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Neutrino Factories: Physics

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Abstract

The recent evidence for neutrino oscillations opens a new and exciting era in neutrino physics. The next generation of accelerator based neutrino oscillation experiments are expected to confirm the nature of the oscillations, and begin to measure some of the associated oscillation parameters. However, these experiments will not be able to completely determine the mixing matrix, determine the pattern of neutrino masses, or search for CP violation in the lepton sector. Therefore we are motivated to consider the neutrino beams that will be needed beyond the next generation of experiments. With this in mind the physics case for a neutrino factory is discussed. It is shown that this new type of neutrino source would enable the crucial questions to be addressed, and perhaps provide enough information to discriminate between Grand Unified Theories, or lead us to an alternative theoretical framework. It is possible that measurements at a neutrino factory will point the way towards an understanding of the origin of quark and lepton flavors.

1 Prologue: Intense muon sources

In recent years there has been much interest in the possibility of developing a new generation of very intense muon sources capable of producing a millimole of muons per year. This interest is well motivated. A very intense muon source producing a bright beam that can be rapidly accelerated to high energies would provide a new tool for particle physics. At present the beam toolkit available for physicists interested in particle interactions at the highest energies is limited to beams of charged stable particles: electrons, positrons, protons, and antiprotons. The development of intense bright μ^+ and μ^- beams would extend this toolkit in a significant way, opening the door for multi-TeV muon colliders, lower energy muon colliders (Higgs factories), muon–proton colliders, etc. In addition, all of the muons decay to produce neutrinos. Hence a new breed of high energy and high intensity neutrino beams would become possible. Finally, there is the prospect of using the low energy (or stopped) muons to study rare processes with orders of magnitude more muons than currently available.

In response to the seductive vision of a millimole muon source an R&D collaboration was formed in the US in 1995, initially motivated by the desire to design a multi-TeV muon collider, and more recently by the desire to design a "neutrino factory" as a step towards a muon collider. The design concepts for a neutrino factory facility are described in the accompanying article written by Andrew Sessler [1]. The motivation for neutrino factories is two-fold. First, the neutrino physics that could be pursued at a neutrino factory is compelling: the subject of this article. Second, a neutrino factory would provide a physics-driven project that would facilitate the development of millimole muon sources: the enabling technology for so many other goodies, including muon colliders.

2 Why do we need a new neutrino source ?

Results from the Superkamiokande experiment [2] (SuperK) have yielded convincing evidence for a deficit of muon-type neutrinos (ν_μ) in the atmospheric neutrino flux. This deficit varies with the zenith angle of the incident neutrinos, and hence varies with the distance between the source and the detector. The natural interpretation of this result is that the missing ν_μ have oscillated into ν_X as they traversed the distance L between their point of production in the atmosphere and the detector. The final state flavor ν_X is currently believed to be ν_τ since (i) the appropriate region of parameter space for $\nu_\mu \to \nu_e$ oscillations is already excluded by the CHOOZ experiment [3], and (ii) oscillations into a sterile neutrino ν_S are excluded at the 99% Confidence Level by other SuperK measurements.

The SuperK results open a new and exciting era in neutrino physics. Neutrino oscillation experiments are no longer searches for a phenomenon that may or may not exist. The experimental sensitivity required to measure oscillations is now known, and the great thing is that $\nu_\mu \to \nu_X$ oscillations are within reach of the next generation of accelerator based experiments. Why is this exciting ? The reason is that, since neutrinos oscillate, they must have mass, requiring either the existence of right handed neutrinos (Dirac masses) or lepton number violation (Majorana masses), or both. Hence, neutrino oscillations cannot be accommodated within the Standard Model. The origin of neutrino masses must arise from physics beyond the Standard Model. Theories that describe physics beyond the Standard Model at Grand Unified scales (GUTs) predict patterns of oscillation parameters (mixing angles and neutrino masses). Comprehensive measurements of neutrino oscillations can therefore discriminate between GUTs. Note that GUTs also "predict" the number of quark and lepton generations. Perhaps neutrino oscillation measurements will help us understand why there are three families. In addition, precision neutrino oscillation measurements can determine, or put stringent limits on, CP violation in the lepton sector. So it appears that we now have, within reach of a new generation of accelerator based experiments, an exciting window on physics at the GUT scale, CP violation in the lepton sector, the origin of neutrino masses and, perhaps, the origin of quark and lepton flavors.

As if this were not motivation enough for detailed neutrino oscillation studies, there is more. First, there is the long standing solar neutrino problem: a deficit of neutrinos from the sun compared to the predictions of the Standard Solar Model. This discrepancy might also be due to neutrino oscillations, in this case the oscillations $\nu_e \to \nu_x$. In the next few years results from the SNO [4] and KamLAND [5] experiments are expected to strengthen the evidence for (or reject) solar neutrino oscillations. If accelerator based experiments can subsequently measure all of the parameters associated with neutrino oscillations we may very well resolve the solar neutrino problem. Second, there is evidence for $\nu_\mu \to \nu_e$ and $\overline{\nu}_\mu \to \overline{\nu}_e$ oscillations from an accelerator experiment (LSND [6]) at Los Alamos. The problem here is that the splittings between the participating neutrino mass eigenstates needed to explain the atmospheric neutrino deficit, the solar neutrino deficit, and the LSND result, are all different. If all three results are due to neutrino oscillations we need three different mass splittings. However, we know of only three neutrino flavors, which can accommodate at most two mass splittings. There is the shocking possibility that there are additional neutrino flavors: sterile neutrinos. This leads us to a further motivation for detailed neutrino oscillation studies, namely to determine whether light sterile neutrinos exist.

With all of these incentives, we can ask: What neutrino beams will be needed in the future to determine all of the oscillation parameters, constrain GUT scale theories, learn about CP violation in the lepton sector, resolve the solar neutrino problem, and determine whether there are light sterile neutrinos? In the following sections we will see that we will certainly need higher intensity beams than already foreseen. We will also need beams propagating through the Earth over baselines of several thousand kilometers, and it is probably essential, and certainly highly desirable, that we have ν_e and $\overline{\nu}_e$ beams in addition to ν_μ and $\overline{\nu}_\mu$ beams.

3 Why neutrino factories ?

Conventional neutrino beams are produced from a beam of charged pions decaying in a long (typically several hundred meters) decay channel. If positive (negative) pions are selected, the result is an almost pure ν_μ ($\overline{\nu}_\mu$) beam from $\pi^+ \to \mu^+\nu_\mu$ ($\pi^- \to \mu^-\overline{\nu}_\mu$) decays, with a small O(1%) component of ν_e from three body kaon decays. The ν_e component is not large enough to be useful for $\nu_e \to \nu_X$ measurements. Hence, if we want ν_e and $\overline{\nu}_e$ beams we will need a different sort of neutrino source.

An obvious way to try to get ν_e and $\overline{\nu}_e$ beams is to exploit the decays $\mu^+ \to e^+\nu_e\overline{\nu}_\mu$ and $\mu^- \to e^-\nu_\mu\overline{\nu}_e$. To create a neutrino beam with sufficient intensity for a new generation of oscillation experiments will require a very intense muon source. With a millimole of muons per year we can imagine producing high energy beams containing $O(10^{20})$ neutrinos and antineutrinos per year. However, to achieve this a large fraction f of the muons must decay in a channel that points in the desired direction. Muons live 100 times longer than charged pions. Since the decay fraction f must be large we cannot use a linear muon decay channel unless we are prepared to build one that is tens of kilometers long. A more practical solution is to inject the muons into a storage ring with long straight sections. The useful decay fraction f is just the length of the straight section divided by the circumference of the ring. It has been shown that $f \sim 0.3$ is achievable [7]. The resulting muon storage ring is sufficiently compact that it can be tilted downwards at a large angle so that the neutrino beam can pass through the Earth [8], and very long baseline experiments ($L \sim O(10^4)$ km) can be imagined.

Thus the "neutrino factory" concept [8, 9] is to create a millimole/year muon source, rapidly accelerate the muons to the desired storage ring energy, and inject them into a storage ring with a long straight section that points in the desired direction. For discussion it is useful to define two types of neutrino factory: "entry–level" and "high–performance". An entry–level neutrino factory [10] can be thought of as a "low" intensity "low" energy neutrino factory that we may (or may not) wish to build as a step towards the high–performance machine. We will take as typical parameters for an entry–level scenario a 20 GeV or 30 GeV storage ring delivering $O(10^{19})$ muon decays per year in the beam forming straight–section. With a 50 kt detector having a detection efficiency of 50% an effective entry–level data sample would be $O(10^{21})$ kt–decays after a few years of running. Typical parameters for a high–performance scenario would be a 50 GeV ring delivering $O(10^{20})$ muon decays per year in the beam forming straight–section, yielding data samples $O(10^{22})$ kt–decays after a few years of running.

Neutrino factories would provide [8, 11]:

(i) **ν_e and $\overline{\nu}_e$ beams**, as well as ν_μ and $\overline{\nu}_\mu$ beams !

(ii) **High event rates.** With 2×10^{20} muon decays per year in the beam–forming straight section of a 50 GeV neutrino factory the ν_μ event rates in a distant detector would be about a factor of 60 higher than the corresponding rates for the next generation of conventional beams (NUMI at FNAL for example). These neutrino factory rates would yield tens of thousands of ν_μ events per year within a reasonable sized detector on the other side of the Earth ($L \sim 10000$ km). In addition, a near–detector a few hundred meters from the end of the beam–forming straight section of a 50 GeV neutrino factory would measure of the order of a million events per year per kg ! This fantastic rate would enable a revolution in non–oscillation neutrino experiments, which could be based on silicon pixel targets, polarized hydrogen targets, and detectors with fine segmentation and good particle identification.

(iii) **Narrow ν and $\overline{\nu}$ energy spectra.** Neutrinos from a neutrino factory have a much narrower energy spectrum than provided by a conventional "wide–band" beam. Hence, a neutrino factory beam can be thought of as being "narrow band".

(iv) **Low systematic uncertainties.** Since the muon decay spectrum is very well known, the systematic uncertainties on the flux and spectrum of neutrinos at a distant experiment are expected to be significantly less than the corresponding uncertainties for a conventional beam. This would be expected to improve the ultimate precision of ν_μ disappearance measurements.

(v) **Polarization.** In the forward direction the ν_e flux at a neutrino factory is sensitive to the polarization of the muons in the storage ring. Hence, by controlling the polarization the ν_e component within the initial beam can be varied. In principle this could be very useful, although a compelling case for muon polarization has yet to be demonstrated in a detailed analysis.

Thus, compared with the next generation of conventional neutrino beams, neutrino factories offer the prospect of higher intensity neutrino and antineutrino beams containing ν_e as well as ν_μ, lower systematic uncertainties, a narrower beam energy distribution, and perhaps beam composition control via polarization. In addition the intensity increase would initiate a revolution in non–oscillation experiments. Its easy, therefore, to understand the current interest in neutrino factories.

4 Neutrino oscillations

Before we can discuss the physics potential of oscillation experiments at a neutrino factory we must first consider the theoretical framework used to describe neutrino oscillations. We know of three neutrino flavors: ν_e, ν_μ, and ν_τ. Within the framework of three–neutrino oscillations, the flavor eigenstates are related to the mass eigenstates by a 3×3 unitary matrix U_{MNS} [12]:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} . \tag{1}$$

In analogy with the CKM matrix, U_{MNS} can be parameterized in terms of three mixing angles θ_{ij} and a complex phase δ:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} , \tag{2}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. If the neutrinos are Majorana, there are two extra phases, but these do not affect oscillations. The evolution of the neutrino flavor states in vacuum is described by:

$$i\,\frac{d\nu_\alpha}{dt} = \sum_\beta \left(\sum_j U_{\alpha j} U^\star_{\beta j} \frac{m_j^2}{2E_\nu} \right) \nu_\beta \, . \tag{3}$$

Hence, the flavor oscillations are driven by the differences in the squares of the masses m_j. It is convenient to define:

$$\Delta m^2_{ij} \equiv m_i^2 - m_j^2 \, . \tag{4}$$

Oscillation probabilities depend upon the time–of–flight (and hence the baseline L), the Δm^2_{ij}, and U_{MNS} (and hence $\theta_{12}, \theta_{23}, \theta_{13}$, and δ).

The oscillation probabilities inferred from the atmospheric neutrino, solar neutrino, and LSND measurements can be used to constrain the oscillation parameters. For the time being we set aside the LSND oscillation results (which have not yet been confirmed by other

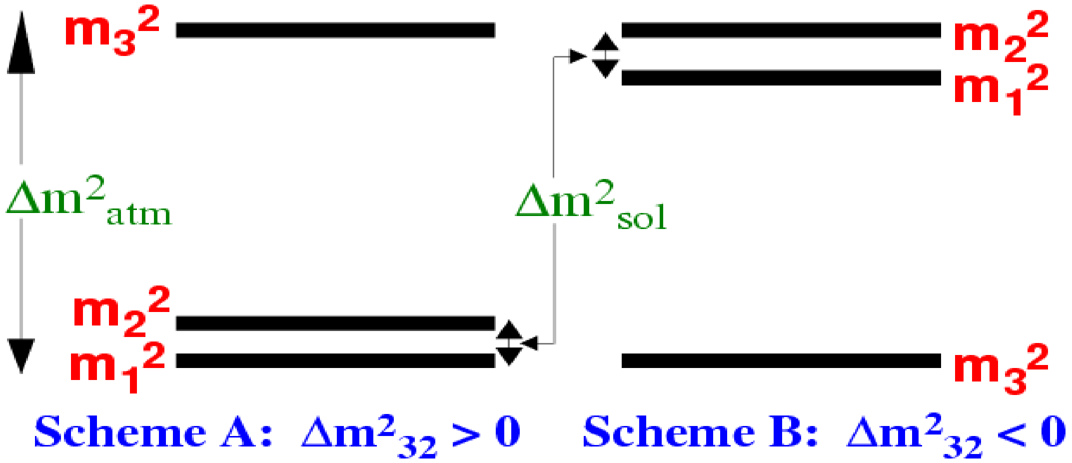

Figure 1: Alternative neutrino mass patterns that are consistent with neutrino oscillation explanations of the atmospheric and solar neutrino deficits.

experiments), and identify Δm^2_{21} and Δm^2_{32} as respectively the splittings that drive the solar and atmospheric neutrino oscillations. The atmospheric neutrino measurements imply that $|\Delta m^2_{32}|$ = (1.5 - 7) $\times 10^{-3}$ eV^2 with an oscillation amplitude $\sin^2 2\theta_{atm} > 0.8$. There are four regions of parameter space consistent with the solar neutrino measurements: (a) MSW Small Mixing Angle (SMA): $|\Delta m^2_{21}| = (4-10) \times 10^{-6}$ eV^2 with amplitude $\sin^2 2\theta_{sol} = 0.001 - 0.01$, (b) MSW Large Mixing Angle (LMA): $|\Delta m^2_{21}| = (1.5 - 10) \times 10^{-5}$ eV^2 with amplitude $\sin^2 2\theta_{sol} \sim 0.8$, (c) MSW Long Wavelength (LOW): $|\Delta m^2_{21}| = (7 - 20) \times 10^{-8}$ eV^2 with amplitude $\sin^2 2\theta_{sol} \sim 0.9$, and (d) Vacuum oscillations (VO): $|\Delta m^2_{21}| = (0.5-8) \times 10^{-10}$ eV^2 with amplitude $\sin^2 2\theta_{sol} \sim 0.9$. Recent preliminary solar neutrino results from SuperK seem to favor the LMA solution [13], but it is perhaps too early to draw strong conclusions from this. In any event, it is evident that $|\Delta m^2_{21}| \ll |\Delta m^2_{32}|$. However, we don't know whether m_3 is greater than or less than m_2, and hence there are two viable patterns for the neutrino mass spectrum (Fig. 1).

How are neutrino oscillation measurements used to determine the oscillation parameters? To gain some insight it is useful to consider the oscillation probabilities $P(\nu_\alpha \to \nu_\beta)$ using the approximation that oscillations driven by the small Δm^2_{21} are neglected. This approximation is valid for long–baseline accelerator experiments. The resulting leading–oscillation probabilities for neutrinos of energy E_ν (GeV) propagating a distance L (km) in vacuum are [14]

$$\begin{aligned} P(\nu_e \to \nu_\mu) &= \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2(1.267 \Delta m^2_{32} L/E_\nu) \,, & (5) \\ P(\nu_e \to \nu_\tau) &= \cos^2\theta_{23} \sin^2 2\theta_{13} \sin^2(1.267 \Delta m^2_{32} L/E_\nu) \,, & (6) \\ P(\nu_\mu \to \nu_\tau) &= \cos^4\theta_{13} \sin^2 2\theta_{23} \sin^2(1.267 \Delta m^2_{32} L/E_\nu) \,. & (7) \end{aligned}$$

The L/E_ν dependence of the oscillation probabilities can be used to determine $|\Delta m^2_{32}|$. However, the oscillating factors $\sin^2(1.267 \Delta m^2_{32} L/E_\nu)$ depend only on the magnitude of Δm^2_{32} and not on its sign. Hence measurements of neutrino oscillations in vacuum cannot distinguish between the two viable mass eigenstate patterns shown in Fig. 1. Fortunately the oscillation probabilities for transitions with a ν_e or $\overline{\nu}_e$ in the initial or final state do depend on the sign of Δm^2_{32} if the neutrinos propagate through matter. We will return to this later. Note that the oscillation amplitudes in Eqs. 5–7 depend upon two mixing angles. It is clearly necessary to measure several oscillation modes to extract all of the mixing angles. Hence ν_e and $\overline{\nu}_e$ (as well as ν_μ and $\overline{\nu}_\mu$ beams) are desirable.

5 What can we learn from ν oscillations ?

The CHOOZ reactor ($\overline{\nu}_e$ disappearance) experiment places a limit on the $\overline{\nu}_e$ oscillation amplitude, yielding $\sin^2 2\theta_{13} < 0.1$. Interpreting the atmospheric neutrino results as evidence for $\nu_\mu \to \nu_\tau$ oscillations gives $\sin^2 2\theta_{23} \sim \sin^2 2\theta_{atm} > 0.8$. The solar neutrino measurements constrain $\sin^2 2\theta_{12} \sim \sin^2 2\theta_{sol} \equiv 4|U_{e1}|^2|U_{e2}|^2$, to be $\sin^2 2\theta_{12} \sim 0.8 - 1$ (LMA, LOW, VO) or $\sin^2 2\theta_{12} = 0.001 - 0.01$ (SMA). Hence we are on the threshold of measuring the three mixing angles, and learning something about the mixing matrix elements that govern neutrino oscillations.

This is exciting because there is a deep connection between the parameters that govern neutrino oscillations and physics at very high mass scales. The first clue to this connection comes from the smallness of the apparent neutrino masses. Direct limits on the electron neutrino mass from the tritium beta decay end point, together with cosmological constraints on the sum of the neutrino masses and the magnitude of the mass splittings obtained from the neutrino oscillation data, imply that all three neutrinos have masses < 2 eV, and are perhaps much smaller than this. If for example the masses are of the same order as the mass splittings, then the heaviest neutrino mass might be O(0.01–0.1) eV. The well known seesaw mechanism [15] provides a natural explanation for the smallness of these masses. If there exist right handed neutrinos ν_R (required by all GUT groups larger than SU(5)), and if lepton number is violated, there will be both Dirac mass (m_D) terms and Majorana mass (m_M) terms in the Lagrangian. The seesaw mechanism then generates light neutrino masses of order m_D^2/m_M. With m_D at the electroweak scale [$m_D \sim O(100$ GeV)] and m_M at the Grand Unification scale (10^{15-16} GeV) neutrino masses in the desired range are natural.

Specific GUT models yield constraints on the neutrino mass eigenstates (m_1, m_2, m_3), and predict the pattern of entries in the mass matrix M (the so called "'texture" of the mass matrix). The effective light neutrino mass matrix M_ν is related by the seesaw formula to the Dirac mass matrix M_N (connecting ν_L and ν_R) and the right–handed Majorana neutrino mass matrix M_R (connecting ν_R and ν_R):

$$M_\nu \; = \; -M_N^T M_R^{-1} M_N \; . \tag{8}$$

The matrices M_N and M_R (and hence M_ν) are predicted by GUT models in their corresponding flavor bases. The light neutrino masses are found by diagonalization of M_ν, where the transformation matrix U_ν between the two bases is just U_{MNS} :

$$U^\dagger_{MNS}\, M_\nu\, U_{MNS} \; = \; \mathrm{diag}(m_1, m_2, m_3) \; , \tag{9}$$

with the charged lepton mass matrix diagonal in its flavor basis (more generally $U_{MNS} = U_L^\dagger U_\nu$). Clearly in the lepton sector U_{MNS} plays the role of the CKM matrix V_{CKM} in the quark sector. Neutrino oscillation measurements constrain the pattern of the elements of U_{MNS} and the pattern of the mass eigenstates (m_1, m_2, m_3), and hence constrain the texture of the mass matrix M_ν which is predicted by GUT models.

We are familiar with the pattern of the CKM matrix elements, as parametrized by Wolfenstein [16]:

$$V_{CKM} \sim \begin{pmatrix} 1-\lambda^2/2 & \lambda & \lambda^3 \\ -\lambda & 1-\lambda^2/2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} , \tag{10}$$

where $\lambda \simeq V_{us} \simeq 0.22$. The atmospheric neutrino measurements, which yield $\sin^2 2\theta_{23} > 0.8$, imply that $U_{\mu 3}$ is large, and hence that U_{MNS} has a different pattern to V_{CKM}. Although this was not a priori expected, in recent months the large $\sin^2 2\theta_{23}$ has provoked a plethora of papers that demonstrate that large mixing in the (23) block of U_{MNS} is not unnatural within the frameworks of a variety of specific GUT models. If U_{MNS} can be completely determined by further oscillation measurements, the resulting constraints on the texture of M_ν will hopefully discriminate between GUT models (or maybe eliminate all of them). These same GUT models also predict proton decay rates and neutrinoless double beta decay rates. Hence, the presence or absence of proton decay and/or neutrinoless double beta decay can be used to further pin down the GUT alternatives.

Neutrino oscillation measurements offer a way of making a direct assault on our understanding of physics at high mass scales. With this in mind, it is difficult to think of neutrino experiments as merely a side–show to the high energy collider experimental program focussed on the origin of electroweak symmetry breaking. Rather, the neutrino oscillation program appears to be the corner stone of an attack on physics at the GUT scale. Over the next ten years the next generation of accelerator based neutrino oscillation experiments are expected to confirm the oscillation interpretation of the atmospheric neutrino deficit measurements, measure $\sin^2 2\theta_{23}$ and $|\Delta m^2_{32}|$ with precisions of about 10%, and find evidence (via $\nu_\mu \to \nu_e$ oscillations) for a finite $\sin^2 2\theta_{13}$ if its value exceeds ~ 0.01. In addition, in the near future further solar neutrino measurements are expected to reduce the viable number of regions of parameter space to one (or none?). This progress will almost certainly keep neutrino oscillations in the limelight for the coming decade. However, to discriminate between GUT models (or point the way to an alternative theoretical framework) we will want to know more. In particular we will want to be sure we have the right oscillation framework (three–flavor?), precisely measure (or put stringent limits on) all of the U_{MNS} elements, and determine whether there is significant CP violation in the lepton sector.

6 Which measurements are important ?

Once U_{MNS} has been measured we may find that it conforms to some recognizable pattern. With what precision will we want to measure U_{MNS}? We will clearly want to know which elements are approximately 0 or 1. Since GUT predictions have uncertainties associated with the evolution from high mass scales to low mass scales, the difference between 0 and some sufficiently small number ϵ, or between 1 and $(1-\epsilon)$, is unlikely to discriminate between GUTs. Lacking any guidance for the size of the GUT uncertainties ϵ we will recklessly seek guidance from V_{CKM} on the required precision with which we want to know U_{MNS}. Noting that some elements of V_{CKM} differ from unity by as little as O(0.01) and some elements differ from 0 by as little as O(0.01), we are motivated to measure all elements of U_{MNS} with a precision O(0.01). With this goal in mind, in ten years time the big neutrino oscillation GUT questions that will need to be answered to pin down U_{MNS} and the pattern of neutrino masses, and hence discriminate between GUT models, are likely to be:

(Q1) If $\nu_\mu \to \nu_e$ has not been observed, then how small is $\sin^2 2\theta_{13}$? Is it $O(10^{-2})$? Is it smaller than 10^{-3} ? If $\nu_\mu \to \nu_e$ has been observed, then precisely how big ($\pm 10\%$) is $\sin^2 2\theta_{13}$?

(Q2) What is the pattern of neutrino masses (Fig. 1 scheme A or scheme B) ?

(Q3) Is there CP violation in the lepton sector, and how big is the phase δ ?

(Q4) How close ($\pm$ few %) is $\sin^2 2\theta_{23}$ to 1 ?

(Q5) If we are left with the SMA solar solution, then precisely how big ($\pm 10\%$) is $\sin^2 2\theta_{12}$? If we are left with the LMA, LOW, or VO solar solutions, then how close ($\pm$ few %) is $\sin^2 2\theta_{12}$ to 1 ?

(Q6) Do neutrino oscillations involve only 3 flavors, or are there light sterile neutrinos ? If in a few years time the totality of the solar, atmospheric, and accelerator data suggests the participation of sterile neutrinos, this question goes to the top of the list.

The following describes how experiments at a neutrino factory can answer these questions.

7 Determining $\sin^2 2\theta_{13}$

The next generation of long baseline accelerator experiments [17, 18, 19, 20] are expected to observe $\nu_\mu \to \nu_e$ if $\sin^2 2\theta_{13} > 0.01$, about an order of magnitude below the presently excluded region. If $\sin^2 2\theta_{13}$ is smaller than this, then $|U_{e3}| < 0.05$. The question will then be, is U_{e3} just small, or is it very small [$|U_{e3}| < O(0.01)$ in which case $\sin^2 2\theta_{13} \sim 4U_{e3}^2 \leq O(10^{-4})$] ? To address this question we would need to improve the $\sin^2 2\theta_{13}$ sensitivity by about two orders of magnitude. Hence we would like to be able to observe $\nu_\mu \to \nu_e$ or $\nu_e \to \nu_\mu$ oscillations if $\sin^2 2\theta_{13} > 0.0001$

At a neutrino factory $\nu_e \to \nu_\mu$ oscillations are the preferred mode for probing small $\sin^2 2\theta_{13}$. Consider a neutrino factory in which positive muons are stored. The initial neutrino beam contains $\overline{\nu}_\mu$ and ν_e. In the absence of oscillations charged current (CC) interactions of the $\overline{\nu}_\mu$ in a far detector will produce positive muons, i.e. muons of the same sign as those stored in the ring. In the presence of $\nu_e \to \nu_\mu$ oscillations there will also be ν_μ CC interactions in the detector, producing negative muons, i.e. muons of opposite charge to those stored in the ring. Hence, the experimental signature for $\nu_e \to \nu_\mu$ oscillations is the appearance of "wrong–sign" muons.

In a long baseline neutrino factory experiment the expected number of wrong–sign muon events will depend on the oscillation amplitude which, to a good approximation, is proportional to $\sin^2 2\theta_{13}$ (Eq. 5). The other relevant oscillation parameters ($\sin^2 2\theta_{23}$ and $|\Delta m^2_{32}|$) will be known ($\pm 10\%$) after the next generation of accelerator based experiments. The $\sin^2 2\theta_{13}$ sensitivity depends upon the number of muons that have decayed in the beam–forming straight section N_{dec}, the muon energy E_μ, the baseline L, the detector mass M_{det}, and the detector efficiency, resolutions, and backgrounds. Detailed simulations that include

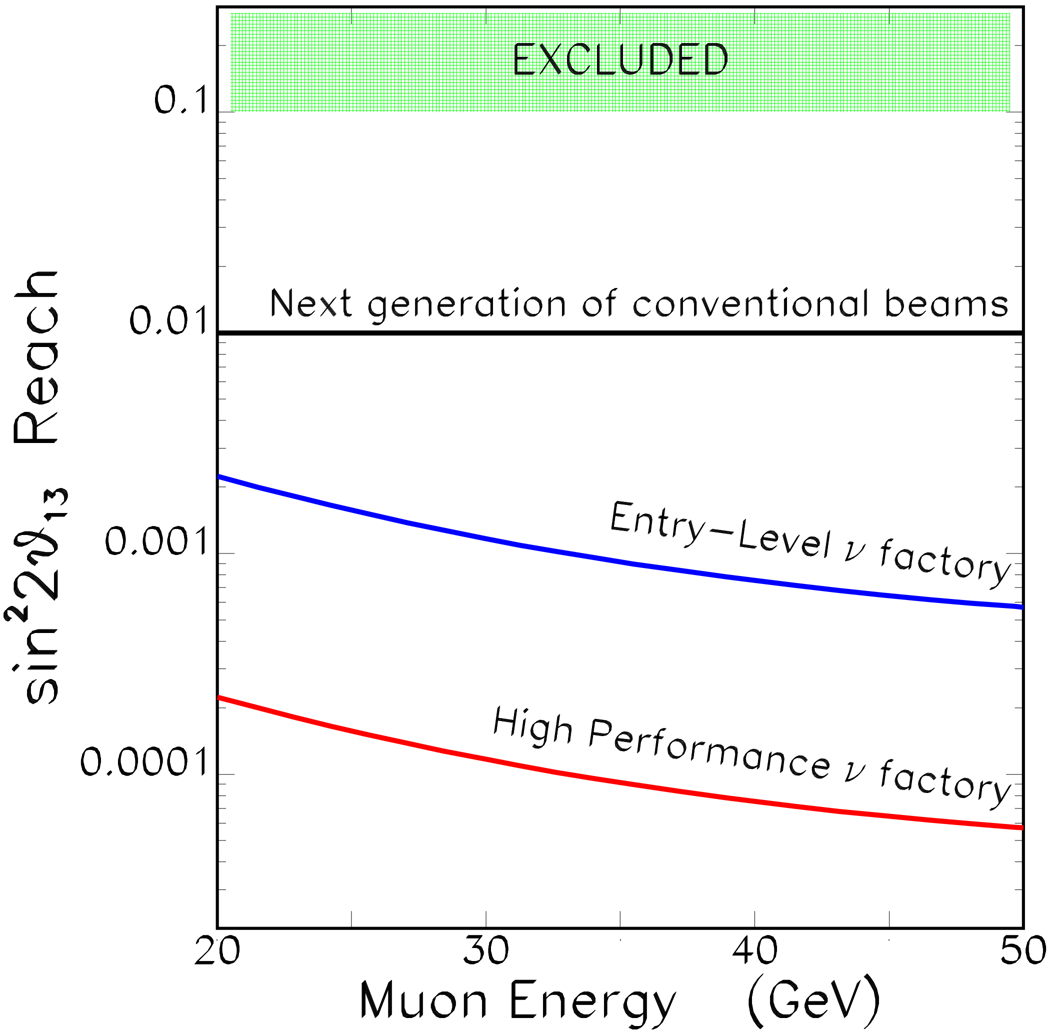

Figure 2: Limiting $\sin^2 2\theta_{13}$ sensitivity for the observation of $\nu_\mu \to \nu_e$ oscillations expected at the next generation of conventional neutrino beams compared, as a function of muon energy, with the corresponding sensitivities for the observation of $\nu_e \to \nu_\mu$ oscillations at entry-level (10^{21} kt–decays) and high–performance (10^{22} kt-decays) neutrino factories. The neutrino factory calculations are for $L = 2800$ km, $\Delta m^2_{32} = 0.0035$ eV^2, and $\sin^2 2\theta_{23} = 1$. Figure based on calculations presented in Ref. [10].

detector efficiencies, resolutions, and backgrounds, have explored the sensitivity as a function of $N_{dec} \times M_{det}$, E_μ, and L.

To illustrate the anticipated limiting $\sin^2 2\theta_{13}$ sensitivity at a neutrino factory consider a 30 GeV muon storage ring pointing at a detector at $L = 7400$ km, and let $N_{dec} \times M_{det} = 2 \times 10^{21}$ kt–decays (corresponding to an entry–level scenario). It has been shown [21] that, for values of $|\Delta m^2_{32}|$ in the center of the preferred SuperK range, the absence of a wrong–sign muon signal in this entry–level scenario would result in an upper limit on $\sin^2 2\theta_{13}$ of a few $\times 10^{-3}$. Similar results have been obtained for $L = 2800$ km [10]. The limiting sensitivity is shown for $L = 2800$ km as a function of neutrino factory energy in Fig. 2. The limiting $\sin^2 2\theta_{13}$ sensitivity at a 30 GeV neutrino factory delivering a factor of 10 more muon decays/year improves to better than 2×10^{-4}. At a high performance 50 GeV neutrino factory the limiting $\sin^2 2\theta_{13}$ sensitivity would be better than 10^{-4} [22].

We conclude that, if no $\nu_\mu \to \nu_e$ signal is observed by the next generation of long baseline experiments, and therefore $|U_{e3}| < O(0.05)$, a search for $\nu_e \to \nu_\mu$ oscillations at an entry–level neutrino factory would facilitate an order of magnitude improvement in the sensitivity to a finite $\sin^2 2\theta_{13}$, probing $\sin^2 2\theta_{13}$ at the 10^{-3} level. Hence, the entry–level experiment would either make a first observation of $\nu_e \to \nu_\mu$ oscillations or significantly improve the limits on $|U_{e3}|$. In either case we would want to upgrade the performance of the neutrino factory to precisely measure, or probe smaller values of, $\sin^2 2\theta_{13}$. A 50 GeV high performance neutrino factory would enable $\nu_e \to \nu_\mu$ oscillations to be observed for values of $\sin^2 2\theta_{13}$ as small as 10^{-4}.

What if the next generation of long baseline accelerator experiments observes a $\nu_\mu \to \nu_e$ signal ? In this case, $\sin^2 2\theta_{13} > 0.01$, and depending on its exact value the experiments would be expected to have observed from a few to a few tens of signal events. The question then becomes, what is the precise value of $\sin^2 2\theta_{13}$ ($\pm 10\%$)? To address this question will require O(100) signal events (or more if there is significant background). An entry–level neutrino factory, providing an order of magnitude improvement in sensitivity with negligible backgrounds, would be expected to determine $\sin^2 2\theta_{13}$ with the desired precision, and would be able to exploit the substantial signal to determine the pattern of neutrino masses !

8 The pattern of neutrino masses

How can we distinguish between the two mass splitting patterns in Fig. 1? Fortunately the oscillation probabilities for transitions involving a ν_e or $\overline{\nu}_e$ are modified if the neutrinos propagate through matter, and the modification depends upon the sign of Δm^2_{32} [23, 24].

In the leading oscillation approximation the probability for $\nu_e \to \nu_\mu$ oscillations in matter of constant density $\rho(x)$ and electron fraction $Y_e(x)$, is given by:

$$P(\nu_e \to \nu_\mu) = s^2_{23} \sin^2 2\theta^m_{13} \sin^2 \Delta^m_{32}\,, \tag{11}$$

where

$$\sin^2 2\theta^m_{13} = \frac{\sin^2 2\theta_{13}}{\left(\frac{A}{\Delta m^2_{32}} - \cos 2\theta_{13}\right)^2 + \sin^2 2\theta_{13}} \tag{12}$$

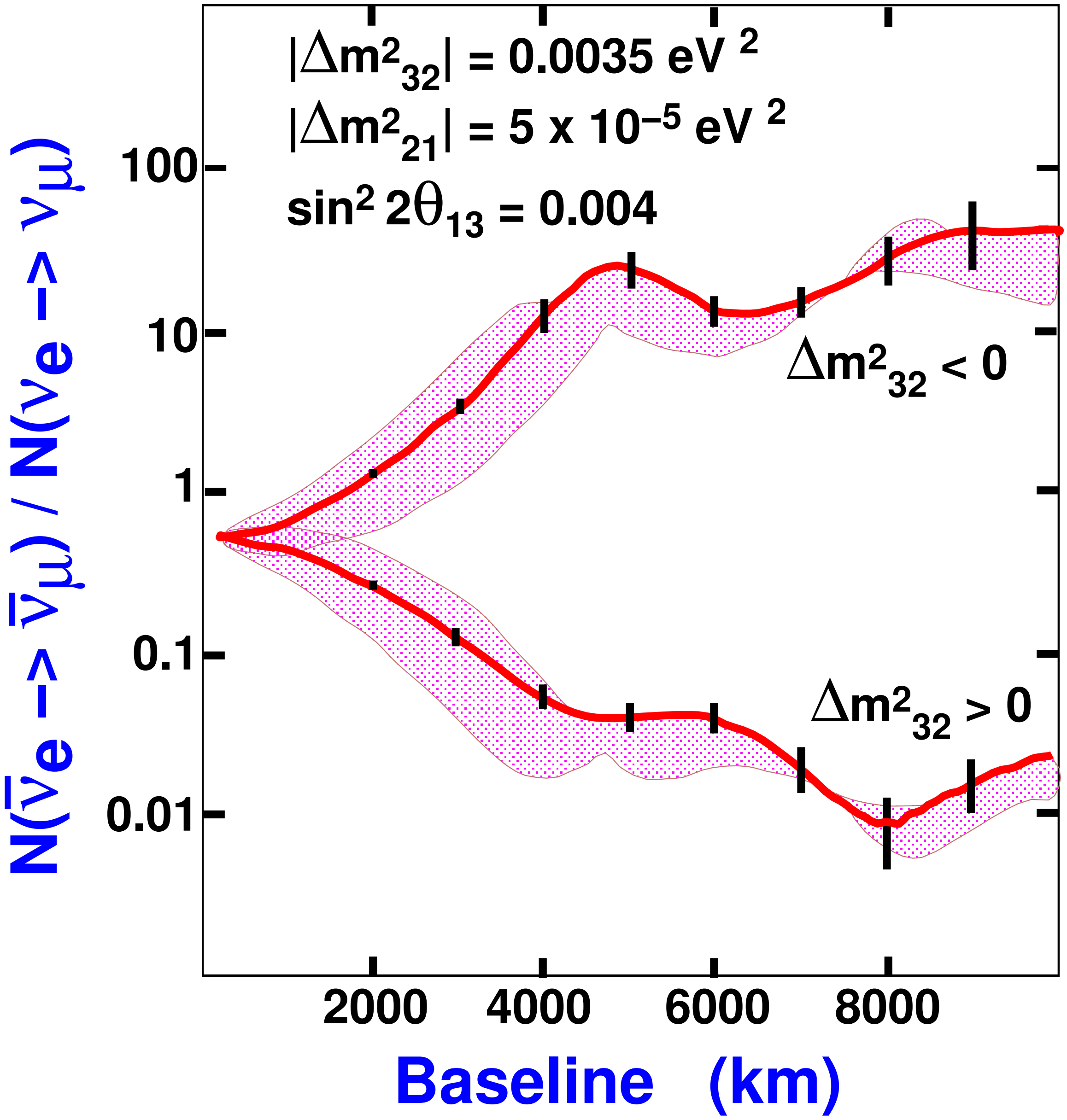

Figure 3: Predicted ratios of wrong–sign muon event rates when positive and negative muons are stored in a 20 GeV neutrino factory, shown as a function of baseline. A muon measurement threshold of 4 GeV is assumed. The lower and upper bands correspond respectively to schemes A and B in Fig. 1. The widths of the bands show how the predictions vary as the CP violating phase δ is varied from $-\pi/2$ to $+\pi/2$, with the thick lines showing the predictions for $\delta = 0$. The statistical error bars correspond to a high–performance neutrino factory yielding a data sample of 10^{21} decays with a 50 kt detector. Figure based on calculations presented in Ref. [10].

and

$$\Delta^m_{32} = \frac{1.27 \Delta m^2_{32}\,(\mathrm{eV}^2)\,L\,(\mathrm{km})}{E_\nu\,(\mathrm{GeV})} \sqrt{\left(\frac{A}{\Delta m^2_{32}} - \cos 2\theta_{13}\right)^2 + \sin^2 2\theta_{13}}\,, \tag{13}$$

and A is the matter amplitude:

$$A = 2\sqrt{2}\, G_F\, Y_e \rho E_\nu = 1.52 \times 10^{-4}\,\mathrm{eV}^2\, Y_e\, \rho\,(\mathrm{g/cm}^3) E_\nu\,(\mathrm{GeV})\,. \tag{14}$$

For $\bar\nu_e \to \bar\nu_\mu$ oscillations, the sign of A is reversed in Eqs. (12) and (13). For $\sin^2 2\theta_{13} \ll 1$ and $A \sim \Delta m^2_{32} > 0$ ($-A \sim \Delta m^2_{32} < 0$), $P(\nu_e \to \nu_\mu)$ is enhanced (suppressed) and $P(\bar\nu_e \to \bar\nu_\mu)$ is suppressed (enhanced) by matter effects. Thus a comparison of the $\nu_e \to \nu_\mu$ CC rate with the $\bar\nu_e \to \bar\nu_\mu$ CC rate discriminates between the two signs of Δm^2_{32}.

To illustrate how well the sign of Δm^2_{32} can be determined at a neutrino factory, consider an experiment downstream of a 20 GeV neutrino factory. Let half of the data taking be with μ^+ stored, and the other half with μ^- stored. In Fig. 3 the predicted ratio of wrong sign muon events $R \equiv N(\overline{\nu}_e \to \overline{\nu}_\mu)/N(\nu_e \to \nu_\mu)$ is shown as a function of baseline for $\Delta m^2_{32} = +0.0035\,\mathrm{eV}^2$ and $-0.0035\,\mathrm{eV}^2$, with $\sin^2 2\theta_{13}$ set to the small value 0.004. The figure shows two bands. The upper (lower) band corresponds to $\Delta m^2_{32} < 0$ (> 0). Within the bands the CP phase δ is varying (more on this later). At short baselines the bands converge, and the ratio $R = 0.5$ since the antineutrino CC cross-section is half of the neutrino CC cross-section. At large distances matter effects enhance R if $\Delta m^2 > 0$ and reduce R if $\Delta m^2 < 0$, and the bands diverge. Matter effects become significant for L exceeding about 2000 km. The error bars indicate the expected statistical uncertainty on the measured R with a data sample of 5×10^{22} kt-decays. With these statistics, the sign of Δm^2_{32} is determined with very high statistical significance. With an order of magnitude smaller data sample (entry level scenario) or with an order of magnitude smaller $\sin^2 2\theta_{13}$ the statistical uncertainties would be $\sqrt{10}$ larger, but the sign of Δm^2_{32} could still be determined with convincing precision in a long baseline experiment.

A more detailed analysis [25] has shown that the pattern of neutrino masses could be determined at a 20 GeV neutrino factory delivering a few times 10^{19} (10^{20}) decays per year provided $\sin^2 2\theta_{13} > 0.01$ (0.001). This '$\sin^2 2\theta_{13}$ "reach" improves with neutrino factory energy ($\sim E_\mu^{3/2}$), and a higher energy neutrino factory could therefore probe the mass pattern for $\sin^2 2\theta_{13}$ smaller than 0.001.

9 CP violation in the lepton sector

The oscillation probabilities $P(\nu_\alpha \to \nu_\beta)$ can be written in terms of CP–even and CP–odd contributions:

$$P(\nu_\alpha \to \nu_\beta) \;=\; P_{\mathrm{CP-even}}(\nu_\alpha \to \nu_\beta) + P_{\mathrm{CP-odd}}(\nu_\alpha \to \nu_\beta)\,, \tag{15}$$

where

$$\begin{aligned} P_{\mathrm{CP-even}}(\nu_\alpha \to \nu_\beta) &= P_{\mathrm{CP-even}}(\bar\nu_\alpha \to \bar\nu_\beta) \\ &= \delta_{\alpha\beta} - 4\textstyle\sum_{i>j}\, Re\, (U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}) \sin^2(\frac{\Delta m^2_{ij} L}{4E_\nu}) \\ P_{\mathrm{CP-odd}}(\nu_\alpha \to \nu_\beta) &= -P_{\mathrm{CP-odd}}(\bar\nu_\alpha \to \bar\nu_\beta) \\ &= 2\textstyle\sum_{i>j}\, Im\, (U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}) \sin(\frac{\Delta m^2_{ij} L}{2E_\nu}) \end{aligned} \tag{16}$$

Hence, if there is CP violation in the lepton sector it might be observable at a neutrino factory [26] by comparing $\nu_e \to \nu_\mu$ with $\overline{\nu}_e \to \overline{\nu}_\mu$ probabilities, which we have seen can be done by measuring wrong–sign muon production when respectively μ^+ and μ^- are stored (Fig. 3). However, CP violation requires that (at least) two mass splittings contribute to the oscillations. This is why the sensitivity to CP violation (shown by the bands in Fig. 3) vanishes at $L \sim 7000$ km, the distance at which $\sin^2(1.267\Delta m^2_{32}L/E_\nu) \to 0$ for neutrinos from a 20 GeV storage ring. The baseline must be chosen carefully! The modification to R also becomes harder to measure in a long baseline experiment as the contribution from the sub–leading scale decreases (i.e. for small Δm^2_{21} or small oscillation amplitude). Within the framework of three–flavor oscillations with the two Δm^2 scales defined by the atmospheric and solar neutrino deficits, CP violation is only likely to be observable at a neutrino factory if the LMA solar solution defines the correct region of parameter space and $|\Delta m^2_{21}|$ is not too small. Interestingly, the LMA solution seems to be favored by the most recent SuperK data, but we must wait a little longer to see whether this is confirmed. Finally, to have an observable CP violating rate $P(\nu_e \to \nu_\mu)$ must not be too small, which means that $\sin^2 2\theta_{13}$ must not be too small.

In the example shown in Fig. 3, with $\sin^2 2\theta_{13} = 0.004$, it is apparent that if L is chosen to be 3000–4000 km, the predicted ratio R varies significantly as the value of δ varies from 0 to $\pm\pi/2$. We might therefore suspect that with this value of $\sin^2 2\theta_{13}$ a high–performance neutrino factory would enable us to observe CP violation and determine δ. However, before we can conclude this we must consider backgrounds and systematics, including the correlations between the fitted oscillation parameters that arise when all parameters are allowed to vary. Fortunately detailed studies have been made [22], including backgrounds and global fits to all of the observed neutrino and antineutrino distributions. For $\sin^2 2\theta_{13}$ as small as 0.005, a 50 GeV high–performance neutrino factory could distinguish $\delta = 0$ from $\pi/2$ provided $|\Delta m^2_{21}| > 2 \times 10^{-5}$ eV^2. With larger $|\Delta m^2_{21}|$ a reasonable measurement of δ can be made ($\sigma_\delta \sim \pm 15^\circ$ if $\Delta m^2_{21} = 1 \times 10^{-4}$ eV^2 and $\sin^2 2\theta_{13} = 0.005$, for example).

We conclude that if the LMA solution turns out to be the correct solution to the solar neutrino deficit problem and $|\Delta m^2_{21}| > 2 \times 10^{-5}$ then CP violation would be observable at a high performance neutrino factory provided $\sin^2 2\theta_{13}$ is larger than ~ 0.005.

10 Precise measurement of $\sin^2 2\theta_{23}$ and $|\Delta m^2_{32}|$

How close is $\sin^2 2\theta_{23}$ to 1 ? In a long baseline experiment using a ν_μ beam, if the baseline L is close to the first oscillation maximum, the oscillations $\nu_\mu \to \nu_x$ will produce a dip in the observed ν_μ spectrum. The position of the dip is determined by $|\Delta m^2_{32}|$, and its depth is determined by $\sin^2 2\theta_{23}$. A fit to the spectra measured by the next generation of long baseline accelerator experiments, with $L = 730$ km, is expected to yield Δm^2_{32} and $\sin^2 2\theta_{23}$ with precisions of about 10% [18, 19].

It has been shown [21] that a few years of running at an entry–level 30 GeV neutrino factory with $L = 7400$ km would (i) yield a comparable statistical precision on the determination of $\sin^2 2\theta_{23}$, with a smaller systematic uncertainty arising from the uncertainty on the neutrino flux, and (ii) improve the precision on $|\Delta m^2_{32}|$ to about 1%. A high-performance 30 GeV neutrino factory would enable $\sin^2 2\theta_{23}$ to be measured with a precision of about 5%. A systematic study to optimize L and E_μ for these measurements has not been performed, and hence it may be possible to improve on these precisions with optimal choices.

11 Determining $\sin^2 2\theta_{12}$ and $|\Delta m^2_{21}|$

It will be a challenge to directly measure the sub–leading oscillation parameters $\sin^2 2\theta_{12}$ and $|\Delta m^2_{21}|$ in long–baseline accelerator experiments since the associated oscillation probabilities tend to be very small. For example, with $\Delta m^2_{21} = 10^{-5}$ eV^2, $L = 10^4$ km, and $E_\nu = 1$ GeV, the oscillating factor in the transition probabilities is given by $\sin^2(1.267\Delta m^2 L/E_\nu) = 0.016$. Hence oscillations driven by the leading scale (Δm^2_{32}) tend to dominate unless the associated amplitude is very small. This could be the case for $\nu_e \to \nu_\mu$ oscillations if $\sin^2 2\theta_{13}$ is very small or zero. As an example, with $\sin^2 2\theta_{13} = 0$, $\sin^2 2\theta_{12} = 0.8$, and $\Delta m^2_{21} = 5 \times 10^{-5}$ eV^2, it has been shown [10] that $\nu_e \to \nu_\mu$ oscillations might be observed at a high performance neutrino factory with $L \sim 3000$ km, but would require background levels to be no larger than $O(10^{-5})$ of the total CC rate. If $\sin^2 2\theta_{12} << 1$ or $\Delta m^2_{21} < 10^{-5}$ eV^2 the oscillation rate would appear to be too low to observe even at a high–performance neutrino factory.

We conclude that, within the framework of three flavor oscillations that give rise to the atmospheric and solar neutrino deficits, direct observation of oscillations driven by the sub–leading scale, and hence direct measurement of $\sin^2 2\theta_{12}$ and $|\Delta m^2_{21}|$, might be feasible at a high–performance neutrino factory, but only if the LMA solution correctly describes the solar neutrino deficit and $\sin^2 2\theta_{13}$ is zero (or very small).

12 The potential for surprises

So far we have considered only three–neutrino oscillations with the Δm^2_{ij} chosen to account for the solar and atmospheric neutrino deficits. What if:

(i) The LSND oscillation results are confirmed?

(ii) The solar neutrino deficit has nothing to do with oscillations?

(iii) There are more than three flavors participating in the oscillations (light sterile neutrinos)?

(iv) Neutrino oscillation dominates the solar and atmospheric deficit results, but is not the whole story (e.g. neutrino decay, ...)?

Although it is tempting to apply Occam's razor, and neglect these exciting possibilities, we must remember that neutrino oscillations require physics beyond the Standard Model, and we might be in for some surprises.

The best way of ensuring that we have the right oscillation framework, and are not missing any additional new physics, is to measure, as a function of L/E_ν, all of the oscillation modes (appearance and disappearance, neutrinos and antineutrinos) that we can, and then check for overall consistency of the oscillation parameters. With a conventional neutrino beam the modes that can in principle be measured are (a) ν_μ disappearance, (b) $\overline{\nu}_\mu$ disappearance, (c) $\nu_\mu \to \nu_\tau$, (d) $\overline{\nu}_\mu \to \overline{\nu}_\tau$ (e) $\nu_\mu \to \nu_e$, and (f) $\overline{\nu}_\mu \to \overline{\nu}_e$. At a neutrino factory all of these can be measured, plus the additional modes: (g) ν_e disappearance, (h) $\overline{\nu}_e$ disappearance, (i) $\nu_e \to \nu_\tau$, (j) $\overline{\nu}_e \to \overline{\nu}_\tau$ (k) $\nu_e \to \nu_\mu$, and (l) $\overline{\nu}_e \to \overline{\nu}_\mu$.

To illustrate the power of the additional measurements at a neutrino factory we consider one example: suppose the LSND oscillation results have been confirmed and we wish to discriminate between three–neutrino oscillations (describing the LSND and atmospheric results, with the solar neutrino deficit due to something else) or four–neutrino oscillations with three active flavors and one sterile neutrino (describing LSND, atmospheric, and solar neutrino results). It has been shown that [27] in the four–neutrino case, if three-neutrino oscillations are incorrectly assumed the parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{13}$ and δ determined by short baseline $\nu_e \to \nu_\tau$ and $\nu_e \to \nu_\mu$ measurements at a neutrino factory would be inconsistent with the same parameters determined from $\nu_\mu \to \nu_\tau$ measurements. Hence, the additional oscillation modes that can be probed at a neutrino factory offer discrimination between different hypotheses. Note that in both these three–flavor and four–flavor cases CP violation might be observable in both $\nu_e \to \nu_\mu$ and $\nu_\mu \to \nu_\tau$ oscillations at a short baseline neutrino factory experiment, even at an entry level neutrino facility !

In general any surprise that forces us to depart from the minimal three–neutrino (solar plus atmospheric neutrino deficit) oscillation framework will increase the need to explicitly measure (or place stringent limits on) all of the appearance and disappearance channels. Hence surprises are likely to strengthen, rather than weaken, the already strong case for a neutrino factory!

13 Questions about staging

We hope that in a few years time the R&D needed for a neutrino factory will be complete. What neutrino–factory should we then propose? The physics potentials for entry–level and high–performance neutrino factories are compared in Table 1 with the corresponding potential for the next generation of long–baseline oscillation experiments. There appears to be a strong physics case for a high–performance facility that can deliver a few $\times 10^{20}$ useful muon decays per year. If we believe we can obtain the required resources for this, and can build a high performance factory without first building a more modest facility to climb the technical learning curve, then that is what we should propose to do. However, cost and/or technical considerations may make staging necessary.

Fortunately there are a variety of possible staging options. As an intermediate step towards a high–performance neutrino factory we can consider (i) a proton accelerator system of the type needed for a neutrino factory, but used to drive a conventional neutrino "superbeam", or (ii) an entry–level neutrino factory. The superbeam facility might also include the neutrino factory target station and pion decay channel, providing an intense stopped muon source and, downstream of the decay channel, an intense low energy neutrino beam.

The pros and cons for any given staging strategy will depend upon the results from the next generation of neutrino oscillation experiments. This dependence is illustrated in Fig. 4 which shows a first "strawman" attempt at constructing a physics scenario dependent decision tree. If the LSND oscillation results are confirmed by the MiniBooNE experiment [28] the immediate big questions are likely to be: What is the oscillation framework? Do light sterile neutrinos participate in the oscillations? Is there significant CP violation in the lepton sector? These questions can be addressed by an entry–level neutrino factory, and hence if

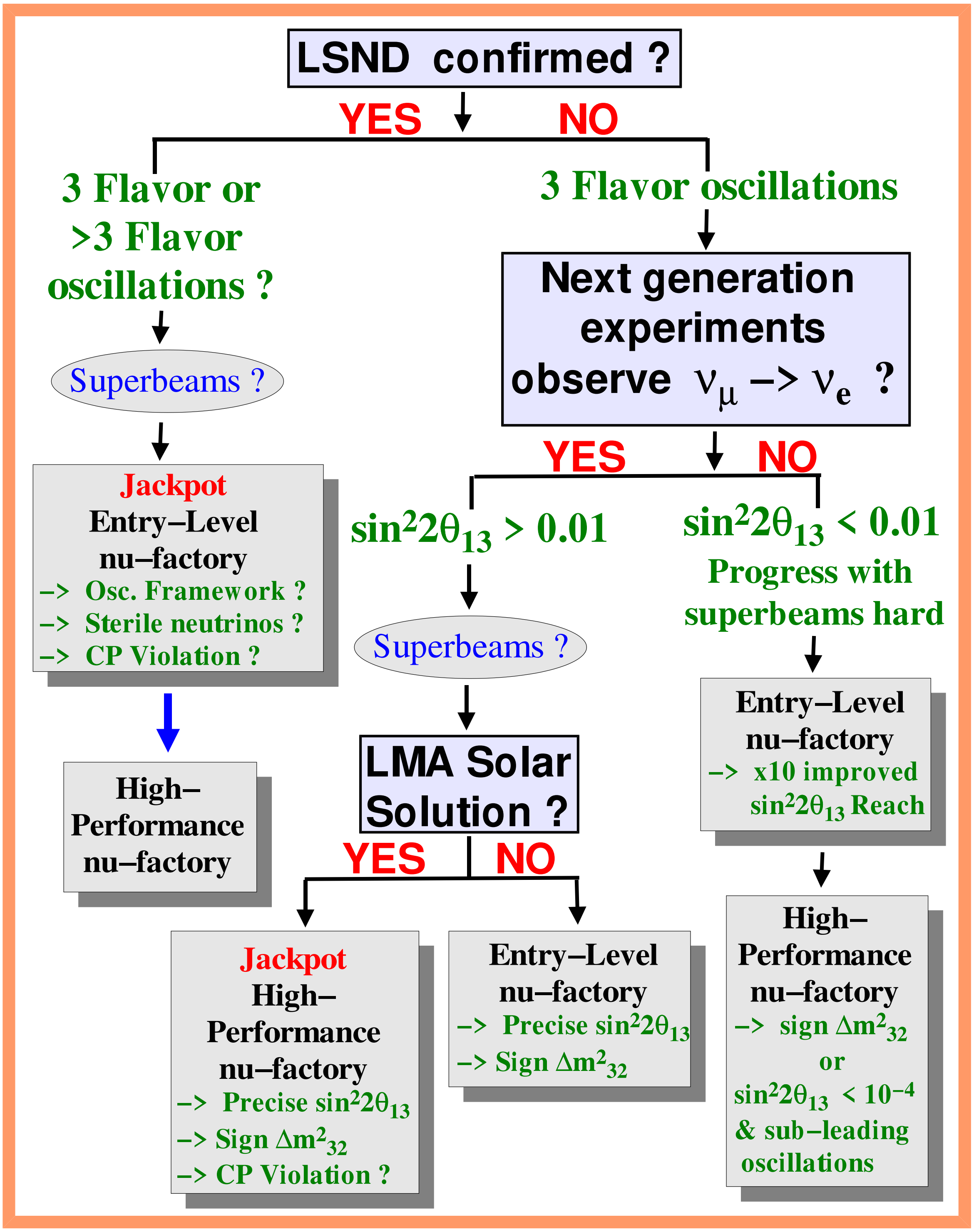

Figure 4: Staging scenario decision tree; a first attempt.

Table 1: Comparison of oscillation physics measurements at the next generation of conventional accelerator based long-baseline experiments with the corresponding programs at entry-level and high–performance neutrino factories.

	Next Generation Conventional	Entry Level	High Performance
$\sin^2 2\theta_{13}$ reach	0.01	10^{-3}	10^{-4}
Δm^2_{32} sign	NO	if $\sin^2 2\theta_{13} > 0.01$	if $\sin^2 2\theta_{13} > 0.001$
CP Violation	NO	NO	if $\sin^2 2\theta_{13} > 0.005$ and $\lvert\Delta m^2_{21}\rvert > 2 \times 10^{-5}$ eV^2
$\sin^2 2\theta_{23}$ precision	10%	10%	<5%
$\lvert\Delta m^2_{32}\rvert$ precision	10%	10%	<1%
sub-leading oscillations ?	NO	NO	if LMA and $\sin^2 2\theta_{13} \leq$ few $\times 10^{-5}$

LSND results are confirmed then neutrino factories will hit the physics jackpot! Beyond an entry–level facility there will probably be so much to measure and sort out that a high–performance factory would be desired. Superbeams might also be proposed to try to make some progress even before a neutrino factory could be built, although whether these high intensity conventional beams can address any of the central questions requires further study.

If the LSND oscillation results are not confirmed it seems likely that three–flavor oscillations will be accepted as the right phenomenological framework. In this case, the preferred staging strategy will probably depend upon whether the next generation of long baseline accelerator experiments observe, or do not observe, $\nu_\mu \to \nu_e$ oscillations, and whether SNO and KamLAND results select, or do not select, the LMA solar neutrino solution. If $\nu_\mu \to \nu_e$ oscillations are observed, then $\sin^2 2\theta_{13} > 0.01$. If in addition the LMA solution correctly describes the solar neutrino deficit, then a high–performance neutrino factory hits the physics jackpot, addressing the pressing questions: Is there significant CP violation in the lepton sector? What is the sign of Δm^2_{32}? What is the precise value of $\sin^2 2\theta_{13}$? In this scenario it might also be possible to make some progress with superbeams, but this requires further study.

In the remaining scenarios (the LMA solution does not describe the solar neutrino deficit and/or $\sin^2 2\theta_{13} < 0.01$) progress on determining the mixing matrix elements and neutrino mass spectrum will be harder, but neutrino factories still offer the possibility of learning more, and may indeed offer the only way of probing very small values of $\sin^2 2\theta_{13}$.

14 Non-Oscillation Physics

Although neutrino oscillations provide the primary motivation for the development of a neutrino factory, we should not neglect the other neutrino physics that could be pursued at a very intense high energy neutrino source. A high–performance neutrino factory would produce beams a few hundred meters downstream of the storage ring that are a factor O(10^4) more intense than existing conventional neutrino beams! This would have a tremendous impact on non–oscillation neutrino physics. For example, we can imagine the use of silicon pixel targets, or hydrogen and deuterium polarized targets, together with compact high–granularity detectors with good particle identification. Some examples of experiments that might be attractive at a neutrino factory have been discussed in Ref. [11]:

- Precise measurements of the detailed structure of the nucleon for each parton flavor, including the changes that occur in a nuclear environment.
- A first measurement of the nucleon spin structure with neutrinos.
- Charm physics with several million tagged particles. Note that charm production becomes significant for neutrino factory energies above 20 GeV.
- Precise measurements of Standard Model parameters: α_s, $\sin^2\theta_W$, and the V_{CKM} matrix elements.
- Searches for exotic phenomena such as neutrino magnetic moments, anomalous couplings to the tau–lepton, and additional neutral leptons.

The physics opportunities at neutrino factories are clearly not limited to neutrino oscillations.

15 Final remarks

In the next few years the particle physics community must decide which neutrino physics facilities should be proposed for the era beyond the next generation of experiments. The recent evidence for neutrino oscillations vastly increases the motivation for a large scale endeavor. Neutrino factories offer the possibility of completely determining the mixing matrix and the pattern of neutrino masses, determining whether there is significant CP violation in the lepton sector, clarifying the oscillation framework, determining whether there are light sterile neutrinos, and consolidating or changing our understanding of solar neutrino oscillations. We can hope, although not guarantee, that neutrino factory measurements will enable us to discriminate between GUTs, or point the way to alternative theories that lead us to an understanding of the origin of quark and lepton flavors. Finally, since neutrino oscillations require physics beyond the Standard Model, there is the very real possibility that something unexpected will be discovered at a neutrino factory.

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