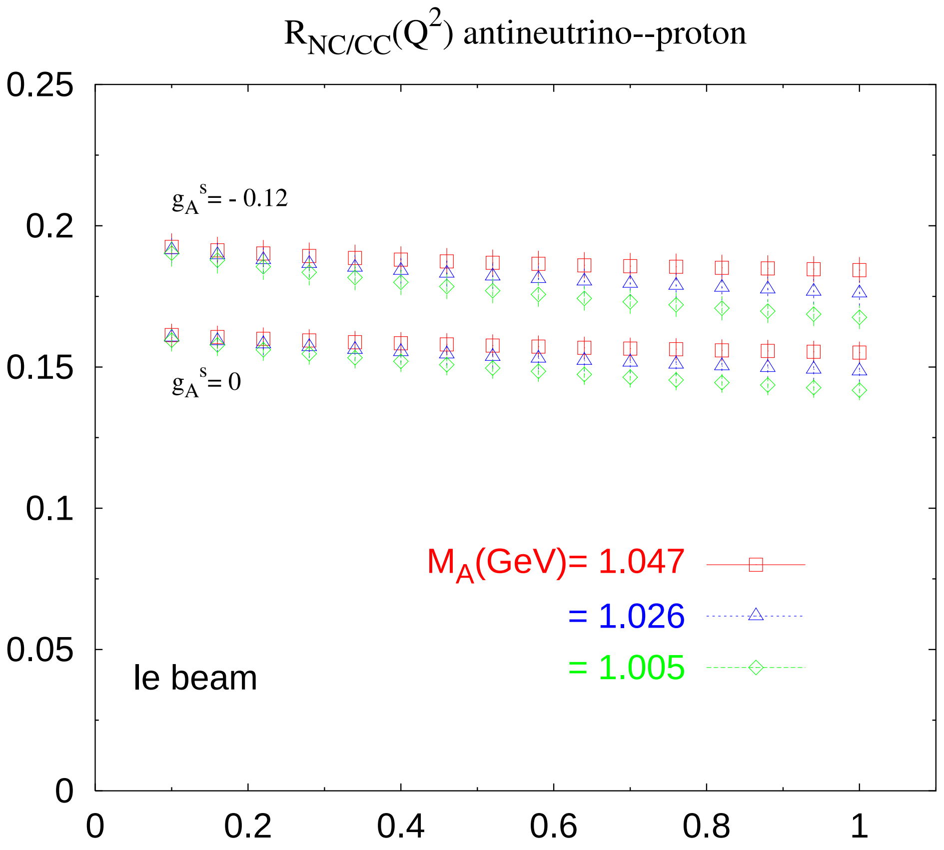


Fig. 10 – Sensitivity of $\bar{\nu}$–proton to vector strange form factors

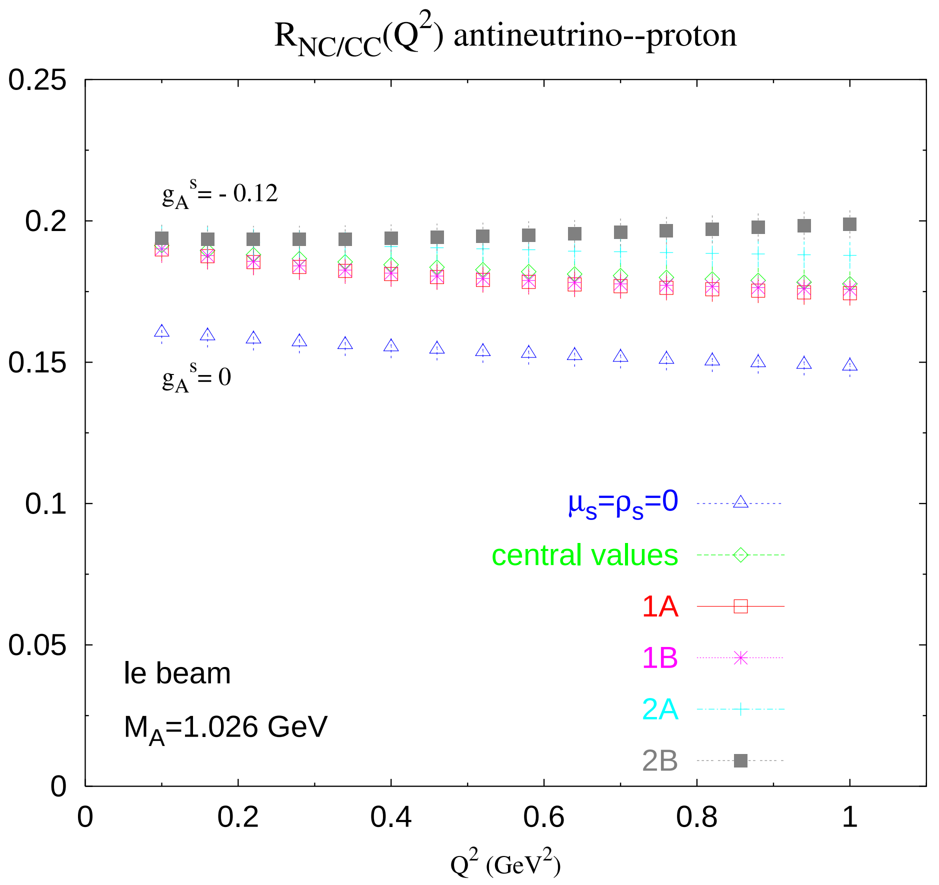


Fig. 11 – Ratio of total cross sections versus g_A^s: sensitivity to μ_s

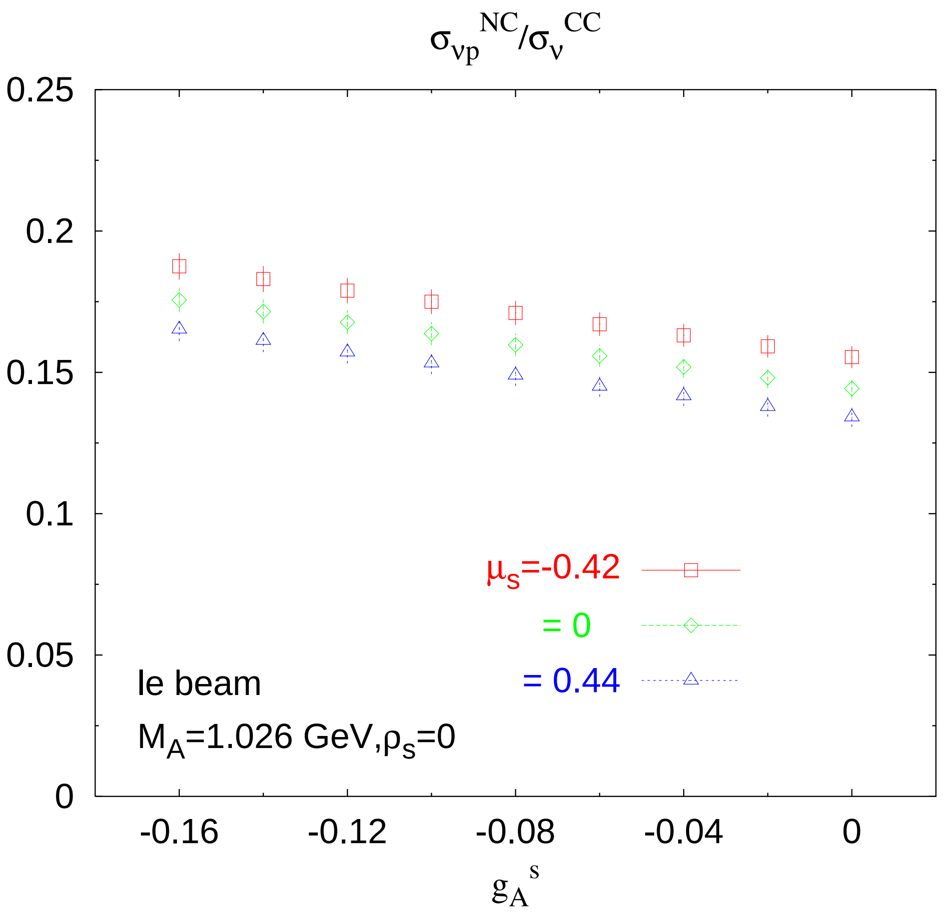


Fig. 12 – Ratio of total cross sections versus g_A^s: elastic versus inelastic on ^{12}C.

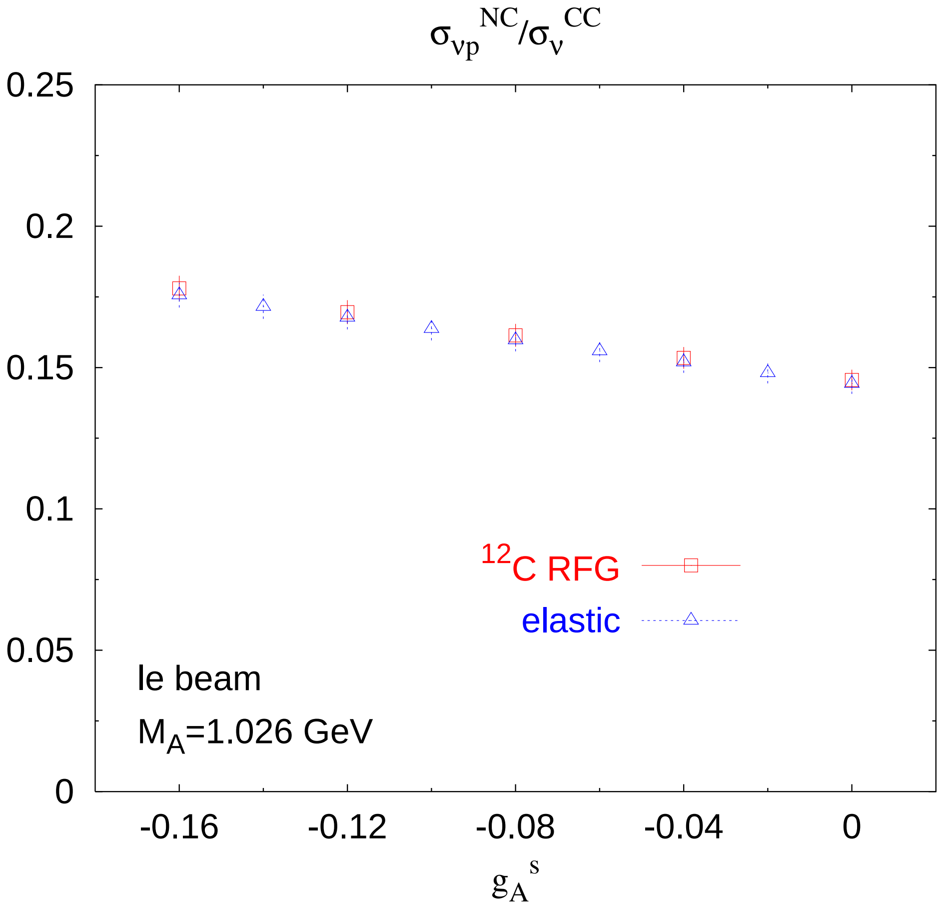


Fig. 13 – Ratio of proton to neutron yields in quasi-elastic NC neutrino scattering on ^{12}C

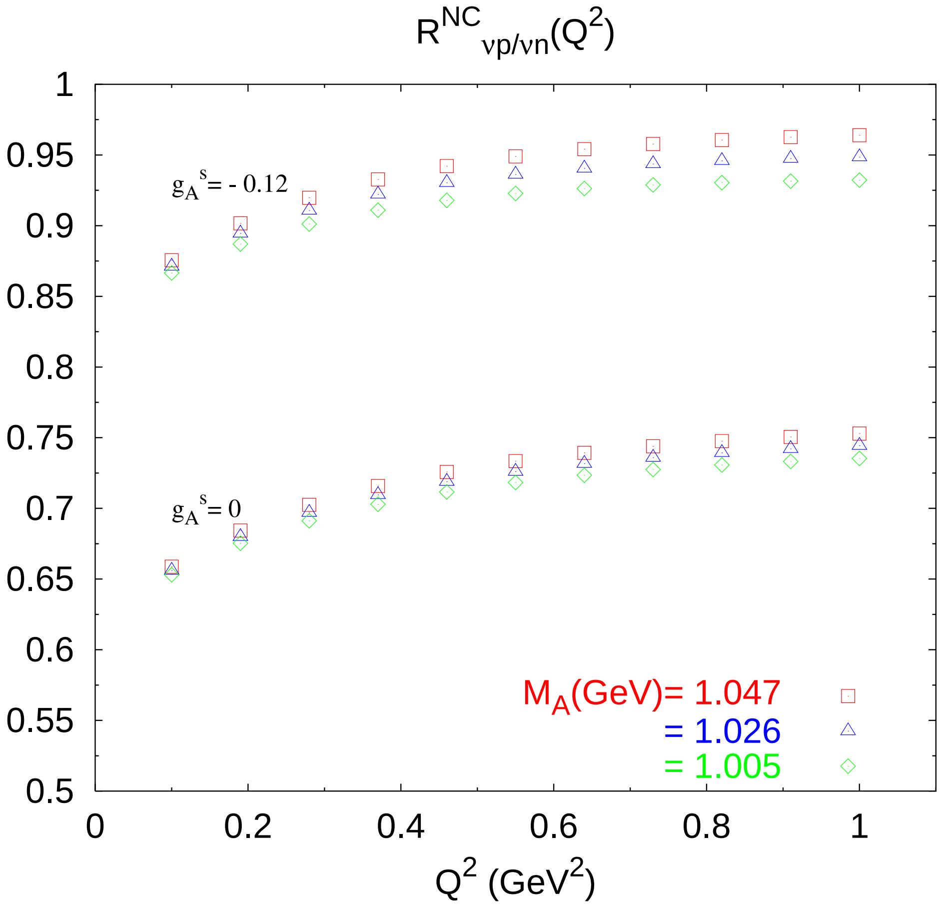


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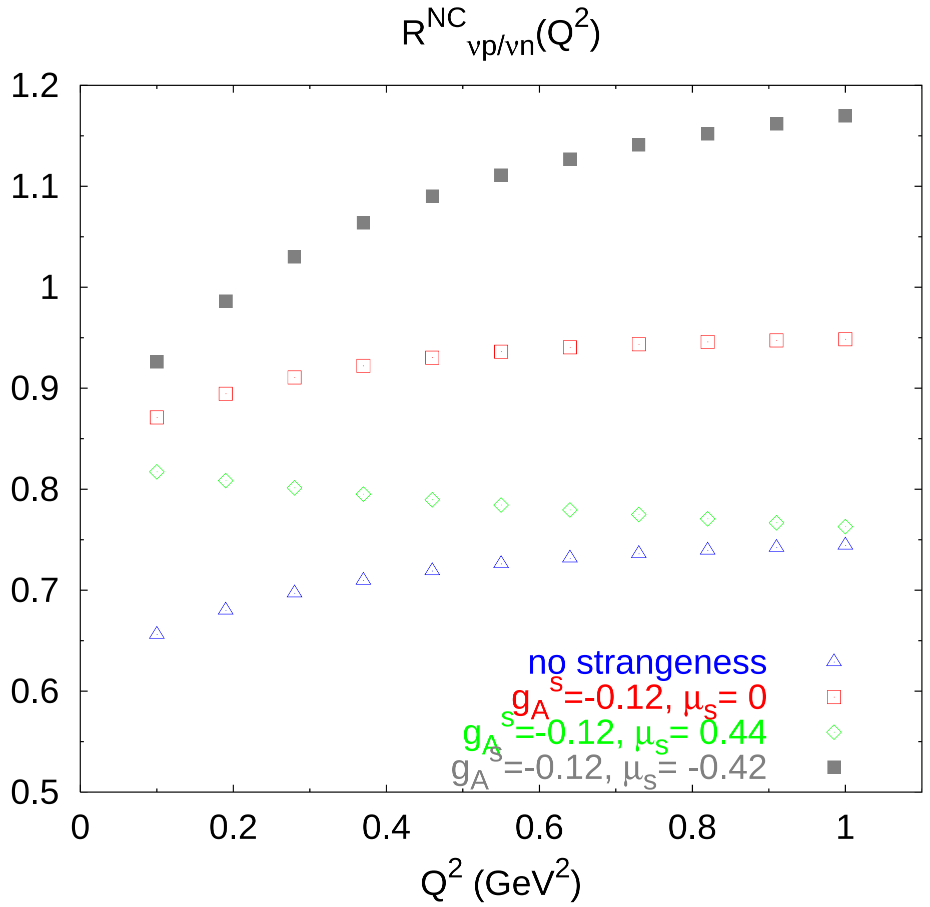
Fig. 14 – Ratio of proton to neutron yields in quasi-elastic NC neutrino scattering on ^{12}C: sensitivity to μ_s



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Elastic scattering on $S = T = 0$ nuclei

Consider the processes

$$\nu\ (\overline{\nu}) + A \longrightarrow \nu\ (\overline{\nu}) + A$$

The axial current, A_α^{NC}, and the isovector part of the vector NC, $V_\alpha^3(1 - 2\sin^2\theta_W)$, do not contribute. Cross sections are given by:

$$\frac{d\sigma_\nu}{dQ^2} = \frac{d\sigma_{\overline{\nu}}}{dQ^2} = \frac{G_F^2}{2\pi}\left(1 - \frac{p\cdot q}{M_A E} - \frac{Q^2}{4E^2}\right)\left[F^{NC}(Q^2)\right]^2$$

with

$$F^{NC}(Q^2) = -2\sin^2\theta_W F(Q^2) - \frac{1}{2}F^s(Q^2)$$

The e.m. FF of the nucleus $F(Q^2)$ can be determined from elastic scattering of unpolarized electrons:

$$\frac{d\sigma_e}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left(1 - \frac{p\cdot q}{M_A E} - \frac{Q^2}{4E^2}\right)\left[F(Q^2)\right]^2 .$$

Hence the strange form factor of the nucleus can be obtained from measurable cross sections:

$$F^s(Q^2) = \pm 2F(Q^2)\times$$
$$\times\left\{\left(\frac{2\sqrt{2}\pi\alpha}{G_F Q^2}\right)\sqrt{\frac{(d\sigma_\nu/dQ^2)}{(d\sigma_e/dQ^2)}} \mp 2\sin^2\theta_W\right\}$$

or, equivalently:

$$F^s(Q^2) = \pm 2 \frac{1}{\sqrt{1 - \frac{p\cdot q}{M_A E} - \frac{Q^2}{4E^2}}} \times$$
$$\times \left\{ \sqrt{\frac{2\pi}{G_F^2}\frac{d\sigma_\nu}{dQ^2}} \mp 2\sin^2\theta_W \sqrt{\frac{Q^4}{4\pi\alpha^2}\frac{d\sigma_e}{dQ^2}} \right\}.$$

The observation of the process of the scattering of neutrino on nuclei requires the measurement of the small recoil energy of the final nucleus. It could be easier to detect the process of scattering of neutrinos and electrons on nuclei if the nucleus undergoes a transition to excited states, for example:

$$\begin{aligned} \nu + {}^4\mathrm{He} &\longrightarrow \nu + {}^4\mathrm{He}^* \\ e + {}^4\mathrm{He} &\longrightarrow e + {}^4\mathrm{He}^* \end{aligned}$$

where ${}^4\mathrm{He}^*$ is the excited state of ${}^4\mathrm{He}$ with with $S = 0$ and $T = 0$ and excitation energy of 20.1 MeV. This state can decay into p and radioactive ${}^3\mathrm{H}$.

Conclusions

- The experiments of ν-proton NC and CC scattering are highly interesting for the determination of $\Delta s \equiv g_A^s$.
- Problems of interference with strange vector form factors can be resolved by complementary experiments (PV electron scattering)
- If feasible, $\bar{\nu}$ scattering would offer relevant and complementary information and:
- would allow the determination of the neutrino asymmetry (a unique tool for unambiguous determination of Δs)

For reference, see:

W.M. Alberico, S.M. Bilenky and C. Maieron, Phys. Rep. 358 (2002) 227;
also: W.M. Alberico, et al., Z. Physik C 70 (1996) 463; Nucl. Phys. A623 (1997) 471; Phys. Lett. B438 (1998) 9; Nucl. Phys. A651 (1999) 277.

Fig. 15 – Sensitivity to flux.

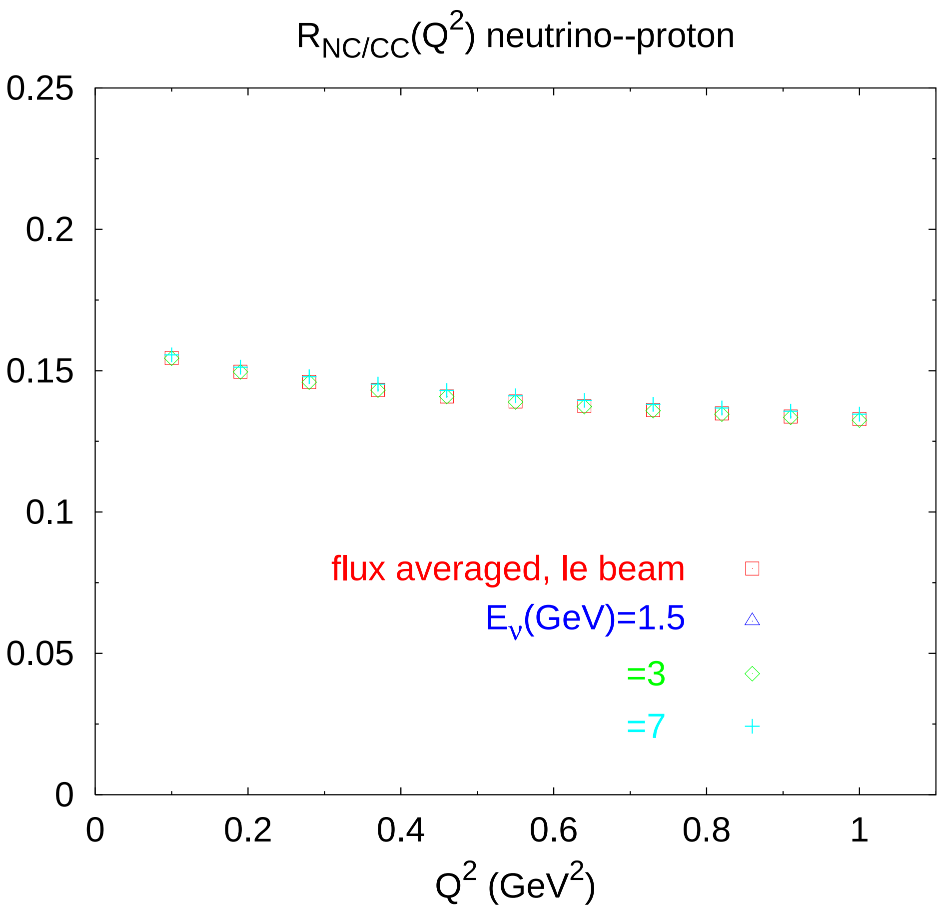

Fig. 16 – Elastic versus Fermi gas (for ^{12}C, with $p_F = 225$ MeV, $\epsilon_B = 25$ MeV)

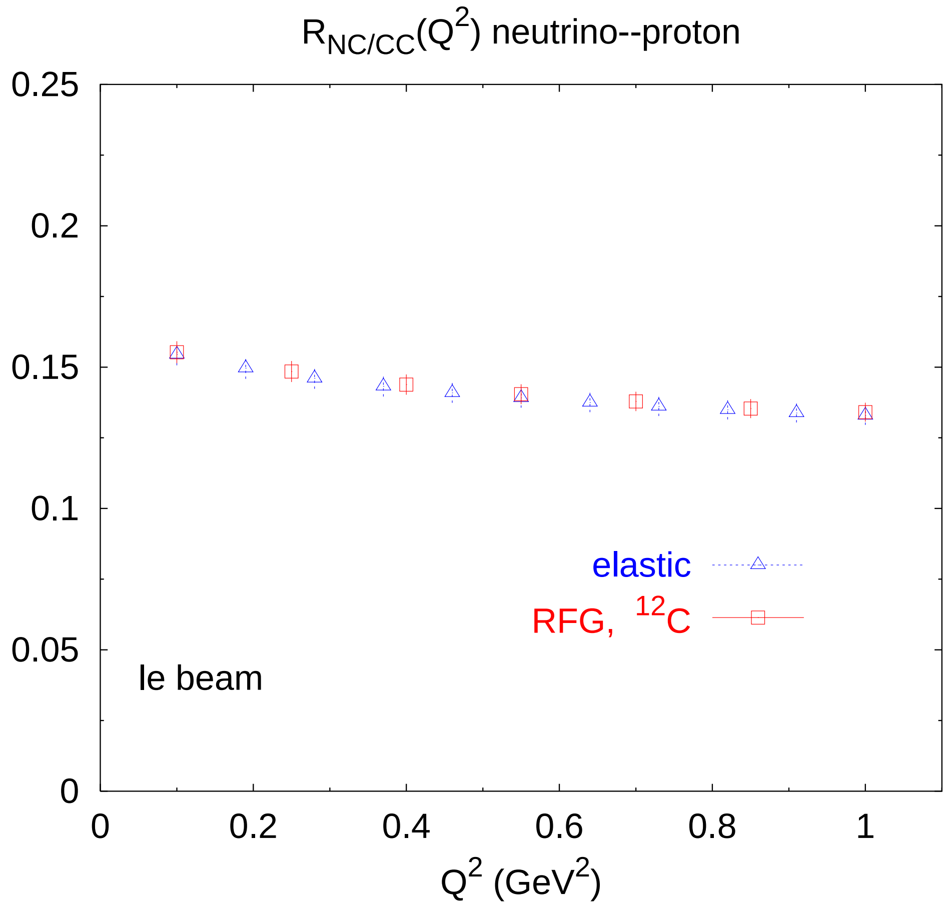

Fig. 17 – Ratio of total cross sections versus g_A^s: sensitivity to the axial cutoff

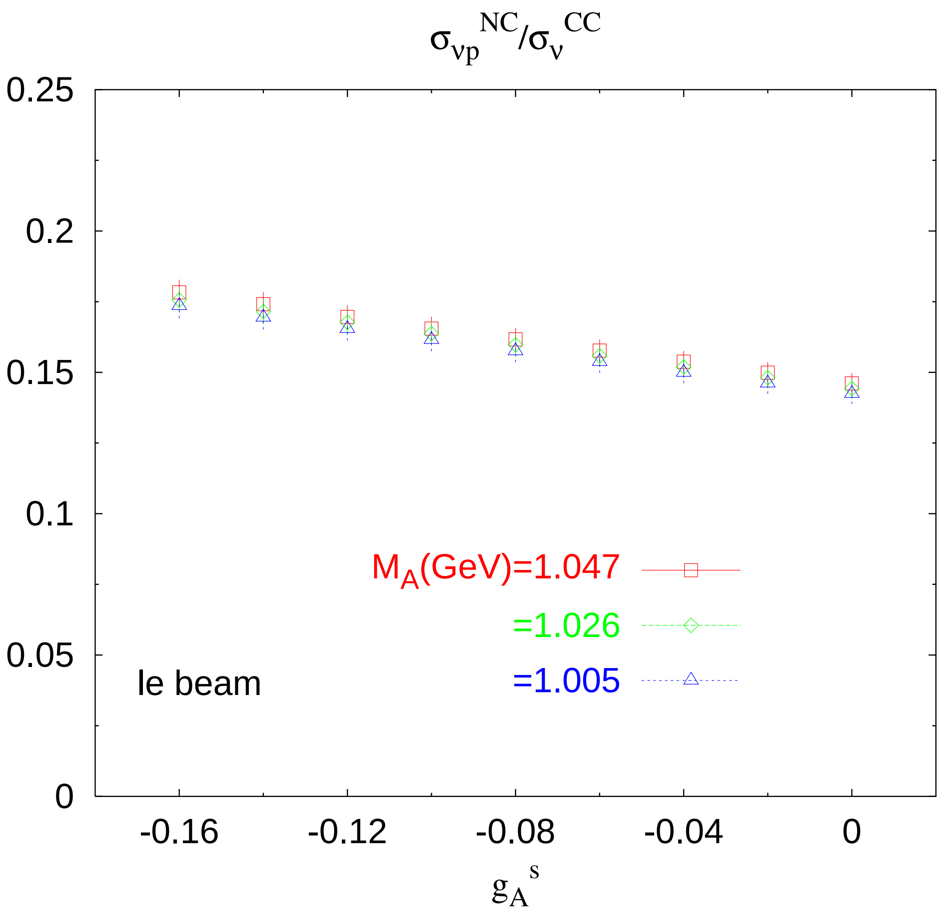

Fig. 18 – Ratio of total cross sections versus g_A^s: sensitivity to strange axial cutoff

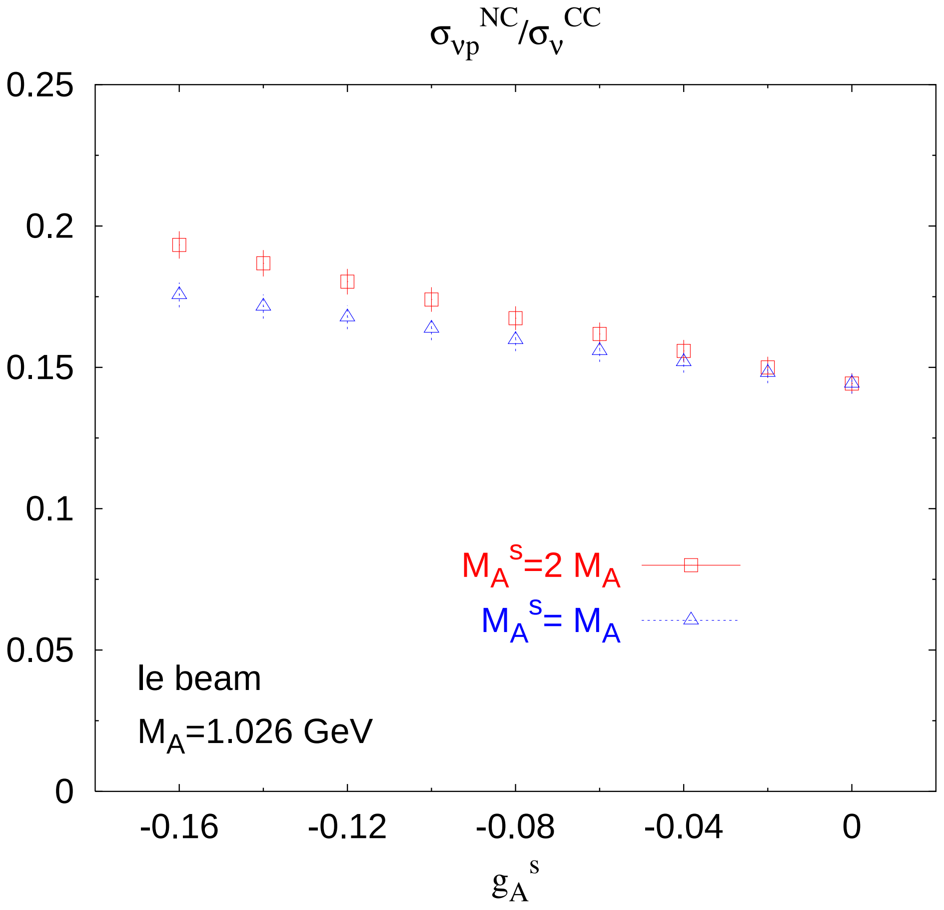

Fig. 19 – Ratio of proton to neutron yields in quasi-elastic NC neutrino scattering on ^{12}C: "elastic" versus RFG

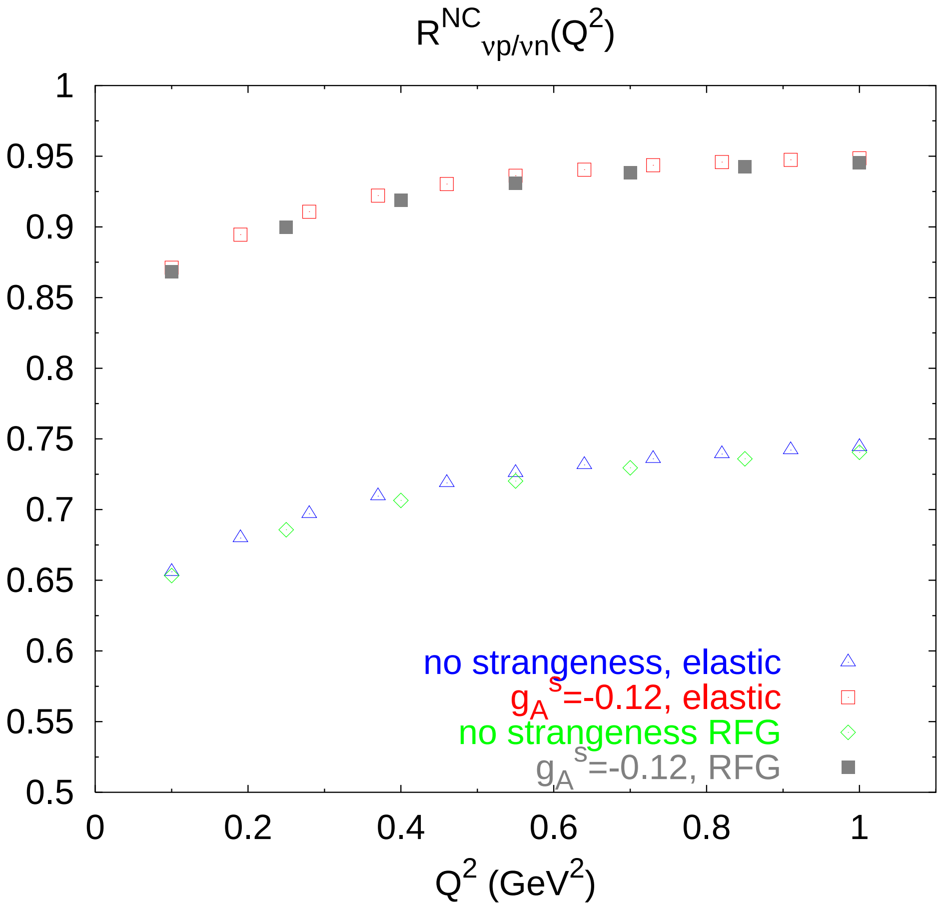

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Fig. 20 –Schematic representation for the amplitude, in Born approximation, of the neutrino–nucleus scattering.