Theory of
Muon $g-2$ and Muon EDM

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1. Muon $g-2$
   – introduction
   – prediction from the Standard Model
   – impact on New Physics search

2. Muon EDM
   – brief review and naive expectations

3. Summary and Outlook

Muon $g-2$

Muon $g-2$ — Introduction

Lepton magnetic moment $\vec{\mu}$:

$$\vec{\mu} = -g\frac{e}{2m}\vec{s}, \qquad (\vec{s} = \frac{1}{2}\vec{\sigma} \ \ \text{(spin)})$$

Anomalous magnetic moment: $a \equiv (g-2)/2$

Historically, ...

★ $g = 2$ (Dirac theory)

★ $a = \alpha/(2\pi)$ for 1-loop QED (Schwinger)

Today, still important, since...

★ One of the **most precisely measured** quantities

$$a_\mu^{\rm exp} = 11\ 659\ 203\ (8) \times 10^{-10}\ [0.7\text{ppm}]$$
$$a_e^{\rm exp} = 11\ 596\ 521.869\ (0.041) \times 10^{-10}\ [3.5\text{ppb}]$$

★ a_μ: **Extremely useful** in **probing/constraining physics beyond the SM**

Why care about a_μ rather than a_e?

Q. Why a_μ, even though...

$$
\begin{aligned}
a_\mu^{\rm exp} &= 11\;659\;203\;(8) \times 10^{-10}\;[0.7{\rm ppm}] \\
a_e^{\rm exp} &= 11\;596\;521.869\;(0.041) \times 10^{-10}\;[3.5{\rm ppb}]
\end{aligned}
$$

A. Because a_μ is **much more sensitive** to New Physics.

New Physics (NP) often induces an MDM-type operator:

$$
\mathcal{L}^{\rm NP} = \frac{e}{4m_\mu} a_\mu^{\rm NP} \overline{\mu}_R \sigma_{\rho\lambda} \mu_L F^{\rho\lambda} + {\rm H.c.}.
$$

$\mathcal{L}^{\rm NP}$: chirality-flipping operator:
$\Longrightarrow \mathcal{L}^{\rm NP} \to 0$ for $m_\mu \to 0$.
$\Longrightarrow a_\mu^{\rm NP} \propto m_\mu^2/\Lambda_{\rm NP}^2$. (cf. $a_e^{\rm NP} \propto m_e^2/\Lambda_{\rm NP}^2$)

The enhancement factor m_μ^2/m_e^2 $(\sim 43,000)$ more than compensates the ratio of the exp. uncertainties, $\delta a_\mu^{\rm exp}/\delta a_e^{\rm exp} \sim 200$.

Recent Developments

- **Feb. 2001**, **new exp.** result (BNL)
  - Diff. between SM and exp: **2.6 σ**
- **Nov. 2001**, **new** evaluation of the **l-by-l** contrib.
  - An overall sign error found in prev. evaluations
  - Diff: 2.6 $\sigma \to$ **1.6 σ**
- **July 2002**, **new exp.** result (BNL)
  - Diff: 1.6 $\sigma \to$ **2.6 σ**
- **Aug 2002 —**, **Reevaluations** of **LO hadronic** contrib. using new $e^+e^- \to \pi^+\pi^-$ data from **CMD-2**
  - Diff: 2.6 $\sigma \to$

    $\to$ **3.0 σ** (e^+e^--based, DEHZ) (**0.9 σ** (τ-based))

    $\to$ **3.3 σ** (e^+e^--based, **HMNT (our group)**)

    $\to$ **2.7 σ** (e^+e^--based, Jegerlehner)
- **Mar 2003**, **Error Found in the CMD-2 data??** (No official comment from CMD-2 yet...)
  - Diff: 3 $\sigma \to$ **2** σ **??**

Standard Model contribution

3 contributions: $a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}$

- **QED** contribution
  - Dominant but known accurately enough

$$a_\mu^{\rm QED} = 11\ 658\ 470.56\ (0.29) \times 10^{-10}$$

- **Electroweak** contribution
  - Small but non-negligible

$$a_\mu^{\rm EW} = 15.4\ (0.2) \times 10^{-10}.$$

- **Hadronic** contribution
  - Less accurately known (pQCD not useful)

$$a_\mu^{\rm had} = 690.4(7.4) \times 10^{-10}.$$

$\Longrightarrow$ next slides...

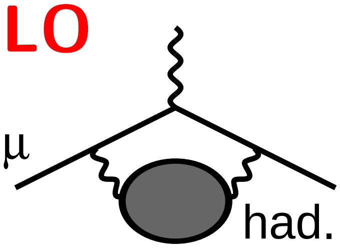
★ **(cf. Exp. :** $a_\mu^{\rm exp} = 11\ 659\ 203\ (8) \times 10^{-10}$**)**

Hadronic contribution

3 contributions:

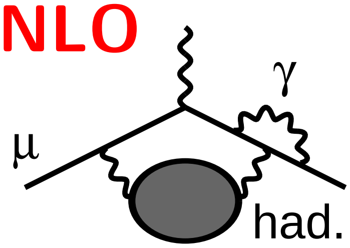
$$a_\mu^{\text{had}} = a_\mu^{\text{had,LO}} + a_\mu^{\text{had,NLO}} + a_\mu^{\text{had,l-by-l}}$$

LO



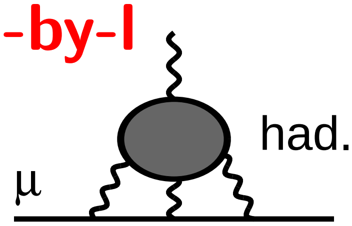
- Dominant, most unknown
  $a_\mu^{\text{had,LO}} = 692.4(6.2) \times 10^{-10}$

NLO



- Less important
  $a_\mu^{\text{had,NLO}} = -10.0(0.6) \times 10^{-10}$

l-by-l

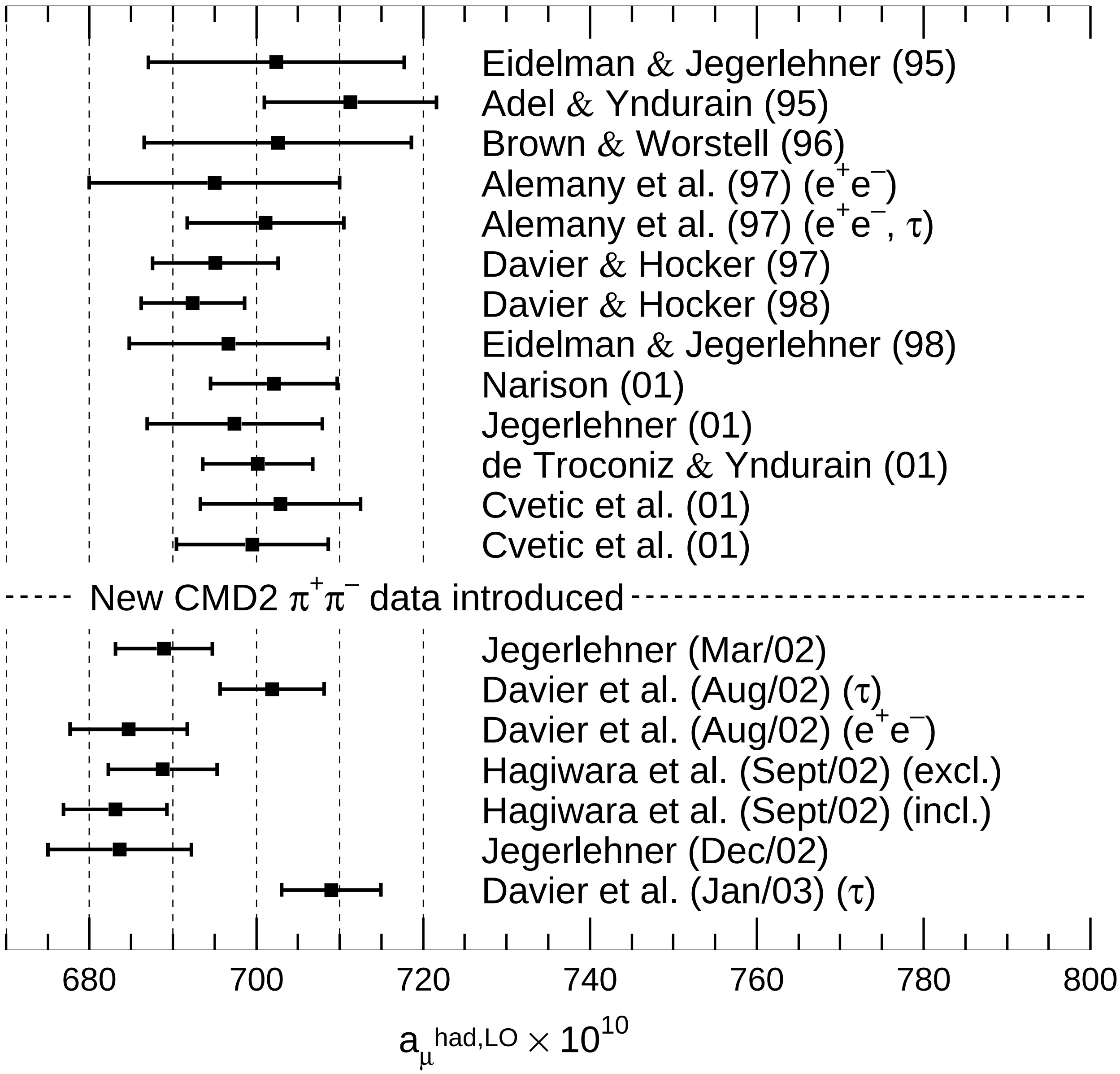


- Small but non-negligible
  $a_\mu^{\text{had,l-by-l}} = 8.0(4.0) \times 10^{-10}$

★ **(cf. Exp. :** $a_\mu^{\text{exp}} = 11\ 659\ 203\ (8) \times 10^{-10}$**)**

Good evaluation of **LO had.** contrib. **vital!**

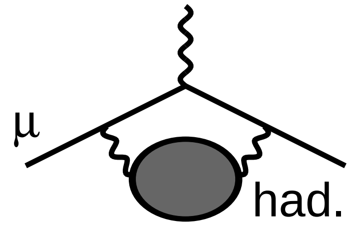
Recent Evaluations of $a_\mu^{\rm had,LO}$



✓ e^+e^--based evaluations — convergent

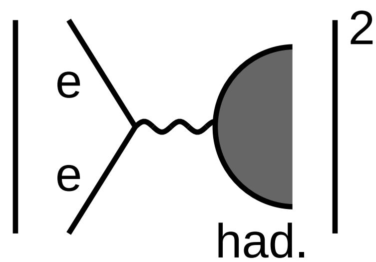
× Diff. between e^+e^--based and τ-based evaluations — must be accounted for !

Evaluating $a_\mu^{\mathrm{had,LO}}$

The diagram to be evaluated: 

pQCD not useful. Use the dispersion relation

$$a_\mu^{\mathrm{LO,had}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\mathrm{th}}}^{\infty} ds\ \frac{1}{s}\hat{K}(s)\sigma_{\mathrm{had}}(s)$$

Weighted integral over 

- The weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$
  – Lower energy region more important

- We have to rely on exp. data for $\sigma_{\mathrm{had}}(s)$
  – Good data crucial

Our Evaluation of $a_\mu^{\rm had,LO}$ and Breakdown

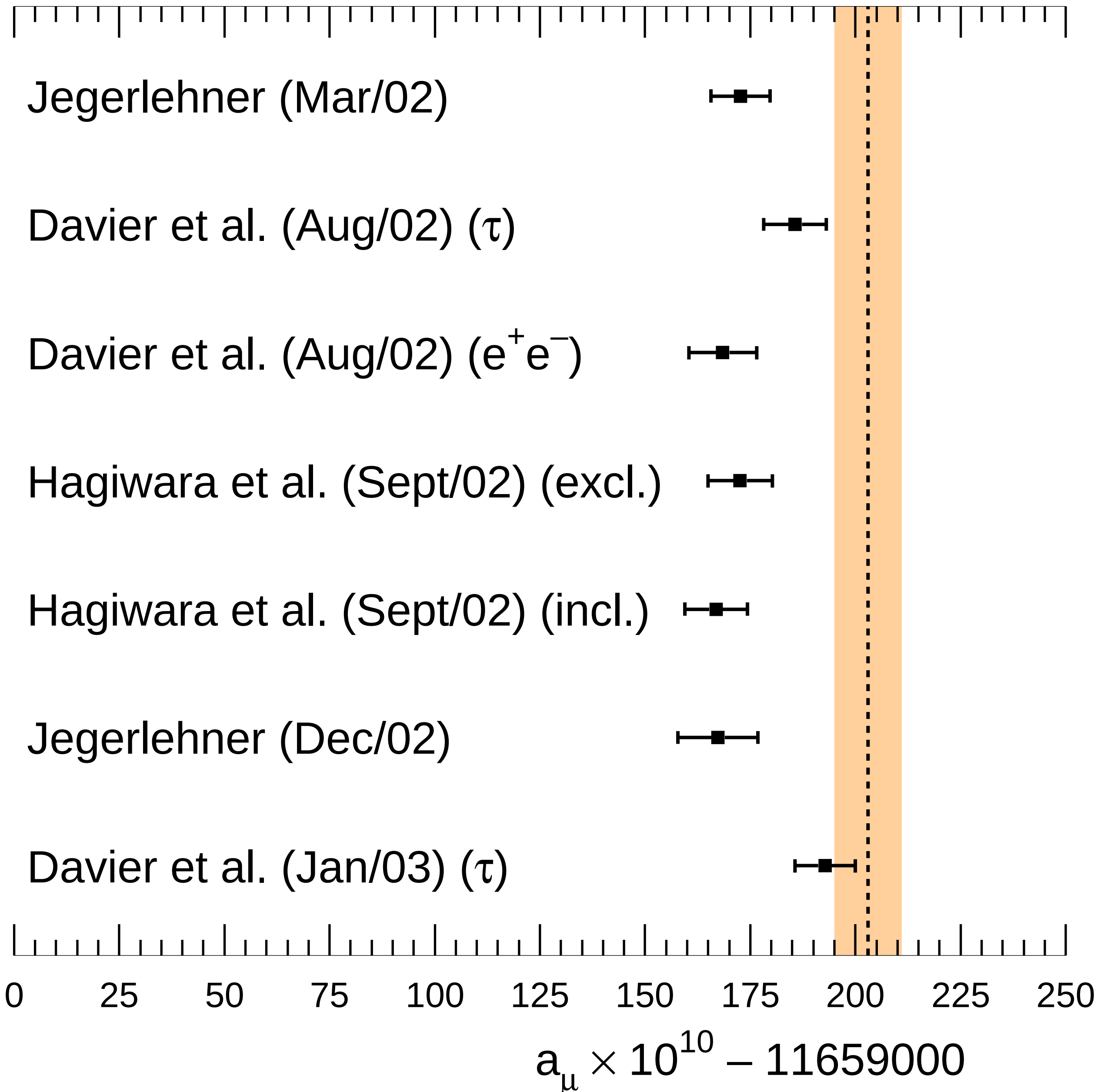
energy range (GeV)	comments	$a_\mu^{\rm had,LO} \times 10^{10}$
$2m_\pi \ldots 0.32$	chiral PT	2.30 ± 0.05
$0.32 \ldots 1.43$	excl. only	596.73 ± 5.18
$1.43 \ldots 2.00$	excl. only	38.14 ± 1.72
	incl. only	32.43 ± 2.46
$2.00 \ldots 11.09$	incl. only	42.09 ± 1.25
J/ψ and $\psi(2S)$	nar. width	7.31 ± 0.43
$\Upsilon(1S-6S)$	nar. width	0.10 ± 0.00
$11.09 \ldots \infty$	pQCD	2.14 ± 0.01
$\sum$ of all	ex-ex-in	688.81 ± 6.17
	ex-in-in	683.11 ± 5.89

(Hagiwara et al., Phys Lett **B557**(2003)69)

- The sum is dominated by the contribution from the low energy region, $\sqrt{s} < 1.4$ GeV. (Roughly 600 out of 700)

- Inconsistency in the data at $1.4 < \sqrt{s} < 2.0$ GeV.

$a_\mu^{\rm had,LO}$ plus the other contrib. to $a_\mu^{\rm SM}$



✓ Our results: **3.3σ** diff. from the exp. for incl. (2.7σ (excl.))

"New Physics" Contributions?

Where does the deviation

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 36.1(\pm 10.9)\times 10^{-10} \quad \text{(for 'incl.')}$$
$$(30.4(\pm 11.1)\times 10^{-10} \quad \text{(for 'excl.')})$$

come from? IF New Physics, from what scale?

Parametrize **New Physics** (NP) contributions by $\Lambda_{\rm NP}$, so that

$$\mathcal{L}^{\rm NP} = \frac{e}{4m_\mu}\frac{m_\mu^2}{\Lambda_{\rm NP}^2}\overline{\mu}_R\sigma_{\rho\lambda}\mu_L F^{\rho\lambda} + {\rm H.c.}.$$

and $a_\mu^{\rm NP} = m_\mu^2/\Lambda_{\rm NP}^2$. In order for New Physics to accommodate $a_\mu^{\rm exp} - a_\mu^{\rm SM}$, $\Lambda_{\rm NP}$ should be

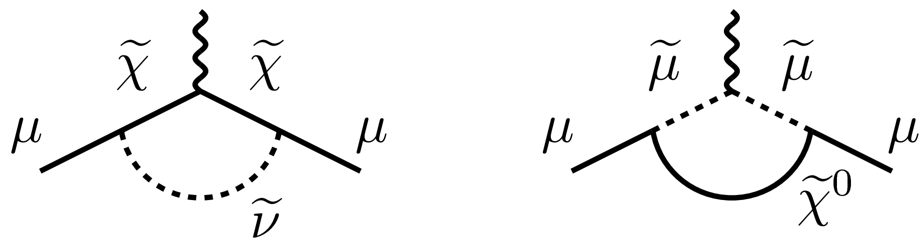
$$\Lambda_{\rm NP} = 1.4 - 2.8 \ ({\rm TeV}) \quad \text{(for 'incl.' data set)}$$
$$(1.5 - 3.7 \ ({\rm TeV}) \quad \text{(for 'excl.' data set)})$$

(**VERY rough estimates**)

SUSY Contributions?

SUSY contribution is very roughly given by

$$|a_\mu^{\rm SUSY}| = \frac{\alpha(M_Z)}{8\pi \sin^2\theta_W} \frac{m_\mu^2}{\widetilde{m}^2} \tan\beta \left(1 - \frac{4\alpha}{\pi} \ln \frac{\widetilde{m}}{m_\mu}\right)$$



Numerically,

$$a_\mu^{\rm SUSY} = ({\rm sgn}\mu) \times 13 \times 10^{-10} \left(\frac{100{\rm GeV}}{\widetilde{m}}\right)^2 \tan\beta$$

In order to be $14.3 \leq a_\mu^{\rm SUSY} \times 10^{10} \leq 57.9$ (2 σ range),

$$\widetilde{m} = 150 - 670 \ {\rm GeV} \quad \text{(for 'incl.' data set)}$$

$$(160 - 890 \ {\rm GeV} \quad \text{(for 'excl.' data set))}$$

for $\tan\beta = 10 - 50$. (**Rough estimates**)

Muon EDM

Muon EDM — Introduction

★ **Definition** of the fermion **EDM** d_f

$$\mathcal{H} = -d_f \boldsymbol{\sigma} \cdot \mathbf{E}, \quad (\text{or } \mathcal{L} = -\frac{i}{2} d_f \overline{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu})$$

★ $d_f \neq 0 \Longrightarrow \not{P}$ and $\not{T}$ ($\simeq \not{CP}$, from the CPT theorem)

★ Current **experimental limit** and near future **improvements**

$$d_\mu = (3.7 \pm 3.4) \cdot \mathbf{10^{-19}} e\text{cm} \; (\to 10^{-\mathbf{24}} e\text{cm} \text{ (BNL, J-PARC)}),$$
$$(\to 10^{-\mathbf{26}} e\text{cm} \text{ (PRISM, } \nu\text{-Factory)}),$$

(**cf.** The **Standard Model** predictions: **far below** $d_\mu \sim 2 \times \mathbf{10^{-38}} e\text{cm}$)

★ **If discovered** in the near future experiments, a clear signal of **New Physics**!!

New Physics contrib. to the Muon EDM

New Physics contribution to the **muon EDM**:
VERY model-dependent

Here is a **VERY rough** estimate

EDM and $g-2$: **Similar** operator

$$\text{EDM}: \qquad \mathcal{L} = -\frac{i}{2} d_\mu^{\rm NP} \overline{\mu} \sigma_{\rho\lambda} \gamma_5 \mu F^{\rho\lambda}$$

$$g-2: \qquad \mathcal{L} = \frac{e}{4m_\mu} a_\mu^{\rm NP} \overline{\mu} \sigma_{\rho\lambda} \mu F^{\rho\lambda}$$

If we assume

$$\left| -\frac{i}{2} d_\mu^{\rm NP} \right| \simeq \frac{e}{4m_\mu} a_\mu^{\rm NP},$$

$d_\mu^{\rm NP} \simeq \mathbf{10^{-22}} e\mathrm{cm}$ for $a_\mu^{\rm NP} \simeq 10 \times 10^{-10}$.

(**VERY rough** estimate)

cf. **Exp.** : $d_\mu^{\rm exp} = (3.7 \pm 3.4) \cdot \mathbf{10^{-19}} e\mathrm{cm}$
($\to \mathbf{10^{-24}} e\mathrm{cm}$ (BNL, J-PARC)...)

Summary and Outlook

Muon $g-2$:

✓ The biggest uncertainty in a_μ: still from the LO hadronic contrib.

★ **Our results**: **3.3σ** diff. between the TH and the exp. for incl. (2.7σ (for "excl.")) ($\Longrightarrow \sim 2\sigma$ level if the CMD-2 data are wrong and are revised)

▶ Waiting for new precise data from the radiative return and the direct measurements at KLOE, BaBar, and Belle

▶ ultimate goal at BNL: $\pm 4.0 \times 10^{-10}$

▶ planned experiment at J-PARC: Another factor of 5-10 improvement expected

Muon EDM:

✓ A very naive expectation is $d_\mu^{\rm NP} \simeq \mathbf{10^{-22}} e{\rm cm}$ for $a_\mu^{\rm NP} \simeq 10 \times 10^{-10} \Longrightarrow$ interesting possibilities at BNL and/or J-PARC which aim at $\mathbf{10^{-24}} e{\rm cm}$.