'Dogbone' RLA – Error Tolerances and Tracking

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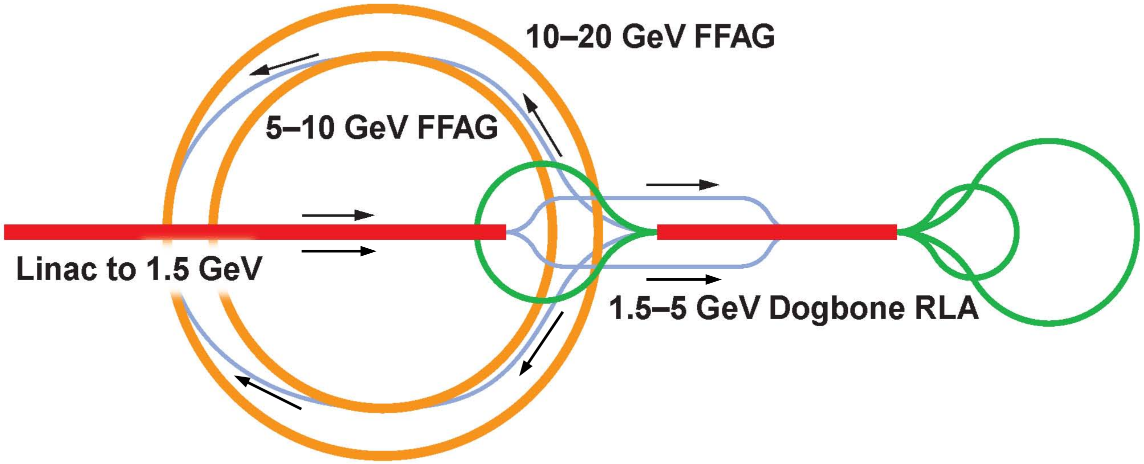
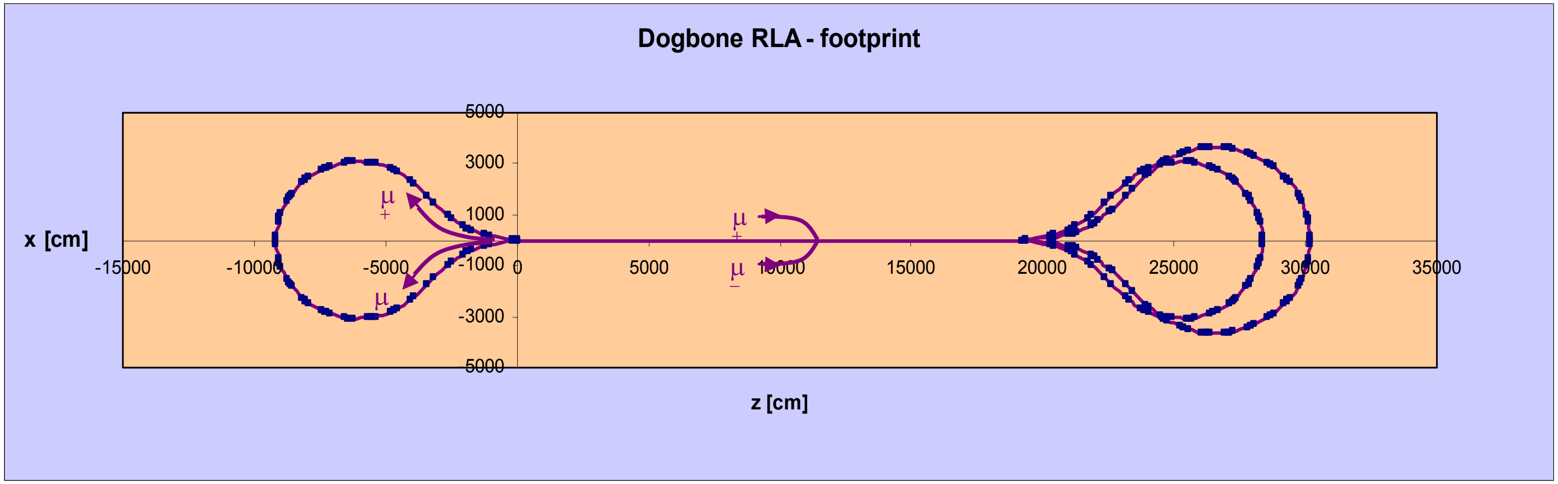
- Symmetric 5 GeV 'Dogbone' RLA – Linear Optics
- Front-to-End Multi-particle Tracking - 30 mm rad Acceptance (normalized)
- Magnet Misalignment Errors – DIMAD Monte Carlo Simulation
- Focusing Errors Tolerance – Betatron Mismatch Sensitivity and Tunability
- Magnet Field Quality Specs – Emittance Dilution due to Nonlinearities

Symmetric Muon Acceleration Complex



- Linear pre-accelerator (273 MeV/c – 1.5 GeV)
- Symmetric 'Dogbone' RLA (allowing to accelerate both μ^+ and μ^- species), 3.5-pass (1.5 – 5 GeV)

Linac Optics – Arcs, multi-pass linac



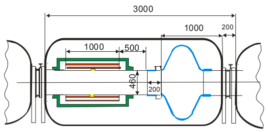
- Multi-pass linac optics additionally constrained by the mirror symmetry of the droplet arcs
  - at the exit/entrance from/to the previous/next linac: the betas are equal and the alphas are of the opposite sign
- Optimized 'bisected' linac was chosen as follows:
  - 90^0 phase advance/cell is set for the 'half pass' linac (1.5-2GeV).
  - as a consequence linac phase advance/cell in the first part of 1-pass drops to about 45^0.
  - to avoid large 'beta beating' one chooses to keep 45^0 phase advance/cell throughout the second part of the linac (Bob Palmer).
  - the phase advance at the end of 2-pass linac drops by another factor of two (22.5^0).
  - the 'beta beating' is rather small on higher passes (2 and 3)

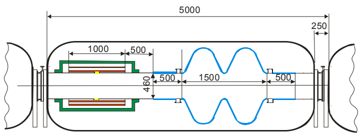
Initial beam emittance/acceptance after cooling at 273 MeV/c

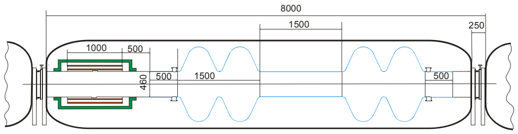
Study IIb		ε_{rms}	**A = (2.5)^2 ε**
normalized emittance: $\varepsilon_x/\varepsilon_y$	mm·rad	**4.8**	**30**
longitudinal emittance: ε_l	mm	**27**	**150**
($\varepsilon_l = \sigma_{\Delta p}\, \sigma_z/m_\mu c$)			
momentum spread: $\sigma_{\Delta p/p}$		**0.07**	**$\pm$0.17**
bunch length: σ_z	mm	**176**	**$\pm$442**

Pre-accelerator – different style cryo-modules

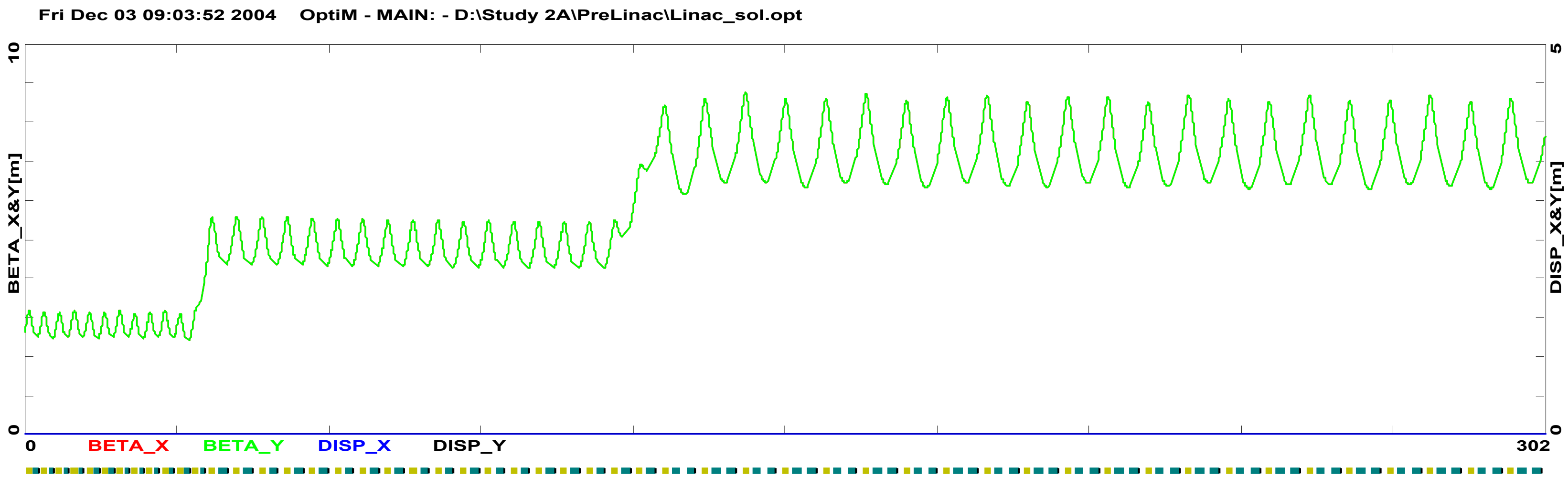
	Short	Medium	Long
Number of periods	12	18	22
Total length of one period	3 m	5 m	8 m
Number of cavities per period	1	1	2
Number of cells per cavity	1	2	2
Cavity accelerating gradient	15 MV/m	15 MV/m	15 MV/m
Real-estate gradient	3.72 MV/m	4.47 MV/m	5.59 MV/m
Aperture in cavities (2*a*)	460 mm	460 mm	460 mm
Aperture in solenoids (2*a*)	460 mm	460 mm	460 mm
Solenoid length	1 m	1 m	1 m
Solenoid maximum field	1.5 T	1.9 T	3.9 T

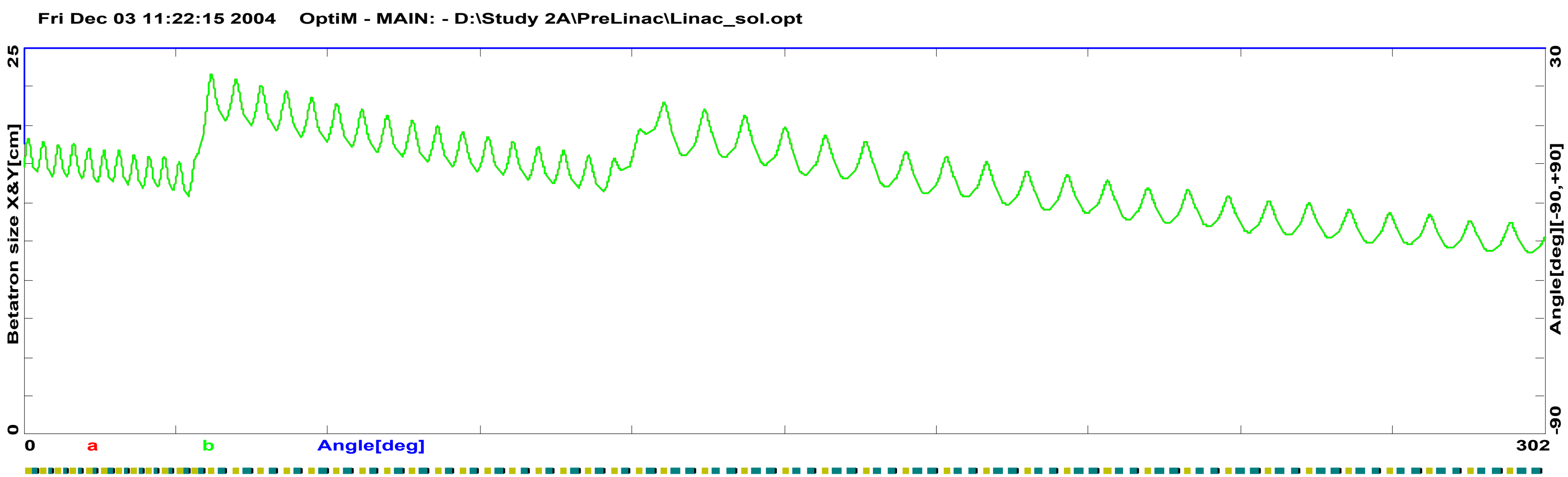




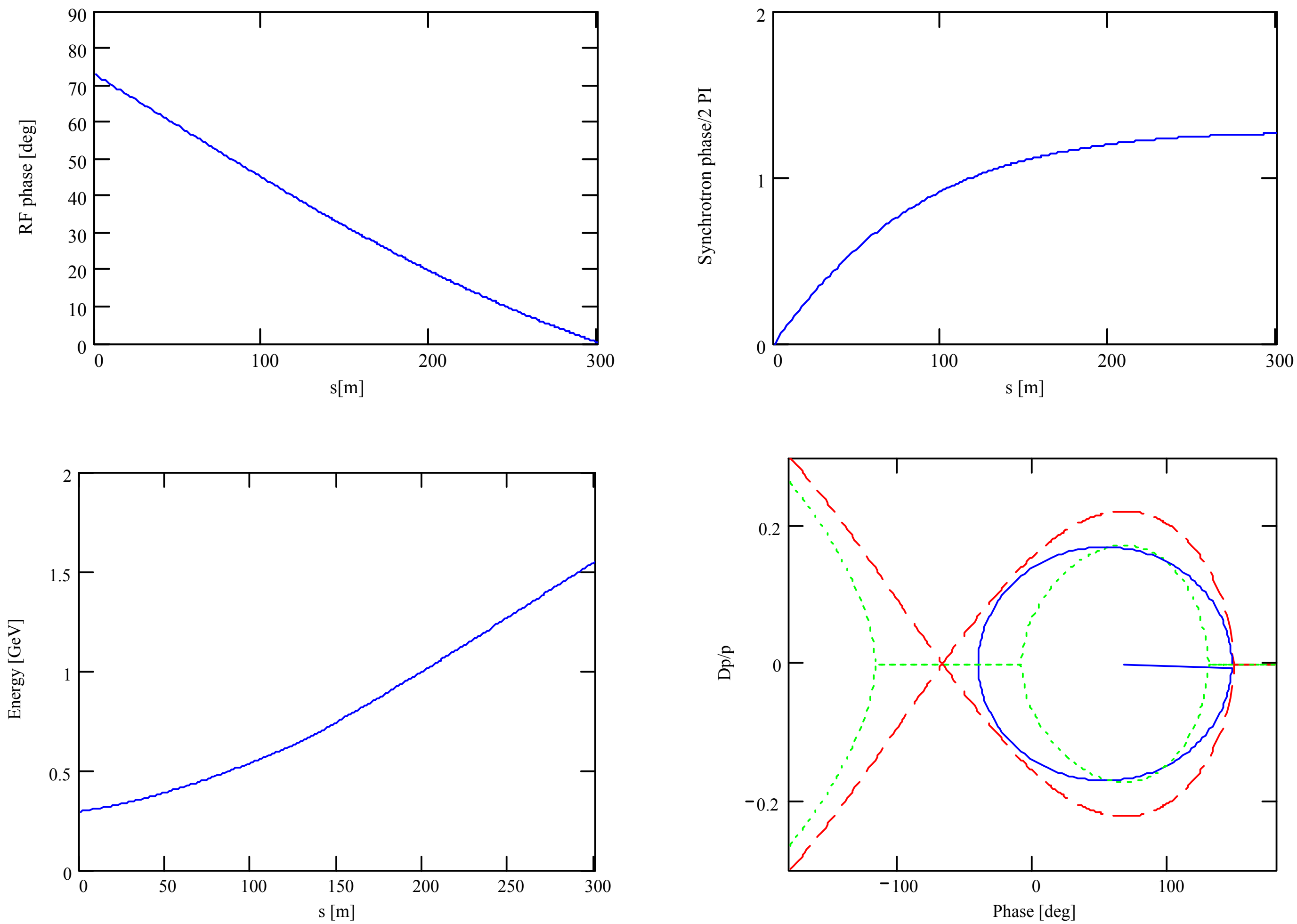


Linear Pre-accelerator – Twiss functions and beam envelope (2.5 σ)

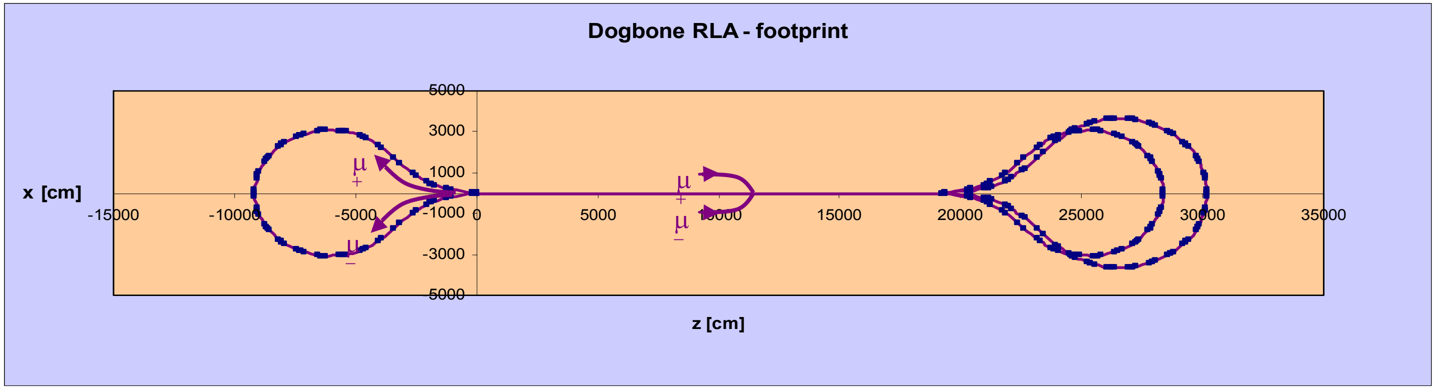


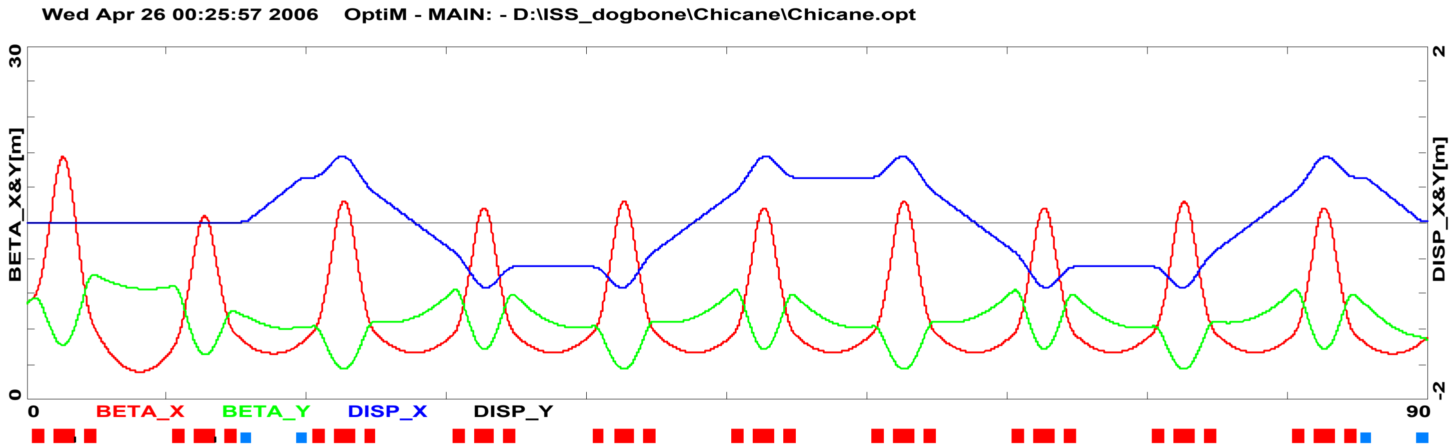


Introduction of synchrotron motion in the initial part of the linac

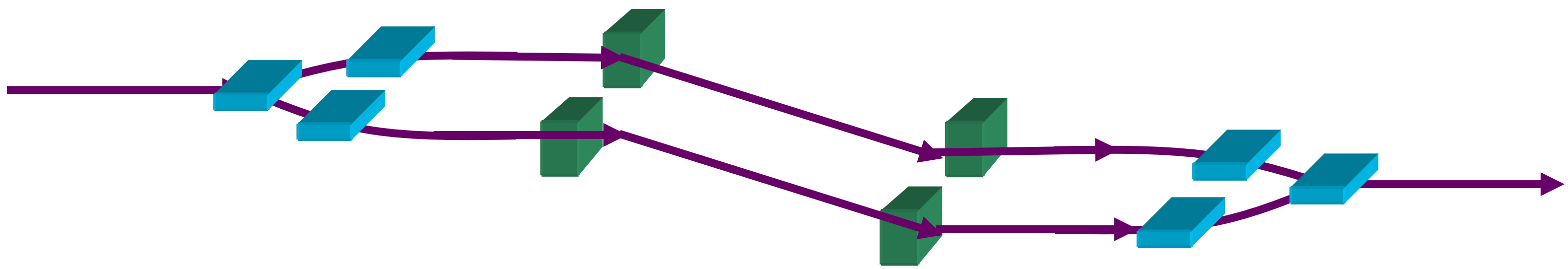


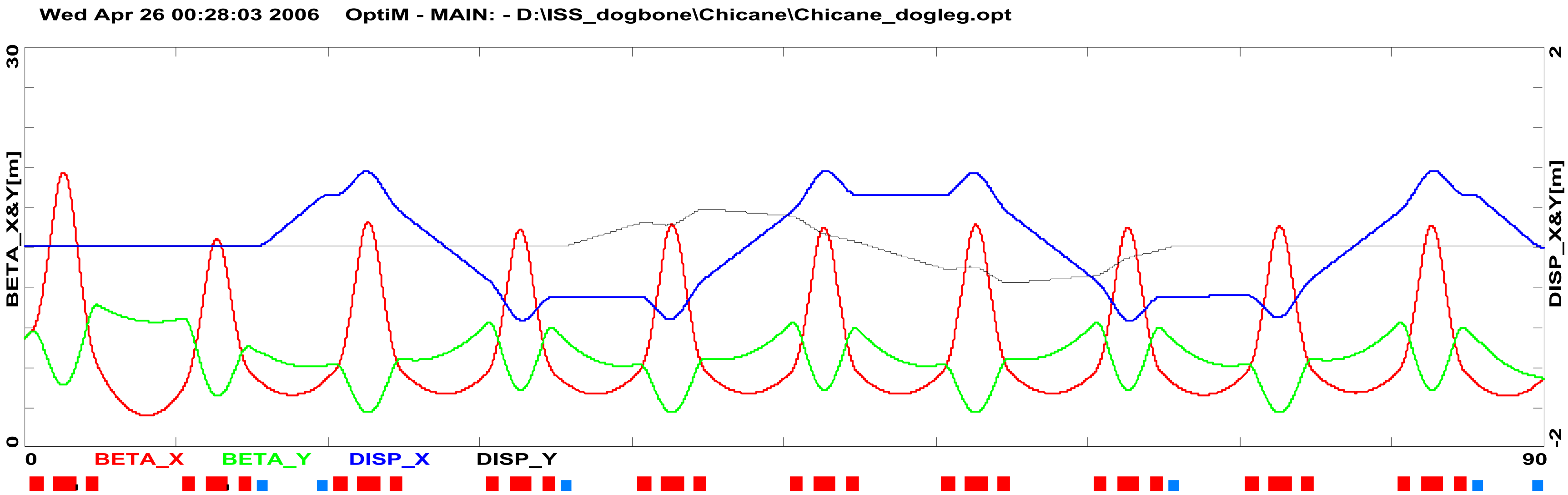
Injection Chicane – both μ^+ and μ^-





Chicane – 'dogleg'

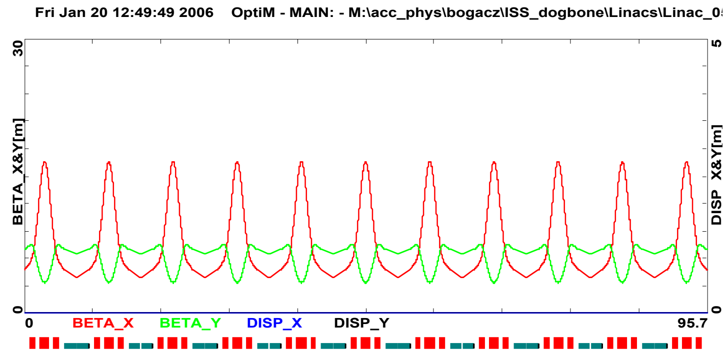




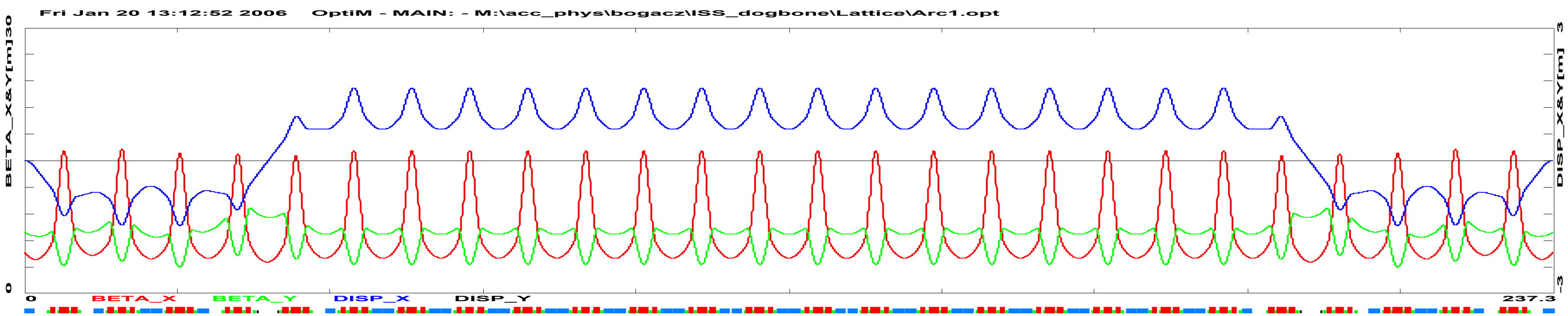
Linac-Arc1-Linac Matching

($\beta_{out} = \beta_{in}$,and $\alpha_{out} = -\alpha_{in}$, matched to the linacs)

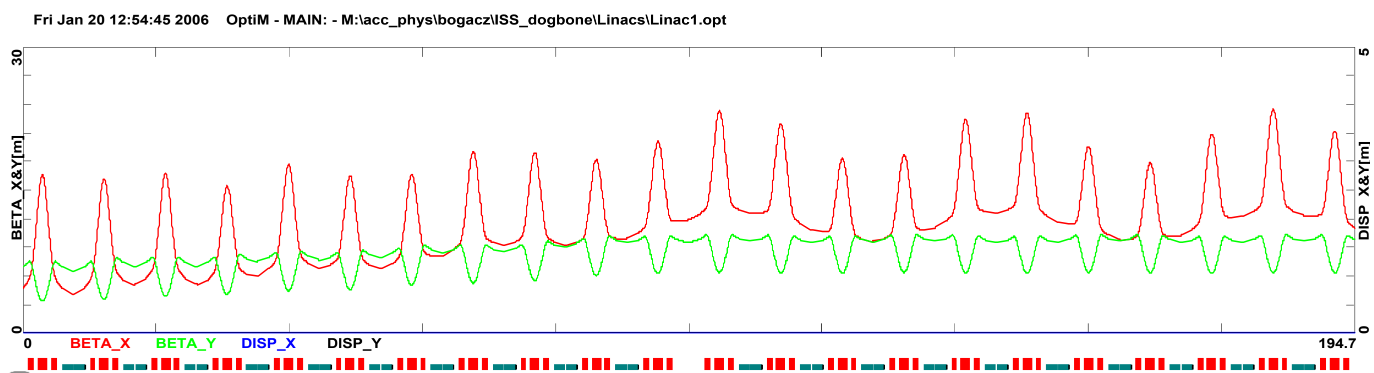
'half pass' (1.5-2GeV)



Arc1 (2GeV)



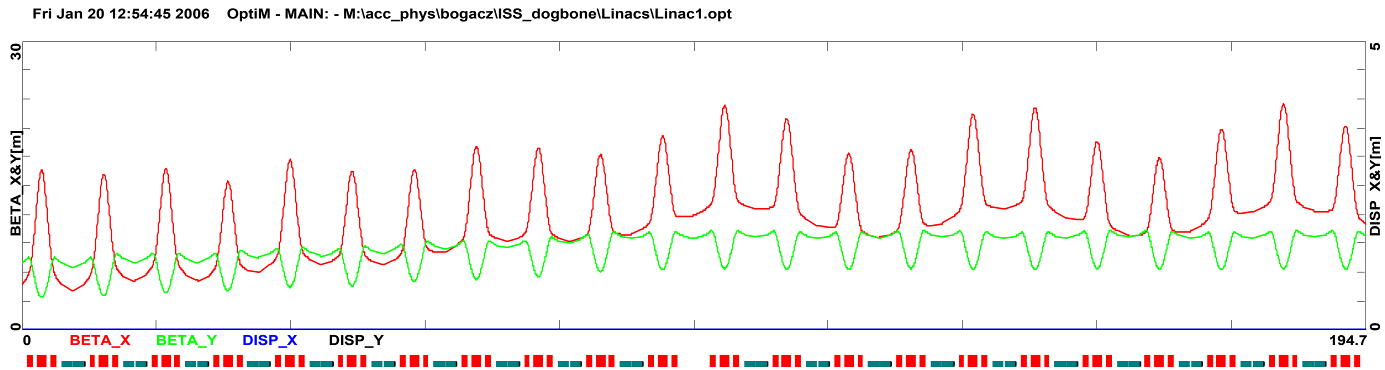
1-pass (2-3GeV)



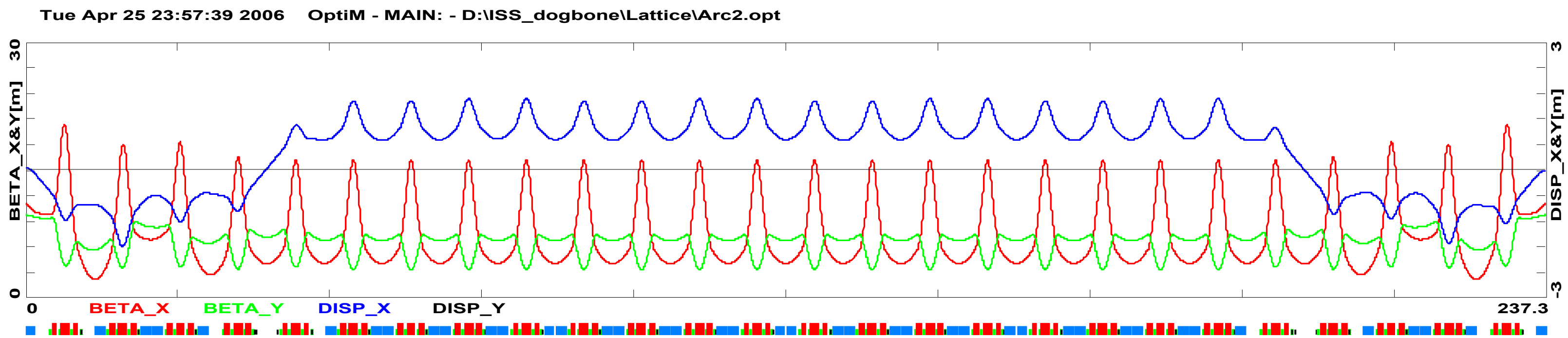
Linac-Arc2-Linac Matching

($\beta_{out} = \beta_{in}$,and $\alpha_{out} = -\alpha_{in}$, matched to the linacs)

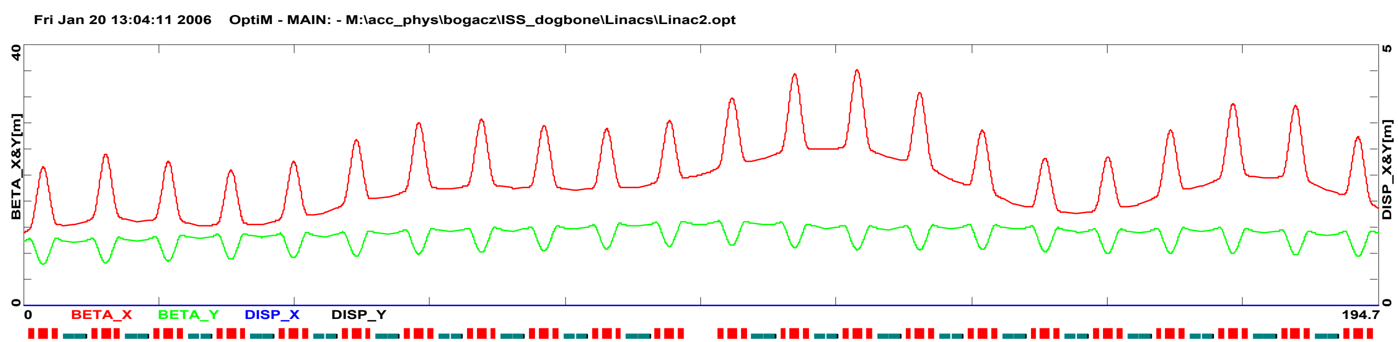
1-pass (2-3GeV)



Arc2 (3GeV)



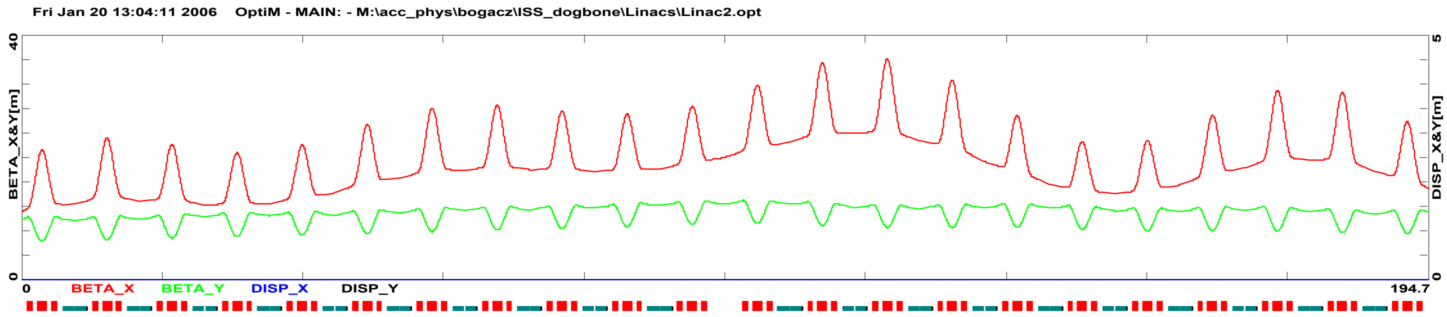
2-pass (3-4GeV)



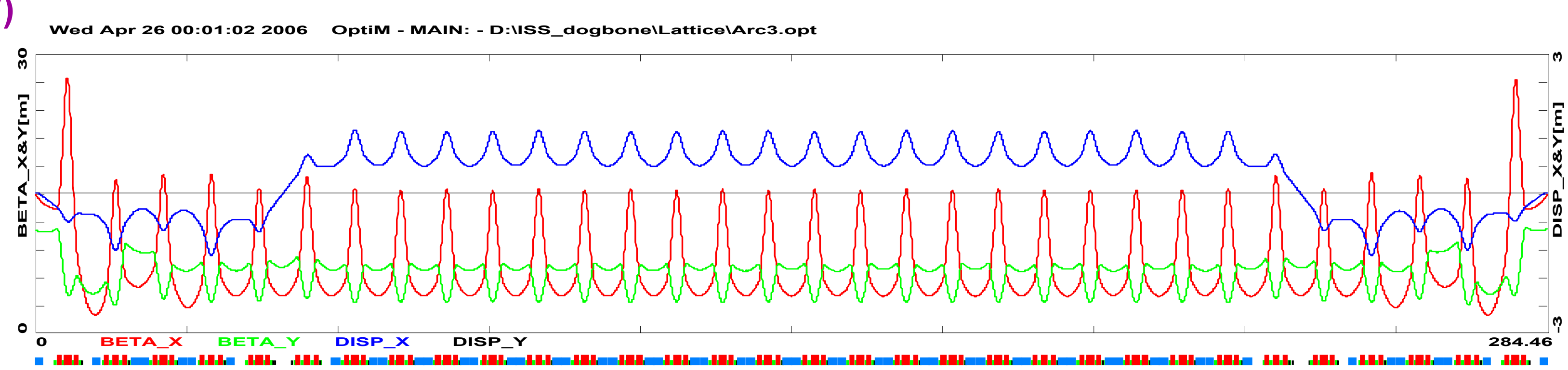
Linac-Arc3-Linac Matching

($\beta_{out} = \beta_{in}$,and $\alpha_{out} = -\alpha_{in}$, matched to the linacs)

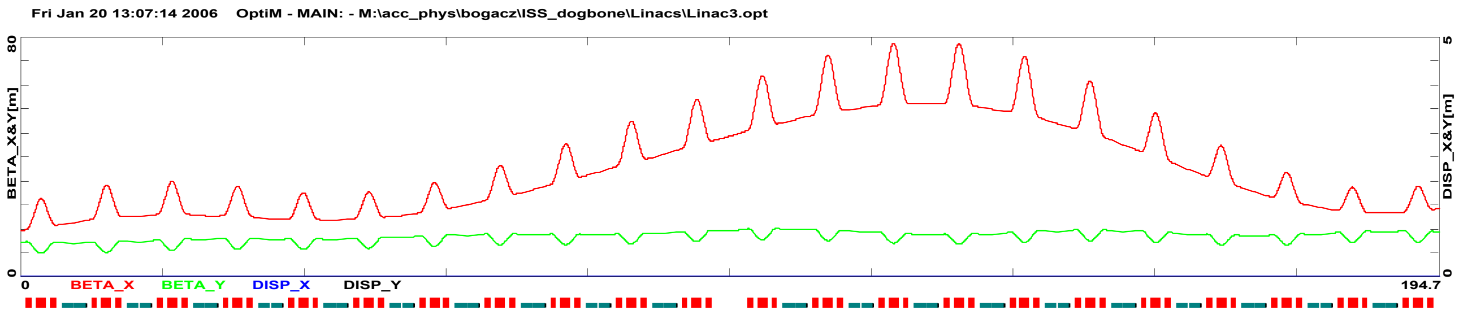
2-pass (3-4GeV)



Arc1 (3GeV)

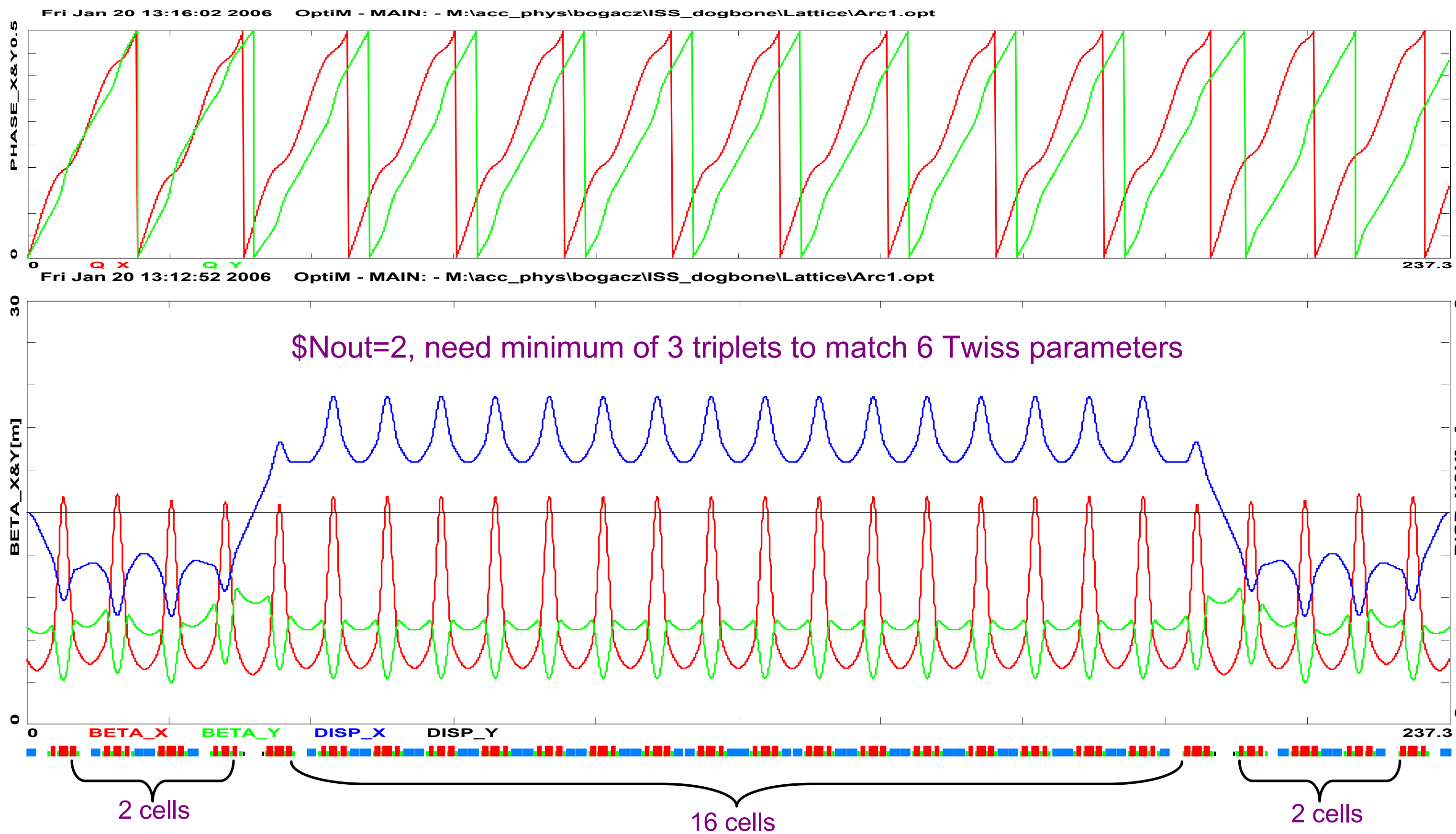


3-pass (4-5GeV)



Arc 1 – Mirror-symmetric Optics

($\beta_{out} = \beta_{in}$,and $\alpha_{out} = -\alpha_{in}$, matched to the linacs)



dipoles (2 per cell)

$Lb=150; => 150 cm

$ang0=10.3283 deg

$ang=(90+$ang0)/($Nin-2*$Nout); => 8.36 deg

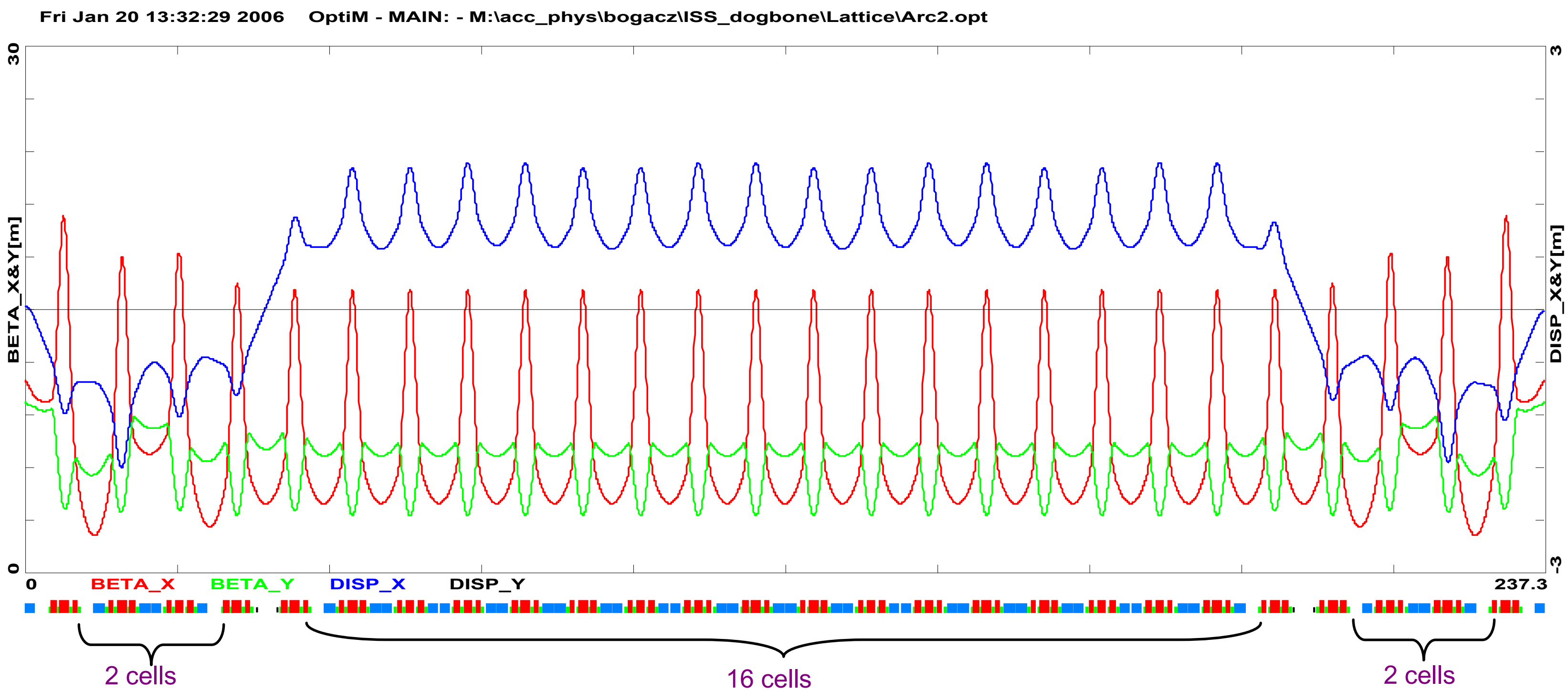
$B=$PI*$Hr*$ang/(180*$Lb); => 6.537 kGauss

quadrupoles (triplet):

L[cm]	G[kG/cm]
68	-0.326
125	0.328
68	-0.326

Arc 2 – Mirror-symmetric Optics

($\beta_{out} = \beta_{in}$,and $\alpha_{out} = -\alpha_{in}$, matched to the linacs)



dipoles:

$Lb=150; => 150 cm

$E=2920.75; => 2920.75 MeV

$ang0=10.3283; => 10.33 deg.

$B0=-$PI*$Hr*$ang0/(180*$Lb); => -12.12 kGauss

$ang=8.3607 deg.

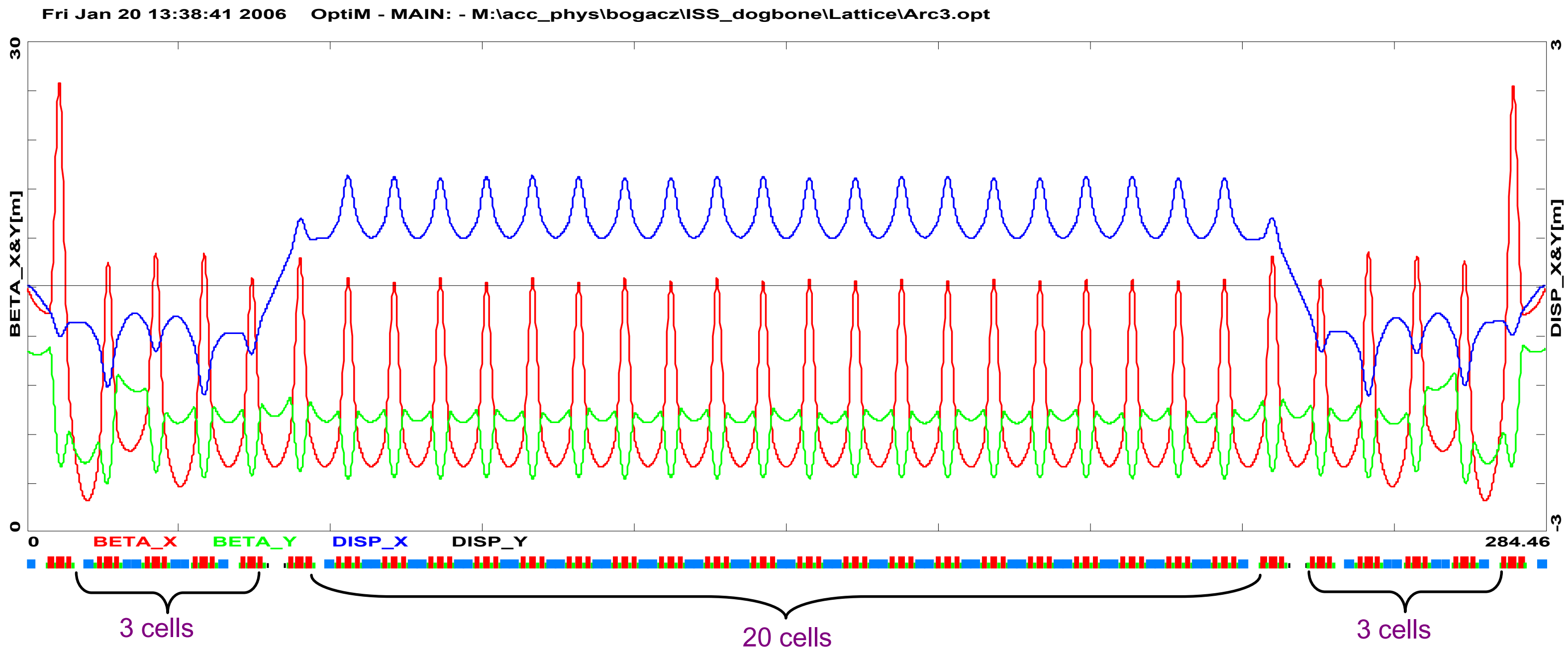
$B=$PI*$Hr*$ang/(180*$Lb); => 9.81 kGauss

quadrupoles (triplet):

L[cm]	G[kG/cm]
68	-0.490
125	0.492
68	-0.490

Arc 3 – Mirror-symmetric Optics

($\beta_{out} = \beta_{in}$,and $\alpha_{out} = -\alpha_{in}$, matched to the linacs)



dipoles:

$E=3929.86 MeV

$B0=-8.0755 kGauss

$ang0= 5.1577 deg

$BP=$PI*$Hr*$ang/(180*$Lb); => 10.64 kGauss

$ang=(90+$ang0)/($Nin-2*$Nout); => 6.797 deg

$Ang_out=$ang0+2*$Nout*$ang; => 45.94 deg

$Ang_in=2*$Nin*$ang; => 271.88 deg

quadrupoles (triplet):

L[cm]	G[kG/cm]
68	-0.6537
125	0.6565
68	-0.6537

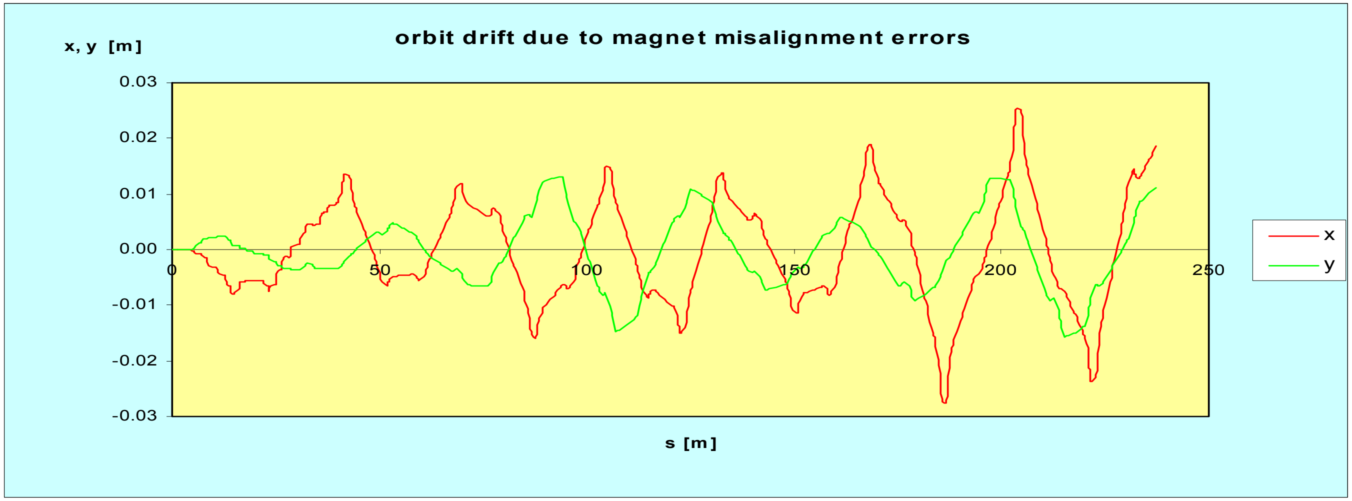
Magnet Misalignment Errors

- Lattice sensitivity to random misalignment errors was studied via DIMAD Monte-Carlo assuming:

quadrupole misalignment errors:

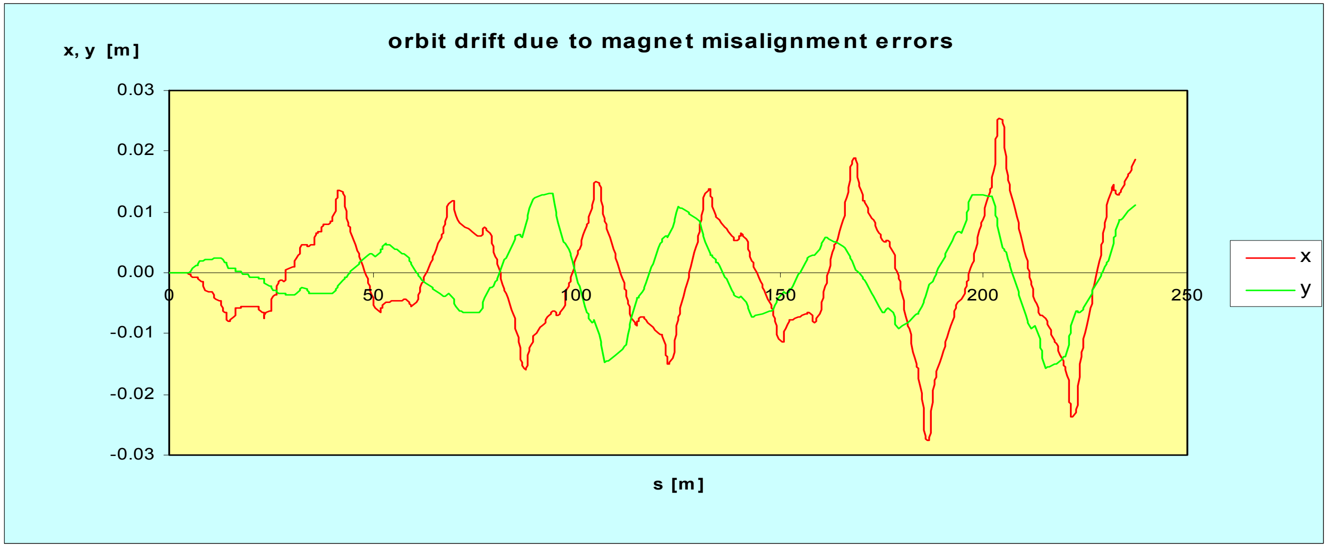
F:	$\sigma_x = \sigma_y = 1$ mm	$(\sigma_{x,y'} = \sigma_{x,y}/L)$	$\sigma_{x'} = \sigma_{y'} = 0.8 \times 10^{-3}$
D:	$\sigma_x = \sigma_y = 1$ mm		$\sigma_{x'} = \sigma_{y'} = 1.47 \times 10^{-3}$

- Gaussian distribution was chosen for individual quad misalignments
- Resulting reference orbit distortion (uncorrected) for Arc 2 is illustrated below



- Similar level of dipole misalignment errors had virtually no effect on random steering

Arc 2 – Magnet Misalignment Errors



RMS Orbit Displacement [m]:

X: 0.9486e-02

y: 0.7003e-02

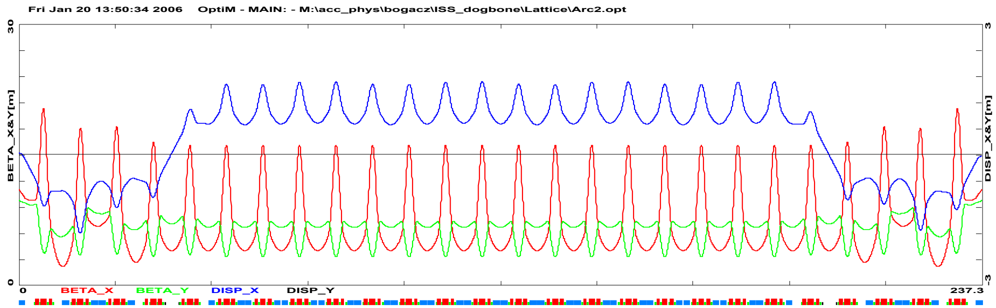
Extr. Orbit Displacement [m]:

X_{max}: 0.2538E-01

X_{min}: -0.2782E-01

y_{max}: 0.1434E-01

y_{min}: -0.1697E-01

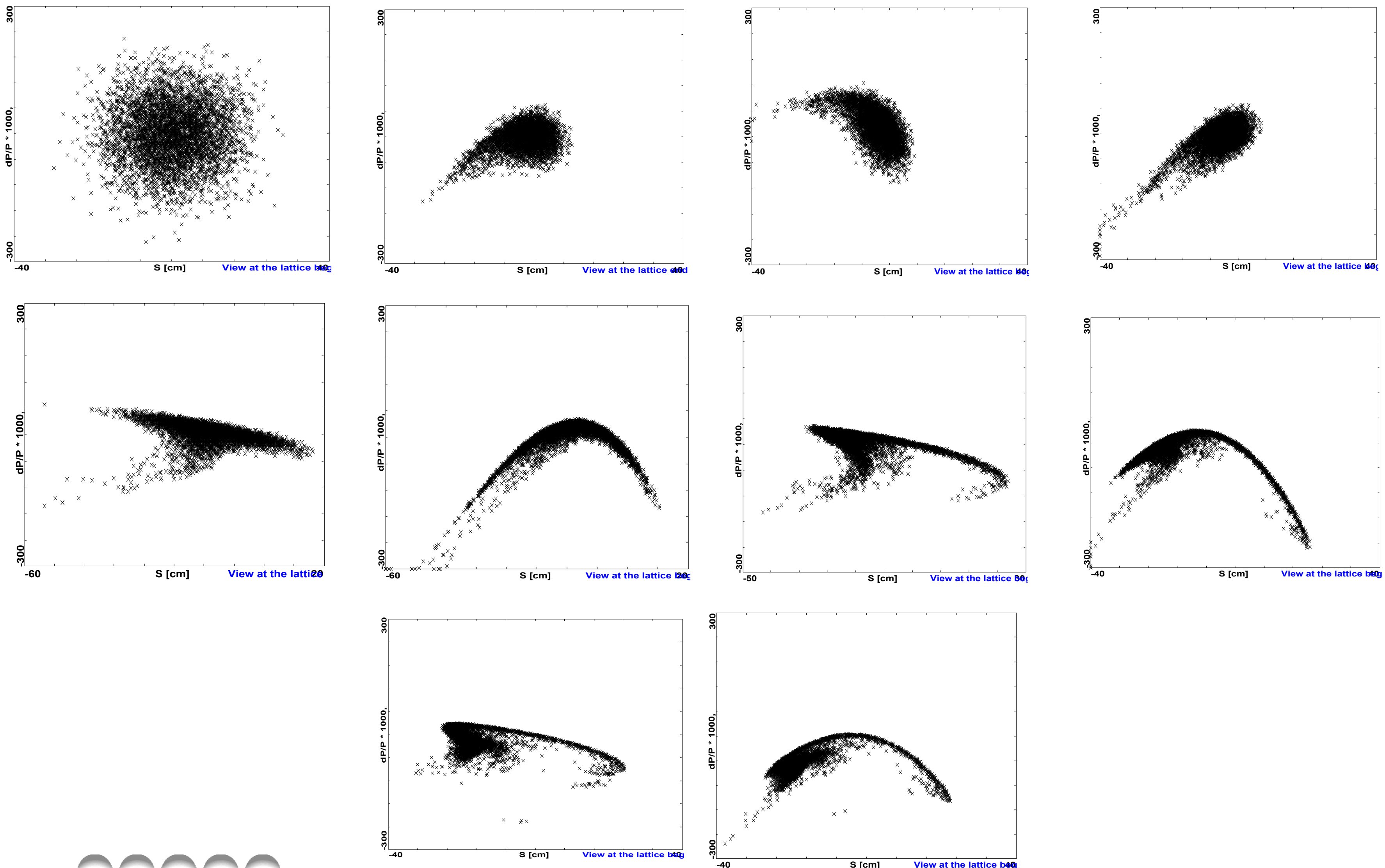


- Same level of orbit drifts due to quad misalignments for other 'Dogbone' segments (Arc 1, 3 and linacs)
- Orbit drifts at the level of ~3 cm can easily be corrected by pairs of hor/vert correctors (2000 Gauss cm each) placed at every triplet girder

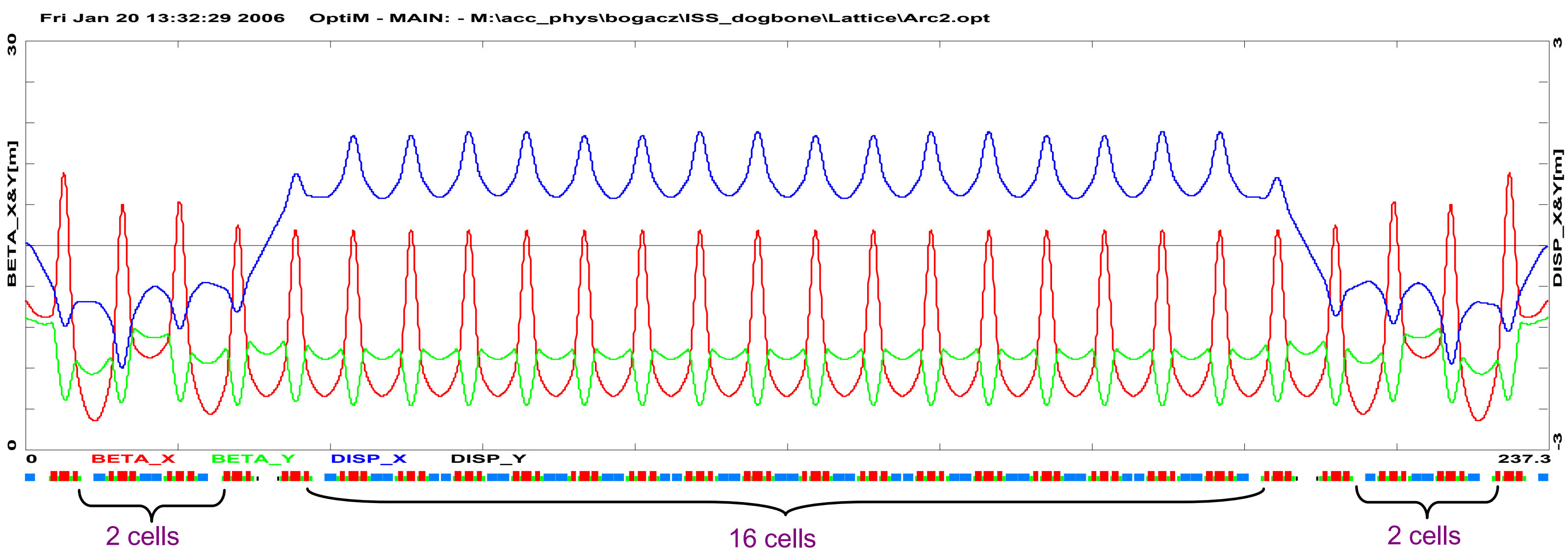
Initial beam emittance/acceptance after cooling at 273 MeV/c

Study IIb		ε_{rms}	**A = (2.5)^2 ε**
normalized emittance: $\varepsilon_x/\varepsilon_y$	mm·rad	**4.8**	**30**
longitudinal emittance: ε_l ($\varepsilon_l = \sigma_{\Delta p}\,\sigma_z/m_\mu c$) momentum spread: $\sigma_{\Delta p/p}$ bunch length: σ_z	mm mm	**27** **0.07** **176**	**150** **±0.17** **±442**

Longitudinal Beam Dynamics – Tracking



Large Momentum Compaction for a 'droplet' arc

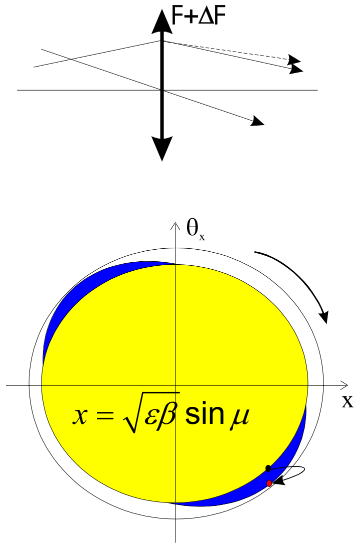


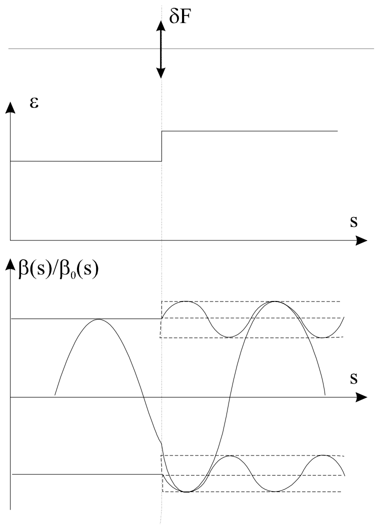
$$M_{56} = -\int \frac{D_x}{\rho}\, ds = -D_x^{dip} \int d\left(\frac{s}{\rho}\right) = -D_x^{dip} \int d\theta_{rad} = -D_x^{dip} \times \theta_{rad}^{tot}$$

$$M_{56} = -\frac{5}{3}\pi \times 1.2\ m = -6.3\ m$$

Cumulative Focusing Errors – Magnet Tolerances

- Focusing 'point' error perturbs the betatron motion leading to the Courant-Snyder invariant change:





- Each source of field error (magnet) contributes the following Courant-Snyder variation

$$\delta\varepsilon = \varepsilon + 2\sqrt{\varepsilon\beta}\cos\mu\,\delta\theta + \beta\delta\theta^2 \quad , \qquad \delta\theta = \sum_{m=1}\delta\phi_m x^m , \qquad \phi_n = \frac{\int G_n dl}{B\rho}\left[cm^{-n}\right] \qquad x = \sqrt{\varepsilon\beta}\sin\mu$$

where, m =1 quadrupole, m =2 sextupole, m=3 octupole, etc

Cumulative Focusing Errors – Magnet Tolerances

- Cumulative mismatch/emittance increase along the lattice (N sources):

$$\varepsilon_N = \varepsilon \prod_{n=1}^{N} \left(1 + 2\beta \sum_{m=1} \left(\sqrt{\varepsilon\beta} \right)^{m-1} \delta\phi_m \cos\mu \ \sin^m \mu \ + \beta^2 \left(\sum_{m=1} \left(\sqrt{\varepsilon\beta} \right)^{m-1} \delta\phi_m \ \sin^m \mu \right)^2 \right) ,$$

- Standard deviation of the Courant-Snyder invariant is given by:

$$\frac{\sigma_\varepsilon}{\varepsilon} = \frac{\sqrt{\langle \delta\varepsilon^2 \rangle - \langle \delta\varepsilon^2 \rangle^2}}{\varepsilon} = \sqrt{\sum_{i=1}^{N} \left[2\beta_i \sum_{m=1} \left(\sqrt{\varepsilon\beta_i} \right)^{m-1} \delta\phi_m \langle \cos\mu \ \sin^m \mu \rangle \ + \beta_i^2 \left\langle \left(\sum_{m=1} \left(\sqrt{\varepsilon\beta_i} \right)^{m-1} \delta\phi_m \ \sin^m \mu \right)^2 \right\rangle \right]}$$

- Assuming uncorrelated errors at each source the following averaging (over the betatron phase) can by applied:

$$\langle \ldots \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\mu \ldots$$

Cumulative Focusing Errors – Magnet Tolerances

- Some useful integrals …. :

$$\left\langle \cos\mu \ \sin^m \mu \right\rangle = 0 \quad ,$$

$$\left\langle \sin^m \mu \right\rangle = \frac{m-1}{m} \left\langle \sin^{m-2} \mu \right\rangle = \begin{cases} 0 & \text{m odd} \\ \dfrac{(m-1)!!}{m!!} & \text{m even} \end{cases}$$

will reduce the coherent contribution to the C-S variance as follows:

$$\frac{\sigma_\varepsilon}{\varepsilon} = \sqrt{\sum_{i=1}^{N} \left[2\beta_i \sum_{m=1} \left(\sqrt{\varepsilon\beta_i}\right)^{m-1} \delta\phi_m \cancel{\left\langle \cos\mu \ \sin^m \mu \right\rangle} + \beta_i^2 \left\langle \left(\sum_{m=1} \left(\sqrt{\varepsilon\beta_i}\right)^{m-1} \delta\phi_m \ \sin^m \mu \right)^2 \right\rangle \right]}$$

- Including the first five multipoles yields:

$$\frac{\sigma_\varepsilon}{\varepsilon} = \sqrt{\sum_{i=1}^{N} \left\{ \beta_i^2 \left[\delta\phi_1^2 \underbrace{\left\langle \sin^2 \mu \right\rangle}_{\frac{1}{2}} + \varepsilon\beta_i \left(\delta\phi_2^2 + 2\delta\phi_1\delta\phi_3 \right) \underbrace{\left\langle \sin^4 \mu \right\rangle}_{\frac{1}{2}\,\frac{3}{4}} + \left(\varepsilon\beta_i\right)^2 \left(\delta\phi_3^2 + 2\delta\phi_1\delta\phi_5 + 2\delta\phi_2\delta\phi_4 \right) \underbrace{\left\langle \sin^6 \mu \right\rangle}_{\frac{1}{2}\,\frac{3}{4}\,\frac{5}{6}} + \ldots \right] \right\}}$$

Cumulative Focusing Errors – Magnet Tolerances

- Beam radius at a given magnet is : $a_i = \frac{1}{2}\sqrt{\varepsilon\beta_i}$

- One can define a 'good fileld radius' for a given type of magnet as: $a = Max(a_i)$

- Assuming the same multipole content for all magnets in the class one gets:

$$\frac{\sigma_\varepsilon}{\varepsilon} = \sqrt{\sum_{i=1}^{N}\frac{1}{2}\beta_i^2} \times \sqrt{\delta\phi_1^{\,2} + \frac{3}{2}a^2\left(\delta\phi_2^{\,2} + 2\delta\phi_1\delta\phi_3\right) + \frac{5}{2}a^4\left(\delta\phi_3^{\,2} + 2\delta\phi_1\delta\phi_5 + 2\delta\phi_2\delta\phi_4\right) + \ldots}$$

- The first factor purely depends on the beamline optics (focusing), while the second one describes field tolerance (nonlinearities) of the magnets:

$$\Phi = \sqrt{\delta\phi_1^{\,2} + \frac{3}{2}a^2\left(\delta\phi_2^{\,2} + 2\delta\phi_1\delta\phi_3\right) + \frac{5}{2}a^4\left(\delta\phi_3^{\,2} + 2\delta\phi_1\delta\phi_5 + 2\delta\phi_2\delta\phi_4\right) + \ldots}$$

Field Error Tolerances – Magnet Specs

- The linear errors, m =1, cause the betatron mismatch – invariant ellipse distortion from the design ellipse without changing its area – no emittance increase.

- By design, one can tolerate some level (e.g. 10%) of Arc-to-Arc betatron mismatch due to the focusing errors, $\delta\phi_1$ (quad gradient errors and dipole body gradient) to be compensated by the dedicated matching quads

$$\left(\frac{\sigma_\varepsilon}{\varepsilon}\right)_{mis} = \sqrt{\frac{1}{2}\sum_{n=1}^{N}\left(\beta_n\delta\phi_1\right)^2} = \sqrt{\frac{1}{2}\Delta\phi_1^2\sum_{n=1}^{N}\left(\beta_n\right)_{quad}^2 + \frac{1}{2}\delta\phi_1^2\sum_{n=1}^{N}\left(\beta_n\right)_{dipole}^2}$$

- The higher, m > 1, multipoles will contribute to the emittance dilution – 'limited' by design via a separate allowance per each segment (Arc, linac) (e.g. 1%)

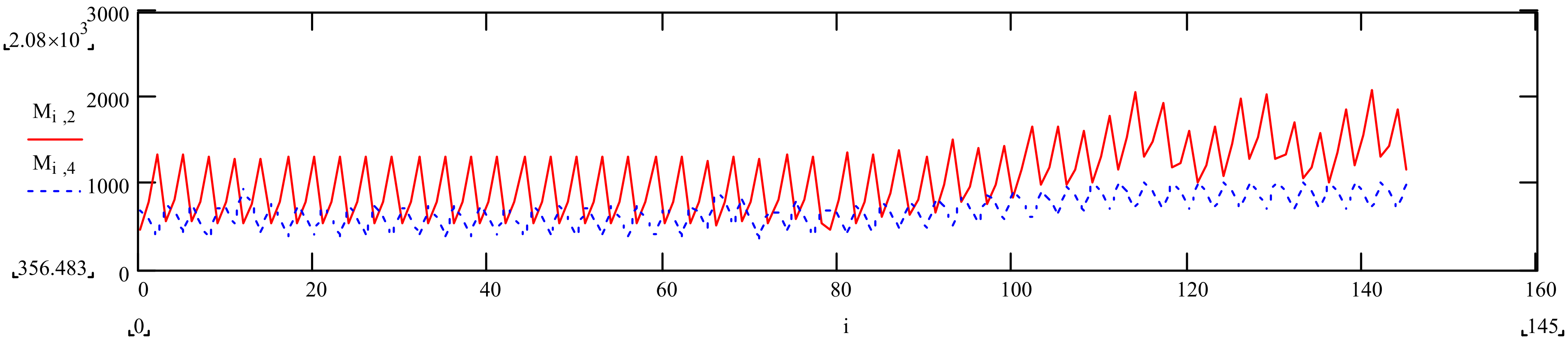
$$\left(\frac{\sigma_\varepsilon}{\varepsilon}\right)_{dil} = \sqrt{\frac{1}{2}\sum_{n=1}^{N}\left(\beta_n\delta\phi\right)^2} = \sqrt{\frac{1}{2}\Delta\phi_{quad}^2\sum_{n=1}^{N}\left(\beta_n\right)_{quad}^2 + \frac{1}{2}\Delta\phi_{dipole}^2\sum_{n=1}^{N}\left(\beta_n\right)_{dipole}^2}$$

$$\Delta\phi = \sqrt{\sum_{n=1}\frac{2n+1}{2}a^{2n}\sum_{i=1}^{n}\delta\phi_i\delta\phi_{2(n+1)-i}}$$

$$\Delta\phi_{quad} = a^2\sqrt{\frac{5}{2}\left(\delta\phi_3^2 + 2\delta\phi_1\delta\phi_5\right) + \frac{9}{2}a^4\left(\delta\phi_5^2 + 2\delta\phi_1\delta\phi_9\right)}$$

$$\Delta\phi_{dipole} = a\sqrt{\frac{3}{2}\delta\phi_2^2 + 5a^2\delta\phi_2\delta\phi_4 + \ldots}$$

- Arc1-Linac1:



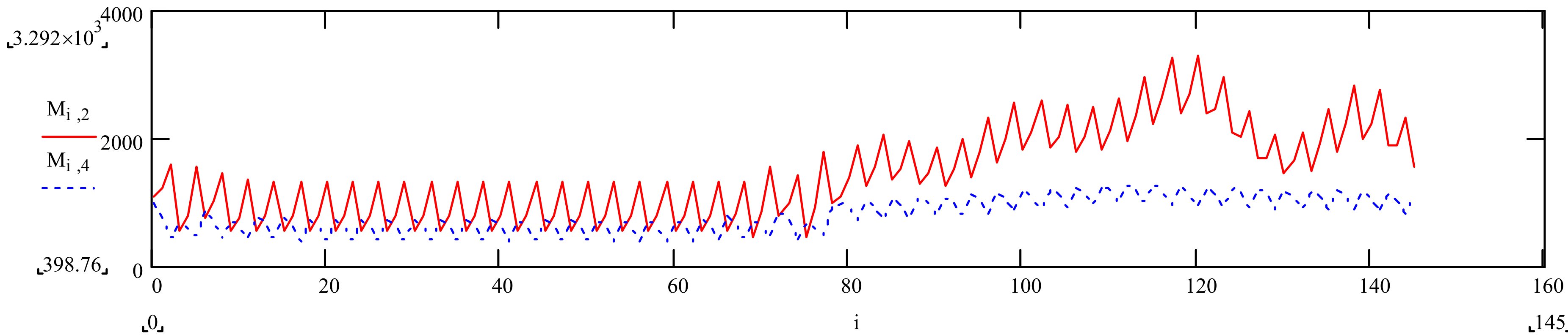
F_{min} = 1 m (GdL=41 kG)

$$\sqrt{\frac{1}{2F_{\min}^{2}}\sum_{n=1}^{N}\beta_n^{2}} \approx 50$$

$$\boxed{\frac{\Phi}{\phi_1^{max}} = 0.002}$$

$$\Phi = \sqrt{\delta\phi_1^{2} + \frac{3}{2}a^{2}\left(\delta\phi_2^{2} + 2\delta\phi_1\delta\phi_3\right) + \frac{5}{2}a^{4}\left(\delta\phi_3^{2} + 2\delta\phi_1\delta\phi_5 + 2\delta\phi_2\delta\phi_4\right) + \ldots}$$

- Arc2-Linac2:



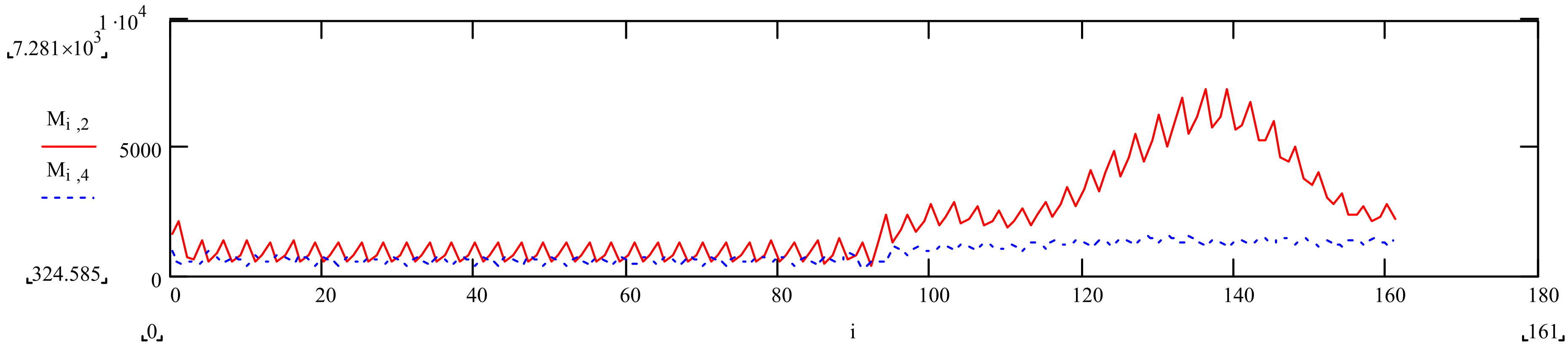
F_{min} = 1.5 m (GdL=65 kG)

$$\sqrt{\frac{1}{2F_{min}^{\ 2}}\sum_{n=1}^{N}\beta_n^{\ 2}} \approx 190$$

$$\boxed{\frac{\Phi}{\phi_1^{max}} = 0.001}$$

$$\Phi = \sqrt{\delta\phi_1^{\ 2} + \frac{3}{2}a^2\left(\delta\phi_2^{\ 2} + 2\delta\phi_1\delta\phi_3\right) + \frac{5}{2}a^4\left(\delta\phi_3^{\ 2} + 2\delta\phi_1\delta\phi_5 + 2\delta\phi_2\delta\phi_4\right) + \ldots}$$

Arc1-Linac3:



F_{min} = 1.46 m (GdL=92 kG)

$$\sqrt{\frac{1}{2F_{min}^{\;2}}\sum_{n=1}^{N}\beta_n^{\;2}} \approx 168$$

$$\boxed{\frac{\Phi}{\phi_1^{max}} = 0.002}$$

$$\Phi = \sqrt{\delta\phi_1^{\;2} + \frac{3}{2}a^2\left(\delta\phi_2^{\;2} + 2\delta\phi_1\delta\phi_3\right) + \frac{5}{2}a^4\left(\delta\phi_3^{\;2} + 2\delta\phi_1\delta\phi_5 + 2\delta\phi_2\delta\phi_4\right) + \ldots}$$

Summary

- Symmetric ‘Dogbone’ RLA (allowing to accelerate both μ^+ and μ^- species), 3.5-pass (1.5 – 5 GeV) scheme – Complete linear Optics
  - multi-pass linac optics – optimized focusing profile (tolerable phase ‘slippage’)
  - mirror-symmetric droplet’ Arc optics based on constant phase advance/cell (90^0)
- Front-to-End Multi-particle Tracking – **30 mm rad** normalized acceptance, 5 % particle loss
- Magnet misalignment error analysis (DIMAD Monte Carlo on the above lattice) shows quite manageable level of orbit distortion for ~1 mm level of magnet misalignment error.
- Great focusing errors tolerance for the presented lattice – 10% of Arc-to-Arc betatron mismatch limit sets the quadrupole field spec at 0.1%