Theoretical Study on the Lepton Flavor Violating μ-e Conversion in Nuclei

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based on Phys. Rev. D **66**, 096002 (2002)

Plan

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1. Introduction

the Standard Model ―――― **no LFV**

↓ Massive neutrinos?
Hierarchy problem?
Charge quantization?

Extentions of the SM ―――― **LFV Search**

Right handed neutrinos
Supersymmetry
Grand Unification

$\mu \to e\gamma$
μ-e **conversion**
$\mu \to 3e$
$\tau \to \mu\gamma$ **etc...**

Experimental Limits on the Branching Ratios

	Present	**Future**
$\mu \to e\gamma$	1.2×10^{-11} MEGA (1999)	10^{-14} (PSI) MEG 10^{-15} (J-PARC) [redacted]
μ-e **conversion**	6.1×10^{-13} SINDRUM II (1998)	2×10^{-17} (MECO) 10^{-18} [redacted] (J-PARC)
$\tau \to \mu\gamma$	1.0×10^{-6} Belle (2001)	10^{-8} (Belle)
$\mu^+ \to e^+e^+e^-$	1.0×10^{-12}	

μ-e Conversion in nuclei:
(Neutrinoless muon capture)

$$\mu^- + {}^A_Z\mathrm{N}_i \rightarrow e^- + {}^A_Z\mathrm{N}_f$$

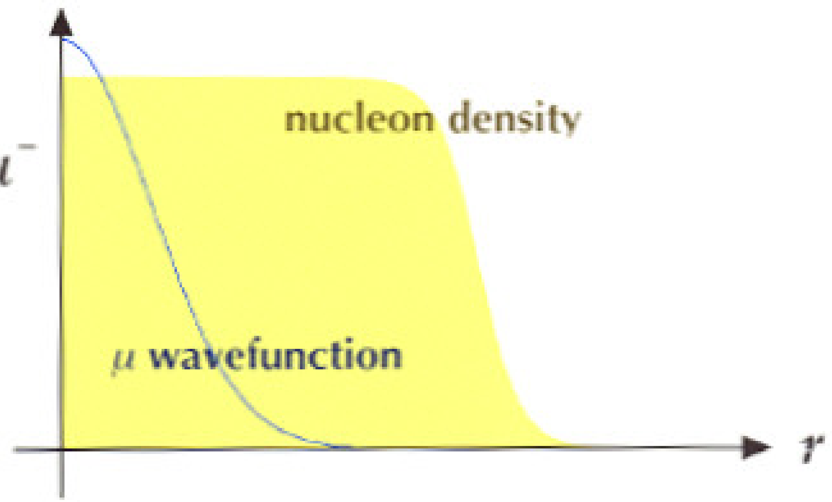
$$\left(\textit{cf.}\ \text{Ordinary muon capture:} \quad \mu^- + {}^A_Z\mathrm{N}_i \rightarrow \nu_\mu + {}_{Z-1}^{\ A}\mathrm{N}_f \right)$$

Coherent process is dominant.

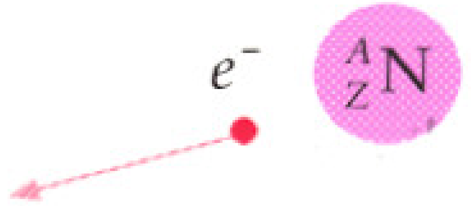
${}^A_Z\mathrm{N}_i = {}^A_Z\mathrm{N}_f$ = Ground state

► a) Initial state — muonic atom

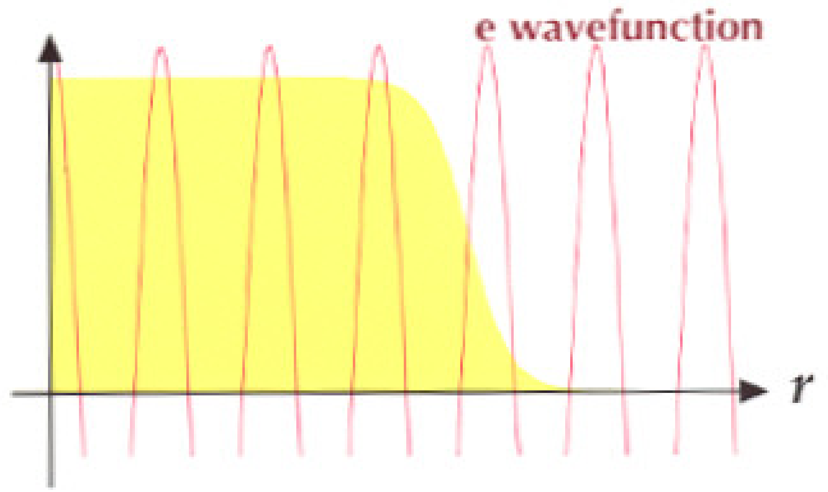
$$\left(a_\mathrm{B}^{(\mu)} \approx \frac{1}{200} a_\mathrm{B}^{(\mathrm{e})} \right)$$



► b) Final state — monochromatic electron

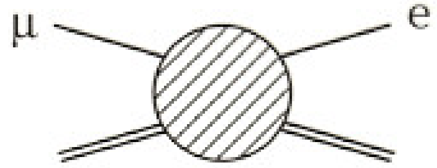


$$E_e = m_\mu - E_\text{binding} - E_\text{recoil}$$
$$\approx m_\mu - E_\text{binding}$$



Why $\mu-e$ conversion?

1. Non-Photonic + Photonic interaction



Contribution from
- 4-Fermi effective interaction
- Photon-penguin interaction

→ More info on THEORY

2. Monochromatic electron signal

Intense beam = Higher precision

→ Clean, precise EXPERIMENT

Why muon ring?

★ Free from π and e in the beam

↓

★ Extinguishes prompt backgrounds:

- Radiative π^- capture

$$\pi^- + (A,Z) \to (A,Z-1) + \gamma$$

$$\hookrightarrow \cancel{e^+}e^-$$

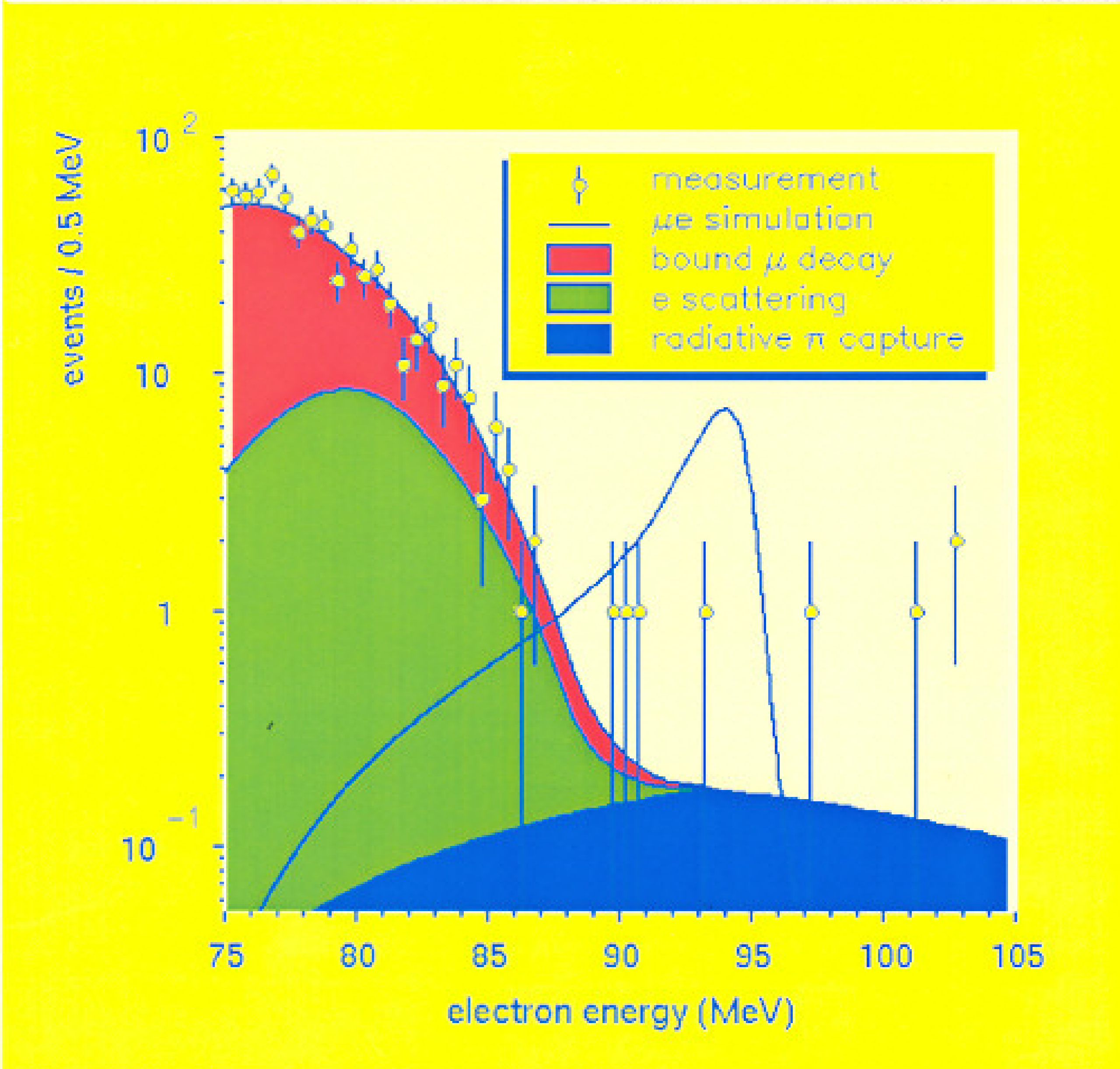
undetected

- e in the beam scattered off the target

Lifts restriction on the selection of target nuclei ! — Need not to worry about the lifetime of muonic atoms.

Electron momentum distribution@SINDRUM

http://sindrum2.web.psi.ch/home/lead92.html **Target: Pb**



Studies on LFV μ-e conversion

	Weinberg & Feinberg (1959)	Shanker (1979)	Czarnezki, Marciano & Melnikov (1998)	Kosmas (2001)	Present work
μ wave function	Constant	Dirac eq.	Dirac eq.	Schrödinger eq.	Dirac eq.
e wave function	Plane wave	Dirac eq.	Dirac eq.	Plane wave	Dirac eq.
q^2	$q^2 = -m_\mu^2$	$q^2 = -m_\mu^2$	Electric fields	$q = m_\mu - E_{bind}$	Electric fields
LFV interaction	Photonic	All	Photonic + Vector	4-Fermi	All
$\rho^{(p,n)}(r)$	Approximate formula	Approximate formula	Experimental data (3 points)	Experimental data	Experimental data

We will be able to choose the target nucleus.

Which is the best, then,

- to discover the $\mu - e$ conversion?
- to investigate the new physics, physics beyond the Standard Model?

Thorough investigation is necessary.

2. Methods

Proton distribution in nuclei

Input

Charge distribution = Proton distribution

Maxwell equation
Dirac equation — ***Relativistic***

Wave functions of initial μ & final e

Proton & neutron distribution in nuclei

Input

Overlap integrals appearing in the μ-e transition amplitude

Output

μ-e **transition amplitude**

μ-e **transition rate**

Input data

Proton density in the nucleus: $\rho^{(p)}(r)$

--- Electron scattering experiments

Determined very precisely (within a few %).
Various nuclei are measured.

Neutron density in the nucleus: $\rho^{(n)}(r)$

--- Scattering experiments (proton, α, $\pi^{\pm}$)
--- Spectroscopy (π atom, $\overline{\mathrm{p}}$ atom)

(Relatively) poorly determined.
Not many data are available.

We also present the result obtained from $\rho^{(n)}(r) = \rho^{(p)}(r)$ assumption.

No justification for heavy nuclei.

Provides the tendency of the Z-dependence of the conversion amplitude/rate, thanks to the large data set.

LFV interaction Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\rm int} = & -\frac{4G_{\rm F}}{\sqrt{2}}\left(m_\mu A_R \bar{\mu}\sigma^{\mu\nu}P_L e F_{\mu\nu} + m_\mu A_L \bar{\mu}\sigma^{\mu\nu}P_R e F_{\mu\nu} + \text{h.c.}\right) && \text{Dipole (photonic)}\\
& -\frac{G_{\rm F}}{\sqrt{2}}\sum_{q=u,d,s}\Bigg[\left(g_{LS(q)}\bar{e}P_R\mu + g_{RS(q)}\bar{e}P_L\mu\right)\bar{q}q && \text{Scalar}\\
& \quad +\left(g_{LP(q)}\bar{e}P_R\mu + g_{RP(q)}\bar{e}P_L\mu\right)\bar{q}\gamma_5 q && \text{Pseudoscalar}\\
& \quad +\left(g_{LV(q)}\bar{e}\gamma^\mu P_L\mu + g_{RV(q)}\bar{e}\gamma^\mu P_R\mu\right)\bar{q}\gamma_\mu q && \text{Vector}\\
& \quad +\left(g_{LA(q)}\bar{e}\gamma^\mu P_L\mu + g_{RA(q)}\bar{e}\gamma^\mu P_R\mu\right)\bar{q}\gamma_\mu\gamma_5 q && \text{Axial vector}\\
& \quad +\frac{1}{2}\left(g_{LT(q)}\bar{e}\sigma^{\mu\nu}P_R\mu + g_{RT(q)}\bar{e}\sigma^{\mu\nu}P_L\mu\right)\bar{q}\sigma_{\mu\nu}q + \text{h.c.}\Bigg]
\end{aligned}
$$

Tensor

Electric field $$E(r) = \frac{Ze}{r^2}\int_0^r dr'\, r'^2 \rho^{(\mathrm{p})}(r')$$

Electric potential $$V(r) = -e\int_r^\infty dr' E(r')$$

Transition amplitude $$M = \langle f | V | i \rangle$$

Electric field

$$
\begin{aligned}
M = & \frac{4G_{\rm F}}{\sqrt{2}}\int d^3x\left(m_\mu A_R^* \bar{\psi}^{\mu(e)}_{\kappa,W}\sigma^{\alpha\beta}P_R\psi^{(\mu)}_{1s} + m_\mu A_L^* \bar{\psi}^{\mu(e)}_{\kappa,W}\sigma^{\alpha\beta}P_L\psi^{(\mu)}_{1s}\right)\langle N'|F_{\alpha\beta}|N\rangle\\
& +\frac{G_{\rm F}}{\sqrt{2}}\sum_{q=u,d,s}\int d^3x\Bigg[\left(g_{LS(q)}\bar{\psi}^{\mu(e)}_{\kappa,W}P_R\psi^{(\mu)}_{1s} + g_{RS(q)}\bar{\psi}^{\mu(e)}_{\kappa,W}P_L\psi^{(\mu)}_{1s}\right)\langle N'|\bar{q}q|N\rangle\\
& \quad +\left(g_{LP(q)}\bar{\psi}^{\mu(e)}_{\kappa,W}P_R\psi^{(\mu)}_{1s} + g_{RP(q)}\bar{\psi}^{\mu(e)}_{\kappa,W}P_L\psi^{(\mu)}_{1s}\right)\langle N'|\bar{q}\gamma_5 q|N\rangle\\
& \quad +\left(g_{LV(q)}\bar{\psi}^{\mu(e)}_{\kappa,W}\gamma^\alpha P_L\psi^{(\mu)}_{1s} + g_{RV(q)}\bar{\psi}^{\mu(e)}_{\kappa,W}\gamma^\alpha P_R\psi^{(\mu)}_{1s}\right)\langle N'|\bar{q}\gamma_\alpha q|N\rangle\\
& \quad +\left(g_{LA(q)}\bar{\psi}^{\mu(e)}_{\kappa,W}\gamma^\alpha P_L\psi^{(\mu)}_{1s} + g_{RA(q)}\bar{\psi}^{\mu(e)}_{\kappa,W}\gamma^\alpha P_R\psi^{(\mu)}_{1s}\right)\langle N'|\bar{q}\gamma_\alpha\gamma_5 q|N\rangle\\
& \quad +\frac{1}{2}\left(g_{LT(q)}\bar{\psi}^{\mu(e)}_{\kappa,W}\sigma^{\alpha\beta}P_R\psi^{(\mu)}_{1s} + g_{RT(q)}\bar{\psi}^{\mu(e)}_{\kappa,W}\sigma^{\alpha\beta}P_L\psi^{(\mu)}_{1s}\right)\langle N'|\bar{q}\sigma_{\alpha\beta}q|N\rangle\Bigg]
\end{aligned}
$$

$$\langle N|\bar{q}q|N\rangle = Zc_p^{(q)}\rho^{(p)} + (A-Z)c_n^{(q)}\rho^{(n)}$$

$$
\langle N|\bar{q}\gamma^0 q|N\rangle = \begin{cases} 2Z\rho^{(p)} + (A-Z)\rho^{(n)} & \text{for } q=u\\ Z\rho^{(p)} + 2(A-Z)\rho^{(n)} & \text{for } q=d\\ 0 & \text{for } q=s \end{cases}
$$

$$\langle N|\bar{q}\gamma^k q|N\rangle = 0 \quad (k=1,2,3)$$

Transition rate $$\omega_{\rm conv} = |M|^2$$

3. Numerical Results

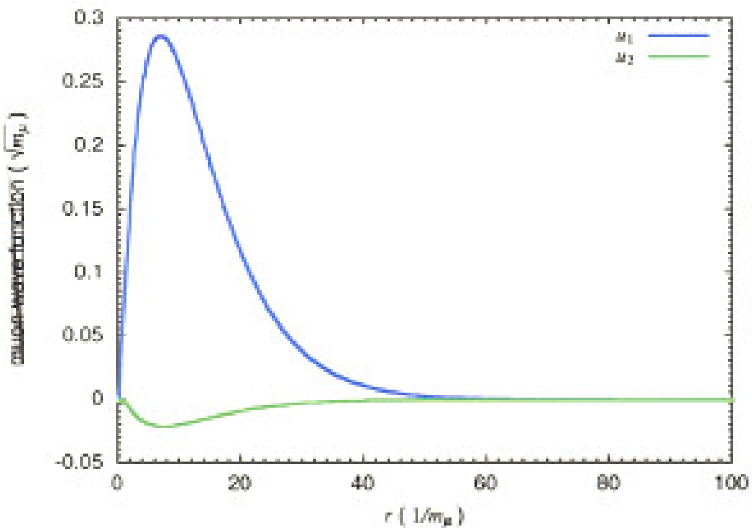
Dirac equation for the radial part of the wave function

Wave function: $\psi_{nlm}(\vec{r}) = \frac{1}{r}\begin{pmatrix} u_1(r)\,\chi_\kappa^m \\ iu_2(r)\,\chi_{-\kappa}^m \end{pmatrix} Y_l^m(\theta,\phi)$

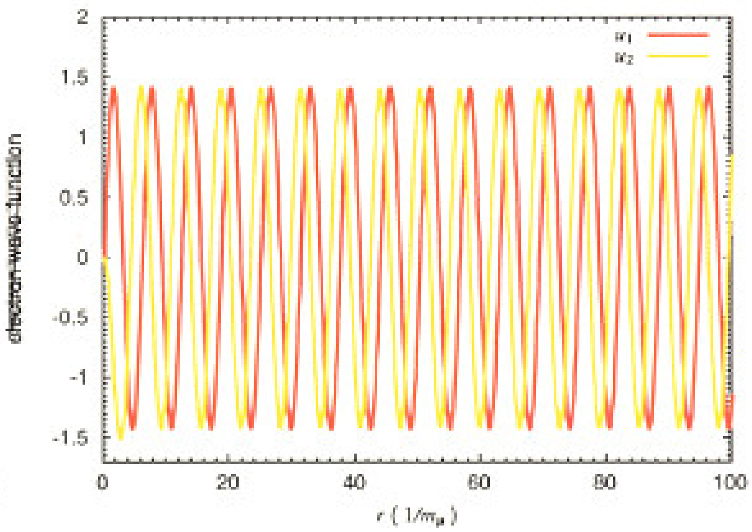
$$\frac{d}{dr}\begin{pmatrix} u_1(r) \\ u_2(r) \end{pmatrix} = \begin{pmatrix} -\kappa/r & W - V + m_\mu \\ -(W - V - m_\mu) & \kappa/r \end{pmatrix}\begin{pmatrix} u_1(r) \\ u_2(r) \end{pmatrix}$$

W :Energy eigenvalue $\qquad \kappa \equiv -(\vec{L}\vec{\sigma} + 1) = -1$ for 1S state

Initial state:
The wavefunction of μ



Final state:
The wavefunction of e

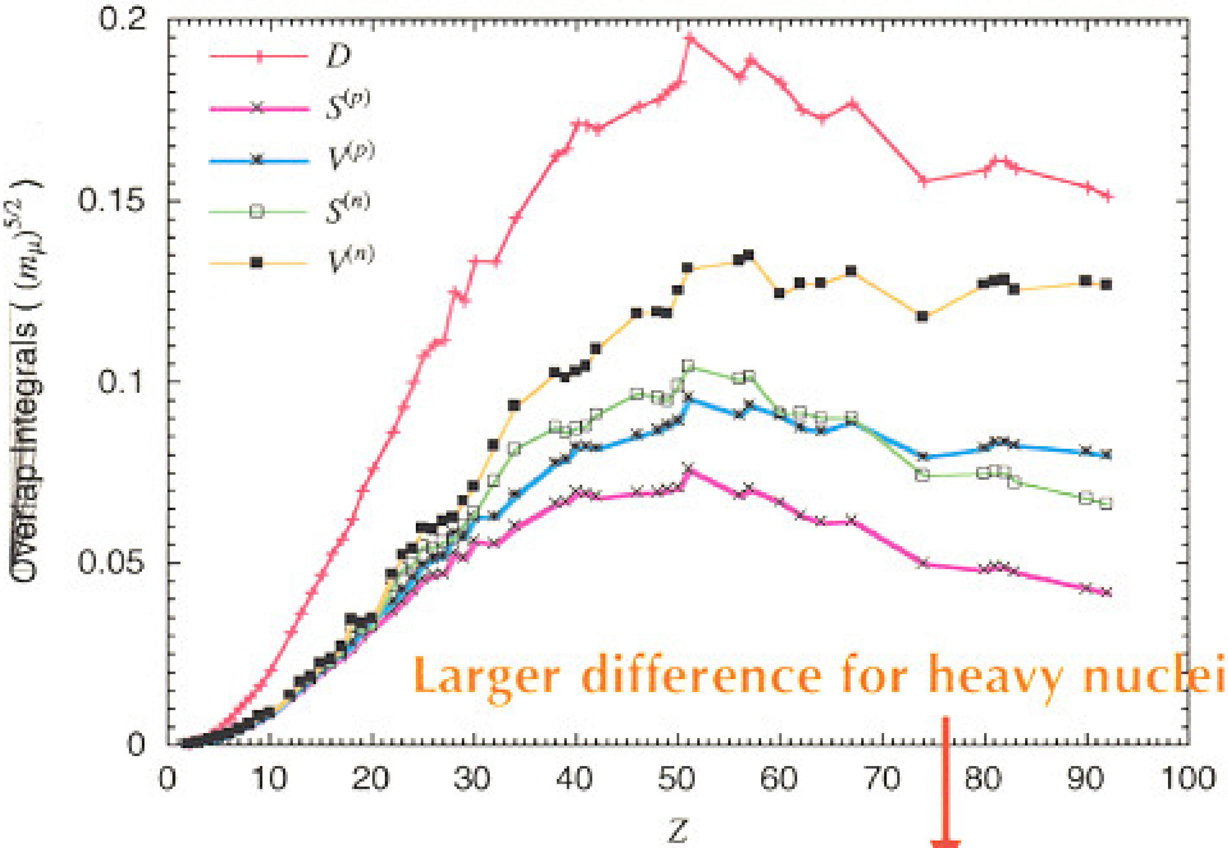


Conversion rate --- written in terms of five Overlap integrals

$$\omega_{\rm conv} = 2G_{\rm F}^2\left|A_{\rm R}^* D + \tilde{g}_{\rm LS}^{(\rm p)} S^{(\rm p)} + \tilde{g}_{\rm LS}^{(\rm n)} S^{(\rm n)} + \tilde{g}_{\rm LV}^{(\rm p)} V^{(\rm p)} + \tilde{g}_{\rm LV}^{(\rm n)} V^{(\rm n)}\right|^2$$
$$+2G_{\rm F}^2\left|A_{\rm L}^* D + \tilde{g}_{\rm RS}^{(\rm p)} S^{(\rm p)} + \tilde{g}_{\rm RS}^{(\rm n)} S^{(\rm n)} + \tilde{g}_{\rm RV}^{(\rm p)} V^{(\rm p)} + \tilde{g}_{\rm RV}^{(\rm n)} V^{(\rm n)}\right|^2$$

Transition amplitudes —— Overlap integrals

"$\rho^{(n)}(r) = \rho^{(p)}(r)$" assumption



Larger difference for heavy nuclei.

Useful to distinguish the models beyond the SM.

$g^- = \frac{u_1}{r}$

$f^- = \frac{u_2}{r}$

$$D = \frac{4}{\sqrt{2}} m_\mu \int_0^\infty dr\, r^2 [E(-r)] \left[g_e^- f_\mu^- + f_e^- g_\mu^- \right]$$

$$S^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr\, r^2 Z \rho^{(p)} \left[g_e^- g_\mu^- - f_e^- f_\mu^- \right]$$

$$S^{(n)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr\, r^2 (A-Z) \rho^{(n)} \left[g_e^- g_\mu^- - f_e^- f_\mu^- \right]$$

$$V^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr\, r^2 Z \rho^{(p)} \left[g_e^- g_\mu^- + f_e^- f_\mu^- \right]$$

$$V^{(n)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr\, r^2 (A-Z) \rho^{(n)} \left[g_e^- g_\mu^- + f_e^- f_\mu^- \right]$$

Conversion branching ratios

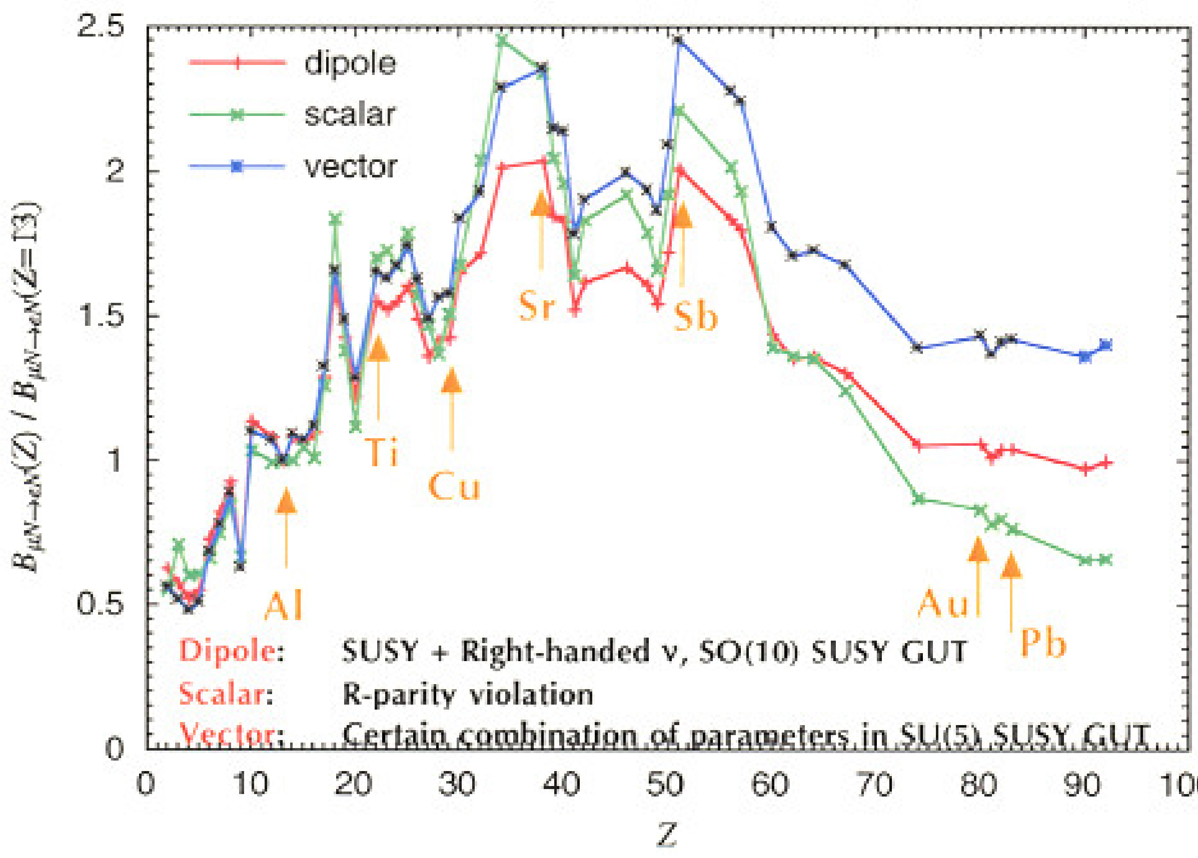
$$\omega_{conv} = 2G_F^2\left|A_R^* D + \tilde{g}_{LS}^{(p)}S^{(p)} + \tilde{g}_{LS}^{(n)}S^{(n)} + \tilde{g}_{LV}^{(p)}V^{(p)} + \tilde{g}_{LV}^{(n)}V^{(n)}\right|^2$$
$$+2G_F^2\left|A_L^* D + \tilde{g}_{RS}^{(p)}S^{(p)} + \tilde{g}_{RS}^{(n)}S^{(n)} + \tilde{g}_{RV}^{(p)}V^{(p)} + \tilde{g}_{RV}^{(n)}V^{(n)}\right|^2$$

LFV

$$\mathrm{Br}\left(\mu^- + {}^Z_A N \to e^- + {}^Z_A N\right) \equiv \frac{\Gamma\left(\mu^- + {}^Z_A \mathrm{N}_i \to e^- + {}^Z_A \mathrm{N}_f\right)}{\Gamma\left(\mu^- + {}^Z_A \mathrm{N}_i \to \nu_\mu + {}^{Z-1}_{\ A} \mathrm{N}'_f\right)}$$

muon capture

"$\rho^{(n)}(r) = \rho^{(p)}(r)$" assumption

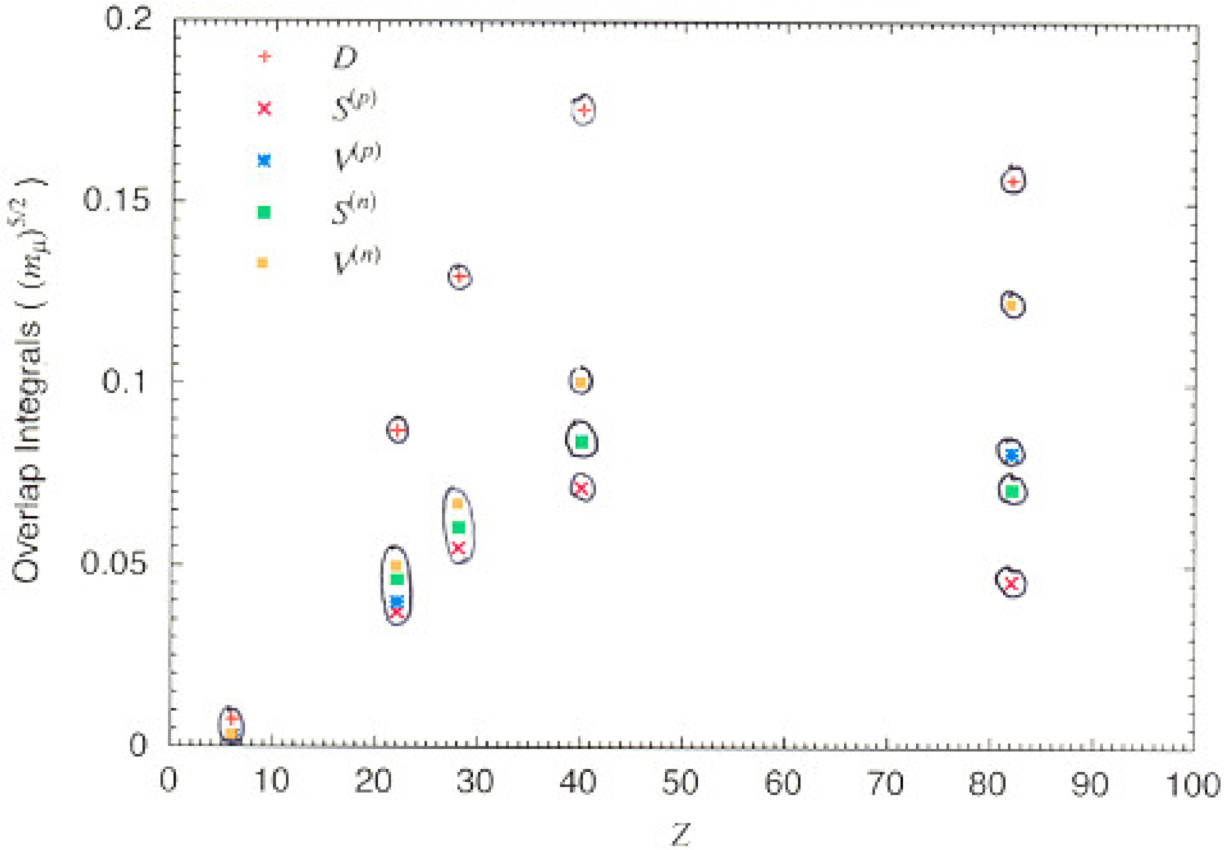


$Z \sim (30 — 60)$ **gives the largest rate.**

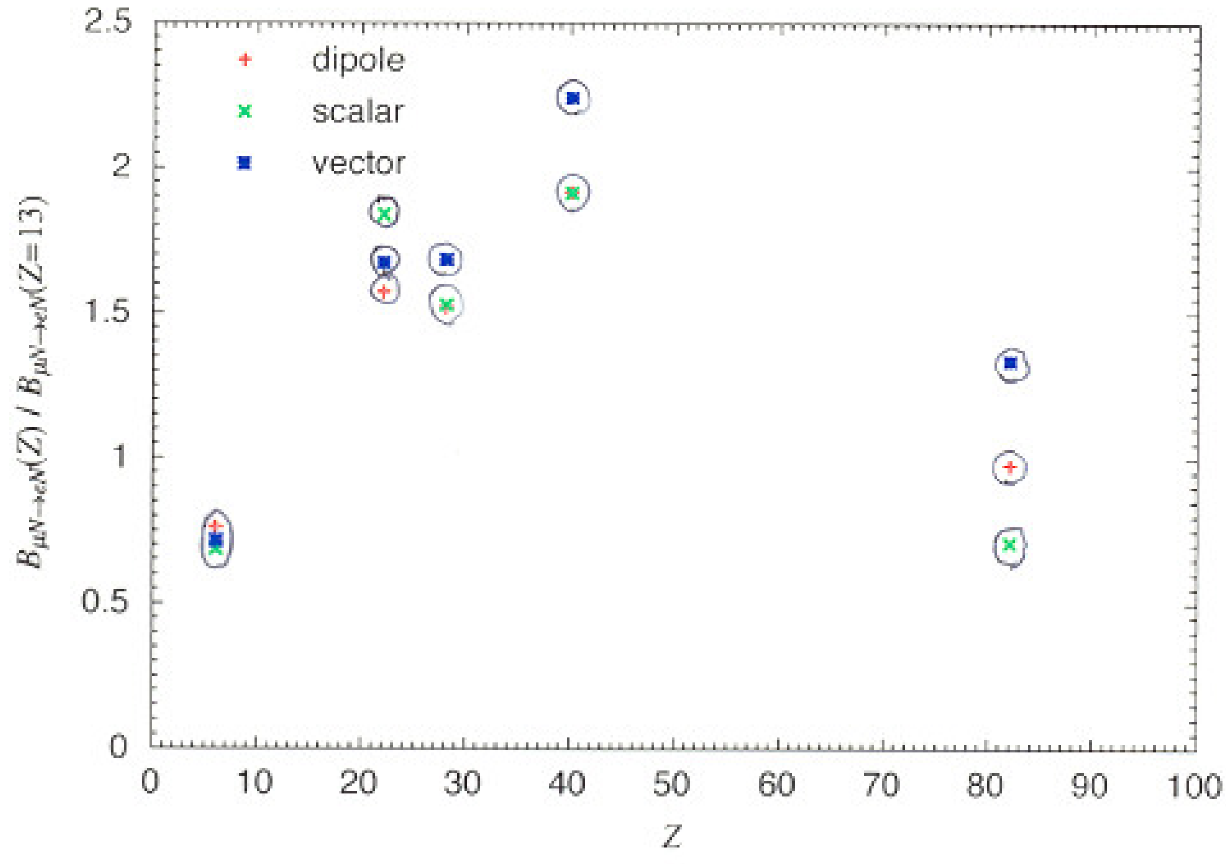
The model of new physics can be distinguished through the measurement of $\mu - e$ rate for several kinds of nuclei.

$\rho^{(n)}(r)$ from the proton scattering experiment

Transition amplitudes —— Overlap integrals

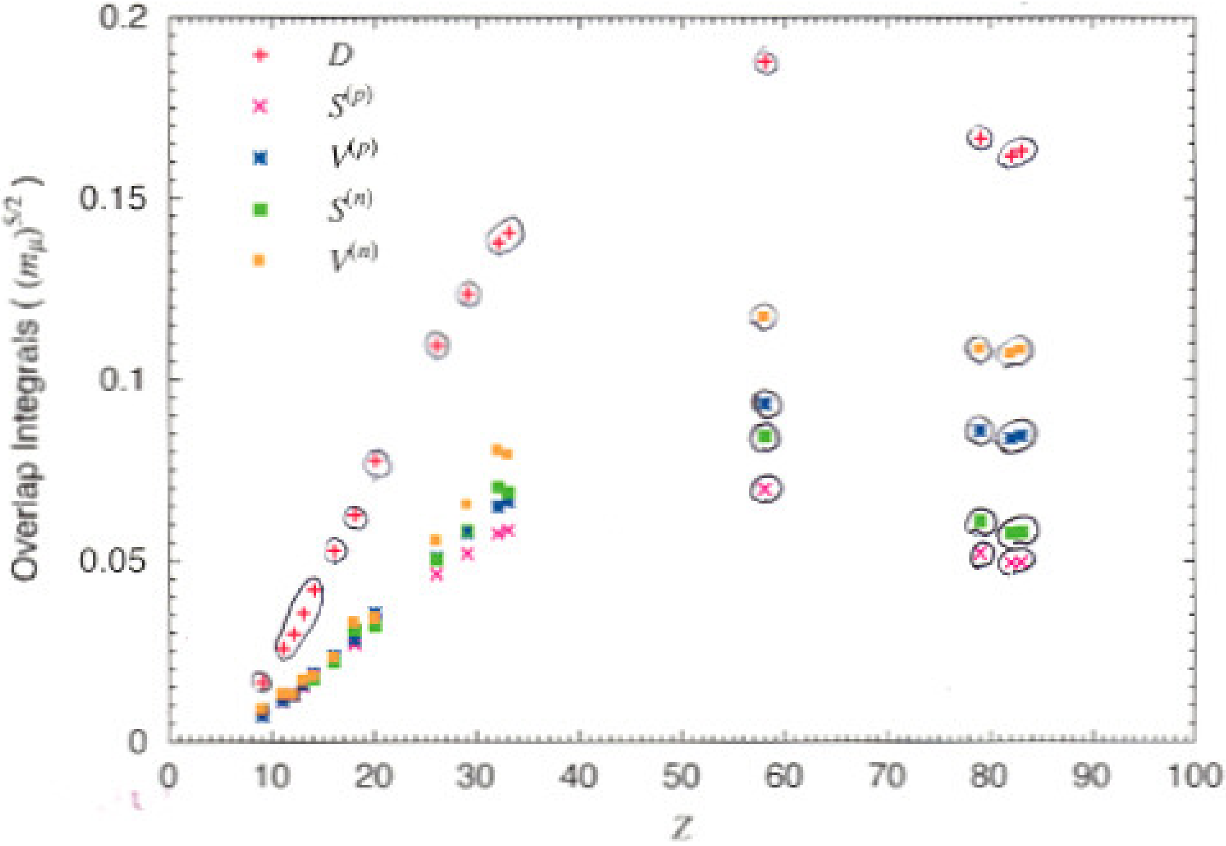


Conversion branching ratios

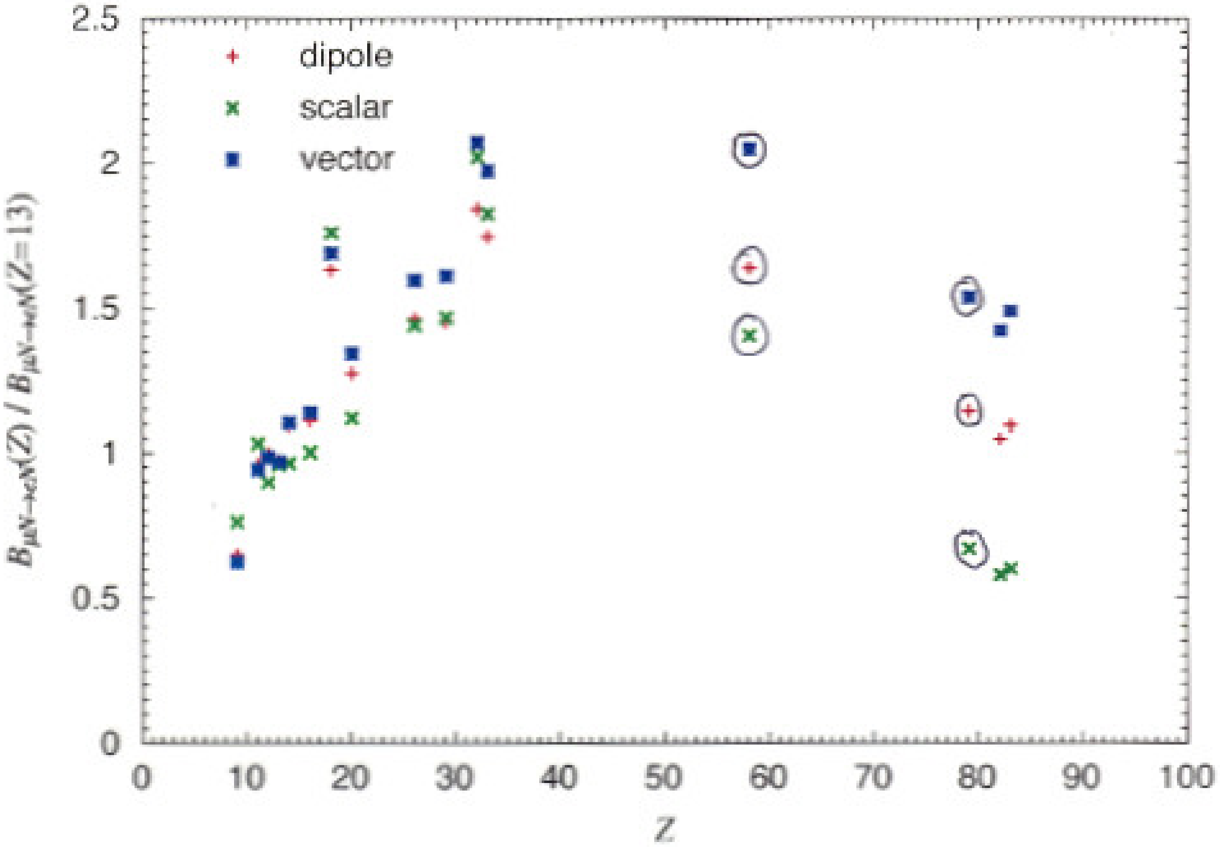


$\rho^{(n)}(r)$ from the pionic atom spectroscopy

Transition amplitudes —— Overlap integrals



Conversion branching ratios



4. Discussion

The ambiguities are chiefly due to the poor knowledge on the **neutron distribution in nuclei**.

Scattering experiments, Spectroscopy, ...

↓

$S^{(n)}$ and $V^{(n)}$suffers from the ambiguity.

- **Light nuclei** $[Z<(\sim 50)]$
  - Agrees anyway within (a few) %.

    (including "$\rho^{(n)}(r)=\rho^{(p)}(r)$" assumption)

- **Heavy nuclei** $[Z>(\sim 50)]$
  - Results of recent scattering experiments on Pb
    → Experimental error leads to **(a few) %** of ambiguity.
  - Spectroscopy gives (10 – 20)% smaller values.

    Spectroscopic methods gives info on the neutron distribution at the ***peripheral*** region, which is ***extrapolated*** into inside the nucleus under a certain assumption.

Neutron density distribution is known for limited species of nucleus at present.

5. Summary

- LFV $\mu - e$ conversion search is clean and informative.
- Muon ring enables the $\mu - e$ conversion search with various target nuclei.
- Nuclei with $Z \sim (30 — 60)$ gives the largest $\mu - e$ conversion rate.
- The measurements with different nuclei are useful to distinguish the theoretical models with LFV.
- Ambiguity is small for light nuclei (within a few %).
- Recent scattering experiments on Pb gives us a hope that ambiguity can be made small for heavy nuclei also.