Optimization of the Pion Capture and Decay Channel

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Overview:

- Objective...
  - ...Maximize muon yield at end of channel
  - ...Find more “physical” field profile
  - ...Rough cost considerations
  - ...Remain (*somewhat*) general!
- Issues considered...
  - ...FS2 Mercury-jet target
  - ...AGS-style proton driver (1 MW @ 24 GeV)
  - ...50 m channel length (target to buncher)
  - ...60 cm, 1.25 T buncher/phase rotator
- Issues *NOT* considered...
  - ...Modifications to target
  - ...Realistic coil configuration
  - ...Energy deposition & radiation

Channel Design:

- Incident proton beam...

  ...1 MW and 24 GeV at 67 mrad

- Mercury-jet target...

  ...1 cm diameter at 100 mrad ($\sim$ 60 cm interaction region)

- Tapered solenoid matching section...

  ...Roughly uniform 20 T at target

  ...Uniform $\sim$ 1 T in decay channel

- Uniform solenoid decay channel...

  ...1.25 - 2.00 T over $\sim$ 40 m

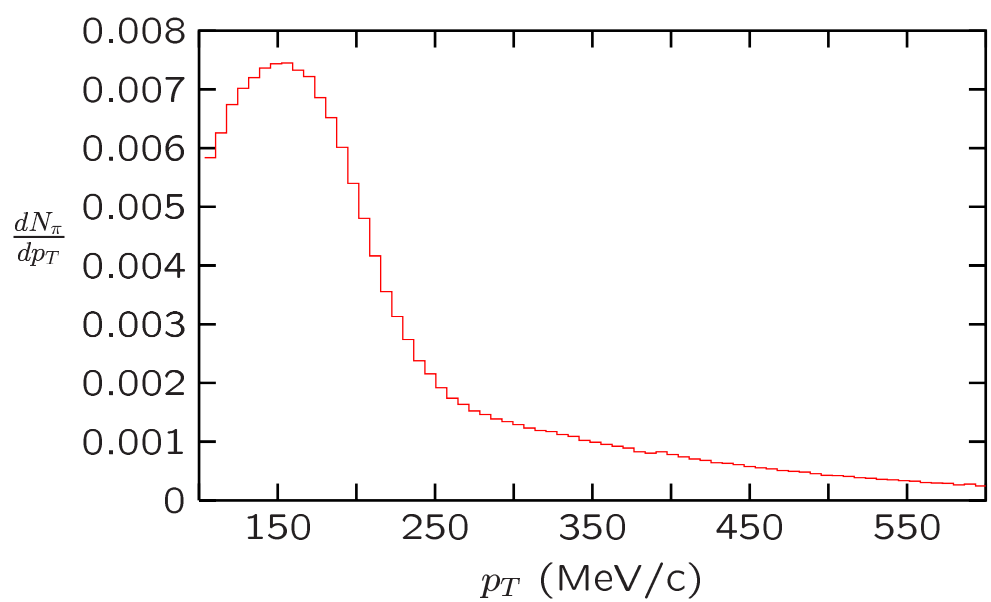
Pion Capture:

- Capture solenoid...

  ...About 1 m long with $B_0 = 20$ T peak field on axis

  ...Captures pions in a $R_0 = 7.5$ cm radius beampipe with

$$p_T < e\,B_0 \left(\frac{R_0}{2}\right) = 225 \text{ MeV/c}$$



Adiabatic Transfer:

- Tapered solenoid matching section...

  ...Constant magnetic flux:

$$\Phi_0 \propto B_0 R_0^2 = B(s) R(s)^2$$

- Change field *slowly*...

  ...Profile should be continuous & monotonic

  ...Minimize curvature and gradient:

$$\left(\frac{\partial B}{\partial s}\right)_{profile} < \left(\frac{\partial B}{\partial s}\right)_{coil}$$

- Field profile design...

  ...Design $R(s)$ and determine on-axis field profile

$$B(s) = B_0 \left(\frac{R_0}{R(s)}\right)^2$$

Designing the Field Profile:

- Our demands...

...Initial and final radii fixed by conservation of flux:

$$R(s_1) = R_1 = 7.5 \text{ cm}$$

and

$$R(s_2) = R_2 = 30 \text{ cm}$$

...Initial and final direction of field fixed for continuity:

$$\frac{\partial R}{\partial s}(s_1) = 0$$

and

$$\frac{\partial R}{\partial s}(s_2) = 0$$

Designing the Field Profile: (*continued*)

- Previous designs...

...Simple choice for radius function:

$$R(s) = \sqrt{\alpha_0 + \alpha_1 s} \;\rightarrow\; B(s) = \frac{\Phi_0}{\pi}\left(\frac{1}{\alpha_0 + \alpha_1 s}\right)$$

where $\alpha_0 = R_1^2 - (R_2^2 - R_1^2)\left(\frac{s_1}{s_2 - s_1}\right)$ and $\alpha_1 = \left(\frac{R_2^2 - R_1^2}{s_2 - s_1}\right)$

...PROBLEM: Discontinuous at $s = s_1$ and $s = s_2$

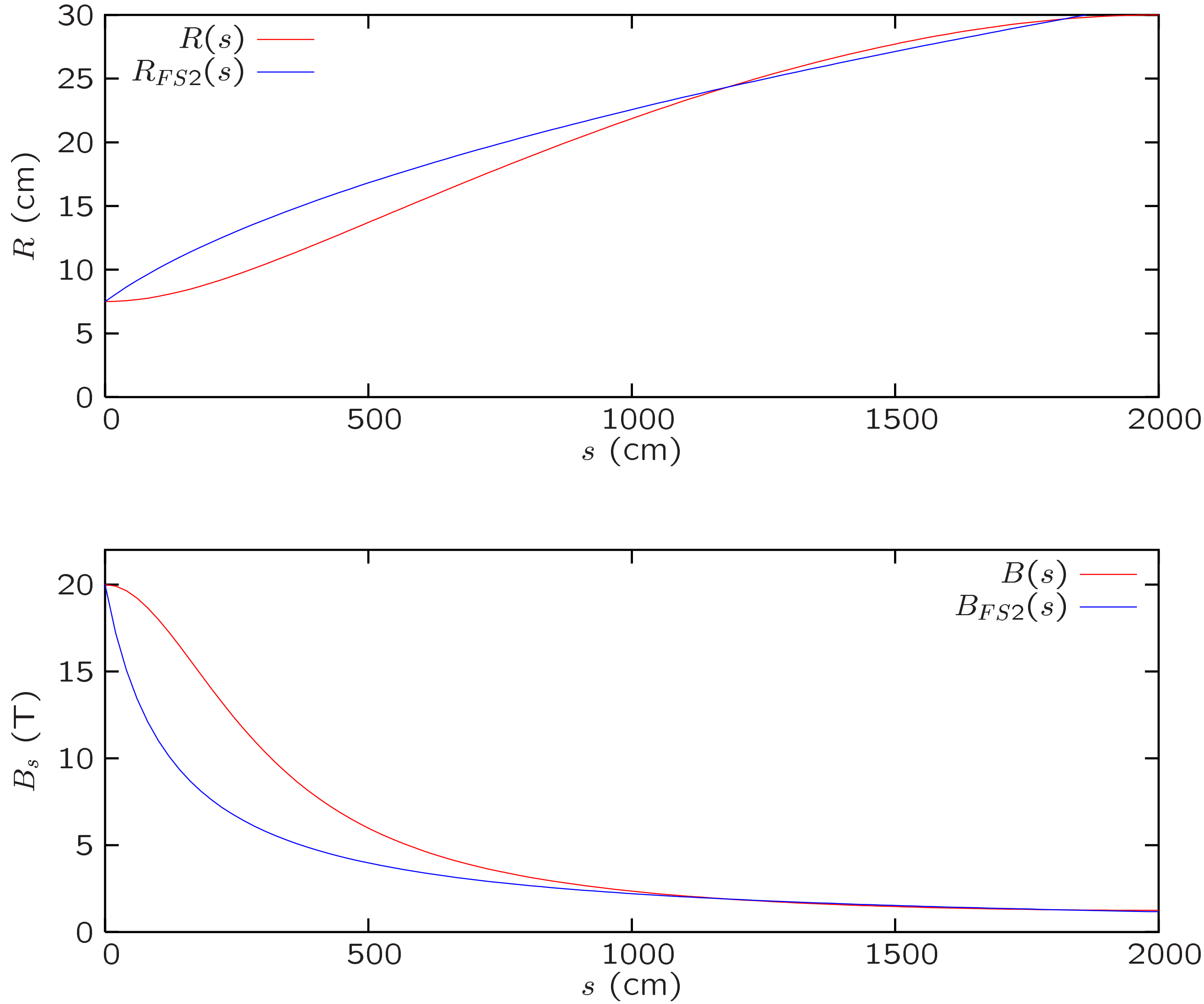
- New design...

...Optimized and continuous radius function:

$$R(s) = \sqrt{\alpha_0 + \alpha_1 s + \alpha_2 s^2 + \alpha_3 s^3}$$

or

$$B(s) = \frac{\Phi_0}{\pi}\left(\frac{1}{\alpha_0 + \alpha_1 s + \alpha_2 s^2 + \alpha_3 s^3}\right)$$


R(s)
R_FS2(s)
R (cm)
s (cm)
B(s)
B_FS2(s)
B_s (T)
s (cm)

Muon Capture from Decay:

- Decay solenoid...

  ...About 40 m long with $B = 1.25$ T uniform field

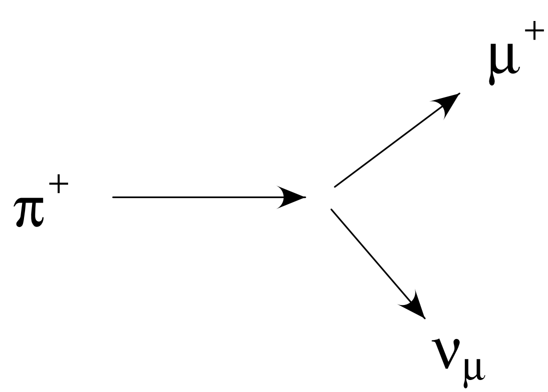
  ...Contains pions in a $R = 30$ cm radius beampipe with

$$p_T < e\,B\left(\frac{R}{2}\right) = 56 \text{ MeV/c}$$

- Pion decay...

  ...Acts like an extended target to the beam!

  ...Gives a transverse momentum kick to the muon



$$\begin{aligned}\langle p_T \rangle &\sim \frac{1}{2} m_\pi c \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \langle \sin^2\theta \rangle \\ &= \frac{1}{4} m_\pi c \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \\ &\approx 15 \text{ MeV/c}\end{aligned}$$

Optimization:

- Tuneable parameters...

  ...Determined from previous analysis with carbon target (i.e., $\lambda = 0$, $k = 2$)

  ...Consider starting taper early ($s_1 < 0$)

- MARS runs...

  ...Various taper lengths: $s_2 - s_1 = 5, 10, 20$ m

  ...Various decay channel field strengths: $B = 1.25, 2$ T (with $R = 30, 23.7$ cm, respectively)

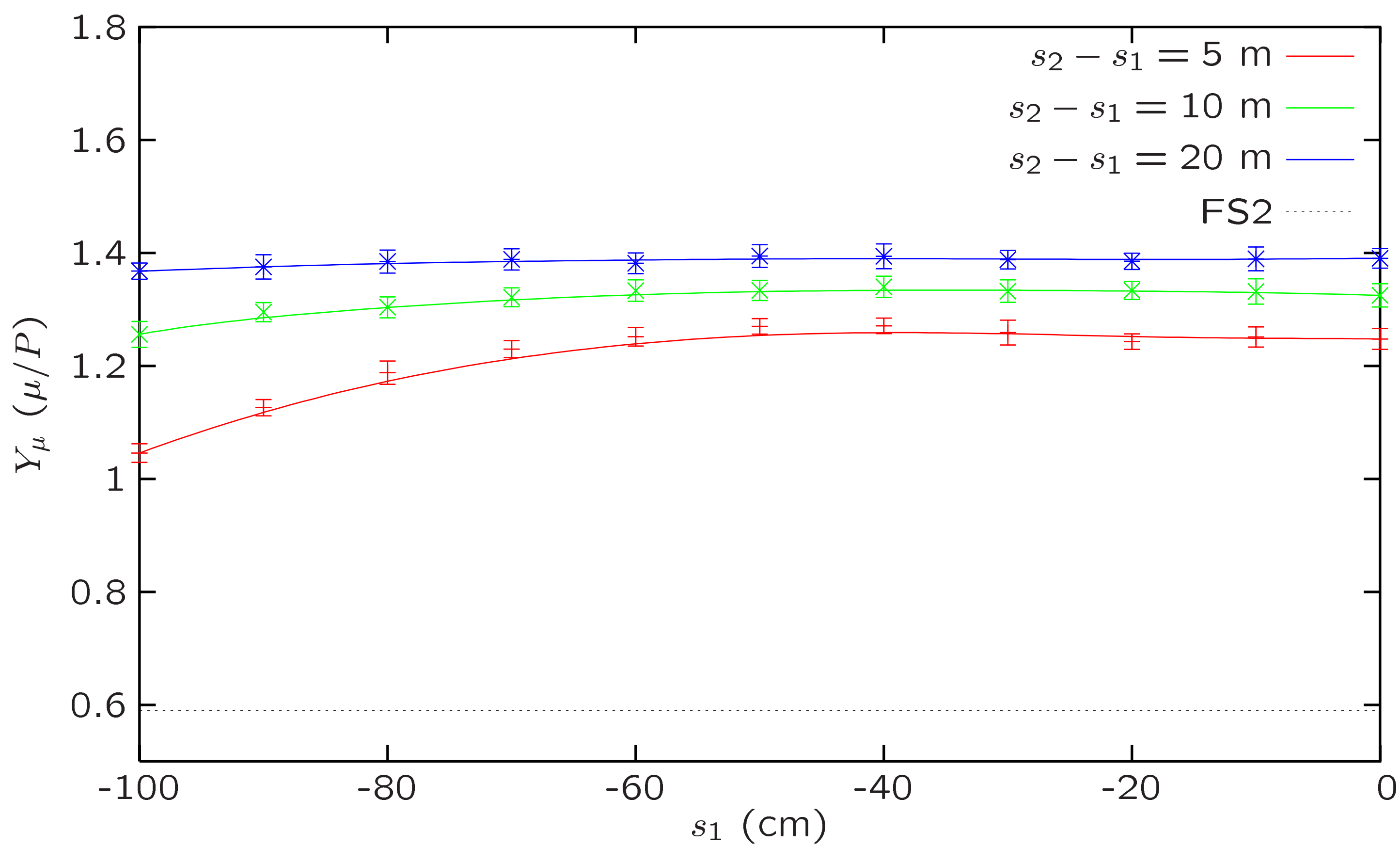
  ...Various starting points for taper: $s_1 = -100 \to 0$ cm

Analysis:

- For $B = 1.25$ T, consider:

  $s_2 - s_1 = 500, 1000, 2000$ cm

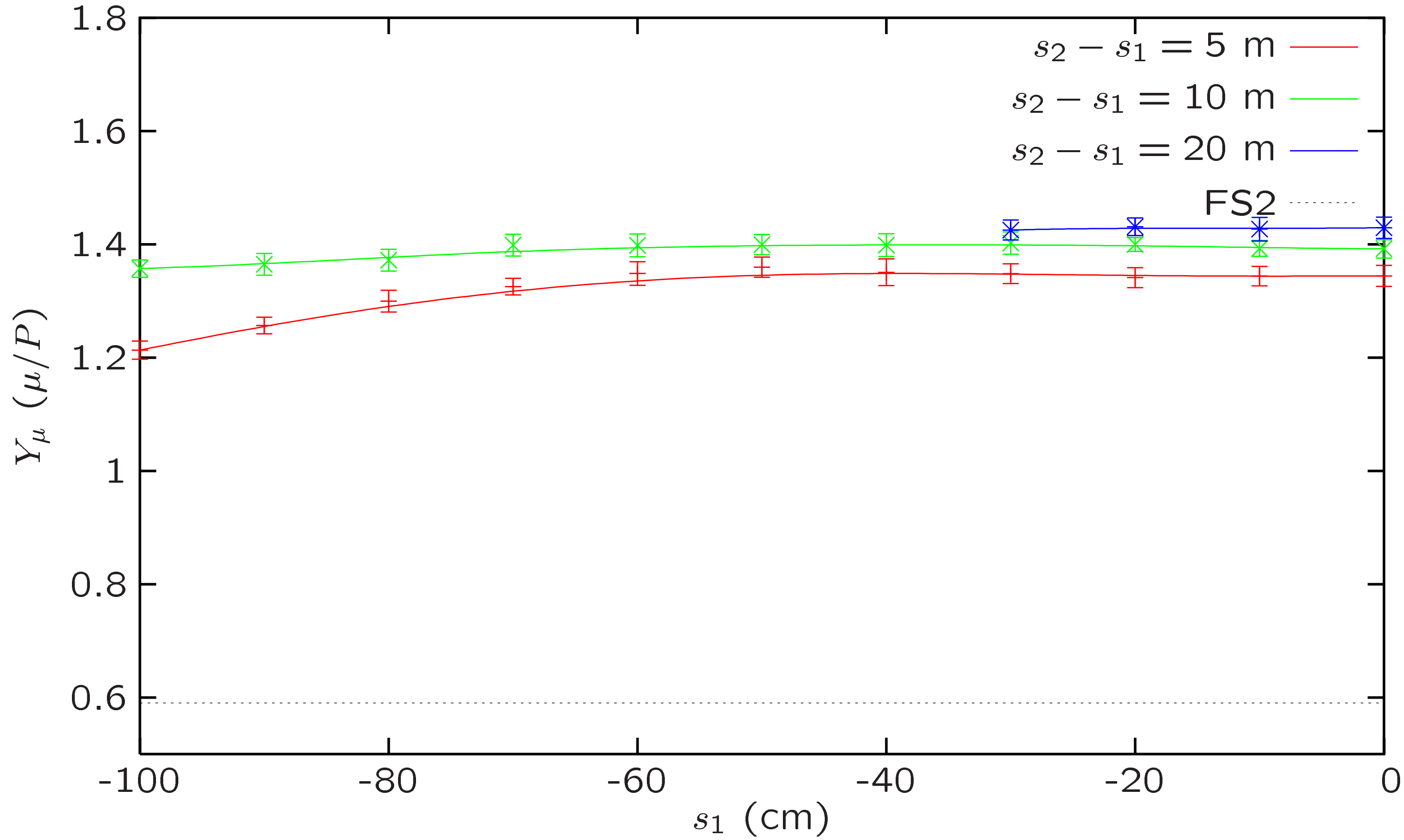
  $s_1 = -100 \to 0$ cm



- For $B = 2.00$ T, consider:

  $s_2 - s_1 = 500, 1000, 2000$ cm

  $s_1 = -100 \rightarrow 0$ cm



Performance-to-cost Ratio:

- Define a merit factor:

$$f = \frac{Y_\mu}{W}$$

where W is the energy stored in the magnetic field

- For FS2:

$$f_{FS2} \approx \frac{0.59\ \mu/P}{15\ \text{MJ}} = 0.039\ \mu/P\ \ \text{MJ}^{-1}$$

- Normalized merit factor:

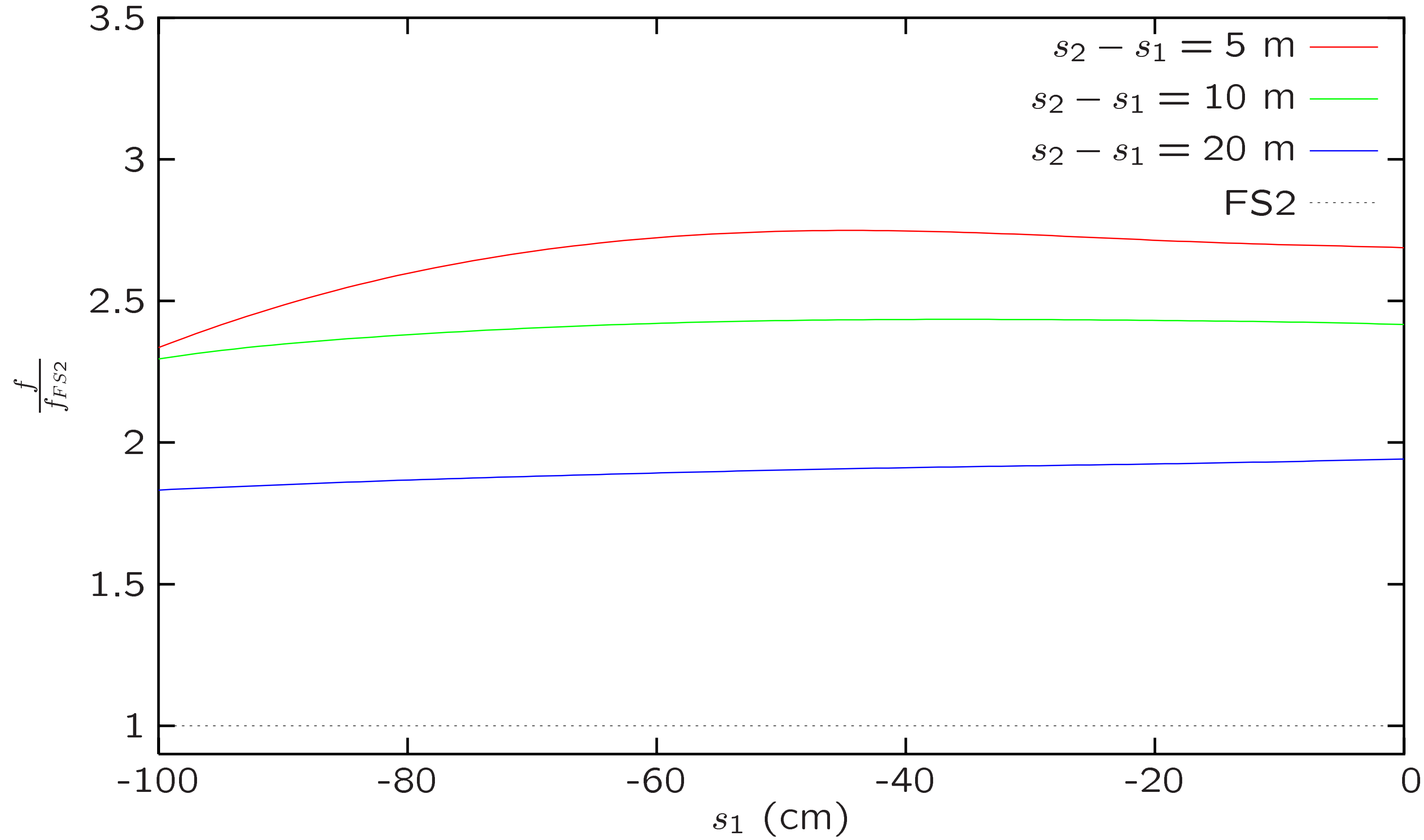
$$f/f_{FS2}$$

should roughly measure comparable performance-to-cost ratio

- For $B = 1.25$ T, consider:

  $s_2 - s_1 = 500, 1000, 2000$ cm

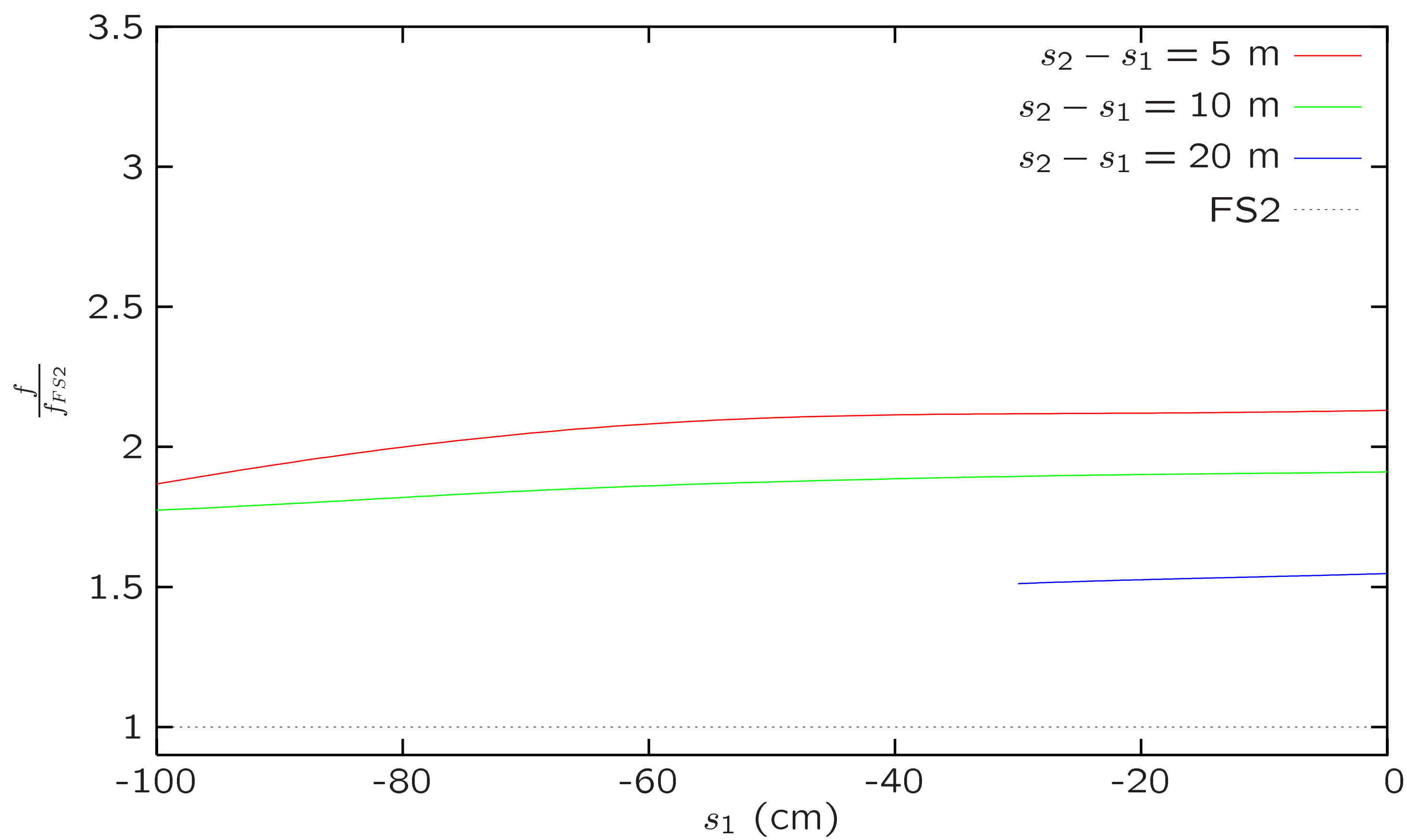
  $s_1 = -100 \to 0$ cm



- For $B = 2.00$ T, consider:

  $s_2 - s_1 = 500, 1000, 2000$ cm

  $s_1 = -100 \rightarrow 0$ cm



Conclusions:

- Performance Optimum:

  ...Long taper: $s_2 - s_1 = 20$ m

  ...Strong field: $B = 2$ T

  ...Start taper after target: $s_1 = 0$ cm

  YIELD: $Y_\mu = 1.43\ \mu/P$

  COST: $\sim 60\%$ more expensive!

- Cost Optimum:

  ...Short taper: $s_2 - s_1 = 5$ m

  ...Low field: $B = 1.25$ T

  ...Start taper in middle of target: $s_1 = -40$ cm

  YIELD: $Y_\mu = 1.30\ \mu/P$

  COST: $\sim 20\%$ less expensive!