

# Muon Acceleration with Scaling FFAG using Harmonic Number Jump

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# Muon Acceleration with FFAG Accelerator

## Scaling FFAG

### advantages

- *no resonance crossing : zero chromaticity*
- *large dynamic aperture*

### problems (issues)

- *not small beam pipe (may not be an issue)*
- *variable rf frequency : broad-band (low frequency & low field)*

## Non-scaling FFAG

### advantages

- *rf acceleration : constant rf frequency (high frequency & high field)*
- *small beam pipe : small momentum compaction*

### problems (issues)

- *resonance crossing*
- *time of flight (path length) for large beam amplitude : cascade rings*

# Scaling FFAG with HNJ(harmonic number jump) Acceleration

## ● Scaling FFAG + HNJ acceleration

### ● constant rf frequency

- *high frequency(200-400MHz) & high field (20MV/m) rf cavity*

### ● good match with phase rotation & non-scaling FFAG

- *Low energy (5-10GeV) muon accelerator as an injector of non-scaling FFAG to avoid path length problem of non-scaling FFAG*
- *Scaling FFAG with HNJ for high energy (10-20GeV) ring*

# HNJ Acceleration

## ● Revolution period for n-th turn

$$\left(\frac{T_n}{T_1}\right) = \left(\frac{C_n / v_n}{C_1 / v_1}\right)$$

● C: circumference, v: particle velocity

## ● Scaling FFAG

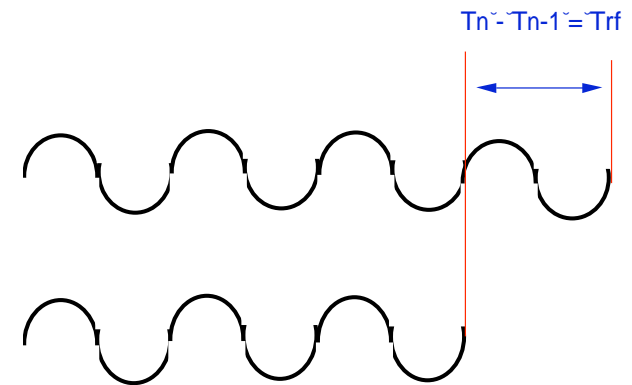
$$\frac{C_n}{C_1} = \left(\frac{p_n}{p_1}\right)^{\frac{1}{k+1}}$$

## ● For muon acceleration ( $v \sim c$ )

● When k increases, or ring size decreases,

- No. of turns decreases.
- Energy gain/turn increases.

● **Need optimization!**



$$T_n - T_{n-1} = Trf \times m$$

$$\frac{C_n}{C_1} = \frac{h_n}{h_1}, \quad p_n = p_1 \left(\frac{h_n}{h_1}\right)^{k+1}, \quad h_n = h_1 + n \times m$$

# Scaling FFAG

## Focusing

### Spiral sector

- *Focusing: body + edge*
- *Small ring size*
- *Rather large edge angle > 70 degree*

### Radial sector

- *Negative bend*
- *doublet, triplet (DFD, FDF)*

## Basic parameters requested

### Ring

- *Energy  $P=5-10\text{GeV}$*
- *$B_{\text{max}} < 2\text{T}$  (Iron magnet :NC or super ferric)*
- *Field index  $k$  : as small as possible*
- *Orbit excursion  $< 1\text{m}$*
- *Beam size : full aperture@10GeV  $< \sim 15\text{cm}$*

### RF

- *RF frequency : 200-400MHz*
- *RF field :  $\sim 10\text{MV/m}$ , Energy gain/m  $> 1.5\text{MeV/m}$*

 We choose “spiral sector”.

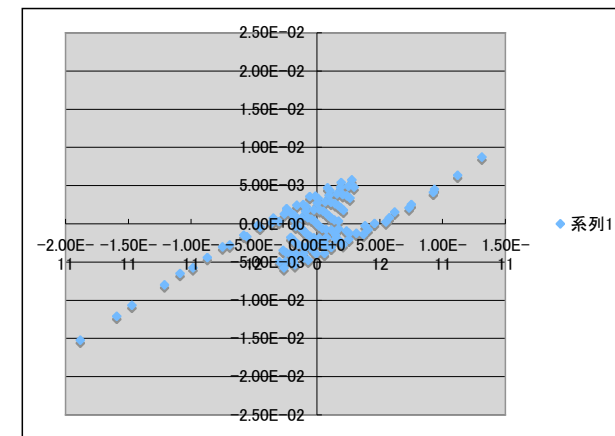
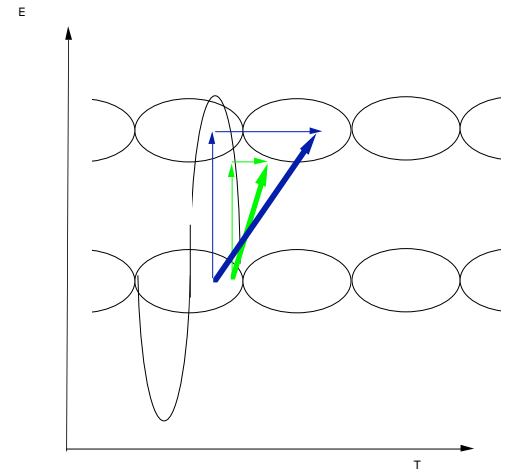
# Issues of HNJ

## Phase acceptance

- Smaller for HNJ cf. synchronized acceleration
- Because energy gain/turn is so large for HNJ that phase slip/turn should be  $2\pi$ . If stable phase is away from  $\pi/2$ , phase slip/turn should be much less than  $2\pi$ .

## Non-linear source dynamic aperture problems in longitudinal direction

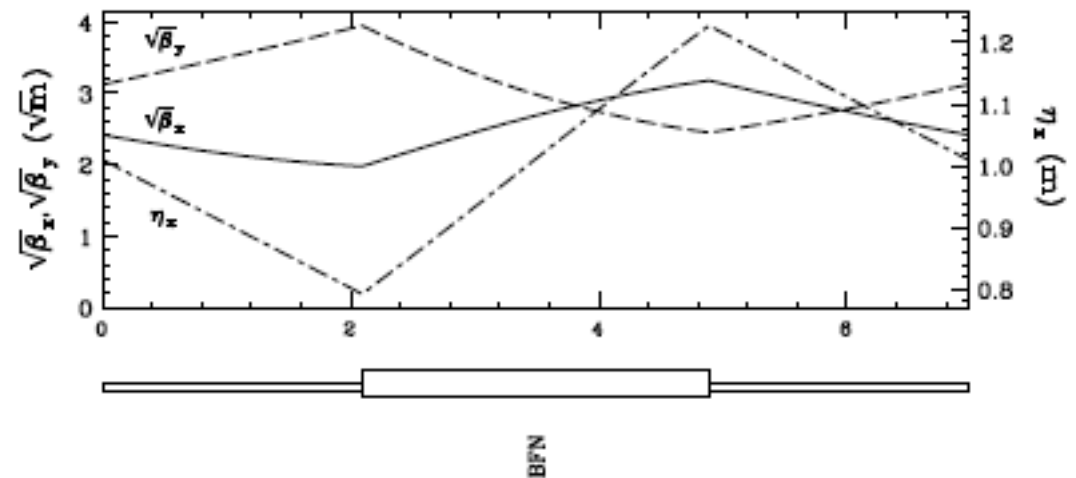
- Sinusoidal rf field contains non-linear components.
- Synchrotron tune is high enough to see non-linear resonances.  $mQ_s = n$



# 5-10GeV scaling FFAG spiral sector - design example

## Ring parameters

- $r=40\text{m}$
- $N=32\text{cells}$
- spiral angle:  $74\text{degree}$
- $B_{\text{max}} \sim 2.1\text{T}$  (p.f.=0.4)
- $k=38$
- Orbit excursion
  - $71.7\text{cm}$
- Beam size(half,  $dp/p=0.03$ ) at  $10\text{GeV}$ 
  - $H: 4.3\text{cm}+3.0\text{cm}=7.3\text{cm}, V=5.2\text{cm}$  @s.s.
  - $H: 5.2\text{cm}+3.6\text{cm}=9.3\text{cm}, V=6.9\text{cm}$  @magnet

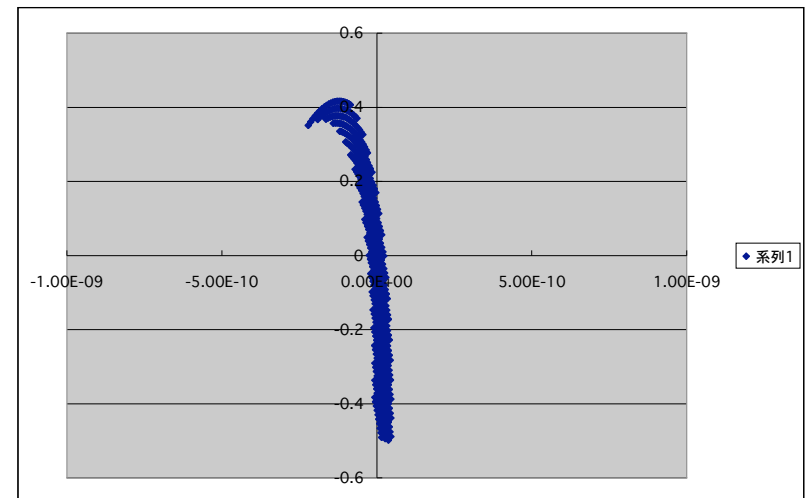
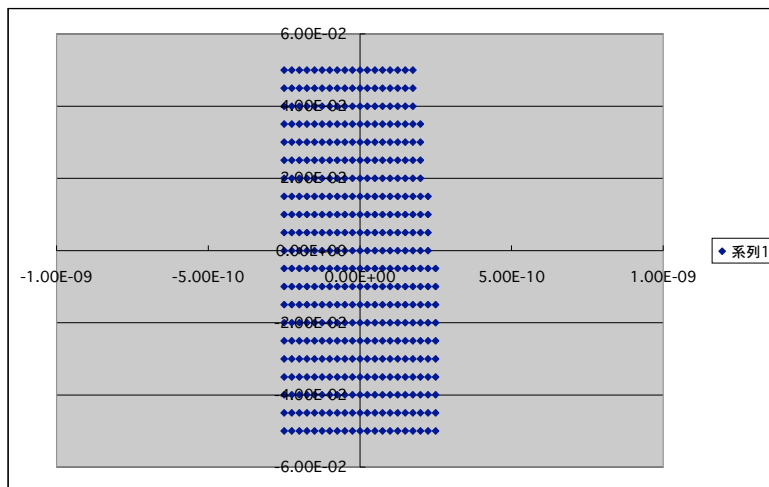
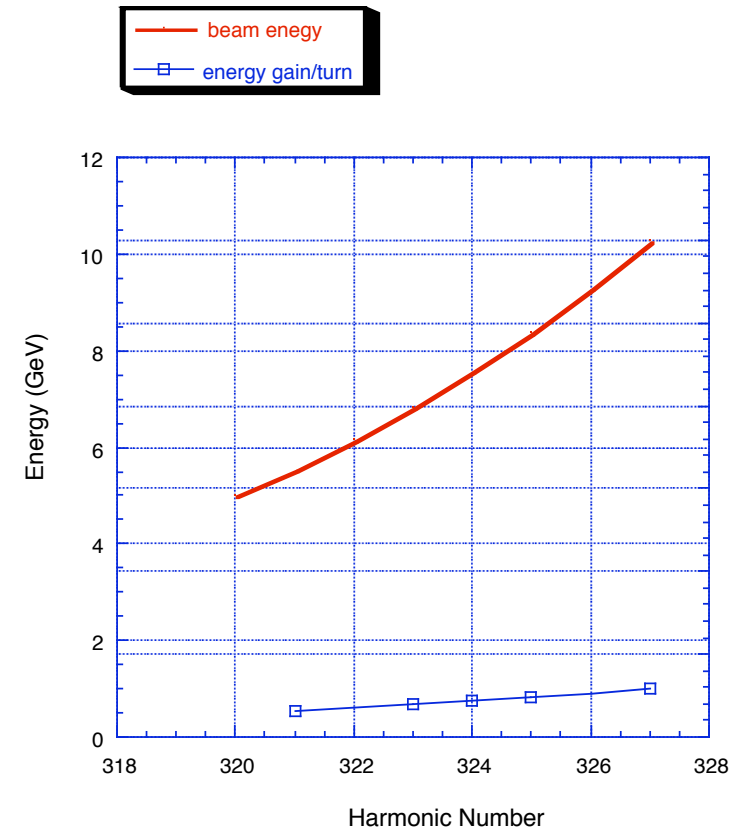


# Spiral FFAG

## 5-10GeV

### Parameters

- r=40m
- k=38
- rf parameters
  - $h=320$
  - $f=400\text{MHz}$
  - $f_{ai\_s}=2\pi/3$
  - $18.8\text{MV/m:4-cell cavity}$





# Spiral FFAG

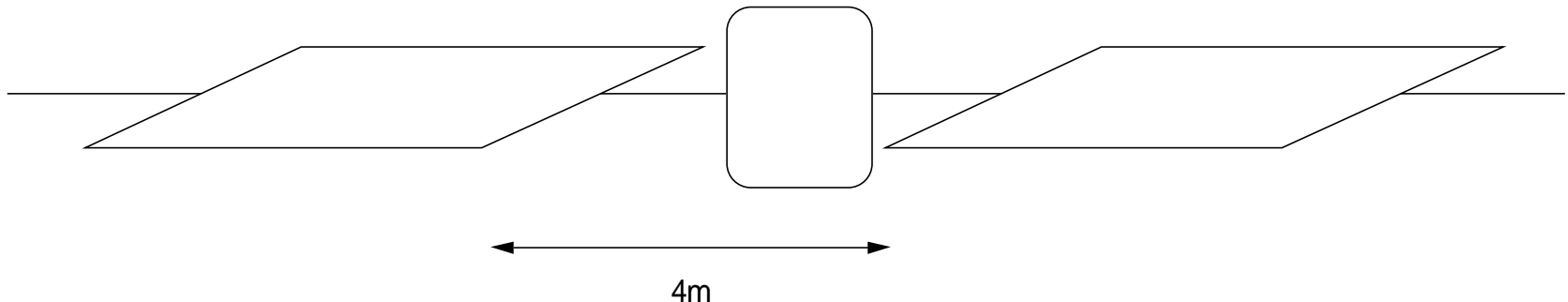
## 5-10GeV

### Lattice

- almost satisfied but more optimization is needed.
  - *k-value:lower, Bmax:lower, packing factor, circumference etc.*

### HNJ acceleration

- seems to have enough acceptance
- frequency of rf cavity
  - *400MHz → 200MHz (depends on lattice design)*
- No. of turns: should be larger >10 turns (now 7turns)
  - *reduce rf voltage 18.8MV/m → 15MV/m*
- **Increase ring radius and reduce k-value**



# Summary

- Scaling FFAG with HNJ acceleration for Muon 5-10GeV (10-20GeV) looks promising but more optimization is needed.
- Flight time problem of non-scaling FFAG may be cured by scaling FFAG.
- Hardwares R&Ds are needed.
  - squashed(?) sc rf cavity
  - large spiral magnet