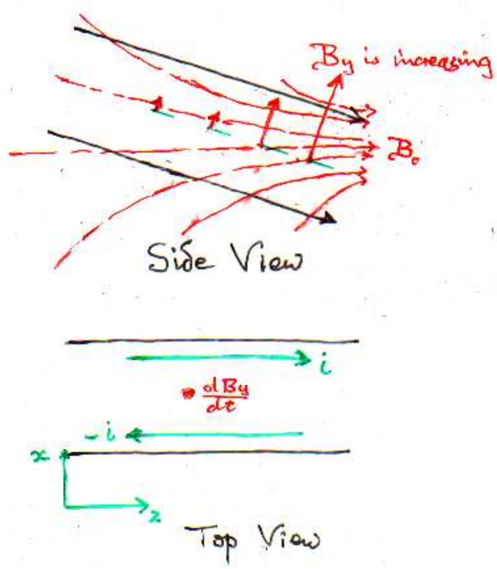
0.1 Induced axial current



Consider a transverse field component B_y
The magnetic flux between transverse positions $-x$ to x and dz is

$$d\Phi_y = 2xdzB_y(z) \tag{1}$$

As a liquid metal jet passes axially down such a field at a velocity $v = dz/dt$, axial voltage gradients will be generated

$$G = x\frac{dB_y}{dz}v \tag{2}$$

If the metal electrical conductivity κ is low enough so that the resulting current has a negligible effect on the field, then the axial current density i_z will be

$$i_z = G\sigma \tag{3}$$

$$i_z \ = xv\kappa\frac{dB_y(0,0,z)}{dz} \tag{4}$$

0.2 Transverse elliptical distortion

If the jet is not on the solenoid axis, the axial induced currents interacting with the transverse fields will generate distorting forces on the jet. These transverse forces per unit volume $dx\ dy\ dz$ are

$$f_x \ = \ i_z \ B_y \ = \ x \ v \ \kappa \ B_y \ \frac{dB_y}{dz} \tag{5}$$

This force will distrot the cross section. Assuming that the liquid in incompresible, we must find the induced pressures and motions $\Delta\vec{r}$ within the cross section that are driven by this force, with the constraint that the diverence of these motions is zero:

$$Div(\Delta\vec{r}) \ = \ 0$$

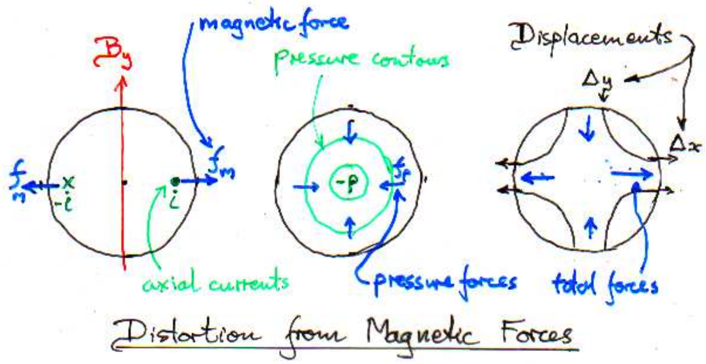
Defining

$$F_o \ = \ v \ \kappa \ B_y \ \frac{dB_y}{dz}$$

so that the magnet force per unit volume:

$$f_x(\text{magnetic}) \ = \ x \ F_o$$

0.2.1 Magnetic Forces



The pressure on the surface of the jet will be independent of azymuth ϕ:

$$p(r_o)_{\text{circular}} = p_{\text{atm}} \ + \ T/r_o \ ,$$

T being the surface tension. If the initial cross section is circular, we can consider pressures within the cross section:

$$p \ = \ p_o \ - \ r^2 \ \frac{F_o}{4}$$

where p_o is set by the constraint the above surface pressure at $r = r_o$. This bulk pressure will induce radial pressure forces:

$$f_r(\text{pressure}) \ = \ \frac{dp}{dr} \ = \ - \ \frac{r \ F_o}{2}$$

so

$$f_x(\text{total}) \;=\; F_o \left(x \;-\; \frac{r\;\cos(\phi)}{2} \right) \;=\; \frac{x\;F_o}{2}$$

$$f_y(\text{total}) \;=\; -\;\frac{F_o\;r\;\sin(\phi)}{2} \;=\; -\;\frac{y\;F_o}{2}$$

and the accelerations:

$$\frac{d^2 x, y}{dz^2} \;=\; \frac{f_{x,y}}{v\;\rho}$$

will give dispacements:

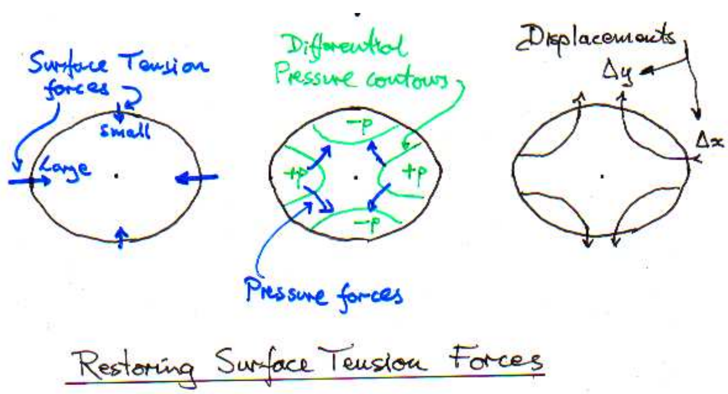
$$\Delta x \;=\; \int\int \frac{f_{x,y}}{v\;\rho}\;dz^2 \;=\; x\;\frac{\kappa}{2\;\rho}\;\int\int\;B_y\;\frac{dB_y}{dz}\;dz^2$$

$$\Delta y \;=\; \int\int \frac{f_{x,y}}{v\;\rho}\;dz^2 \;=\; -\,y\;\frac{\kappa}{2\;\rho}\;\int\int\;B_y\;\frac{dB_y}{dz}\;dz^2$$

Such motions are quadrupole (see fig ***) and, as required, give $Div\;(dx, dy) \;=\; 0$. The resulting ellipticity $\epsilon = \frac{\Delta x_o}{r_o} \;=\; -\,\frac{\Delta y_o}{r_o}$:

$$\epsilon(z) \;=\; \frac{r_o\;\kappa}{2\;\rho}\;\int\int B_y\;\frac{dB_y}{dz}\;dz^2 \tag{6}$$

0.2.2 Surface Tension Forces



Once the cross section becomes somewhat elliptical (we consider only a small ellipticity), then the pressure at the surface is no longer independent of the asymuthal angle ϕ, but is given by:

$$p(r = r_o) \;=\; p_{\text{atm}} \;+\; \frac{T}{r_0}\;(1 \;-\; \epsilon)\;\cos(2\;\phi)$$

Consider, in addition to those given above for the circular case, pressures within the cross section:

$$p(x,y)_{\text{elliptical}} \;=\; p_o \;+\; \frac{T\;\epsilon}{r_o}\;\left(\cos^2(2\;\phi)\;-\;\sin^2(2\;\phi)\right)\;=\;p_o\;+\;\frac{T\;\epsilon}{r_o^3}\;(x^2\;-\;y^2)$$

which has the correct values at $r = r_o$.
Defining

$$S_o \;=\; \frac{2\;T\;\epsilon}{r_o^3},$$

the resulting bulk forces are:

$$f_x(\text{elliptical}) \;=\; \frac{dp}{dx} \;=\; x\;S_o$$

$$f_y(\text{elliptical}) \;=\; \frac{dp}{dy} \;=\; -\,y\;S_o$$

which are, once again, quadrupole forces that will generate quadrupole motions with Div($\vec{dr}$)=0.

$$\Delta x, \Delta y \;=\; \int\int \frac{(x,\;\;-y)}{v\;\rho}\;S_o\;dz^2$$

0.2.3 Magnetic and Surface Tension Forces

Adding these surface tension dispalcements to the forces derived for the circular case:

$$\Delta x, \Delta y \;=\; (x,\;\;-y)\;\int\int\left(\frac{\kappa}{2\;\rho}\;B_y\;\frac{dB_y}{dz}\;+\;\frac{2\;T\;\epsilon}{v\;\rho\;r_o^3}\right)\;dz^2$$

and the resulting ellipticity $\epsilon = \frac{\Delta x_o}{r_o} \;=\; -\frac{\Delta y_o}{r_o}$:

$$\epsilon(z) \;=\; \int\int\left(\frac{r_o\;\kappa}{2\;\rho}\;B_y\;\frac{dB_y}{dz}\;+\;\frac{2\;T\;\epsilon}{v\;\rho\;r_o^2}\right)\;dz^2 \tag{7}$$