An Overview of Neutrino Masses and Mixing in $SO(10)$ Models

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"Fermion masses and mixing and CP violation in SO(10) models with family symmetries"

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What Do We Know about Neutrinos?

- SK:

  $\Delta m^2_{atm} = 2.6 \times 10^{-3} eV^2, \quad \sin^2 2\theta_{atm} = 1.0$

- Solar + KamLAND:

  LMA solution is the unique solution at 4.7σ
  $\Delta m^2_{\odot} = 7.25 \times 10^{-5} eV^2, \quad \tan^2 \theta_{\odot} = 0.45$

- CHOOZ: $\sin\theta_{13} < 0.16$

- LSND: $\nu_\mu \to \nu_e$?

- Neutrinoless double beta decay:
  $< m >_{ee} = (0.05 - 0.86)\ eV$

- WMAP: $m_\nu < 0.69 eV$

- Put aside the LSND result, there are three possibilities to accommodate all experiments

  (i) normal (ii) inverted (iii) degenerate

- To account for LSND result:
  (i) sterile neutrinos
  (ii) CPT violation

$$\mathcal{L}_{cc} = (\overline{\nu}_1, \overline{\nu}_2, \overline{\nu}_3)_L \gamma^\mu U^\dagger_{PMNS} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L W^+_\mu$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad U_{PMNS} = U_{e,L} U^\dagger_{\nu,L}$$

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\delta_{12}} & \\ & & e^{i\delta_{13}} \end{pmatrix}$$

Bi-large mixing pattern translates into,

$$U_{PMNS} = \begin{pmatrix} c_{12} & s_{12} & \epsilon \\ -(s_{12}+c_{12}\epsilon)/\sqrt{2} & (c_{12}-s_{12}\epsilon)/\sqrt{2} & 1/\sqrt{2} \\ (s_{12}-c_{12}\epsilon)/\sqrt{2} & -(c_{12}+s_{12}\epsilon)/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

In other words,

$$\begin{aligned} |\nu_1> &= c_{12}|\nu_e> - \frac{1}{\sqrt{2}} s_{12}(|\nu_\mu> - |\nu_\tau>) - \frac{\epsilon}{\sqrt{2}} c_{12}(|\nu_\mu> + |\nu_\tau>) \\ |\nu_2> &= s_{12}|\nu_e> + \frac{1}{\sqrt{2}} c_{12}(|\nu_\mu> - |\nu_\tau>) - \frac{\epsilon}{\sqrt{2}} s_{12}(|\nu_\mu> + |\nu_\tau>) \\ |\nu_3> &= \frac{1}{\sqrt{2}}(|\nu_\mu> + |\nu_\tau>) + \epsilon|\nu_e> \end{aligned}$$

Evidence of Physics Beyond the Standard Model

- In SM:
  no ν_R
  lepton number conservation
  $\Rightarrow$ neutrinos are massless

- Many Free Parameters:
  fermion masses and mixing angles are parameterized by three arbitrary complex 3×3 Yukawa matrices

- $m_\nu \neq 0 \Rightarrow$ make SUSY GUTs attractive candidates as Beyond the SM Physics
  - gauge coupling constant unification
  - charge quantization explained $\Rightarrow \sin^2\theta_w$
  - provide the ingredients needed to generate neutrino masses a la the see-saw mechanism:

    the existence of ν_R
    the $(B-L)$ symmetry and a heavy scale $M_{(B-L)}$

Small Neutrino Masses

- Seesaw mechanism

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \quad \overset{v_R}{\rightarrow} \quad SU(2)_L \times U(1)_Y \quad \Rightarrow \quad \begin{pmatrix} 0 & 0 \\ 0 & f v_R \end{pmatrix}$$

$$\overset{v_{ew}}{\rightarrow} \quad U(1)_{EM} \quad \Rightarrow \quad \begin{pmatrix} f v_L & h v_{ew} \\ h v_{ew} & f v_R \end{pmatrix}$$

  - Type I seesaw mechanism: without parity

$$\begin{pmatrix} 0 & M_{LR}^T \\ M_{LR} & M_{RR} \end{pmatrix}, \qquad M_\nu^{eff} = -M_{LR} M_{RR}^{-1} M_{LR}^T$$

  - Type II seesaw mechanism: with parity

$$\begin{pmatrix} M_{LL} & M_{LR}^T \\ M_{LR} & M_{RR} \end{pmatrix}, \qquad M_\nu^{eff} = M_{LL} - M_{LR} M_{RR}^{-1} M_{LR}^T$$

Other possibilities:
$\Rightarrow$ harder to understand why m_ν is small

- R-parity violation
- Radiative generated masses
- $SU(2)_L$ triplet

SO(10)

All 15 known fermions and one RH neutrino in each family are unified into a single spinor representation

u	:	$\vert$	$+$	$-$	$+$	$+$	$-$	$>$
u	:	$\vert$	$+$	$-$	$+$	$-$	$+$	$>$
u	:	$\vert$	$+$	$-$	$-$	$+$	$+$	$>$
d	:	$\vert$	$-$	$+$	$+$	$+$	$-$	$>$
d	:	$\vert$	$-$	$+$	$+$	$-$	$+$	$>$
d	:	$\vert$	$-$	$+$	$-$	$+$	$+$	$>$
u^c	:	$\vert$	$-$	$-$	$+$	$-$	$-$	$>$
u^c	:	$\vert$	$-$	$-$	$-$	$+$	$-$	$>$
u^c	:	$\vert$	$-$	$-$	$-$	$-$	$+$	$>$
d^c	:	$\vert$	$+$	$+$	$+$	$-$	$-$	$>$
d^c	:	$\vert$	$+$	$+$	$-$	$+$	$-$	$>$
d^c	:	$\vert$	$+$	$+$	$-$	$-$	$+$	$>$
e	:	$\vert$	$+$	$-$	$-$	$-$	$-$	$>$
ν_e	:	$\vert$	$-$	$+$	$-$	$-$	$-$	$>$
e^c	:	$\vert$	$-$	$-$	$+$	$+$	$+$	$>$
ν^c	:	$\vert$	$+$	$+$	$+$	$+$	$+$	$>$

- Yukawa Sector:

$$16 \otimes 16 = 10_S \oplus 120_A \oplus 126_S$$

$$y^{10}_{ab} = y^{10}_{ba}, \quad y^{120}_{ab} = -y^{120}_{ba}, \quad y^{126}_{ab} = y^{126}_{ba}$$

$$M_u \leftrightarrow M_{\nu_{LR}}, \quad M_d \leftrightarrow M_e$$

- Symmetry Breaking
  - Left-Right Symmetry breaking chain
    $\Rightarrow$ symmetric texture

$$\begin{aligned} SO(10) &\rightarrow SU(4) \times SU(2)_L \times SU(2)_R \\ &\rightarrow SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ &\rightarrow SU(3) \times SU(2)_L \times U(1)_Y \\ &\rightarrow SU(3) \times U(1)_{EW} \end{aligned}$$

  - $SU(5)$ breaking chain
    $\Rightarrow$ lop-sided texture

$$\begin{aligned} SO(10) &\rightarrow SU(5) \\ &\rightarrow SU(3) \times SU(2) \times U(1) \\ &\rightarrow SU(3) \times U(1)_{EW} \end{aligned}$$

- Under SU(5) decomposition:

$$16 = 1 + \overline{5} + 10$$
$$126 = 1 + \overline{5} + 10 + \overline{15} + 45 + \overline{50}$$

Two ways to generate RH neutrino Majorana masses:

– $< \overline{126} >$: $\Delta(B-L) = 2$, $(16)_i (16)_j < \overline{126} >$

R-parity preserved at all energy scales

$$R = (-1)^{3(B-L)+2S}$$

difficult to obtain from heterotic string theories up to level-5 construction

– $< \overline{16} >$: $\Delta(B-L) = 1$, $\frac{(16)_i < \overline{16} > (16)_j < \overline{16} >}{M_{pl}}$

R-parity is broken
need to impose additional discrete symmetry to stabilize dark matter

Type II seesaw: M_{LL} due to coupling to 15 of SU(5) in $\overline{126}$

The flavor structure:

Phenomenological approach:

- texture assumption
- ϵ expansion

Top-down approach:

- family symmetry + Froggatt-Nielsen mechanism

  SM + ν_R: 6 multiplets in one generation

  $$G_F \subset [U(3)]^6$$

  SO(10): single multiplet for each generation

  $$G_F \subset U(3)$$

  - Abelian family symmetry
  - Non-Abelian family symmetry

Less model-dependent approach:

- renormalization group evolution
- "bottom-up" approach in minimal models

Models differ by

- the Higgs content
- how SO(10) breaks down to SM
- additional flavor/discrete symmetries
- the way flavor structure is added
- right-handed neutrino Majorana mass terms

In what follows, we will discuss four types of $SO(10)$ Models:

- Symmetric texture with non-abelian family symmetry
- Lop-sided texture
- RG enhanced large mixing angle
- "Bottom-up" approach in minimal models

Other types of models:

- asymmetric texture
- 3×2 seesaw mechanism:

  require two additional $SO(10)$ singlets

  sign of CP phase in oscillation $\leftrightarrow$ sign of baryonic asymmetry

Questions to be addressed:

- Why m_ν are smaller than masses of the charged fermions?

  SO(10) models either utilize Type I or Type II see-saw mechanism

- How does the bi-large mixing pattern arise? (In other words, how do the two large leptonic mixing angles arise while quark mixing angles are small, in the presence of quark-lepton unification?)

$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \leftrightarrow V_{PMNS} \sim \begin{pmatrix} 1 & 1 & < \lambda \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\lambda \simeq 0.22$$

- What is the ordering of masses: normal hierarchy, inverted hierarchy or nearly degenerate?

- What is the prediction for U_{e3}?

SO(10) Models with Symmetric Texture

Chen, Mahanthappa: a complete model with $SU(2)_H$
Matsuda, Fukayama, Okada; Bando, Obara: pheno. analyses
Buchmuller, Wyler; .....

- SO(10) breaks down through the left-right symmetry breaking chain

- Higgs content:

  SO(10) breaking: 45, 54, 126
  Yukawa sector: 10,126
  Doublet-triplet splitting: 10,45,54,126

- $SU(2)_H$ symmetry: $(\psi_1, \psi_2) \sim 2, \quad \psi_3 \sim 1$
  flavon fields: $< \phi_a >\sim \begin{pmatrix} 0 \\ \phi_2 \end{pmatrix}$, $< S_{ab} >\sim \begin{pmatrix} 0 & s_{12} \\ s_{12} & s_{22} \end{pmatrix}$

$$M_u = M_{\nu_{LR}} \sim \begin{pmatrix} 0 & \epsilon^3 & 0 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ 0 & \epsilon^2 & 1 \end{pmatrix} m_t$$

$$M_{d,e} \sim \begin{pmatrix} 0 & \epsilon^{'3} & 0 \\ \epsilon^{'3} & (1,-3)\epsilon^{'2} & \epsilon^{'2} \\ 0 & \epsilon^{'2} & 1 \end{pmatrix} m_b$$

$$\epsilon \sim 0.085, \quad \epsilon^{'} \sim 0.204$$

The factor (-3) is needed to satisfy the Georgi-Jarlskog relations at the GUT scale

$$m_b \simeq m_\tau, \quad m_s \simeq m_\mu/3, \quad m_d \simeq 3m_e$$

Arise as Clebsch factor if couple to $< \overline{126} >$

- bi-large mixing angles arise because of large hierarchy in M_{RR}:

$$M_{RR} \sim \begin{pmatrix} 0 & x & 0 \\ x & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} v_R, \quad M_{RR}^{-1} \sim \begin{pmatrix} 0 & 1/x & 0 \\ 1/x & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{1}{v_R}$$

Type I seesaw:

$$M_\nu^{eff} = M_{LR} M_{RR}^{-1} M_{LR} \sim \begin{pmatrix} 0 & \epsilon^6/x & 0 \\ \epsilon^6/x & \epsilon^5/x + \epsilon^4 & \epsilon^5/x + \epsilon^2 \\ 0 & \epsilon^5/x + \epsilon^2 & 1 \end{pmatrix} \frac{v_{ew}^2}{v_r}$$

If $x \sim \epsilon^5 \Rightarrow$

$$M_\nu^{eff} \sim \begin{pmatrix} 0 & \epsilon & 0 \\ \epsilon & 1 & 1+\epsilon^2 \\ 0 & 1+\epsilon^2 & 1 \end{pmatrix} + \mathcal{O}(\epsilon^4)$$

$\Rightarrow$ hierarchical masses: $m_{\nu_1} : m_{\nu_2} : m_{\nu_3} = \epsilon : \epsilon : 1$

- prediction for $\sin\theta_{13}$ is related to

$$\sin\theta_{13} \sim \left(\frac{\Delta m_\odot^2}{\Delta m_{atm}^2}\right)^{1/2} \sim \epsilon$$

$$\epsilon \sim \mathcal{O}(0.1) \Rightarrow \text{LMA}$$

SO(10) Models with Lop-sided Texture

Albright, Barr: a complete model with $U(1)_H$ symmetry
Babu, Pati, Wilczek: ν mass matrix and proton decay analysis
.....

- $SO(10)$ breaks down to SM through the $SU(5)$ breaking chain
- Higgs content of the model: $10, 16, 45, 54$
- consider the operator:

$$\lambda(16_i 16_{H_1})(16_j 16_{H_2})$$

$< 16_{H_1} >$: breaks $SO(10)$ down to $SU(5)$
$< 16_{H_2} >$: breaks the EW symmetry

The resulting mass terms:

$$\lambda(\overline{5}_i)(10_j) < 1_{H_1} >< \overline{5}_{H_2} > \quad \sim \quad \lambda(d^c_{L,i} d_{L,j} + e_{L,i} e^c_{L,j}) v_d$$

For $(i, j) = (2, 3)$:

$$(\overline{d}_{R,2}\ \overline{d}_{R,3}) \begin{pmatrix} 0 & \lambda \\ 0 & 0 \end{pmatrix} \begin{pmatrix} d_{L,2} \\ d_{L,3} \end{pmatrix} v_d$$

$$+(\overline{e}_{R,2}\ \overline{e}_{R,3}) \begin{pmatrix} 0 & 0 \\ \lambda & 0 \end{pmatrix} \begin{pmatrix} e_{L,2} \\ e_{L,3} \end{pmatrix} v_d, \qquad \lambda \sim \mathcal{O}(1)$$

$$\Rightarrow \quad M_d = M_e^T$$

- When other operators are included,

$$\begin{aligned}
M_{u,\nu_{LR}} &= \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & (1/3,1)\epsilon \\ 0 & -(1/3,1)\epsilon & 1 \end{pmatrix} \cdot m_u \\
M_d &= \begin{pmatrix} \eta & \delta & \delta' e^{i\phi} \\ \delta & 0 & \lambda + \epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix} \cdot m_d \\
M_e &= \begin{pmatrix} \eta & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \lambda + \epsilon & 1 \end{pmatrix} \cdot m_d
\end{aligned}$$

- Large mixing in $U_{e,L}$ $\Rightarrow$ large mixing in $U_{d,R}$
  $\Rightarrow$ Large atmospheric mixing
  Large branching ratio for LFV processes, e.g. $\mu \to e\gamma$

- large solar mixing angle: special choice of M_{RR}

$$M_{\nu_{RR}} = \begin{pmatrix} c^2\eta^2 & -b\epsilon\eta & a\eta \\ -b\epsilon\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \cdot \Lambda_R$$

  Type I seesaw:

$$M_\nu^{eff} = \begin{pmatrix} 0 & -\epsilon & 0 \\ -\epsilon & 0 & 2\epsilon \\ 0 & 2\epsilon & 1 \end{pmatrix} m_u^2/\lambda_R, \text{accidental cancellation}$$

- the value for U_{e3} is predicted to be small

SO(10) Models with RG Enhanced Mixing Angles

Mohapatra, Parida, Rajasekaran; ......

$$\frac{dm_\nu}{dt} = -\{\kappa_u m_\nu + m_\nu P + P^T m_\nu\}$$

$$P \simeq -\frac{1}{32\pi^2}\frac{h_\tau^2}{\cos^2\beta} diag(0,0,1), \qquad \kappa_u \simeq \frac{1}{16\pi^2}[\tfrac{6}{5}g_1^2 + 6g_2^2 - 6\frac{h_t^2}{\sin^2\beta}]$$

$$\begin{aligned}
\frac{d\ m_i}{dt} &= -4P_\tau m_i U_{\tau\nu_i}^2 - m_i\kappa_u, \quad (i = 1, 2, 3) \\
\frac{d\ s_{23}}{dt} &= -2P_\tau c_{23}^2(-s_{12}U_{\tau\nu_1}\nabla_{31} + c_{12}U_{\tau\nu_2}\nabla_{32}) \\
\frac{d\ s_{13}}{dt} &= -2P_\tau c_{23}c_{13}^2(c_{12}U_{\tau\nu_1}\nabla_{31} + s_{12}U_{\tau\nu_2}\nabla_{32}) \\
\frac{d\ s_{12}}{dt} &= -2P_\tau c_{12}(c_{23}s_{13}s_{12}U_{\tau\nu_1}\nabla_{31} - c_{23}s_{13}c_{12}U_{\tau\nu_2}\nabla_{32} \\
&\qquad +U_{\tau\nu_1}U_{\tau\nu_2}\nabla_{21})
\end{aligned}$$

$$\nabla_{ij} \equiv \frac{(m_i + m_j)}{(m_i - m_j)}$$

- Conditions at the GUT scale:
  - nearly degenerate mass pattern: $m_1 \lesssim m_2 \lesssim m_3$
  - same Majorana CP phases

  $\Rightarrow \nabla_{ij} \rightarrow$ large, driving s_{12}, s_{23} large
  $\Rightarrow$ corrections on m_i small

- Assuming PMNS matrix = CKM matrix at the GUT scale

  Start with $s_{12}^0 \simeq \lambda$, $s_{23}^0 \simeq \mathcal{O}(\lambda^2)$, and $s_{13}^0 \simeq \mathcal{O}(\lambda^3)$:

  $\sin^2 2\theta_{atm} = 0.99, \quad \sin^2 2\theta_\odot = 0.87, \quad \sin\theta_{13} = 0.08$

- These GUT scale conditions can be understood in Type II seesaw mechanism, with M_{LL} term dominates,

$$M_{LL} \sim I \cdot m_{LL} \rightarrow \text{degenerate masses}$$
$$M_{LR} M_{RR}^{-1} M_{LR}^T \rightarrow \text{mixing matrix} \sim V_{CKM}$$

- The U_{e3} is also amplified by the RG flow

“Bottom-up” Approach in Minimal SO(10) Models

Fukuyama, Kikuchi, Okada: Type I seesaw

Bajc, Senjanovic, Vissani; Goh, Mohapatra, Ng: Type II seesaw

......

- A minimal Higgs sector:

  $\{10, 126, 45, 54\}$:

  break EW symm., fermions masses:10, 126
  break SO(10):45, 54, 126
  required by Type II seesaw to give Δ_L mass: 54

- The following mass relations

$$\begin{aligned} M_u &= f < 10 > +h < \overline{126} > \\ M_d &= f < 10 > +h < \overline{126} > \\ M_e &= f < 10 > -3h < \overline{126} > \\ M_{\nu_{LR}} &= f < 10 > -3h < \overline{126} > \end{aligned}$$

- The small neutrino masses are explained by the Type II see-saw mechanism with the assumption that the LH Majorana mass term dominates over the usual Type I see-saw term:

$$M_\nu^{eff} = M_{\nu,LL} - M_{LR} M_{RR}^{-1} M_{LR}^T$$

  The mass terms $M_{\nu,LL}$ and $M_{\nu,RR}$ are both due to the coupling to $\overline{126}$, thus

$$M_{\nu,LL} \sim h \frac{v_{ew}^2}{v_r}, \quad M_{\nu,RR} \sim h v_R$$

- In this minimal scheme, we have the following sum-rule

$$M_\nu^{eff} = c(M_d - M_e)$$

- The down-type quark and charged lepton mass matrices can be parameterized in terms of Wolfenstein parameter as

$$M_{b,\tau} \sim \begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} m_{b,\tau}$$

- For some value of $\tan\beta$, the deviation from $b-\tau$ unification at the GUT scale is

$$m_b(M_{GUT}) - m_\tau(M_{GUT}) \simeq \mathcal{O}(\lambda^2) m_\tau$$

which leads to a bi-large mixing pattern in M_ν

$$\begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix} c m_\tau$$

small $\tan\beta$ is preferred

- The predictions of this model are testable:

$$\sin^2\theta_{23} < 0.9, \qquad \sin^2\theta_{12} > 0.9$$

- The prediction of this model for the value of $|U_{e3}|$ is large: This is closely related to the large atmospheric mixing due to the following sum-rule,

$$\tan 2\theta_{13} \sim \frac{M_{\nu,13}}{M_{\nu,33}} \simeq \frac{\lambda^3 m_\tau}{m_b(M_{GUT}) - m_\tau(M_{GUT})} \sim \lambda$$

An explicit model with Symmetric Textures

Chen and Mahanthappa, PRD62, 113007; PRD65, 053010; hep-ph/0212375, to appear in PRD

- in order to uniquely specify the Yukawa superpotential without any unwanted terms $\Rightarrow Z_4 \times Z_4$ symmetry.

- Field content:
  - matter fields

$$\psi_a \sim (16,2)^{i\,i} \quad (a=1,2), \qquad \psi_3 \sim (16,1)^{-\,-}$$

  - Higgs fields:

$$(10,1): \qquad T_1^{+\,+}, \quad T_2^{-\,i}, \quad T_3^{-i\,i}, \quad T_4^{-\,-}, \quad T_5^{i\,-}$$

$$(\overline{126},1): \qquad \overline{C}^{-\,-}, \quad \overline{C}_1^{+\,+}, \quad \overline{C}_2^{i\,i}$$

  - Flavon fields:

$$(1,2): \qquad \phi_{(1)}^{-i\,+}, \quad \phi_{(2)}^{-\,+}, \quad \Phi_{(1)}^{+\,+}, \quad \Phi_{(2)}^{i\,i}$$

$$(1,\overline{2}): \qquad \overline{\phi}_{(1)}^{i\,+}, \quad \overline{\phi}_{(2)}^{-\,+}, \quad \overline{\Phi}_{(1)}^{+\,+}, \quad \overline{\Phi}_{(2)}^{-i\,-i}$$

$$(1,3): \qquad S_{(1)}^{i\,+}, \quad S_{(2)}^{+\,+}, \quad \Sigma^{-\,-}$$

$$(1,\overline{3}): \qquad \overline{S}_{(1)}^{-i\,+}, \quad \overline{S}_{(2)}^{+\,+}, \quad \overline{\Sigma}^{-\,-}$$

$$(1,1): \qquad P_{(1)}^{i\,i}, \quad P_{(2)}^{+\,+}$$

- Complete Superpotential:

$$W = W_{Dirac} + W_{\nu_{RR}}$$

$$\begin{aligned} W_{Dirac} &= W^{y}_{Dirac} + W^{'}_{Dirac} \\ W_{\nu_{RR}} &= W^{y}_{\nu_{RR}} + W^{'}_{\nu_{RR}} \end{aligned}$$

$$\begin{aligned} W^{y}_{Dirac} &= \psi_3\psi_3 T_1 + \tfrac{1}{M}\psi_3\psi_a \left(T_2\phi_{(1)} + T_3\phi_{(2)}\right) \\ &\quad + \tfrac{1}{M}\psi_a\psi_b \left(T_4 + \overline{C}\right) S_{(2)} + \tfrac{1}{M}\psi_a\psi_b T_5 S_{(1)} \\ W^{'}_{Dirac} &= \phi_{(1)}\overline{S}_{(1)}\phi_{(2)} + \phi_{(2)}\overline{S}_{(1)}\phi_{(1)} \\ &\quad + \overline{\phi}_{(1)} S_{(1)}\overline{\phi}_{(2)} + \overline{\phi}_{(2)} S_{(1)}\overline{\phi}_{(1)} \\ &\quad + X_{S_1} S_{(1)}\overline{S}_{(1)} + \phi_{(2)}\overline{S}_{(2)}\phi_{(2)} + \overline{\phi}_{(2)} S_{(2)}\overline{\phi}_{(2)} \\ &\quad + X_{S_2} S_{(2)}\overline{S}_{(2)} + X_{\phi_1}\phi_{(1)}\overline{\phi}_{(1)} + X_{\phi_2}\phi_{(2)}\overline{\phi}_{(2)} \\ W^{y}_{\nu_{RR}} &= \psi_3\psi_3\overline{C}_1 + \tfrac{1}{M}\psi_3\psi_a(\Phi_{(1)}\overline{C}_2 + \Phi_{(2)}\overline{C}_{(1)}) \\ &\quad + \tfrac{1}{M}\psi_a\psi_b\Sigma\overline{C}_1 \\ W^{'}_{\nu_{RR}} &= P_{(2)}(\Phi_{(1)}\overline{\Phi}_{(1)} - \Delta^2) + (X_{\Phi_2} + P_{(2)})\Phi_{(2)}\overline{\Phi}_{(2)} \\ &\quad + \Phi_{(2)}\overline{\Sigma}\Phi_{(2)} + \overline{\Phi}_{(2)}\Sigma\overline{\Phi}_{(2)} \\ &\quad + (X_\Sigma + P_{(2)})\Sigma\overline{\Sigma} + P_{(1)}\Phi_{(1)}\overline{\Phi}_{(2)} \end{aligned}$$

- The vacuum alignment in the flavon sector is achieved:

$$
\begin{aligned}
\left\langle \phi_{(1)} \right\rangle &= \begin{pmatrix} \epsilon' \\ 0 \end{pmatrix}, & \left\langle \phi_{(2)} \right\rangle &= \begin{pmatrix} 0 \\ \epsilon \end{pmatrix} \\
\left\langle S_{(1)} \right\rangle &= \begin{pmatrix} 0 & \epsilon' \\ \epsilon' & 0 \end{pmatrix}, & \left\langle S_{(2)} \right\rangle &= \begin{pmatrix} 0 & 0 \\ 0 & \epsilon \end{pmatrix}
\end{aligned}
$$

$$
\langle \Phi_1 \rangle = \begin{pmatrix} \delta_1 \\ 0 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ \delta_3 \end{pmatrix}, \quad \langle \Sigma \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \delta_2 \end{pmatrix}
$$

- Mass matrices:

$$
\begin{aligned}
M_{u,\nu_{LR}} &= \begin{pmatrix} 0 & 0 & \left\langle 10_2^+ \right\rangle \epsilon' \\ 0 & \left\langle 10_4^+ \right\rangle \epsilon & \left\langle 10_3^+ \right\rangle \epsilon \\ \left\langle 10_2^+ \right\rangle \epsilon' & \left\langle 10_3^+ \right\rangle \epsilon & \left\langle 10_1^+ \right\rangle \end{pmatrix} \\
&= \begin{pmatrix} 0 & 0 & r_2\epsilon' \\ 0 & r_4\epsilon & \epsilon \\ r_2\epsilon' & \epsilon & 1 \end{pmatrix} M_U
\end{aligned}
$$

$$
\begin{aligned}
M_{d,e} &= \begin{pmatrix} 0 & \left\langle 10_5^- \right\rangle \epsilon' & 0 \\ \left\langle 10_5^- \right\rangle \epsilon' & (1,-3) \left\langle \overline{126}^- \right\rangle \epsilon & 0 \\ 0 & 0 & \left\langle 10_1^- \right\rangle \end{pmatrix} \\
&= \begin{pmatrix} 0 & \epsilon' & 0 \\ \epsilon' & (1,-3)p\epsilon & 0 \\ 0 & 0 & 1 \end{pmatrix} M_D
\end{aligned}
$$

$$
\begin{aligned}
M_{\nu_{RR}} &= \begin{pmatrix} 0 & 0 & \left\langle \overline{126}_2'^0 \right\rangle \delta_1 \\ 0 & \left\langle \overline{126}_2'^0 \right\rangle \delta_2 & \left\langle \overline{126}_2'^0 \right\rangle \delta_3 \\ \left\langle \overline{126}_2'^0 \right\rangle \delta_1 & \left\langle \overline{126}_2'^0 \right\rangle \delta_3 & \left\langle \overline{126}_1'^0 \right\rangle \end{pmatrix} \\
&= \begin{pmatrix} 0 & 0 & \delta_1 \\ 0 & \delta_2 & \delta_3 \\ \delta_1 & \delta_3 & 1 \end{pmatrix} M_R
\end{aligned}
$$

$\Rightarrow$

$$
\begin{aligned}
Y_{u,\nu_{LR}} &= \begin{pmatrix} 0 & 0 & a \\ 0 & be^{i\theta} & c \\ a & c & 1 \end{pmatrix} d \\
Y_{d,e} &= \begin{pmatrix} 0 & ee^{-i\xi} & 0 \\ ee^{i\xi} & (1,-3)f & 0 \\ 0 & 0 & 1 \end{pmatrix} h
\end{aligned}
$$

$$M_{\nu_{RR}} = \begin{pmatrix} 0 & 0 & \delta_1 \\ 0 & \delta_2 & \delta_3 \\ \delta_1 & \delta_3 & 1 \end{pmatrix} M_R, \quad \delta_i = f_i(a,b,c,t)$$

effective neutrino mass matrix:

$$M_{\nu_{LL}} \simeq \begin{pmatrix} 0 & 0 & t \\ 0 & 1 & 1+t^{3/2} \\ t & 1+t^{3/2} & 1 \end{pmatrix} \frac{d^2 v_u^2}{M_R}, \quad t \ll 1$$

$\Rightarrow$ bimaximal mixing, hierarchical masses

RGE Analysis

- Input parameters at $M_z = 91.187 GeV$:

$$
\begin{aligned}
m_u &= 2.32\ MeV (2.33^{+0.42}_{-0.45}) \\
m_c &= 677\ MeV (677^{+56}_{-61}) \\
m_t &= 182\ GeV (181^{+}_{-}13) \\
m_e &= 0.485\ MeV (0.486847) \\
m_\mu &= 103\ MeV (102.75) \\
m_\tau &= 1.744\ GeV (1.7467) \\
|V_{us}| &= 0.222 (0.219 - 0.224) \\
|V_{ub}| &= 0.0039 (0.002 - 0.005) \\
|V_{cb}| &= 0.036 (0.036 - 0.046)
\end{aligned}
$$

- Correspond to parameters at $M_{GUT} = 1.03 \times 10^{16}\ GeV$:

$$
\begin{aligned}
&a = 0.00246, \quad b = 3.50 \times 10^{-3}, \quad c = 0.0320 \\
&d = 0.650, \quad \theta = 0.110 \\
&e = 4.03 \times 10^{-3}, \quad f = 0.0195, \quad h = 0.0686 \\
&\xi = -0.720
\end{aligned}
$$

$$g_1 = g_2 = g_3 = 0.746, \quad \tan\beta = 10$$

	experimental results extrapolated to M_Z	predictions at M_z
$\frac{m_s}{m_d}$	17∼25	25
m_s	$93.4^{+11.8}_{-13.0} MeV$	$85.66 MeV$
m_b	$3.00 \pm 0.11 GeV$	$3.147 GeV$
$\|V_{ud}\|$	0.9745−0.9757	0.9751
$\|V_{cd}\|$	0.218−0.224	0.2218
$\|V_{cs}\|$	0.9736−0.9750	0.9744
$\|V_{td}\|$	0.004−0.014	0.005358
$\|V_{ts}\|$	0.034−0.046	0.03611
$\|V_{tb}\|$	0.9989−0.9993	0.9993
J^q_{CP}	$(2.71 \pm 1.12)\times 10^{-5}$	1.748×10^{-5}
$\sin 2\alpha$	−0.95 − 0.33	−0.8913
$\sin 2\beta$	$0.59 \pm 0.14 \pm 0.05$ (BaBar)	0.7416
	$0.99 \pm 0.14 \pm 0.06$ (Belle)	
γ	34^0−82^0	34.55^0 ($0.6030 rad$)

LMA solution

- Inputs in the RH ν's sector:

$$\Delta m^2_{atm} = 2.78 \times 10^{-3}\, eV^2$$
$$\Delta m^2_{\odot} = 7.25 \times 10^{-5}\, eV^2$$

$\Rightarrow$

$$t = 0.35, \quad M_R = 5.94 \times 10^{12} GeV$$
$$\delta_1 = 0.00119$$
$$\delta_2 = 0.000841 \times e^{i\,(0.220)}$$
$$\delta_3 = 0.0211 \times e^{-i\,(0.029)}$$

- Predictions:

$$m_{\nu_1} = 0.00363\, eV$$
$$m_{\nu_2} = 0.00926\, eV$$
$$m_{\nu_3} = 0.0535\, eV$$

$$|U_{MNS}| = \begin{pmatrix} 0.787 & 0.599 & 0.149 \\ 0.508 & 0.496 & 0.705 \\ 0.350 & 0.629 & 0.694 \end{pmatrix}$$

$$\sin^2 2\theta_{atm} \equiv \frac{4|U_{\mu\nu_3}|^2|U_{\tau\nu_3}|^2}{(1-|U_{e\nu_3}|^2)^2} = 1$$
$$tan^2\theta_{atm} \equiv \frac{|U_{\mu\nu_3}|^2}{|U_{\tau\nu_3}|^2} = 1.03$$
$$\sin^2 2\theta_{\odot} \equiv \frac{4|U_{e\nu_1}|^2|U_{e\nu_2}|^2}{(1-|U_{e\nu_3}|^2)^2} = 0.93$$
$$tan^2\theta_{\odot} \equiv \frac{|U_{e\nu_2}|^2}{|U_{e\nu_1}|^2} = 0.58$$
$$\sin^2\theta_{13} \equiv |U_{e\nu_3}|^2 = 0.022$$

- CP violation measures:

$$J_{CP}^{l} \equiv Im\{U_{11}U_{12}^{*}U_{21}^{*}U_{22}\} = -0.00690$$

$$(\alpha_{31}, \alpha_{21}) = (0.490, -2.29)$$

- $\beta\beta_{0\nu}$ decay matrix element $|<m>|$:

$$\begin{aligned}
|<m>|^2 &= m_1^2|U_{e1}|^4 + m_2^2|U_{e2}|^4 + m_3^2|U_{e3}|^4 \\
&+2m_1m_2|U_{e1}|^2|U_{e2}|^2\cos\alpha_{21} \\
&+2m_1m_3|U_{e1}|^2|U_{e3}|^2\cos\alpha_{31} \\
&+2m_2m_3|U_{e2}|^2|U_{e3}|^2\cos(\alpha_{31}-\alpha_{21})
\end{aligned}$$

$$|<m>| = 2.22 \times 10^{-3}\ eV$$

Current bound: $|<m>| = (0.05 - 0.86)\ eV$

- The heavy right-handed neutrinos:

$$\begin{aligned}
M_1 &= 1.72 \times 10^{7} GeV \\
M_2 &= 2.44 \times 10^{9} GeV \\
M_3 &= 5.94 \times 10^{12} GeV
\end{aligned}$$

Distinguishing Models

Using $\sin\theta_{13}$ to distinguish models:

	$\sin^2 2\theta_{13}$	$\sin\theta_{13}$
current limit:	10^{-1}	0.16
reactor:	10^{-2}	0.05
conventional beam:	10^{-2}	0.05
superbeam:	3×10^{-3}	2.7×10^{-2}
neutrino factory:	$(5 - 50) \times 10^{-5}$	$(3.5 - 11) \times 10^{-3}$

Model	Flavor Symmetry	$\sin\theta_{13}$
Albright-Barr	U(1)	0.014
Blazek-Raby-Tobe	$U(2) \times U(1)^n$	0.049
Ross-Velasco-Sevilla	SU(3)	0.070
Chen-Mahanthappa	SU(2)	0.149
Kitano-Mimura	$SU(3) \times U(1)$	$\sim \lambda \sim 0.22$
Maekawa	U(1)	$\sim \lambda \sim 0.22$
Mohapatra-Parida -Rajasekaran	RG Enhanced	0.08-0.10
Raby	3×2 seesaw	$\sim m_{\nu_2}/2m_{\nu_3} \sim \mathcal{O}(0.1)$
Goh-Mohapatra-Ng	Minimal Model	0.16

Using three angles of unitarity triangle to distinguish models:

	$\sin 2\alpha$	$\sin 2\beta$	γ
Albright-Barr	-0.2079	0.6428	64^o
Blazek-Raby-Tobe (LMA Solution)	0.94	0.39	47^o
Berezhiani-Rossi (Ansatz B)	0.27	0.75	73^o
Chen-Mahanthappa	-0.8913	0.7416	34.55^o
Raby	0.92	0.50	71.7^o
Experiments	$-0.95 \sim 0.33$	0.79 ± 0.12	$34^o \sim 82^o$

Albright-Barr: $U(1)$
Blazek-Raby-Tobe: $U(2) \times [U(1)]^n$
Berezhiani-Rossi: $SU(3)$
Chen-Mahanthappa: $SU(2)$
Raby: 3×2 seesaw

Some General Observations

- Mass Hierarchy:

  $\Rightarrow$ long baseline experiments with $L > 900\ Km$

  - SO(10) models with $SU(3)_H$, $SU(2)_H$, $U(1)_H$
    SO(10) models based on "bottom-up" approach
    SO(10) models with 3×2 see-saw
    $\Rightarrow$ normal hierarchy
  - models with $L_e - L_\mu - L_\tau$ horizontal symmetry
    $\Rightarrow$ inverted hierarchy
  - SO(10) models with RG enhanced lepton mixing
    models with anarchy
    $\Rightarrow$ nearly degenerate

- Value of U_{e3}:

  - models with symmetric texture
    models based on anarchy
    models with RG enhanced leptonic mixing
    minimal models
    $\Rightarrow$ large $U_{e3} \sim \mathcal{O}(0.1)$
    $\Rightarrow$ conventional/superbeam
  - models with asymmetric texture
    $\Rightarrow$ intermediate U_{e3} value $\sim (0.05 - 0.07)$
    $\Rightarrow$ superbeam
  - models with lop-sided texture
    $\Rightarrow$ small U_{e3}, neutrino factory may be needed

Outlook + Comments:

- To obtain LMA solution generally requires some tuning of parameters in models

- diverse predictions for $\sin\theta_{13}$

  $U_{PMNS} = U_{e,L} U_{\nu}^{\dagger}$

  If both large $\sin^2 2\theta_{atm}$ and $\tan^2\theta_{\odot}$ are due to large elements in U_{ν}, $\sin\theta_{13}$ tends to be large

  On the other hand, if one comes from U_e and the other from U_{ν} (e.g. in lop-sided texture scenario), $\sin\theta_{13}$ can be very small

- prediction for leptonic CP violating phases

  $\Rightarrow$ require more precise knowledge of PMNS matrix and three neutrino masses

- Discovery of proton decay around the corner?