NuFACT03 @ Columbia Univ. NY
June 5-11, 2003

Overview of Degeneracies

SARS in
ν Physics

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(Pathology Department, TMU)

The 4th International Workshop on Neutrino Factories based on Muon Storage Rings

NuFact '02

July 1st - 6th 2002

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CLRC Central Laboratory of the Research Councils
ECFA
PPARC Particle Physics and Astronomy Research Council
IOP The Institute of Physics

5th International Workshop on Neutrino Factories and Superbeams

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Sixth International Workshop on Neutrino Factories, Superbeams, and Reactors (NuFACT04)

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June 17-23, 2004

Osaka University, Osaka, Japan

周辺スポット：江戸時代の宿場であった大内宿（下郷町）。国の重要保存地区に指定されている。

Problem of Parameter Degeneracy

Definition of the Problem:

Precise determination of

$P(\nu_\mu \to \nu_\mu)$

$P(\nu_\mu \to \nu_e)$

$P(\bar{\nu}_\mu \to \bar{\nu}_e)$

at a single baseline and an energy

DOES NOT determine

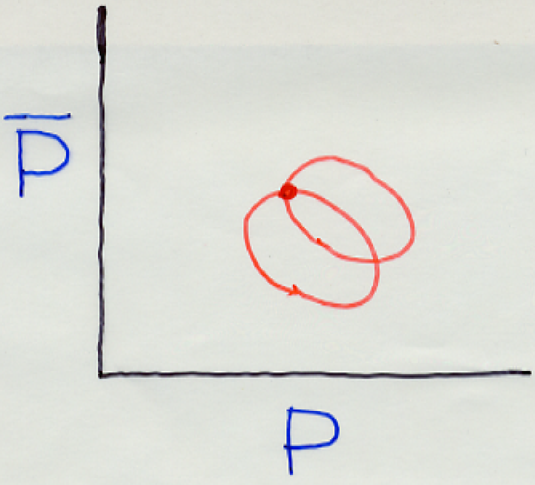
$\theta_{13} \quad \theta_{23} \quad \delta$

uniquely

$\Rightarrow$ maximal 8 fold degeneracy

Why Degeneracy?

- they come from "elliptic nature" of oscillation probability



intrinsic degeneracy

☹

$$P = A\cos\delta + B\sin\delta + C$$

- they are duplicated by discrete transformations

$$\begin{cases} \Delta m^2_{13} \longrightarrow -\Delta m^2_{13} \\ \theta_{23} \longrightarrow \frac{\pi}{2} - \theta_{23} \end{cases}$$

$$\underset{\text{intrinsic}}{2} \times \overset{\pm\Delta m^2}{2} \times \underset{\theta_{23}}{2} = 8$$

disappear if $\theta_{23} = \pi/4$

H.M. H. Nunokawa and S. Parke hep-ph/0208163
Phys. Rev. D66

FIGURES

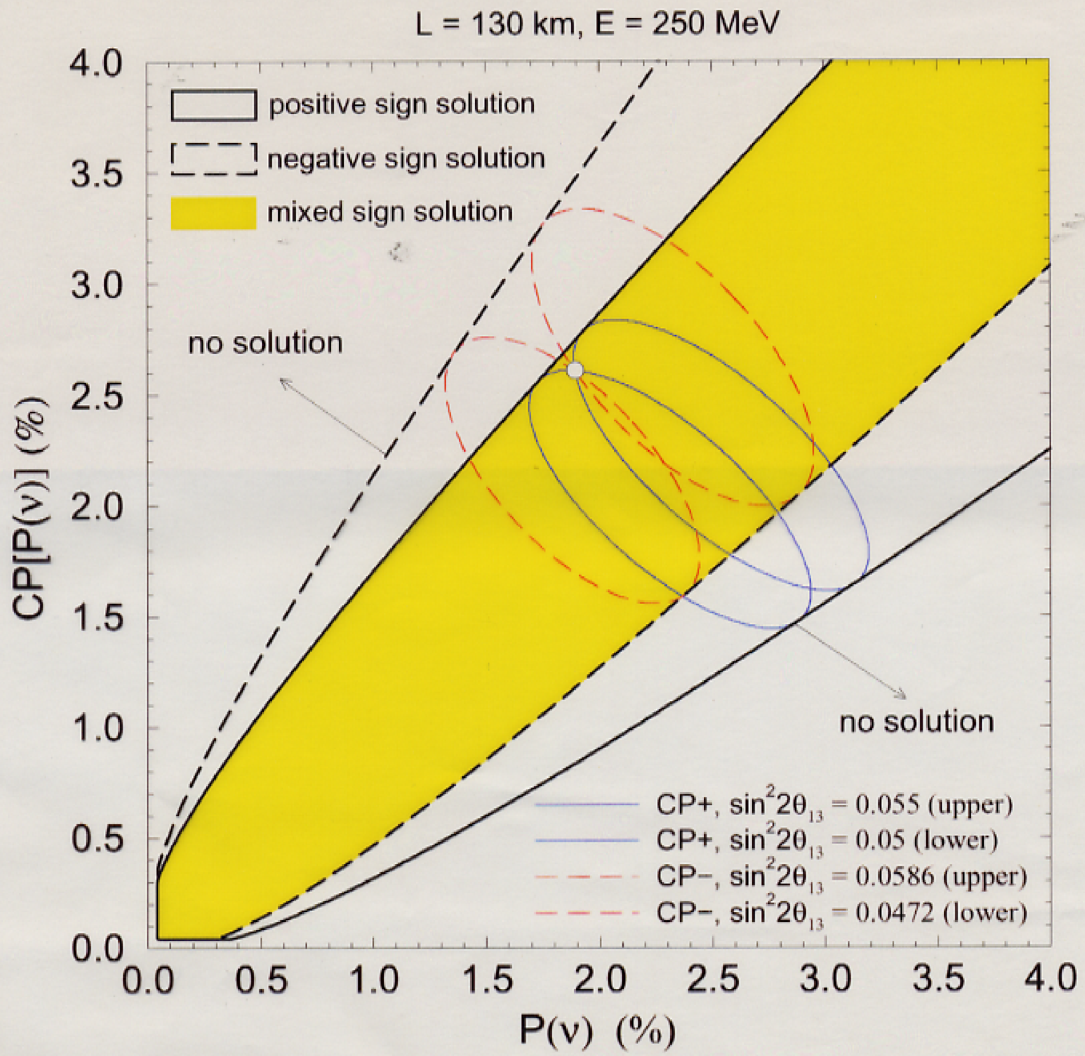


FIG. 1. An example of the degenerate solutions for the CERN-Frejius project in the $P(\nu) \equiv P(\nu_\mu \to \nu_e)$ verses $CP[P(\nu)] \equiv P(\bar{\nu}_\mu \to \bar{\nu}_e)$ plane. Between the solid (dashed) lines is the allowed region for positive (negative) Δm^2_{13} and the shaded region is where solution for both signs are allowed. The solid (dashed) ellipses are for positive (negative) Δm^2_{13} and they all meet at a single point. This is the CP parameter degeneracy problem. We have used a fixed neutrino energy of 250 MeV and a baseline of 130 km. The mixing parameters are fixed to be $|\Delta m^2_{13}| = 3 \times 10^{-3} eV^2$, $\sin^2 2\theta_{23} = 1.0$, $\Delta m^2_{12} = +5 \times 10^{-5} eV^2$, $\sin^2 2\theta_{12} = 0.8$ and $Y_e\rho = 1.5$ g cm^{-3}.

Parameter Degeneracy: The Simplest Form

$\dfrac{\Delta m^2_{12}}{\Delta m^2_{13}} \simeq 0$ $\oplus$ short baseline

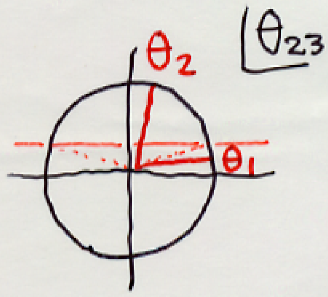
LOW or VAC $\nu_\odot$ solutions

(vacuum approximation OK)

[Notation]

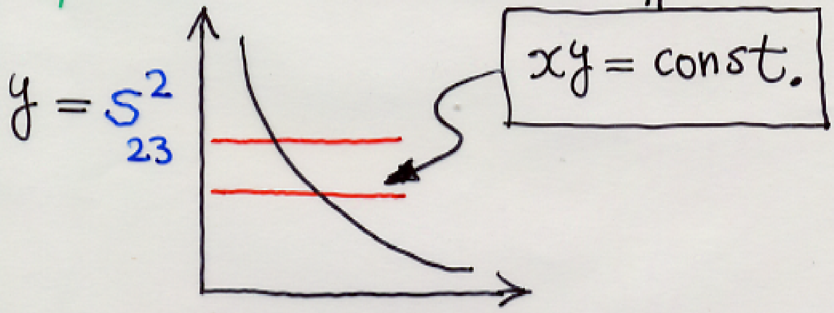
$$\Delta_{13} \equiv \frac{\Delta m^2_{13} L}{2E}$$

$$1 - P(\nu_\mu \to \nu_\mu) = \underline{\sin^2 2\theta_{23}} \sin^2 \frac{\Delta_{13}}{2}$$



$$P(\nu_\mu \to \nu_e) = \underline{s^2_{23} \sin^2 2\theta_{13} \sin^2 \frac{\Delta_{13}}{2}}$$

$\bar{\nu}_\mu \to \bar{\nu}_e$



$x = \sin^2 2\theta_{13}$

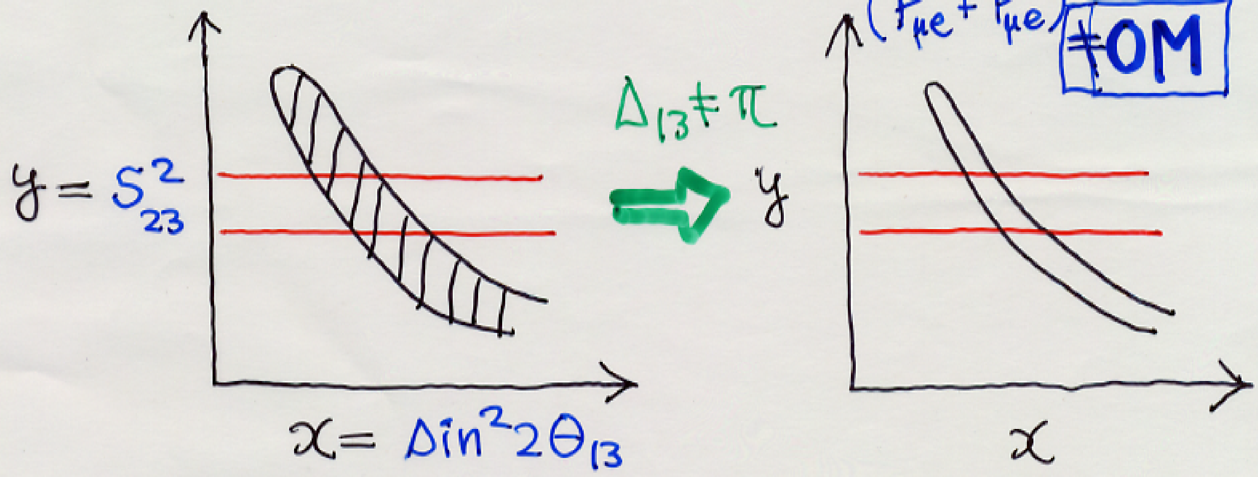
$\boxed{\text{If}}$ $\sin^2 2\theta_{23} = 0.95 \Rightarrow s^2_{23} = \begin{cases} 0.39 \\ 0.61 \end{cases}$ $\pm 20\%$ difference!

Parameter Degeneracy : The Next Simplest Form

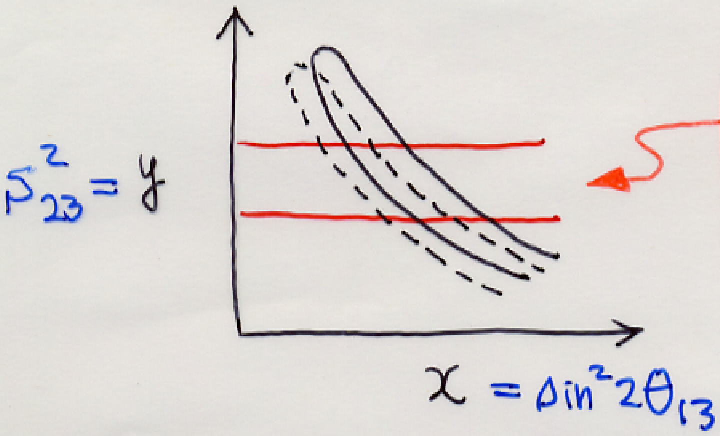
Vacuum approximation $\oplus$ $\frac{\Delta m^2_{12}}{\Delta m^2_{13}} \equiv \varepsilon_r \ll 1$

$0.1 \sim 0.01 \leftarrow$ $\boxed{\text{LMA}}$

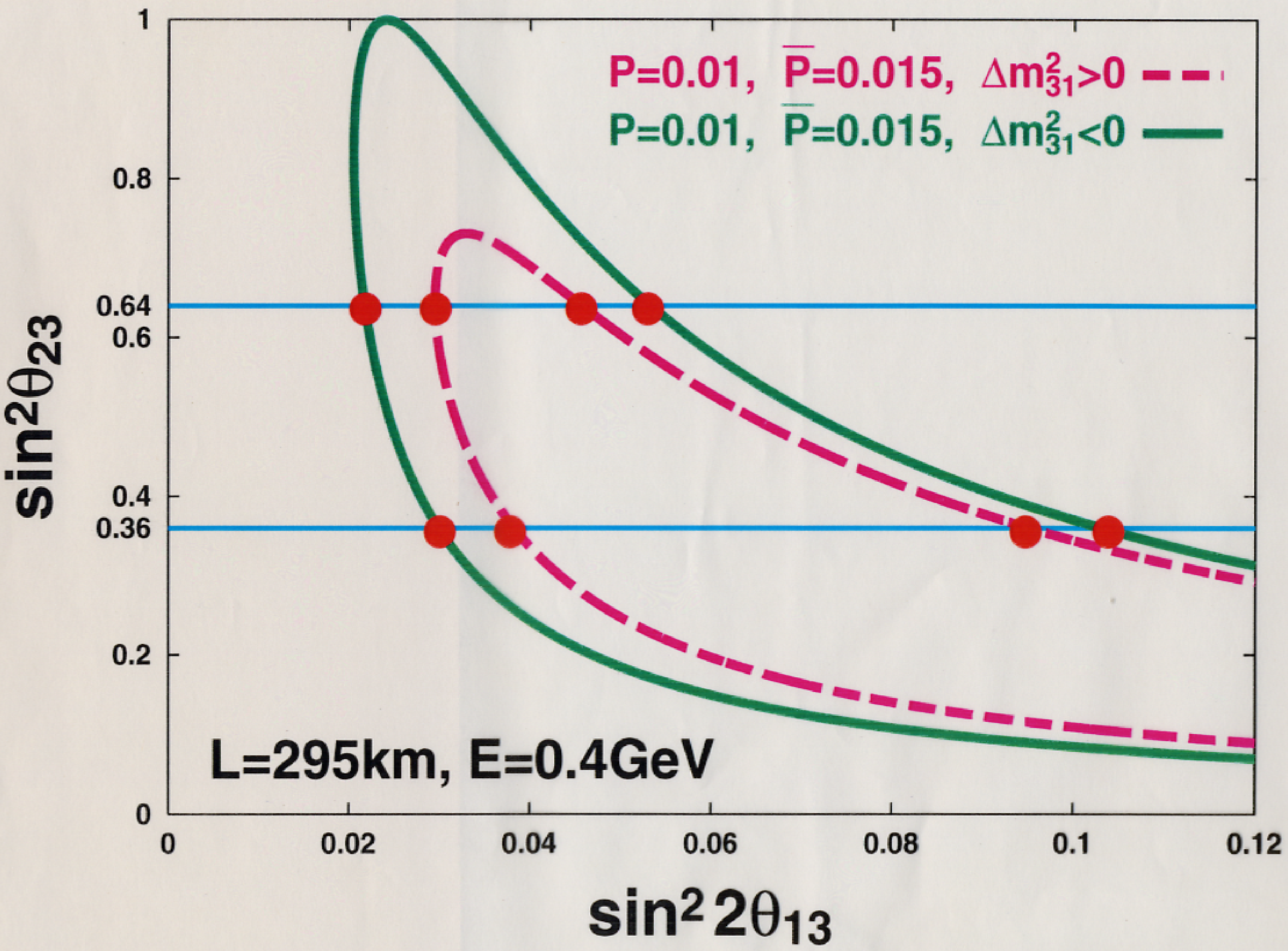
For $\varepsilon_r \neq 0$ there exist $\cos\delta$, $\sin\delta$ terms in $P_{\mu e}$



Matter effect $(\Delta_{13} \neq \pi)$



$\boxed{\text{8-fold degeneracy}}$



$P=0.01$, $\bar{P}=0.015$, $\Delta m^2_{31}>0$
$P=0.01$, $\bar{P}=0.015$, $\Delta m^2_{31}<0$
L=295km, E=0.4GeV
$\sin^2\theta_{23}$
$\sin^2 2\theta_{13}$
0
0.2
0.36
0.4
0.6
0.64
0.8
1
0.02
0.04
0.06
0.08
0.1
0.12

Oscillation Probabilities

$$\left(\begin{array}{l}\nu_\mu \to \nu_e \\ \bar{\nu}_\mu \to \bar{\nu}_e\end{array}\right.$$

$$P_+ = X_+ \theta^2 + Y_+ \theta \cos\left(\delta + \frac{\Delta_{13}}{2}\right) + P_\odot$$

$$\bar{P}_+ = X_- \theta^2 - Y_- \theta \cos\left(\delta - \frac{\Delta_{13}}{2}\right) + P_\odot$$

$$P_- = X_- \theta^2 + Y_- \theta \cos\left(\delta - \frac{\Delta_{13}}{2}\right) + P_\odot$$

$$\bar{P}_- = X_+ \theta^2 - Y_+ \theta \cos\left(\delta + \frac{\Delta_{13}}{2}\right) + P_\odot$$

$$\left\{\begin{array}{l}
\theta \equiv \theta_{13} \simeq s_{13} \qquad \Delta_{13} \equiv \frac{\Delta m_{13}^2}{2E} L \\
X_\pm \equiv 4 s_{23}^2 \left(\frac{\Delta_{13}}{\Delta_{13} \mp aL}\right) \sin^2\left(\frac{\Delta_{13} \mp aL}{2}\right) \\
Y_\pm \equiv \pm 2 \sin 2\theta_{12} \sin 2\theta_{23} \Delta_{12} \left(\frac{\Delta_{13}}{\Delta_{13} \mp aL}\right) \\
\qquad \times \sin\left(\frac{aL}{2}\right) / aL \times \sin\left(\frac{\Delta_{13} \mp aL}{2}\right)
\end{array}\right.$$

Relationship between Degenerate Solutions : Same Δm^2-sign degeneracy

$$\theta_2 - \theta_1 = \left(\frac{\sin\delta_1 + z\cos\delta_1}{1+z^2}\right)\left(\frac{C^{(+)2} - C^{(-)2}}{C^{(-)}}\right)\sin\left(\frac{\Delta_{13}}{2}\right)$$

$$\simeq \sin 2\theta_{12} \cot\theta_{23} \cos\delta_1 \left(\frac{\Delta_{12}}{2}\right) \cot\left(\frac{\Delta_{13}}{2}\right) + O(aL)$$

$$\delta_2 = \pi - \delta_1 + \underline{\arccos\left[\frac{z^2-1}{z^2+1}\right]}$$

$$\gtrsim 0.01\pi \quad \left(\frac{\pi}{2} < \Delta_{13} < \frac{3\pi}{2}\right)$$

$$z \equiv \frac{C^{(+)}}{C^{(-)}} \tan\left(\frac{\Delta_{13}}{2}\right)$$

$$C^{(\pm)} \equiv \frac{1}{2}\left(\frac{Y_+}{X_+} \mp \frac{Y_-}{X_-}\right)$$

The Mixed Δm^2-sign Degeneracy

$\theta_3 - \theta_1 =$ too complicated to show

$$\sin(\delta_1 - \delta_3) = 2x\sqrt{1-x^2} + O(aL)$$

$O(1)$

$$x \equiv \frac{\sqrt{P} - \sqrt{\bar{P}}}{c_{23} \sin 2\theta_{12} \Delta_{12} \sin\left(\frac{\Delta_{13}}{2}\right)}$$

Easily confuse

CP conservation

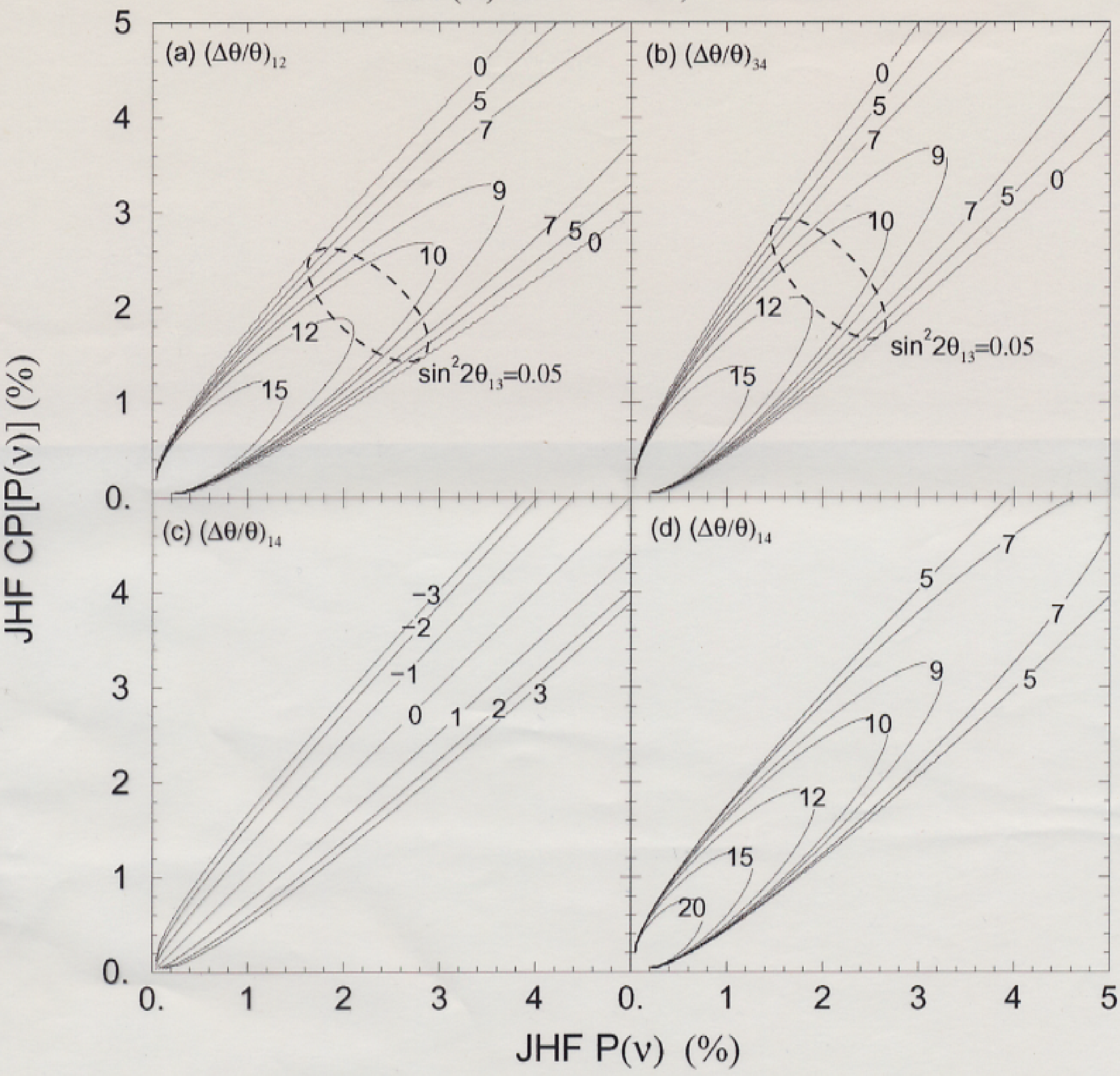
CP violation

$$\left(\frac{\Delta\theta}{\theta}\right)_{ij} \equiv \frac{\theta_i-\theta_j}{\frac{1}{2}(\theta_i+\theta_j)}$$

JHF

LMA I

$\Delta\theta/\theta$ (%) for L =295 km, E = 0.8 GeV

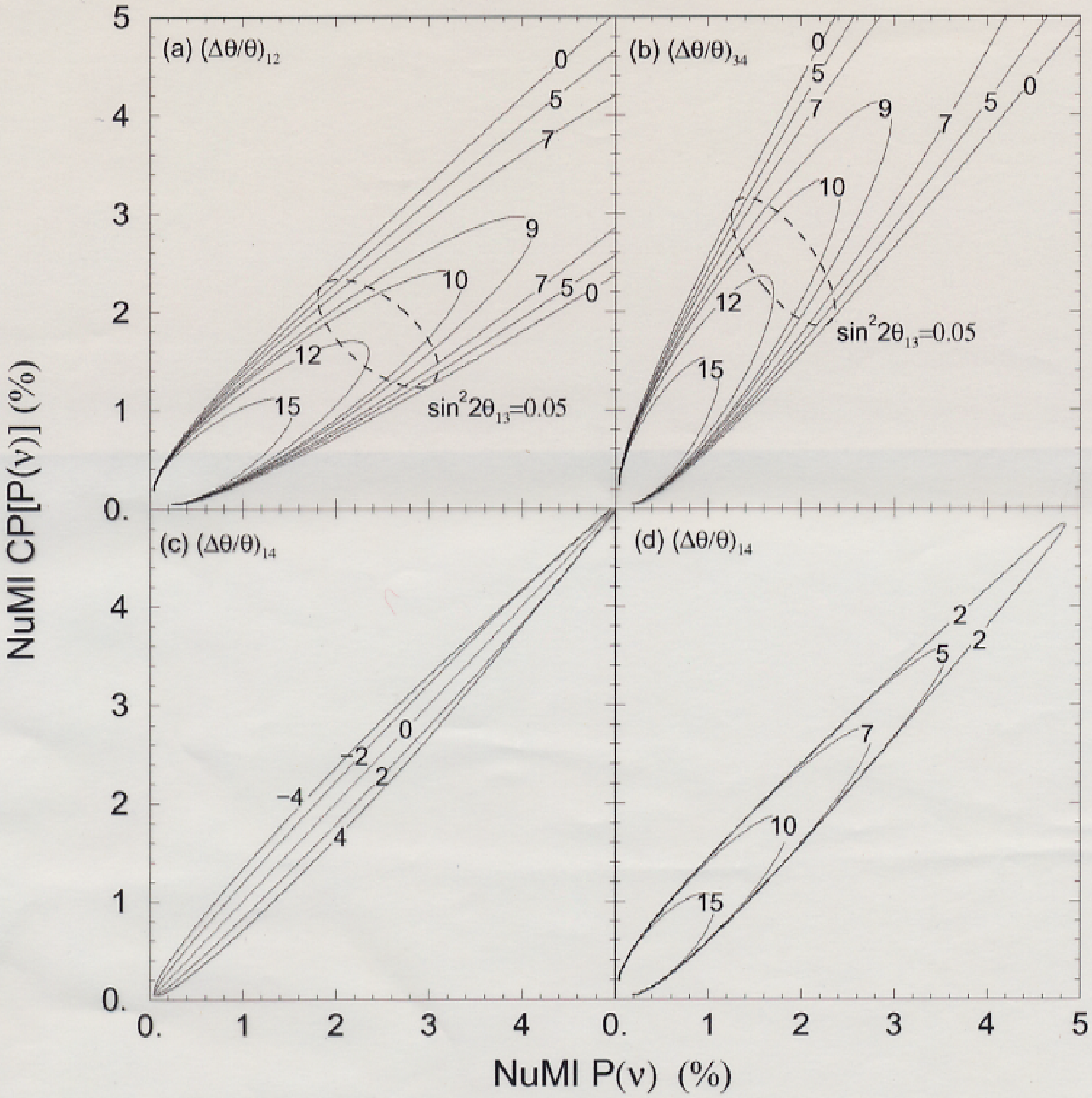


Same mixing parameters and matter density for JHF as in MNP3 (hep-ph/0301210).

$$\left(\frac{\Delta(\sin^2 2\theta)}{\sin^2 2\theta}\right)_{ij} \underset{\theta^2 \ll 1}{\simeq} \frac{\theta_i^2-\theta_j^2}{\frac{1}{2}(\theta_i^2+\theta_j^2)} \underset{\Delta\theta \ll \bar{\theta}}{\simeq} \frac{\theta_i-\theta_j}{\frac{1}{2}(\theta_i+\theta_j)} \times 2$$

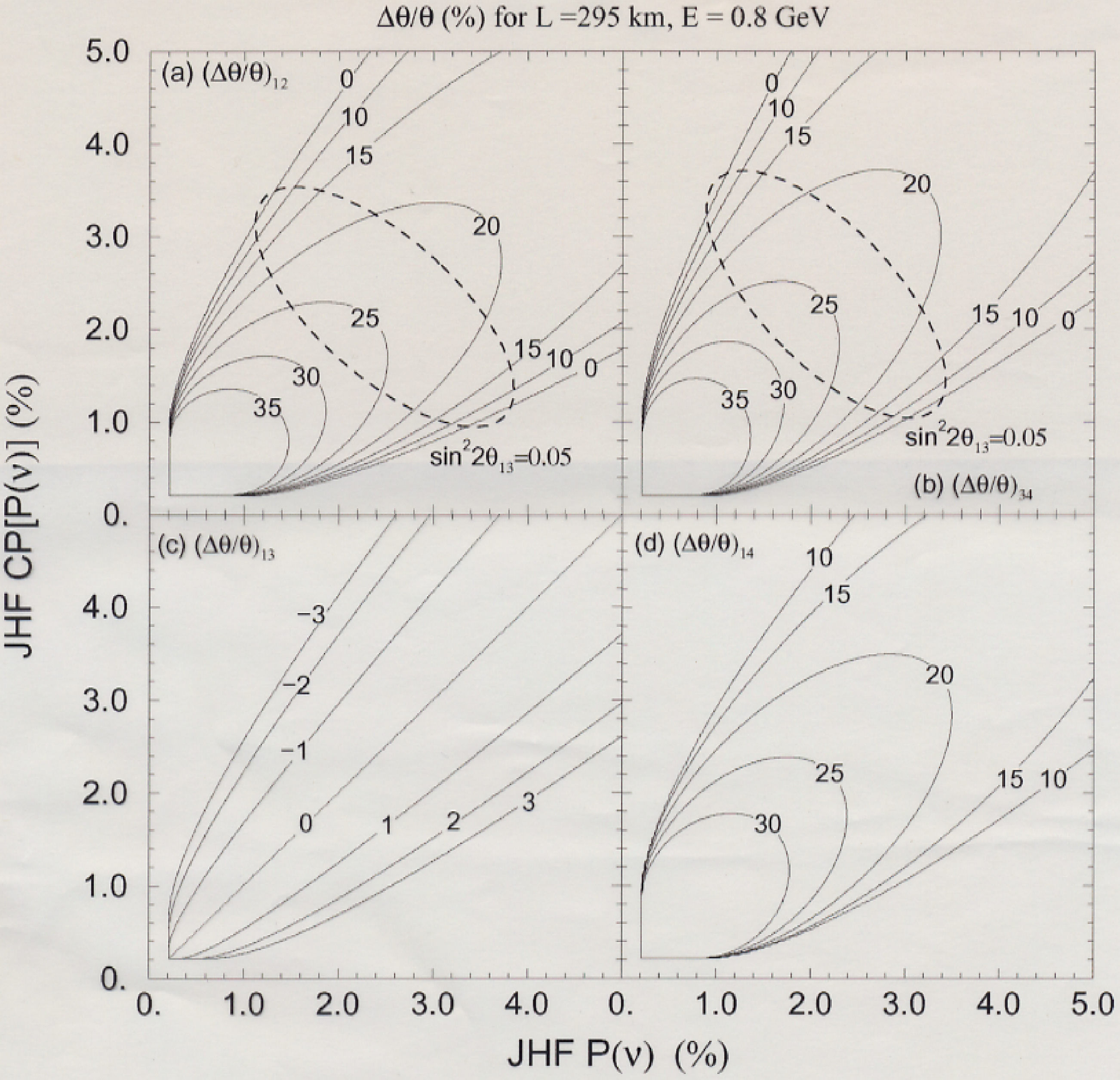
NuMI LMA I

$\Delta\theta/\theta$ (%) for L =732 km, E = 2 GeV



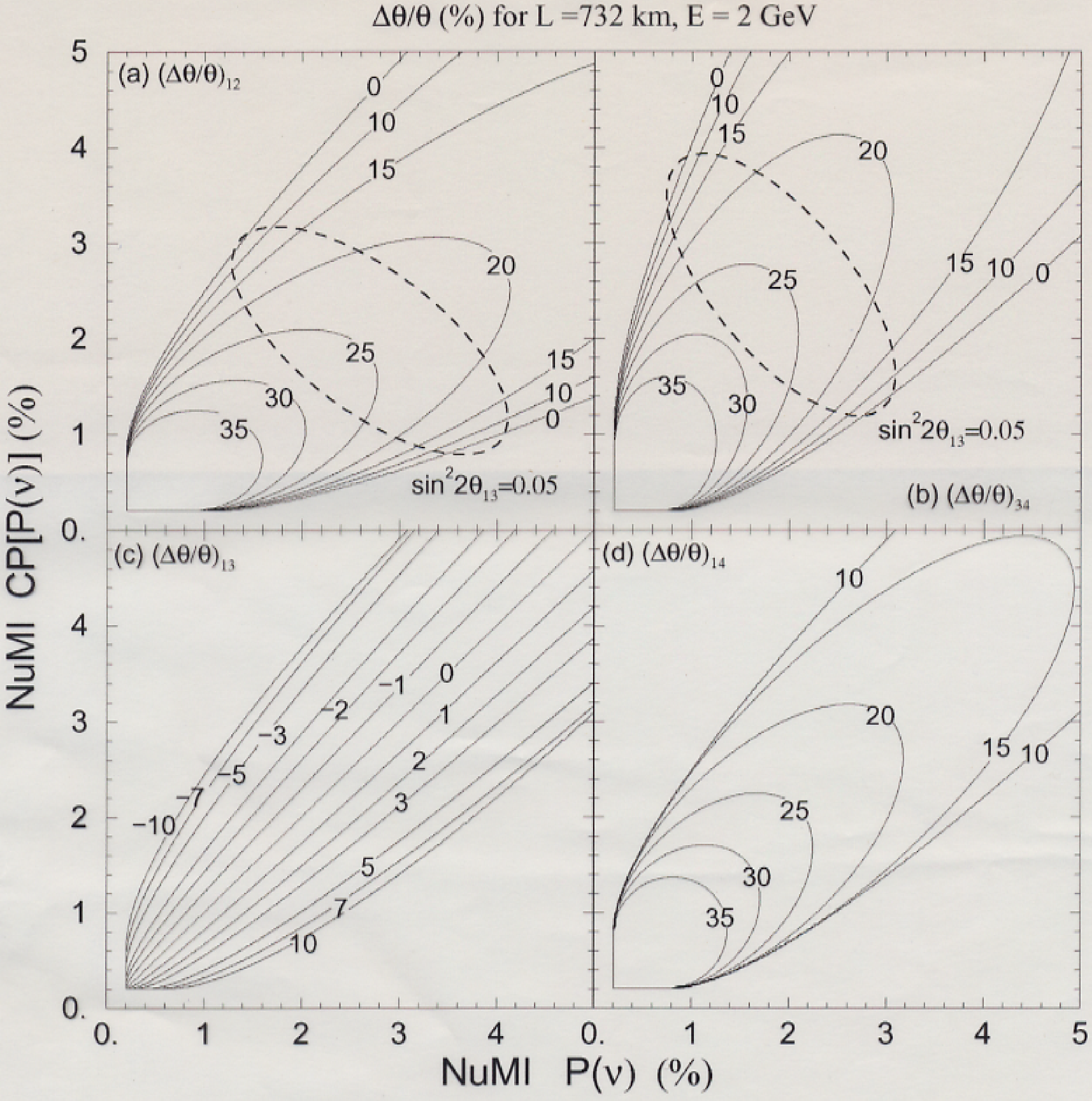
Same mixing parameters and matter density for NuMI as in MNP3 (hep-ph/0301210).

JHF LMA II



Same mixing parameters and matter density for JHF as in MNP3 (hep-ph/0301210), except for $\Delta m^2_{12} = 1.5 \times 10^{-4}$ eV^2 (LMA II)

NuMI LMA II



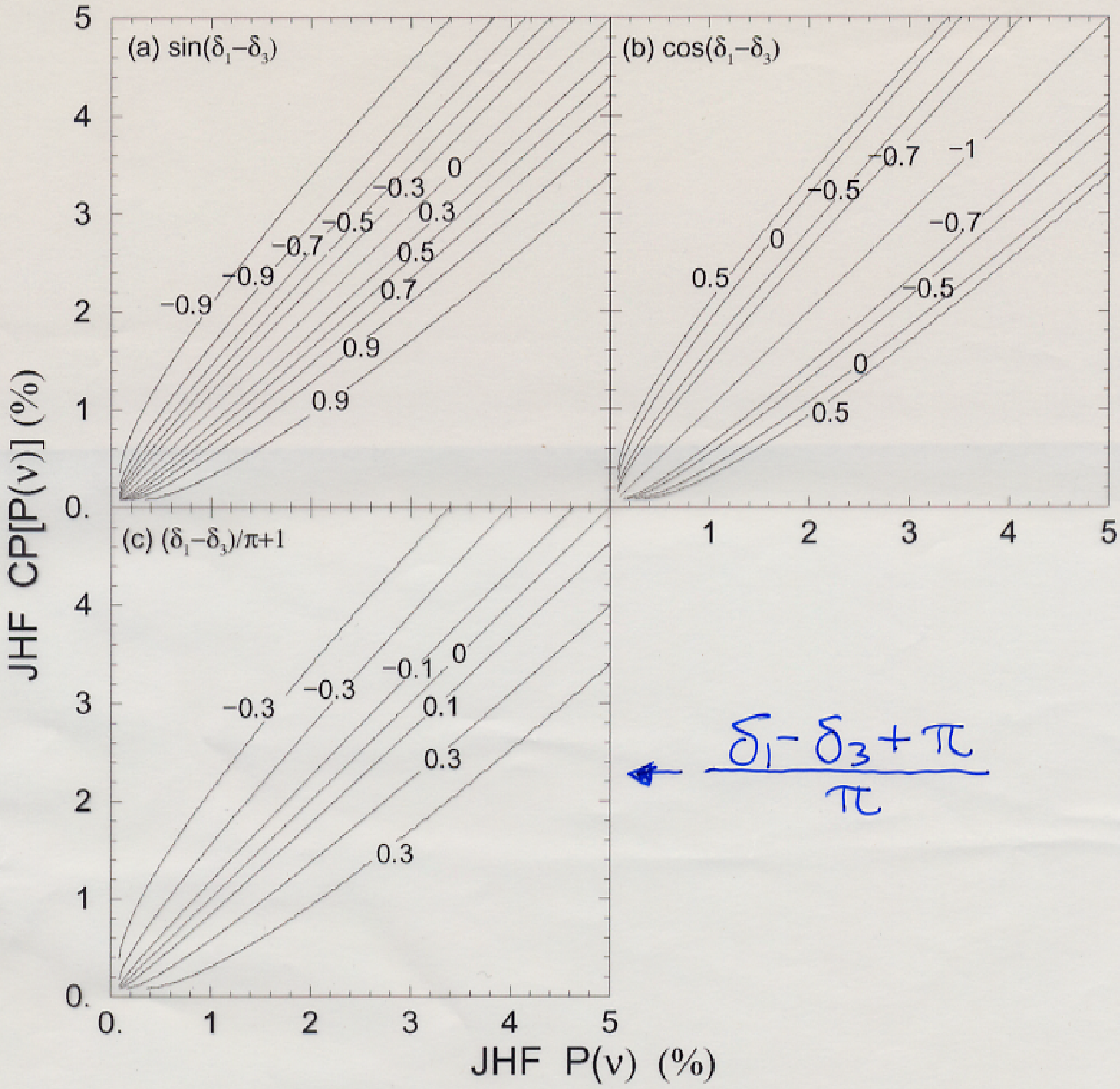
Same mixing parameters and matter density for NuMI as in MNP3 (hep-ph/0301210), except for $\Delta m^2_{12} = 1.5 \times 10^{-4}$ eV^2 (LMA II)

Mixed Δm^2-sign degeneracy

JHF

LMA I

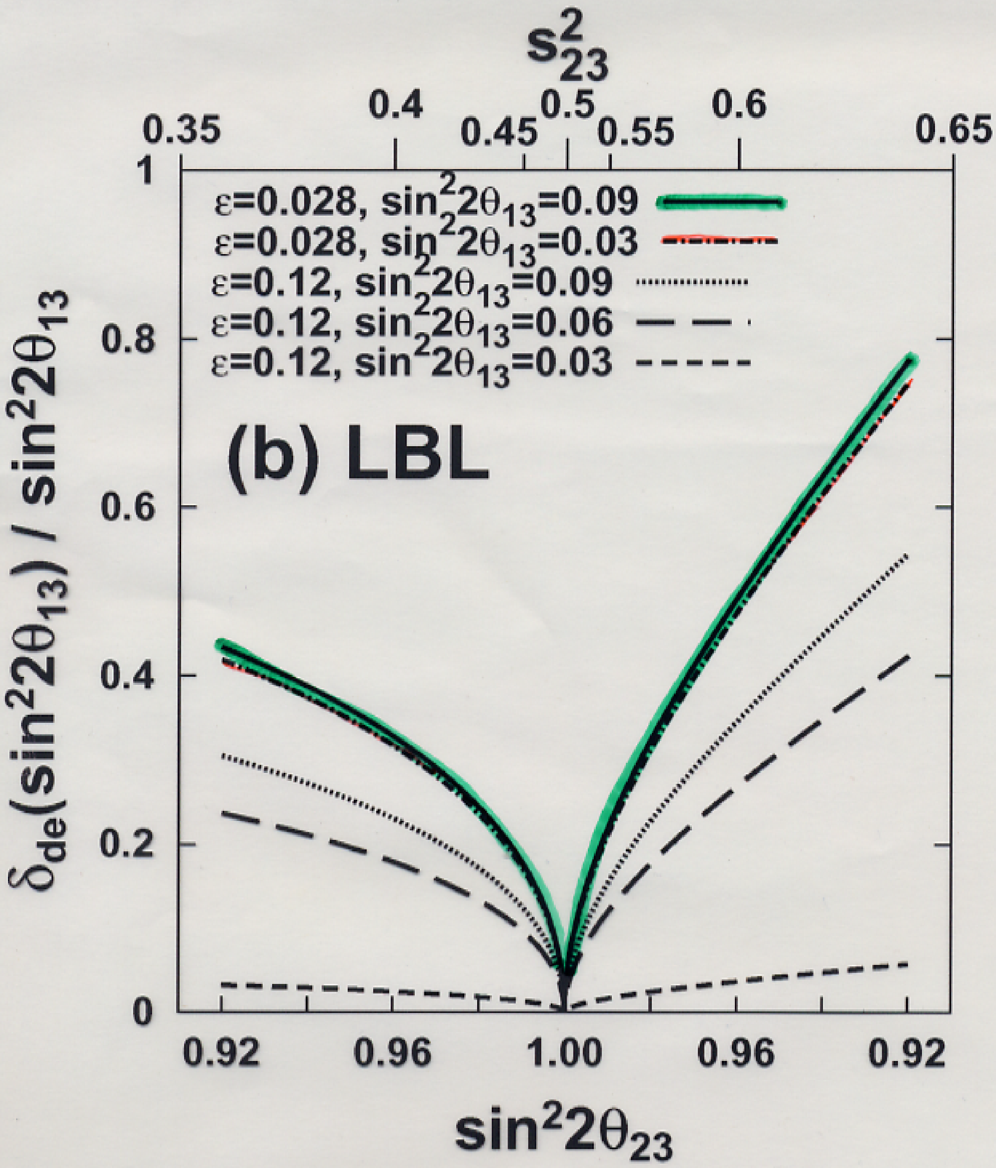
Δθ/θ (%) for L = 295 km, E = 0.6 GeV



$\leftarrow \dfrac{\delta_1 - \delta_3 + \pi}{\pi}$

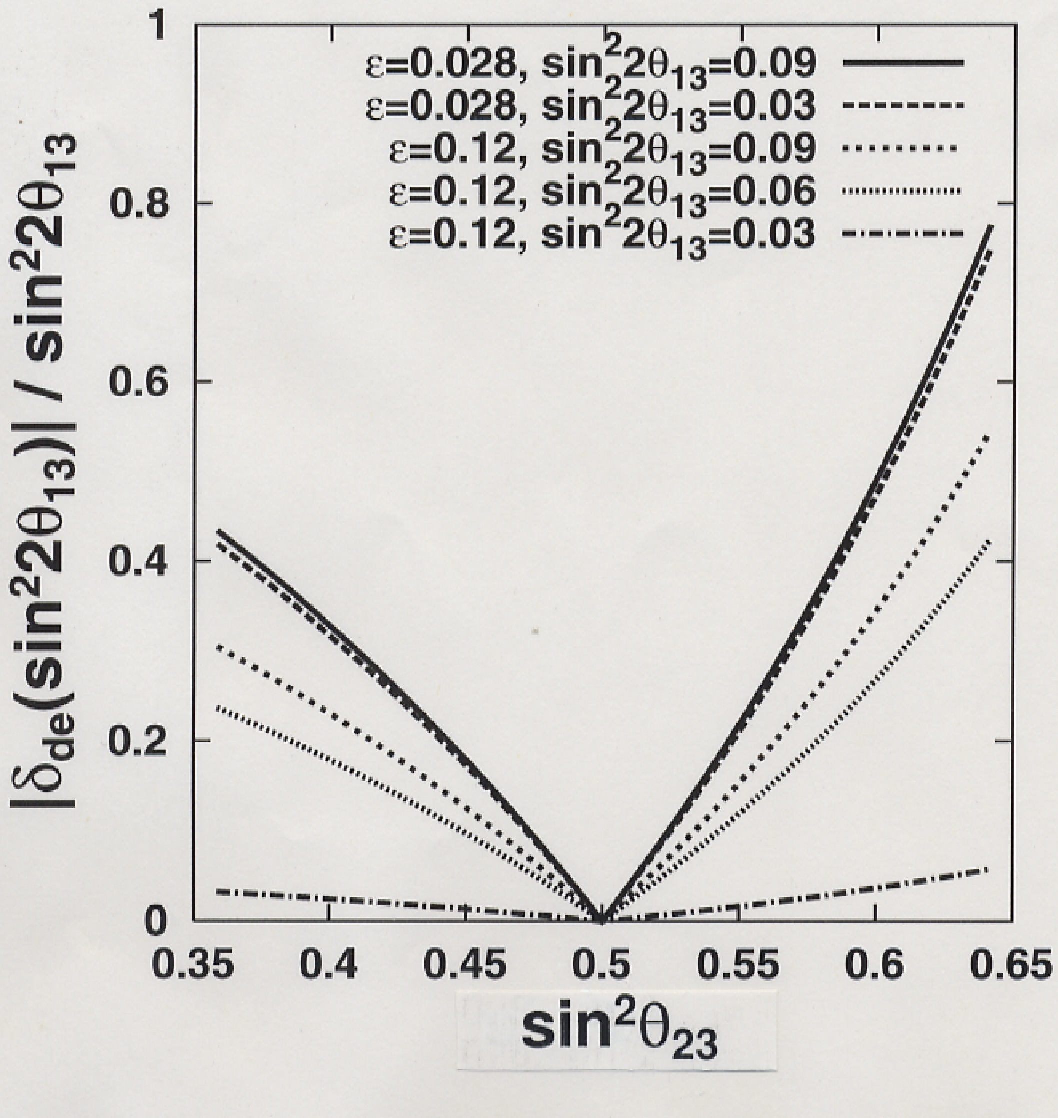
Same mixing parameters and matter density for JHF as in MNP3 (hep-ph/0301210).

$\Delta \sin^2 2\theta_{23}$ is NOT the right variable!



LMA I

s_{23} is the right variable !



T Violation

Cleaner measurement of $\not{CP}$ or $\not{T}$ phase δ

matter effect cannot create $\not{T}$

can create $\not{CP}$

KTY formula

$$P(\mu \to e) = A(a)\cos\delta + B(a)\sin\delta + C(a)$$

$$P(e \to \mu) = A(a)\cos\delta - B(a)\sin\delta + C(a)$$

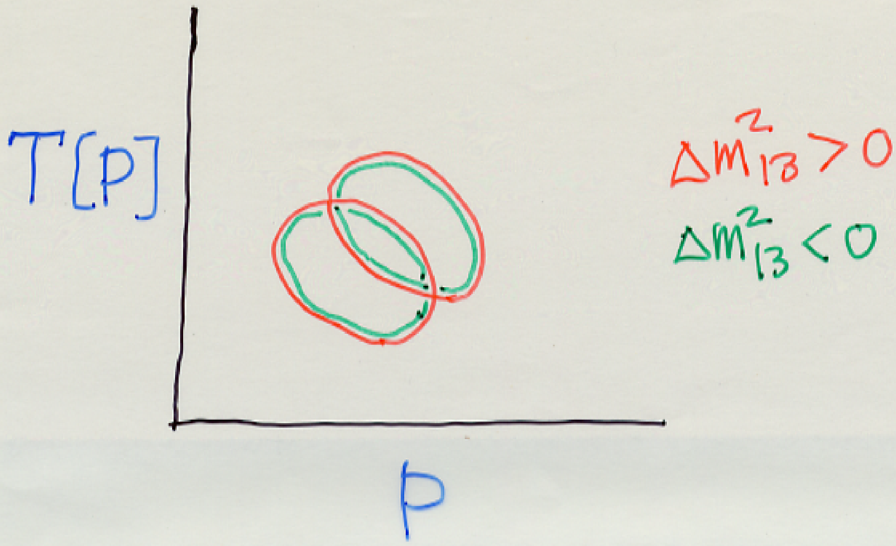
$$\Delta_T P = 2B(a)\sin\delta$$

matter effects modify the coefficient.

<note>

$$P(\bar{\mu} \to \bar{e}) = A(-a)\cos\delta - B(-a)\sin\delta + C(-a)$$

Degeneracy in T measurement



- $\text{sign}(\Delta m^2_{13})$ **cannot** be determined by T measurement at single $\begin{pmatrix} E \\ L \end{pmatrix}$ **no matter how large** the matter effect is !

☹ exact symmetry in P − T[P] system

~~$P[\nu] \to T[P]$~~ under

$$\begin{pmatrix} P_+ \\ T[P]_+ \end{pmatrix} \to \begin{pmatrix} P_- \\ T[P]_- \end{pmatrix} \quad \left(\begin{array}{l} \delta \to \pi - \delta \\ \theta \to \sqrt{\dfrac{X_-}{X_+}}\, \theta \end{array} \right.$$

- No CPV/CPC confusion

How to Solve Degeneracies ?

- **Combining LBL with reactors**

  may resolve θ_{23} degeneracy if $\sin^2 2\theta_{13} \gtrsim 0.03$

  and if departure ~~of~~ from maximal mixing large

- **Combining LBL's with different channel**

  $\nu_\mu \to \nu_e$ & $\nu_e \to \nu_\tau$

  $P_{\nu_\mu \to \nu_e} \propto s_{23}^2$ $P_{\nu_e \to \nu_\tau} \propto c_{23}^2$

- **Combining LBL's with different E/L**

$$\theta_2 - \theta_1 \simeq \sin 2\theta_{12} \cot\theta_{23} \cos\delta_1 \left(\frac{\Delta_{12}}{2}\right) \cdot \underline{\cot\left(\frac{\Delta_{13}}{2}\right)} + aL$$

SPL −0.43

NuFACT +2.68 (30 GeV, 2800 km)

Tuning Beam Energy to Thin Ellipse Limit

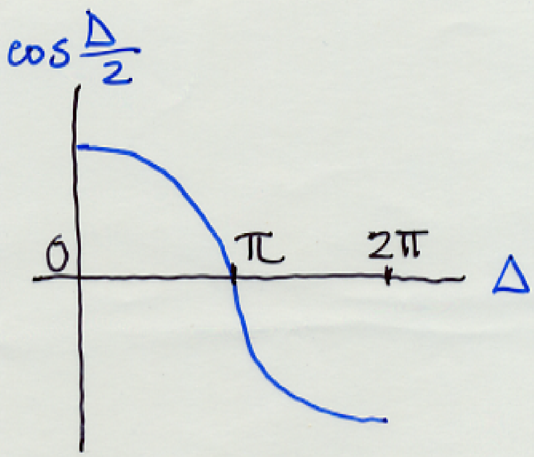
$\neq$ Oscillation maximum

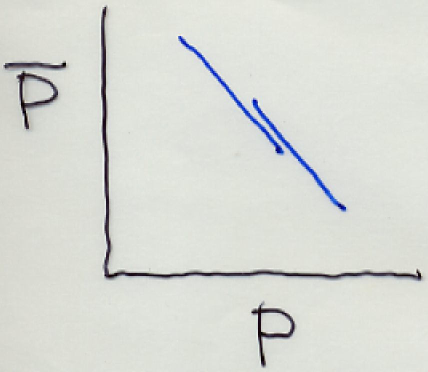
$$* \Delta_{12} \cos\left(\delta \pm \frac{\Delta_{13}}{2}\right) = * \Delta_{12} \cos\delta \cos\frac{\Delta_{13}}{2}$$

$+ \sin\delta$ term

Tune the beam energy

$$\left\langle \phi_\nu(E)\sigma(E)\,\Delta_{12}\cos\frac{\Delta_{13}}{2}\right\rangle = 0$$





$\Rightarrow$ Elimirate intrinsic degeneracy for θ_{13}

$\Rightarrow$ $\delta \leftrightarrow \pi - \delta$ remains

$\sin^2 2\theta_{13} \cdot s_{23}^2$ if 1-st·2nd octant deg.

$\Rightarrow$ another measurement $(P, \bar{P})$ at $\Delta_{13} \neq \pi$ lift remaining degeneracies

One by One
vs.
Everything at once

Strategies

If we know the sign of Δm^2_{13}

4 fold

If θ_{23} maximal

2 fold

MINOS
JHF phase I
can tell !

intrinsic degeneracy !

How to determine sign of Δm^2_{13} ?

in next generation LBL's

sign of Δm^2_{13}

(HM - Nunokawa - Parke)

FIGURES

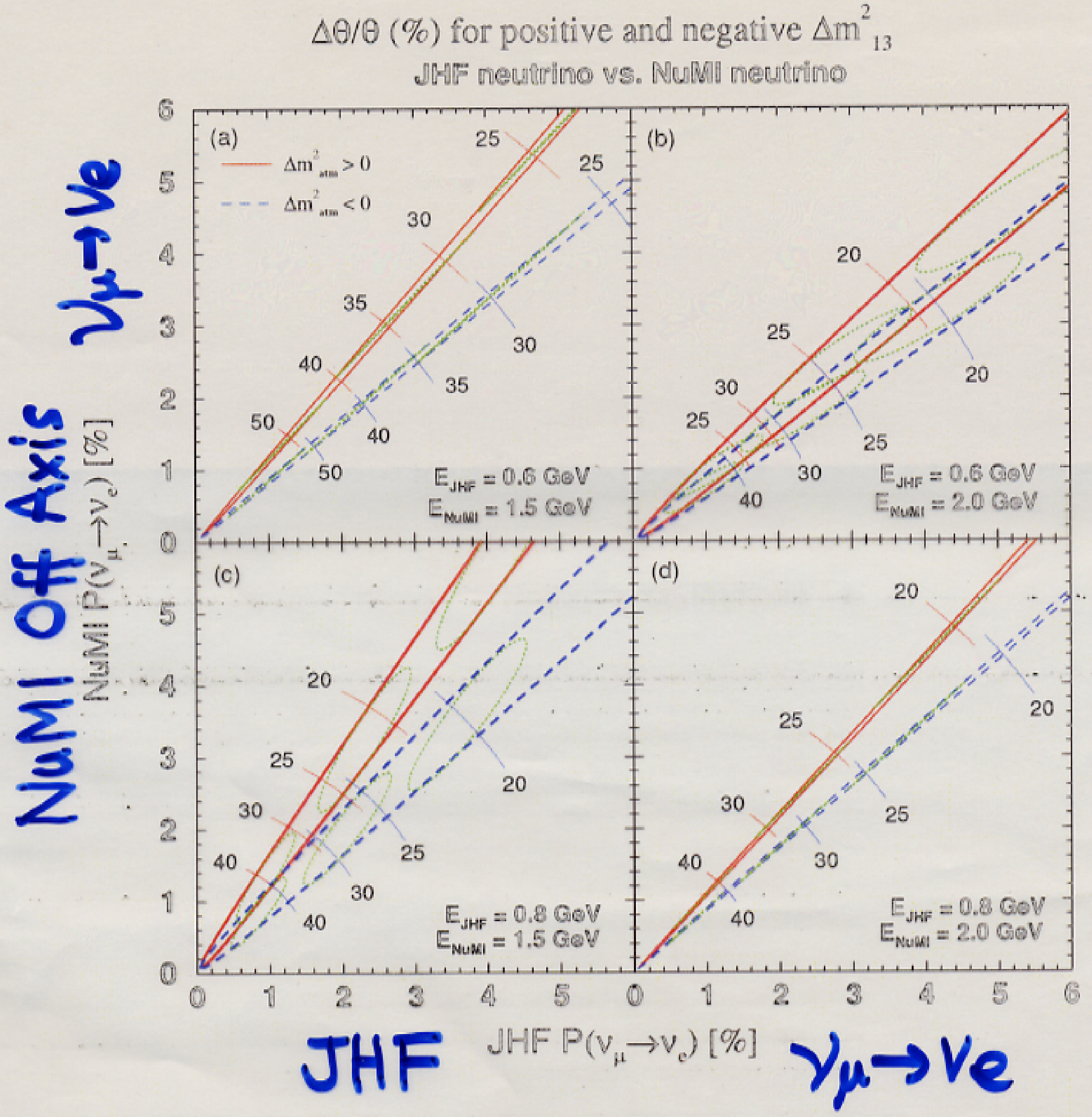


FIG. 1. Allowed range of $P(\nu_\mu \to \nu_e)$ for JHF verses $P(\nu_\mu \to \nu_e)$ for NuMI, which are referred to as "pencils" in the text, are delimited by thick solid (dashed) lines for positive (negative) Δm^2_{13} for the energies (E_{JHF}/GeV, E_{NuMI}/GeV) = (a) (0.6,1.5), (oscillation maximum for both experiments) (b) (0.6,2.0), (c) (0.8,1.5) and (d) (0.8,2.0). In the same plot, the positions for some representative values of the fractional variation across the width of the "pencil" of $\theta \equiv \sin\theta_{13}$; $\Delta\theta/\theta$ [%], indicated by numbers, are shown by thin solid arcs. Inside each allowed region, trajectories corresponding to $\sin^2 2\theta_{13} = 0.02$, 0.05 and 0.09 are plotted by dotted lines. The mixing parameters are fixed to be $|\Delta m^2_{13}| = 2.5\times10^{-3}$ eV^2, $\sin^2 2\theta_{23} = 1.0$, $\Delta m^2_{12} = +7\times10^{-5}$ eV^2 and $\sin^2 2\theta_{12} = 0.85$ whereas θ_{13} and δ are assumed to be unknown. The electron density is fixed to be $Y_e\rho = 1.15$ and 1.4g cm^{-3} for JHF and NuMI experiment, respectively. For JHF and NuMI both anti-neutrinos, the roles of $\Delta m^2_{13} > 0$ and $\Delta m^2_{13} < 0$ are interchanged.

Parameter Degeneracy

$\Rightarrow$ Hard to solve experimentally

$\because$

terms with resolving power is

either

small (apart from $\pm \Delta m^2_{13}$ at large L)

or

hard to detect ($\nu_e \to \nu_\tau$)

What Should We do in coming 5 years ?

Theorists

- Figure out all details of the PD
- develope practical ideas for experimental solution

Experimentalists

- Find $\theta_{13} \neq 0$
- Do NOT worry about PD

Theorists $\oplus$ Experimentalists

- Start collaboration toward the Real solution of PD

Parameter Degeneracy : Summary

- Nature of the desease : ~ understood theoretically

  a bit better Situation Compared with SARS

- Realistic way for experimental resolution still uncovered

what can we do in coming 10, 15, 20 years ?