$\mu^+\mu^-$ COLLIDER REPORT OF A FEASIBILITY STUDY*

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PRESENT STUDIES :*

- (High-energy) $E = 4\,\mathrm{TeV}$ c-of-m;
  $\mathcal{L} = 10^{35}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$

- (Low-energy) $E = 0.5\,\mathrm{TeV}$ c-of-m;
  $\mathcal{L} = 6 \times 10^{33}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$

- (Demonstration) $E = 0.5\,\mathrm{TeV}$ c-of-m;
  $\mathcal{L} = 1 \times 10^{33}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$

* $\mu^+\mu^-$ COLLIDER

WHY STUDY A MUON COLLIDER?

- Synchrotron radiation power is REDUCED

$$P_\gamma[MW] \approx 0.026 E^3 [GeV] I[A] B[T] \left(\frac{m_e}{m_\mu}\right)^4$$

$$\left(\frac{m_e}{m_\mu}\right) = 4.8 \times 10^{-3} \approx \frac{1}{207}$$

Hence, circular accelerator is possible (Size $\approx$ 3 Km)

- High Luminosity

$$\mathcal{L}_\mu \approx \frac{1}{8\pi c^2 \alpha m_\mu r_\mu} \frac{P_{\text{beam}}}{\sigma_r} \frac{n_\gamma}{\gamma} N_{\text{number−of−crossings}}$$

$$\mathcal{L}_\mu = \mathcal{L}_{\text{e}} \left(\frac{m_\mu}{m_e}\right) N_{\text{number−of−crossings}}$$

- Compact machine. μ's can be recirculated in CEBAF-like structures.

- Energy of beam is precisely defined due to small synchrotron radiation.
  Studies of s-channel resonances.

- Full energy of the beam is available for production of new particles.

- Both beams can be partially polarized albeit at the cost of luminosity.

- Energy could be increased over time.
  Beams of muons, neutrinos, kaons possibility of new physics. In particular, neutrino physics, rare kaon and rare muon decay experiments $(\mu \to e + \gamma)$

- $\boxed{\text{THERE ARE DIFFICULTIES !}}$

BRIEF HISTORY

Muon Collider mentioned $\left\{ \begin{array}{l} \text{Tinlow(1960)} \\ \text{Budker(1969)} \\ \text{Skrinsky(1971)} \\ \text{Neuffer(1979)} \end{array} \right.$

Ionization Cooling $\left\{ \text{Skrinsky\&Parkhomchuk(1981)} \right.$

High Luminosity $\left\{ \begin{array}{c} \text{Neuffer\&Palmer(1994)} \\ \vdots \\ \text{Meetings\&Workshops} \end{array} \right.$

- $\mu^+\mu^-$ Collider: A Feasibility Study
  Snowmass (1996)
  BNL–52503 // Fermi Lab–Conf.–96/092 //
  LBNL–38946
  18 Institutions, 83 Collaborators.

- WEB Home page
  URL `http://www.bnl.gov/~cap/mumu/mu_home_page.html`

PROTON SOURCE

- $\boxed{\text{For 4 TeV c-of-m and } \mathcal{L} = 10^{35}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}}$

	30GeV	**10GeV**	
Rep.Rate	15	30	Hz
Protons	10^{14}	10^{14}	/pulse
Bunches	4	2	at target
Protons	2.5×10^{13}	5×10^{13}	/bunch

One possible proton driver consists of:

600 MeV Linac	(BNL	SNS)		
3.6 GeV Booster	(BNL	SNS	lower	f)
30 GeV Driver	(JHP & KAON)			

- $\boxed{\text{For 0.5TeV c-of-m and } \mathcal{L} = 10^{33}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}}$

AGS	30 GeV	10^{14}	2.5Hz
FNAL	8 GeV	3×10^{13}	15Hz

Important Issues in a Proton Driver*

- Production of short bunches (1 ns)

- Stability

*

TARGET AND PION PRODUCTION*

- Target: Cu, Hg (Liquid Ga, lead)

- Studies Π production CODES:
  MARS, DPMJET, ARC

- Π spectrum peaks at low energy ≈ 100 MeV/c, large angles

- Capture with SOLENOID
  Momentum acceptance $p_{\perp}^{max} = 0.5 eBa \approx 300\,\mathrm{MeV/c}$
  Solenoid on target B=20 T, matching section
  $\frac{B_o}{1+\alpha z}$, followed by decay channel 5 T

IONIZATION COOLING (BUDKER, SKRINSKY) *

- Schematic of basic principle
  (see figure)

- $\frac{dE}{dz}$ in Li, Be, LiH reduces both
  longitudinal and transverse momentum
  a subsequent *rf* cavity restore $p_{\|}$

- Combined effect is: beam divergence is reduced. Transverse $\epsilon_n \rightarrow$ decreases.

- MULTIPLE SCATTERING is source of heat
  $\epsilon_n \rightarrow$ increases

†

- Emittance rate of change

$$\frac{d\epsilon_n}{dz} = -\frac{\epsilon_n}{E\beta^2}\left\|\frac{dE}{dz}\right\| + 0.5\beta\gamma\frac{d<\theta^2>}{dz}$$

- MINIMUM EMITTANCE

$$\epsilon_n \approx \frac{0.5E_s^2}{m_\mu c^2}\frac{\beta_\perp}{\beta}\left(L_R\|dE/dz\|\right)^{-1}$$

- Best material for cooling, Li, LiH, Be

Material	ρ [g/cm^3]	dE/dx [MeV/cm]	L_R [cm]	cof.of $\beta_\perp$ [mm mr/cm]
liq.H_2	0.071	0.286	890.	42
liq.He	0.125	0.242	756.	59
LiH	0.82	1.34	102.	78
Li	0.534	0.875	155.	79
Be	1.848	2.95	35.3	103

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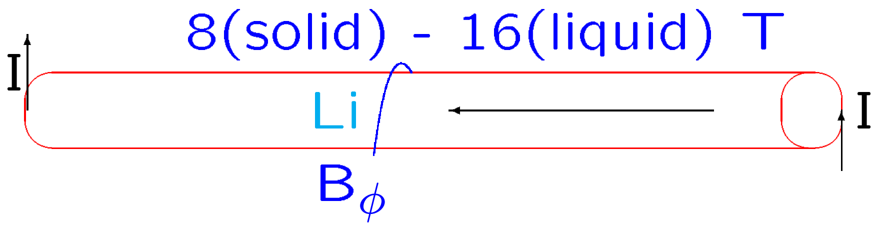
HOW TO GET LOW β^*? *

- Lithium Lens
  Experience at CERN/FNAL/Novosibirsk
  $\beta^* \approx 1\,\mathrm{cm}$ at $100\,\mathrm{MeV}$



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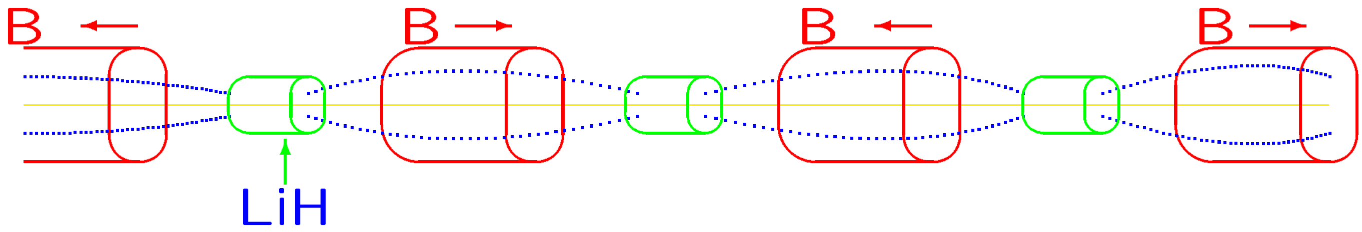
- Alternating Solenoids - FOFO

  $\beta^* \approx 6\,\mathrm{cm}$ at $100\,\mathrm{MeV}$

  $10 - 15\,T$



- Longitudinal cooling

The natural logarithmic raise of $\frac{dE}{dz}$ is TOO WEAK

- Exchange

  Introduce dispersion and use Be or Li WEDGE to reduce longitudinal phase space. ‡

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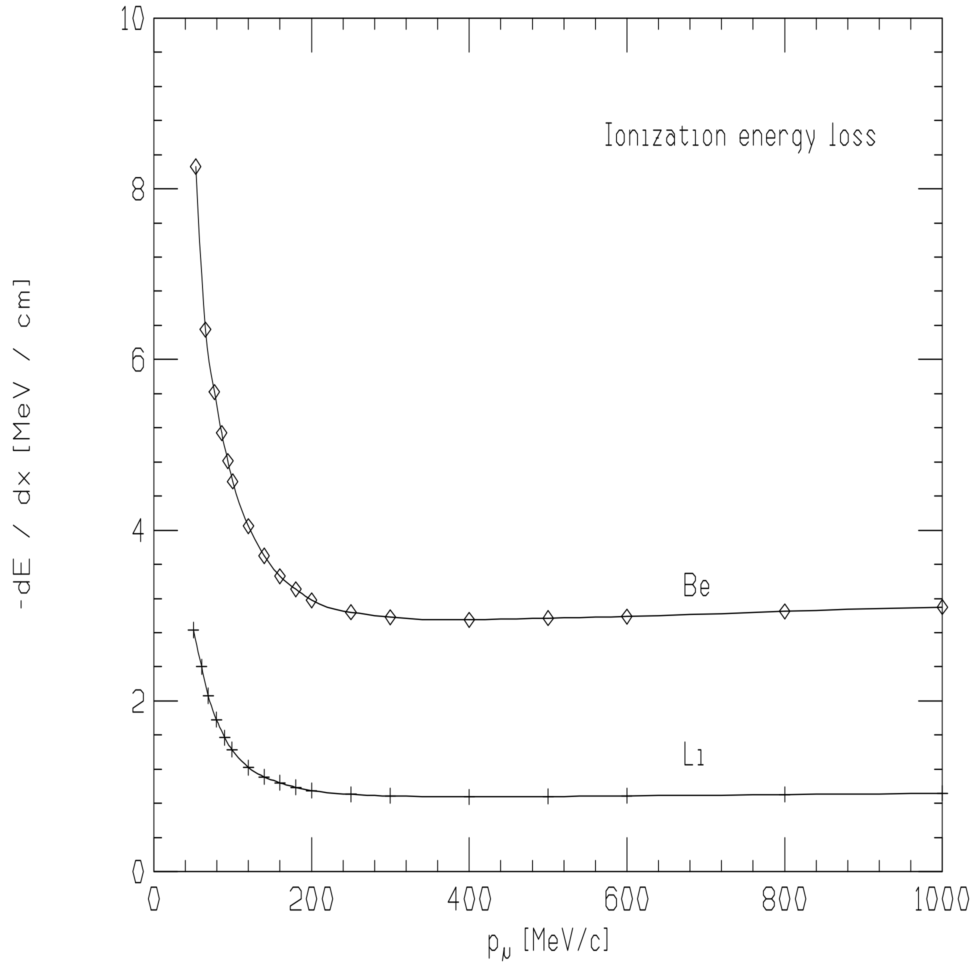

$\frac{dE}{dz}$ as a function of muon momentum for Li and Be

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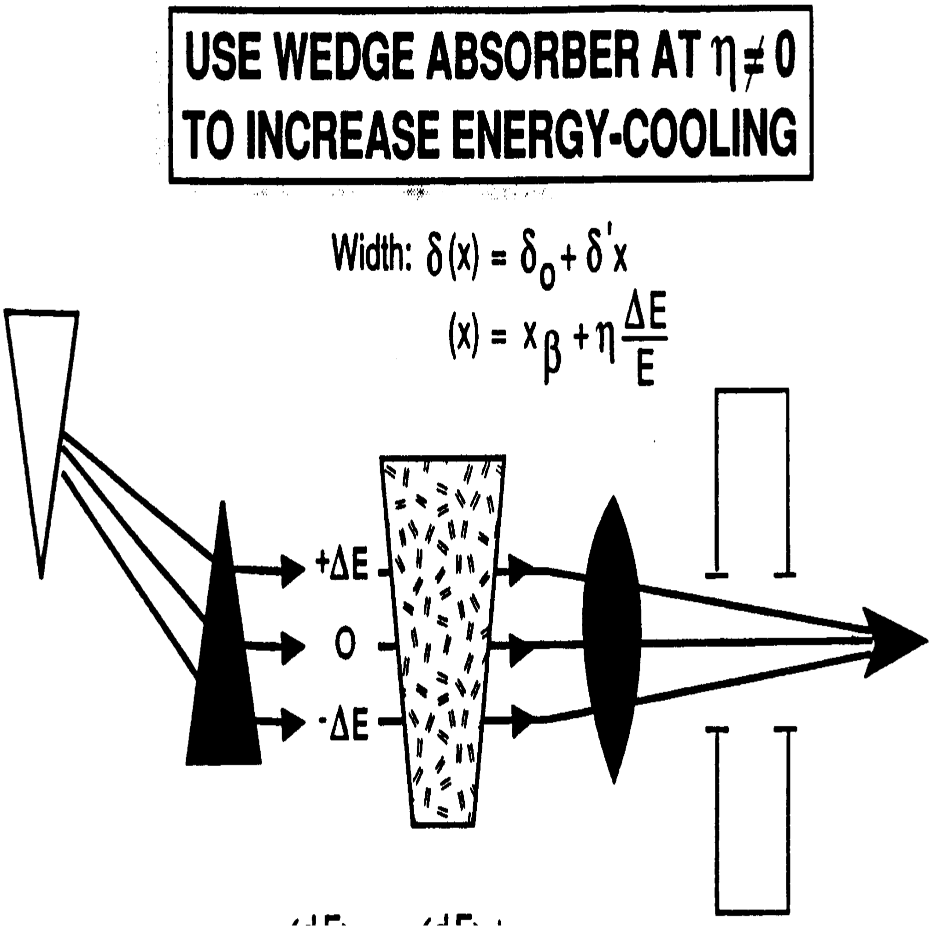

Basic principle of Ionization Cooling using a wedge absorber

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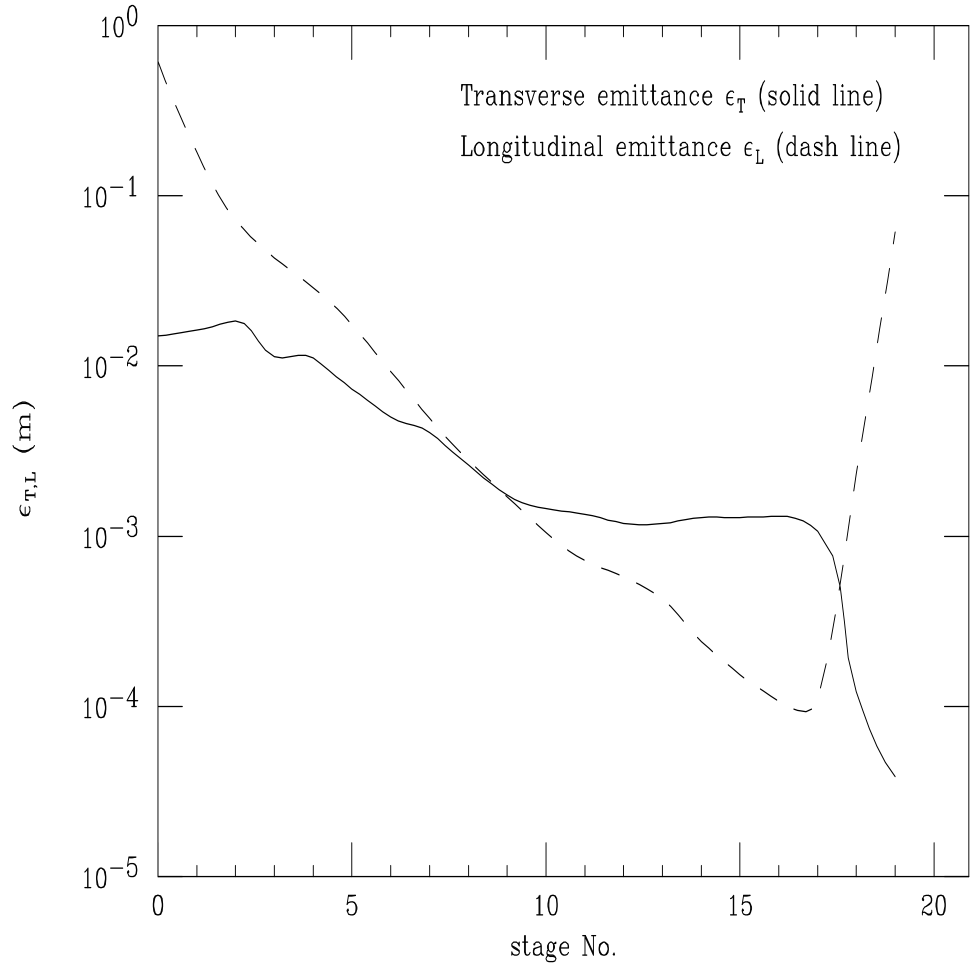
- Summary of the Cooling Section

	Entrance	Exit	
total length		743	m
sections		19	
total acceleration		4.8	GeV
accelerator length		690	m
μ decay loss		45	%
contingency loss		20	%
KE	300	15	MeV
p	392	58	MeV/c
β	0.966	0.481	
$\epsilon_{xN}(rms)$	15000	39	$mm\,mr$
$\epsilon_{zN}(rms)$	61.2	6.0	m %
σ_z	1.50	0.35	m
$\frac{\delta p}{p}$	11.0	31.7	%
μintensity	7.5	3.0	10^{12}/bunch



Normalized transverse and longitudinal emittance as a function of section number in a model cooling system**

** $\mu^+\mu^-$ COLLIDER

ACCELERATION :*

The central difficulty in a $\mu^+\mu^-$ COLLIDER is: **MUON DECAY**. They must be collected, cooled, accelerated and collided within lifetime.

Acceleration system must accommodate fairly large phase space and compress it to match the requirements of the collider

- Lifetime Constraints

  Decay rate $\frac{dN}{ds} = -\frac{1}{L_\mu \gamma}$ $\qquad L_\mu c\tau_\mu \approx 660\,\mathrm{m}$

  Assume low losses $\rightarrow eV'_{\mathrm{rf}} >> 0.16\,\frac{\mathrm{MeV}}{\mathrm{m}}$

- Scenarios

1. Sequence of linacs (VERY EXPENSIVE)

2. Recirculating linacs with multiple arcs (similar to CEBAF) (RELATIVELY EXPENSIVE)

3. Synchrotrons with fast pulsed magnets with long SC linacs (MORE ECONOMICAL)

4. (250 GeV) 4 T pulsed magnets (t=1 msec)

5. (2 TeV) Interlace of fixed 8 T SC dipole magnets with ± 2 T pulsed magnets

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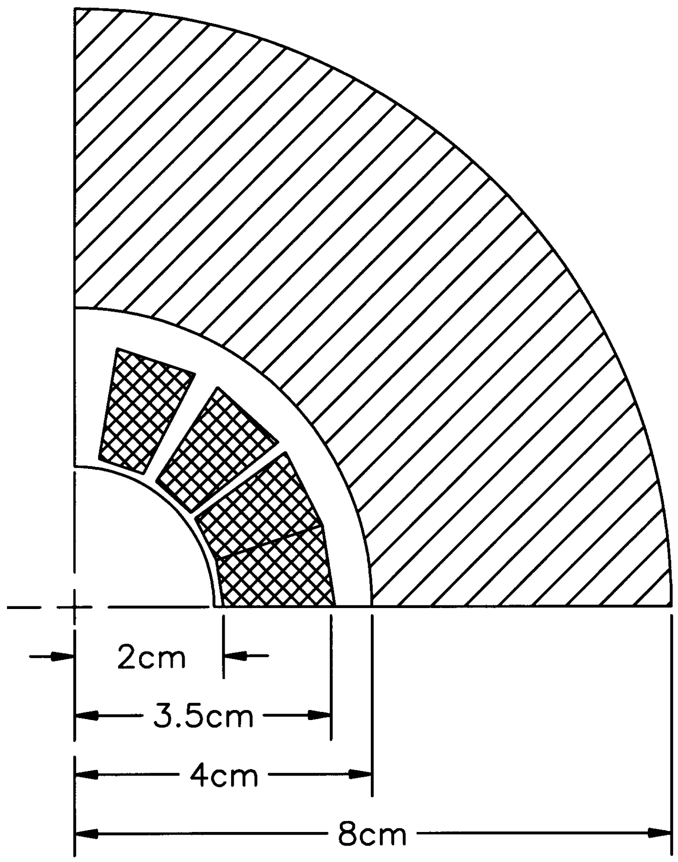
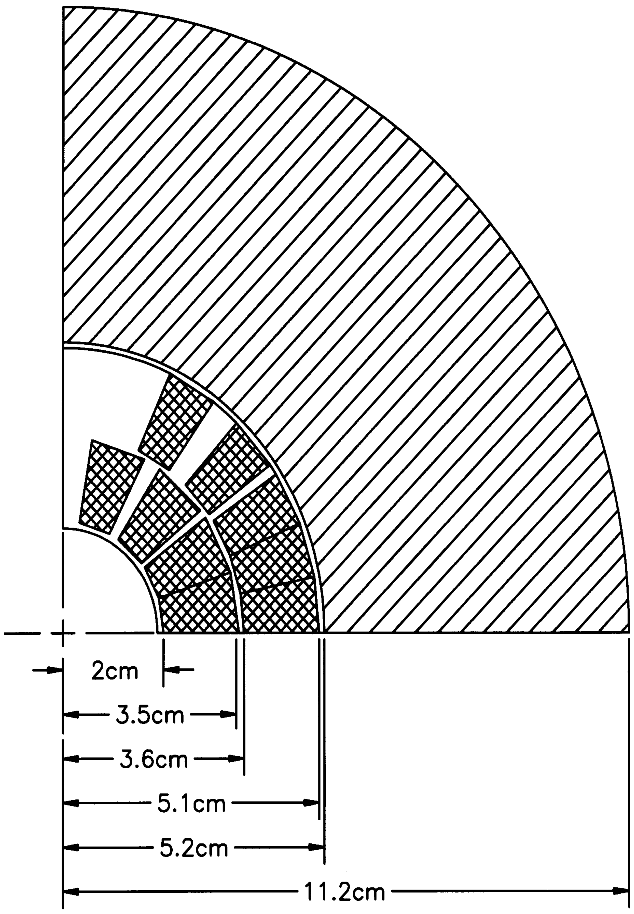


Cross sections of pulsed current dipoles for a μ rapid-cycling accelerator dipole (4 T)



Cross sections of pulsed current dipoles for a collider dipole (6 T).

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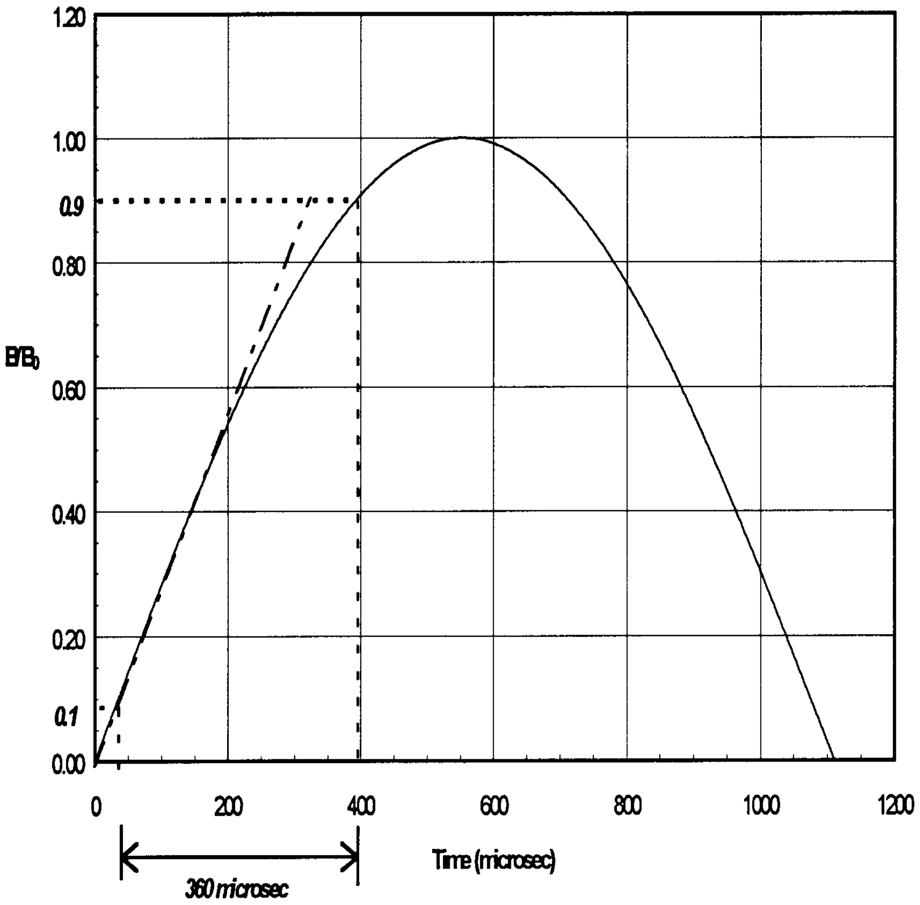

Ramp for rapid-cycling pulsed-dipoles for acceleration to 250 GeV ¶

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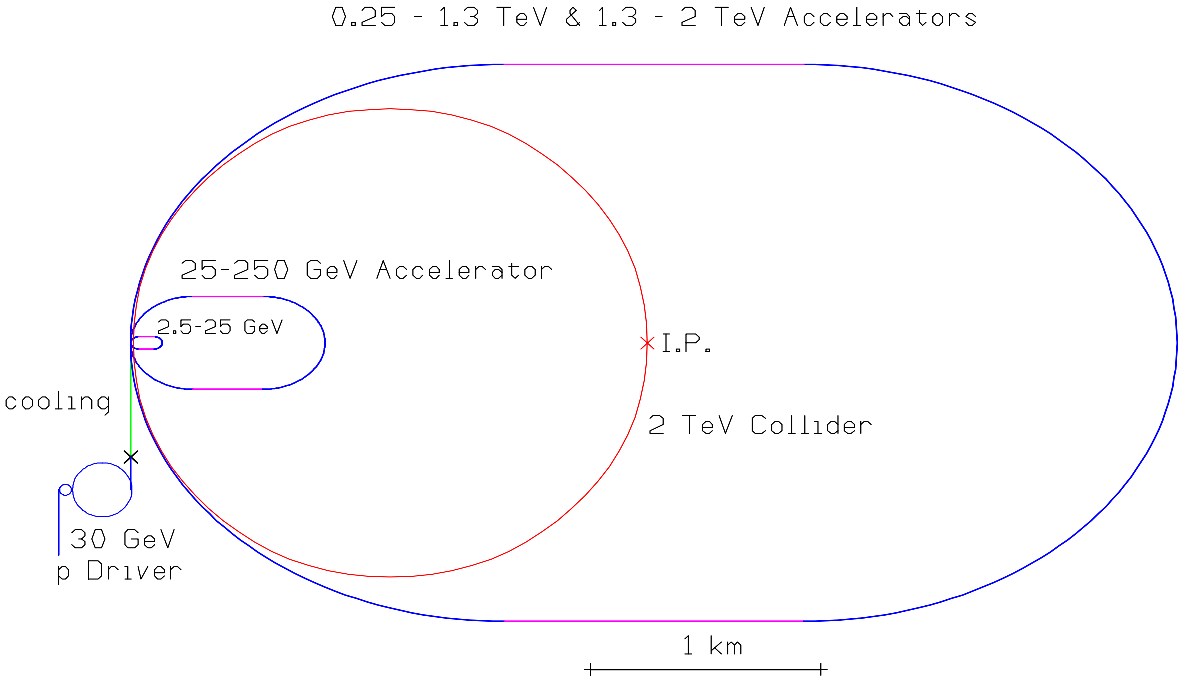
$\mu^+\mu^-$ COLLIDER



A scale drawing of a possible 4 TeV muon collider machine ‖

‖

Parameters for pulsed conductor-dominated accelerator and storage ring dipoles

Parameter unit	Accelerator *Dipole*	Storage Ring *Dipole*
Coil r_{inner} (cm)	2	2
Magnet length (m)	10	10
Field (T)	4	6
Current (kA)	29.5	24.9
Stored energy (kJ)	160	360
Inductance (mH)	0.37	1.2
Coil R (mW)	19	44
Ramp time (μs)	360	
Store Time (μs)		5000
Power supply V (kV)	31.2	1.1
P. into mag. at 2 Hz (kW)	19	452
Power into ring (MW)	2.7	39.4

**

COLLIDER RING :*

- Highest possible bending magnet to maximize No. of turns in the ring before decay

β^*	$3\,mm$
σ_z	$3\,mm$
ϵ_n	$50\,\pi\,mm - mrad$
$\delta = \frac{\Delta p}{p}$	$0.12\,\%$
No. of turns	1000
No. muons	2×10^{12}
No. bunches	2
beam − beam tune shift	0.05

- Isochronous lattice

- IP Local Chromatic Correction is essential

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- Resistive wall impedance instability $\rightarrow$ *BNS damping* with rf quadrupoles is a possible solution

- Momentum compaction, $\alpha \approx 10^{-6}$

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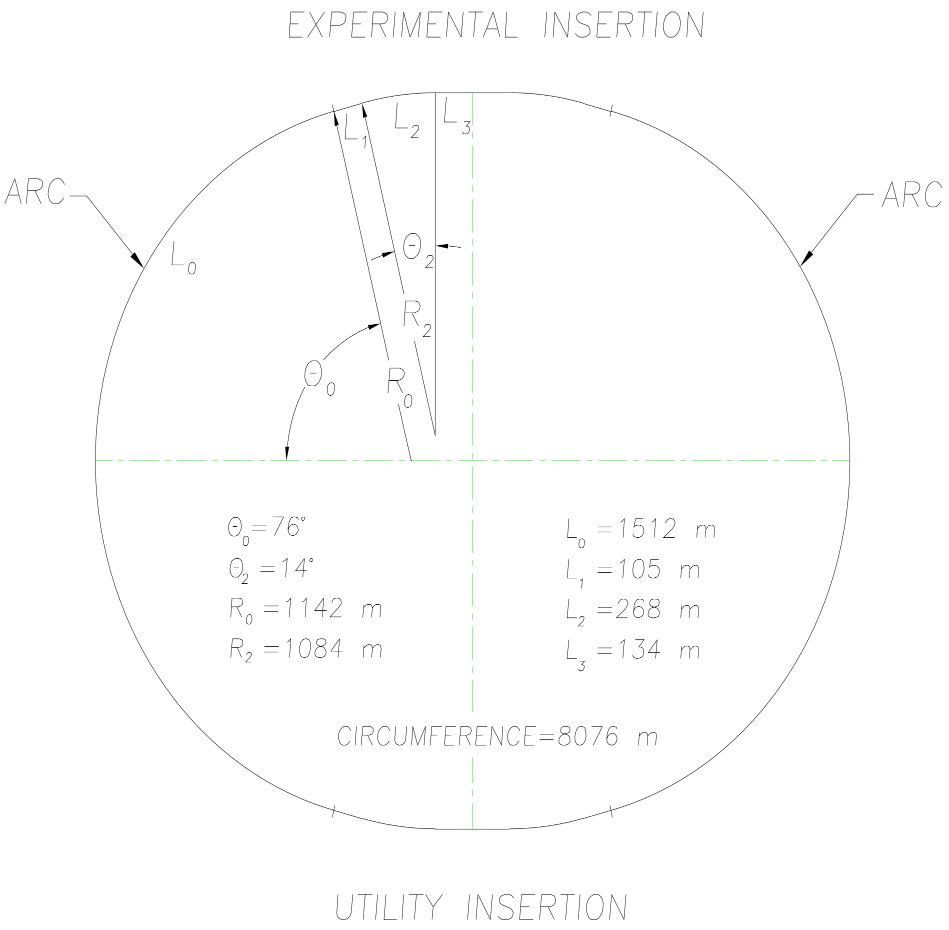


The complete collider ring layout (Garren)

- There are two lattices designed by A. Garren and Oide, neither is totally complete

- Oide's has shown a dramatic increase of the dynamical aperture (100 turns) by including octupoles and decapoles in the chromatic correction section

- At Snowmass a new lattice was designed simpler and equally good properties (C. Johnstone and A. Garren)

‡

MACHINE INDUCED BACKGROUND:*

- Muon Halo
- Muon Decay
- Beam-Beam Interaction

Calculations are done with

GEANT and MARS

Study is just beginning†

†

BACKGROUND FROM μ HALO : Muon halo refers to μ's lost from main bunches but manage to appear at the detector(full energy)

Passing through the calorimeter undergo Deep Inelastic Scattering and deposit clumps of energy (constraints on calorimeter)

SOLUTION: careful injection and collimation

BACKGROUND FROM μ DECAY :‡

$$\mu^- \to e^- + \nu_\mu + \overline{\nu_e}$$

$2 \times 10^{12} \times 2$ decays in 10^3 turns

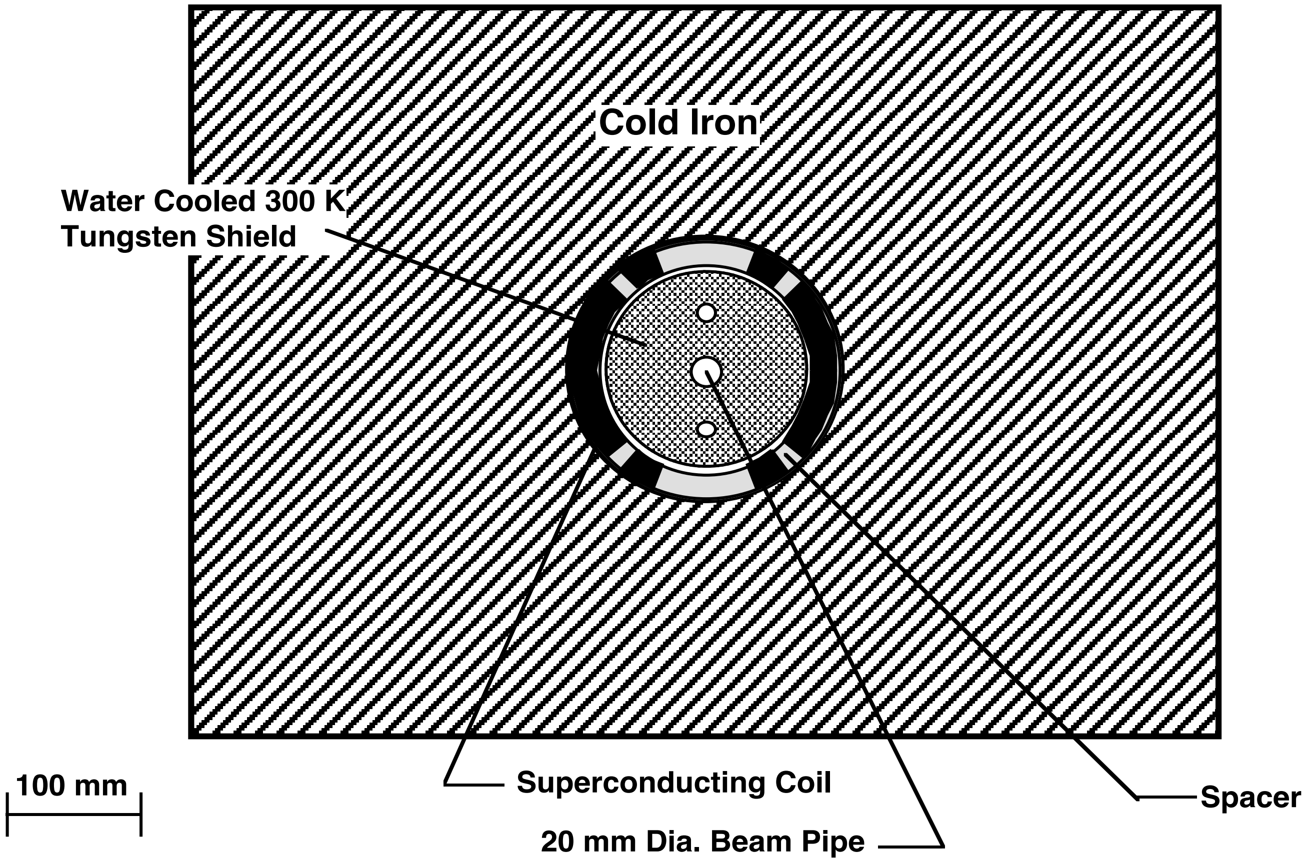
$2 \times 10^9 \times 2$ decays per turn

5×10^5 decays per m

electron synchrotron radiation
High energy electromagnetic
showers (e,γ, neutrons and
charged hadrons)

- Heating of beam pipe $\rightarrow$ 6 cm of Tungsten liner

- background at detector $\rightarrow$ design of W nose cone



Beam Power 38 MW 6 kW/m

Power →pipe 12 MW 2 kW/m

Power → Cold Fe 30 kW 6 W/m

Radiation (after 1 day) on outside of W 100 mR/hr

Radiation (after 1 day) on outside of Fe 1 mR/hr

- Incoherent pair creation e^+e^- due to beam beam interaction ($\sigma \approx 10\, mb \to 3 10^4\, e^+e^-$ per crossing). 90% trapped in tungsten nose cone; only pairs with $30 < E < 100\, MeV$ will enter detector (20° shielding cone angle).
  Solution: Design of nose cone; Skrinsky and P. Chen has suggested plasma (Li jet) at IP ($\sigma \approx 90\, mb$ but most pairs move along beam pipe)

- Electrons generate Bethe-Heitler muon pairs, Deep Inelastic Scattering cause spikes of energy distribution

- hadron background (neutrons) due to photoproduction

STRAWMAN DETECTOR:§

Present state-of-the-art technologies seems to be sufficient to build a detector which will meet the requirements (background: large number of soft particles)

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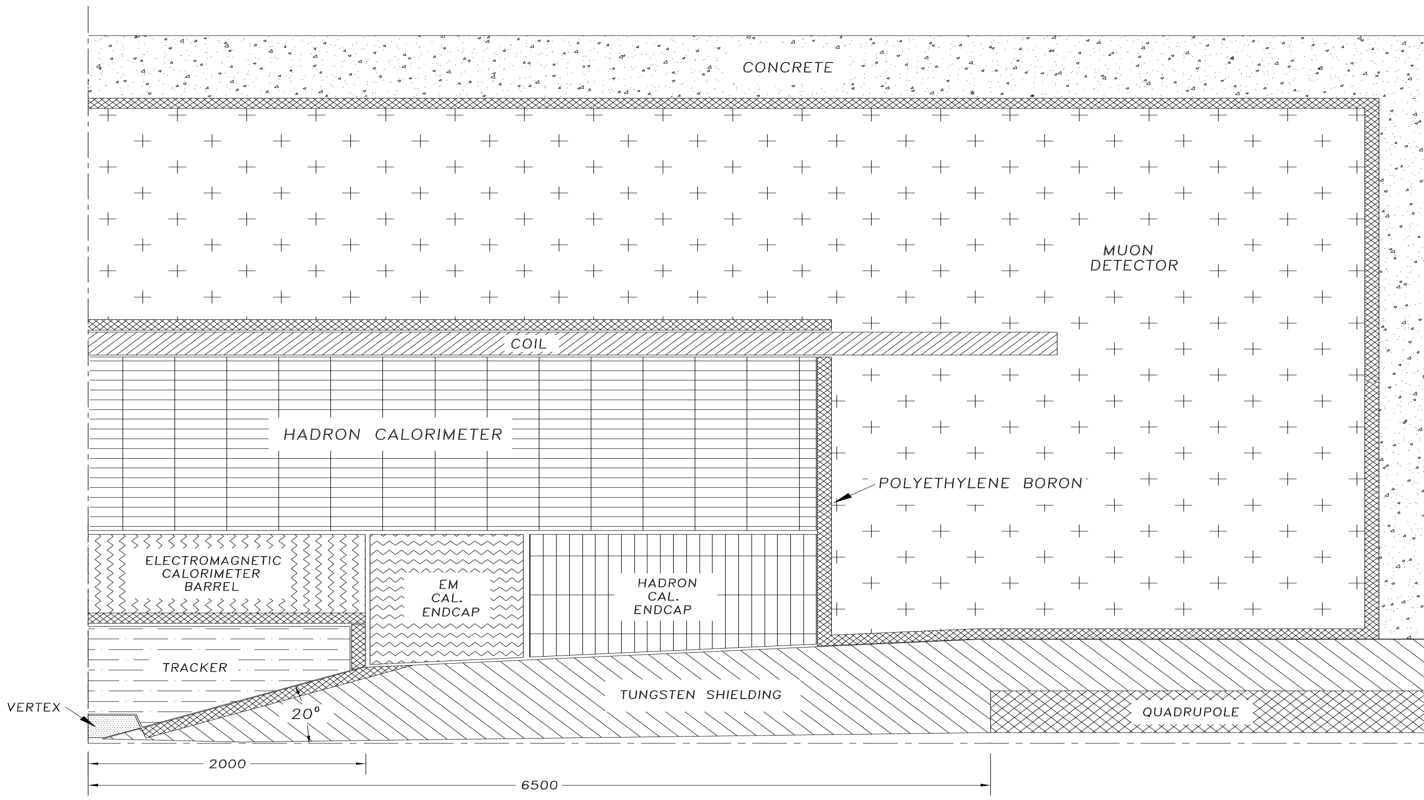

CONCRETE
MUON DETECTOR
COIL
HADRON CALORIMETER
POLYETHYLENE BORON
ELECTROMAGNETIC CALORIMETER BARREL
EM CAL. ENDCAP
HADRON CAL. ENDCAP
TRACKER
VERTEX
20°
TUNGSTEN SHIELDING
QUADRUPOLE
2000
6500

Detector Component	Minimum Resolution/Characteristics
Magnetic Field	Solenoid; $B \geq 2T$
Vertex Detector	b − tagging, small pixels
Tracking	$\Delta p/p^2 \sim 1 \times 10^{-3}(GeV)^{-1}$ at large p
	High granularity
EM Calorimeter	$\Delta E/E \sim 10\%/\sqrt{\mathrm{E}} \oplus 0.7\%$
	Granularity : longitudinal and transverse
	Active depth : $24X_0$
Hadron Calorimeter	$\Delta E/E \sim 50\%/\sqrt{\mathrm{E}} \oplus 2\%$
	Granularity : longitudinal and transverse
	Total depth (EM + HAD) $\sim 7\lambda$
Muon Spectrometer	$\Delta p/p \sim 20\% at 1\ TeV$

Detector Performance Requirements¶

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CONCLUSIONS :*

- ADVANTAGES

  1. Reduced synchr. rad.$\rightarrow$ ring 10^3 turns

  2. Full energy projectile available for new particles production

  3. Both beams partially polariz. (lower $\mathcal{L}$)

  4. Fairly compact (see Fig.). Multipurpose: intense π, K, ν, μ. Possibility of μp collisions. Physics of rare K and μ decays.

  5. Start with 0.5 TeV and progress to 2 TeV over time

- DIFFICULTIES

* $\mu^+\mu^-$ COLLIDER

1. making sufficient μ's cooling, accelerate and collide them before DECAY
2. Problem decay products (magnet heating) and detector (background)

- TECHNICAL DEVELOPMENTS
  1. Demonstrate working cooling system without losses
  2. target high field solenoid
  3. low frequency linacs for phase rotation and cooling
  4. accelerator magnets, shielding and SC rf cavities
  5. Quads at IP

A great deal of progress has been accomplish; however, many questions remains (you may have many more) that require theoretical study as well as R&D on hardware[†]

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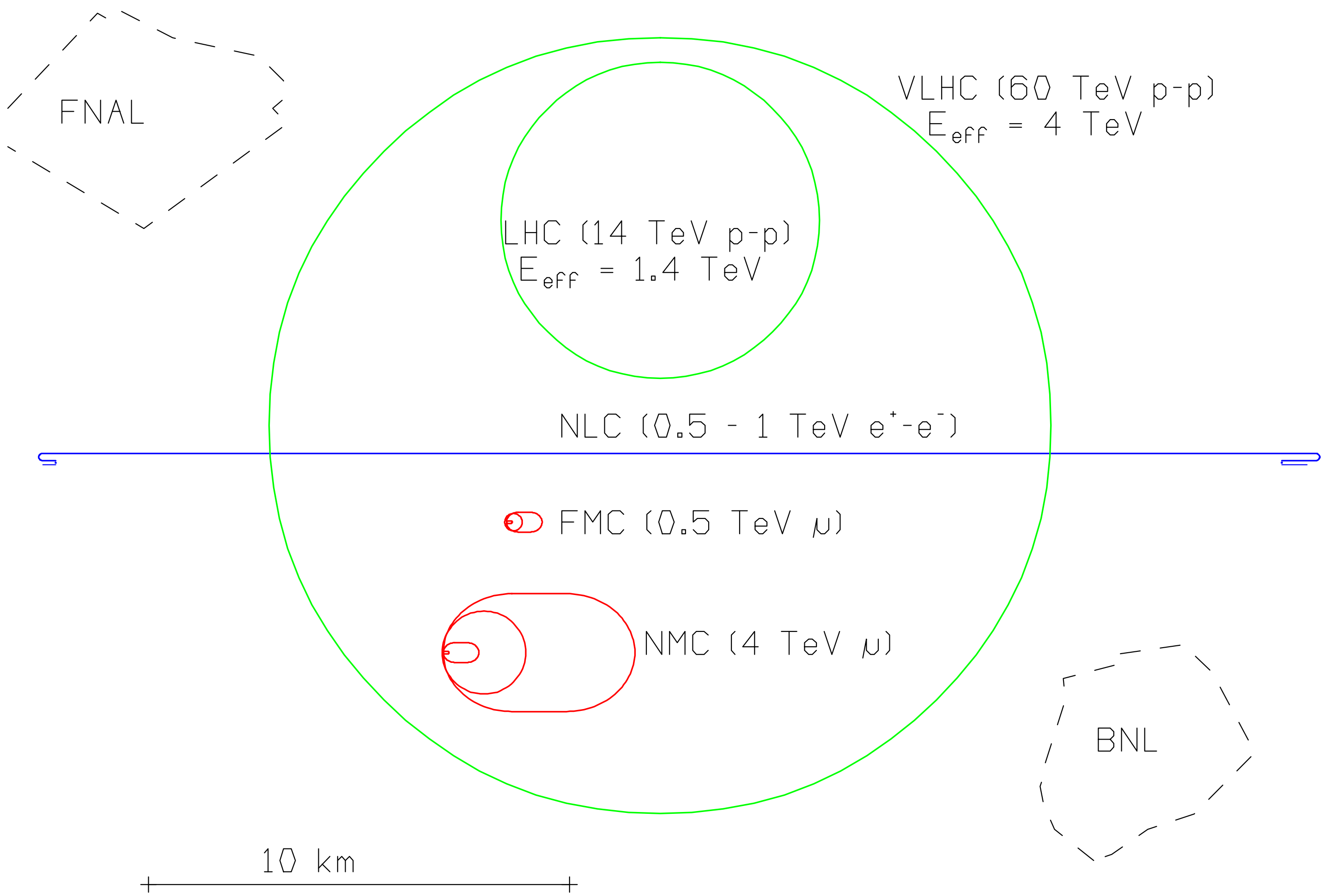

FNAL
VLHC (60 TeV p-p)
Eeff = 4 TeV
LHC (14 TeV p-p)
Eeff = 1.4 TeV
NLC (0.5 - 1 TeV e+-e-)
FMC (0.5 TeV μ)
NMC (4 TeV μ)
BNL
10 km

LUMINOSITY :* $\mathcal{L}$ is defined as the interaction event rate per unit of elementary cross section

$$\#\text{events rate} = \sigma_{\text{elem.}}\mathcal{L}$$

A typical value of cross section in e^+e^- annihilation is set by the point cross section (s center of mass energy)

$$1R = \frac{4\pi}{3}\frac{\alpha^2}{s} \equiv \frac{86.8[\text{fb}]}{s[\text{TeV}^2]}$$

It is reasonable to set a luminosity of 1.5×10^4 events per R per year at 1 TeV, then

$$\mathcal{L}[cm^{-2}s^{-1}] \approx 5.5{\times}10^{33}[cm^{-2}s^{-1}]\left(\frac{E_{eff}[TeV]}{1[TeV]}\right)^2$$

Notice :

Hadron collider $E_{eff} \approx \frac{E_{c-of-m}}{10}$

Lepton collider $E_{eff} = E_{c-of-m}$

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PARTICLE PHYSICS OPPORTUNITIES :*

- The physics capability of $\mu^+\mu^-$ and e^+e^- colliders with the same energy and luminosity are *S*IMILAR
  $\mu^+\mu^-$ collider is a complementary machine to e^+e^- and hadron(HLC) colliders

- s-channel production of Higgs boson
  SM (h$_{SM}$) and MSSM (h^o, H^o, A^o, H$^\pm$)

- Beam-beam interaction is reduced (2 bunches of each sign at 15 Hz and 10^3 turns)

- Finer energy resolution (reduced synchrotron radiation)

*

- Both beams may be polarized albeit with loss of luminosity

- Possiblity of μp collision; sudy rare μ decay, also other beams π, kaons, neutrinos

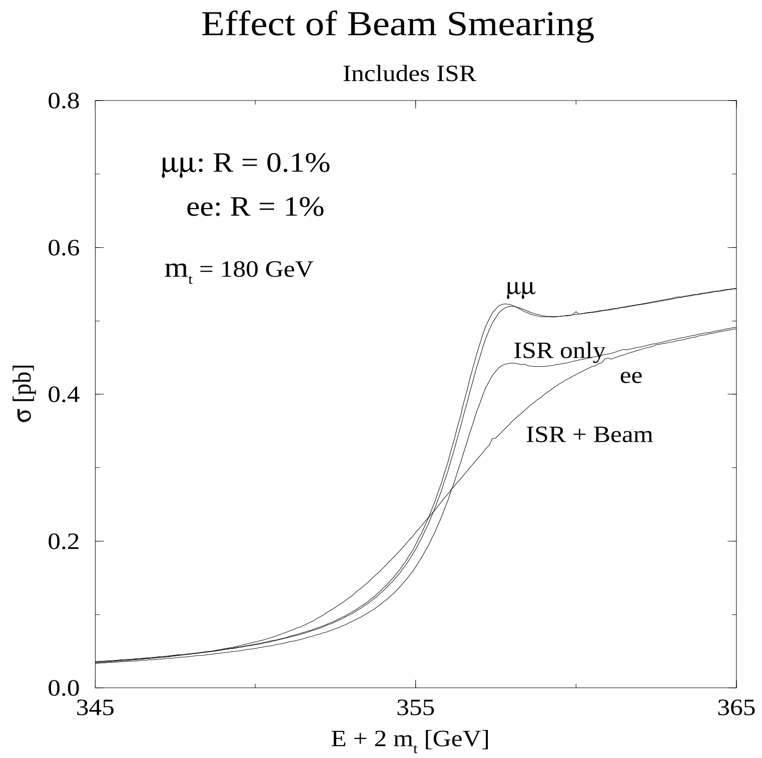
- If SUSY does not exist, a 4 TeV machine may be needed to study the mechanism for electroweak symmetry breaking (W W strong boson scattering)

†

PRECISION THRESHOLD STUDIES TOP QUARK:*

- The $t\bar{t}$ threshold SHAPE determines m_t and Γ_t

- Even a conservative natural beam resolution $R \approx 0.1\,\%$ will increase precision compared with other machines (e^+e^- collider $\rightarrow R \approx 1\,\%$) Initial state radiation (ISR) is reduced



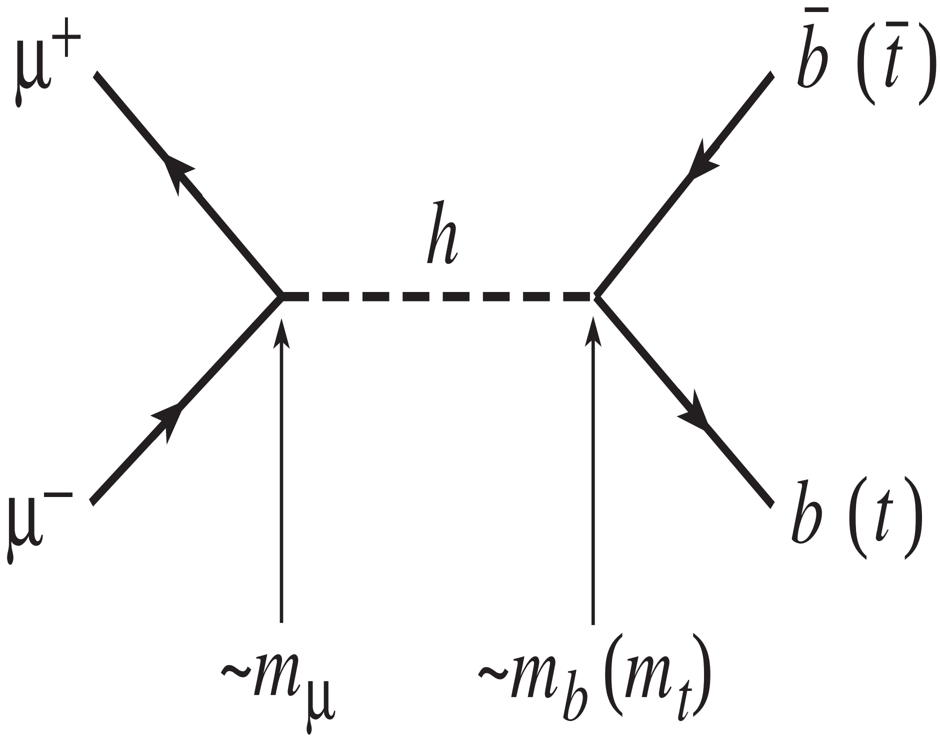
The threshold curves are shown for $\mu^+\mu^-$ and e^+e^- machines including ISR and with and without beam smearing. Beam smearing has only a small effect at a muon collider, whereas at an electron collider the threshold region is significantly smeared. The strong coupling is taken to be $\alpha_s(m_Z) = 0.12$.

- Both measurements of mh, Γ_t and results for top quark will allow consistency tests of EWSB theory

s-CHANNEL HIGGS PHYSICS:*

Standard Model (SM)$\rightarrow$ one Higgs boson (h)

Minimal Super Symmetric Model (MSSM)$\rightarrow$
5 Higgs bosons (h^o, H^o, A^o, $H^{\pm}$)



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- Energy of machine has to be adjusted to mh

- Energy spread of machine R. The $\sqrt{s}$ rms Gaussian spread $\sigma_{\sqrt{s}}$(natural spread) has to be smaller or order of Γ_h
  to be sensitive to $\Gamma_{h_{SM}} \rightarrow R \approx 0.01\,\%$

- Requirements:

  1. Luminosity $\mathcal{L} > 10^{33}\,[cm^{-2}s^{-1}]$ at $\sqrt{s} \approx m_h$
  2. Excellent energy resolution $R \approx 0.01\,\%$
  3. Ability to adjust machine energy accurately and quickly over an interval of several GeV

PARAMETERS OF LOWER ENERGY $\mu^+\mu^-$ COLLIDERS*

$180 \oplus 180$ GeV; $350 \oplus 350$ GeV ; $750 \oplus 750$ GeV

We can use SIMPLE SCALING RELATIONS to determine the main parameters.

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SCALING

SIMPLE ARGUMENT

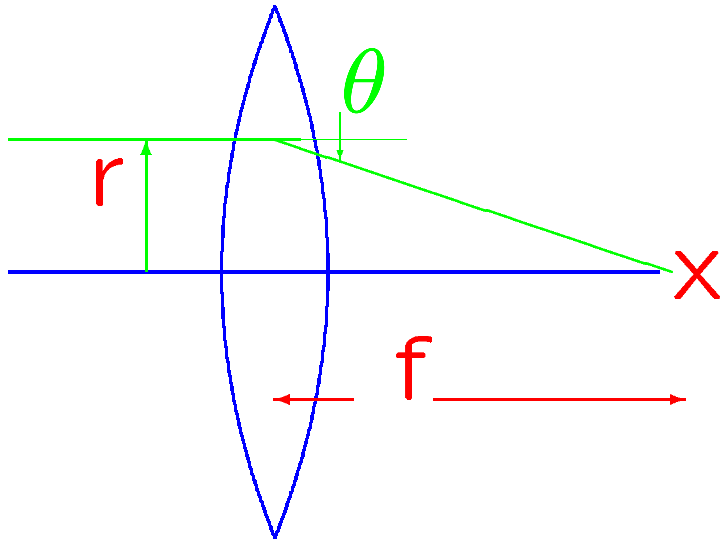
Assume a focusing system (thin-quadrupole)

$$\theta = -rkl$$

$$k = 0.3(\frac{B}{a})\frac{1}{p}[\frac{T/m}{GeV/c}]$$

$l \equiv$ eff. length of Quad

$a \equiv$ aperture Quad



Main assumption:
Length of Quad is kept in a constant ratio with the focal length

$$\boxed{l = \alpha f}$$

Substituting we get

$$\boxed{f^2 = \frac{p}{\alpha 0.3(\frac{B}{a})}}$$

New constraints: keep

apertures fixed

keep gradients fixed

LENGTHS SCALE AS $\sqrt{\gamma}$

How the emittance scale?

- $\theta = \frac{a}{f} ==> \gamma^{-1/2}$

- $\theta = \sqrt{\frac{\epsilon_N}{\gamma\beta^*}} ==>$

- $\boxed{\epsilon_N \propto \beta^*}$

- 6-D emittance

  $\epsilon_6 = \epsilon_{xN}\epsilon_{yN}\frac{\Delta p}{p}\sigma_z p$

- Fixed Energy spread ==>

- $\beta^* \propto \epsilon_N \propto \gamma^{-1/3}$

LUMINOSITY

$$\mathcal{L} = \frac{N_\mu^2 n_b n_{eff} f_{rep} \gamma}{4\pi\epsilon_N \beta^*}$$

$$\boxed{\propto \gamma^{5/3}}$$

keeping constant

N_μ, n_b, n_{eff} and f_{rep}

Tune Shift

Beam-beam tune shift

$$\xi = \frac{r_\mu n_\mu}{4\pi\epsilon_N} ==>$$

$$\boxed{\xi \propto \gamma^{1/3}}$$

Additional assumption:

Keep beam-beam tune shift fixed

$$N_\mu \propto \gamma^{-1/3}$$

Now if current is kept constant

$$\boxed{\mathcal{L} \propto \gamma^{4/3}}$$

Summary of Scaling Relations

$Energy$	4	0.36	0.7	1.5
$Length \propto \gamma^{1/2}$	6.5	1.9	2.65	3.9
$\theta_{4\sigma} \propto \gamma^{1/2}$	4	14	10	7
$aper. \propto const.$	12	12	12	12
$\beta^* \propto \gamma^{-1/3}$	3	7	5.5	4.2
$\epsilon_N \propto \gamma^{-1/3}$	50	113	91	70
$\mathcal{L} \propto \gamma^{4/3}$	1	0.038	0.09	0.25
$N_\mu \propto \gamma^{-1/3}$	2	4.5	3.6	2.8

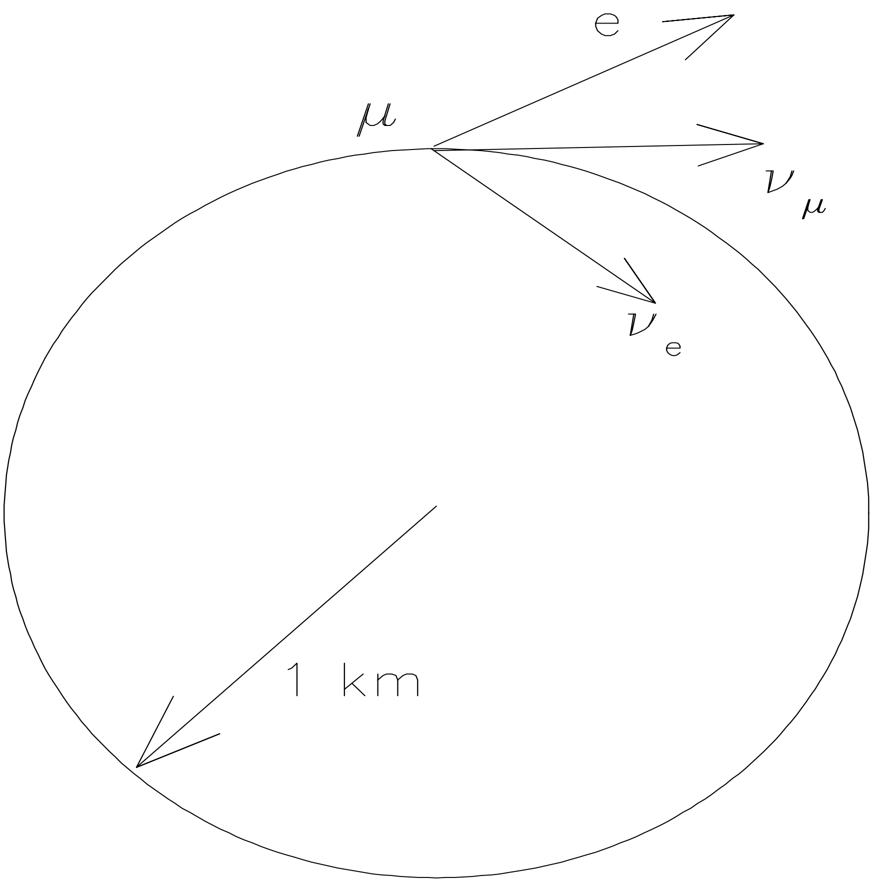
$Energy[TeV]$ $\quad$ $Length[m]$

$aperture[cm]$ $\quad$ $\beta^*[mm]$

$\epsilon_N[\pi \times mm - mrad]$ $\quad$ $N_\mu \times 10^{12}$

$\mathcal{L}[10^{35} \times cm^{-2}s^{-1}]$

Neutrino-Induced Radiation Hazard



Muon Decay

$$\mu^- ==> \nu_\mu + \overline{\nu}_e + e^-$$

$$\mu^+ ==> \overline{\nu}_\mu + \nu_e + e^+$$

Typical opening angle:

$$\theta_\nu \approx \frac{1}{\gamma_\mu}$$

For 2 TeV muons

$$\theta_\mu \approx 50\,\mu rad$$

ν INTERACTIONS (DIS)

TeV-scale NEUTRINOS interact primarily through Deep Inelastic Scattering (DIS) on nucleons (N)

$$\nu_{\substack{\mu \\ e}} + N \longrightarrow$$

$$\substack{\mu \\ e} + Hadrons\,(75\%)$$

$$\nu + Hadrons\,(25\%)$$

Producing the particles:

$Hadron$ $showers$	$meters$	$\approx 40\%$ $of\ E_T$
$Electromag.$ $showers$	$meters$	$\approx 20\%$ $of\ E_T$
$Muons$	km	$\approx 20\%$ $of\ E_T$
$Neutrino$		$\approx 20\%$ $of\ E_T$

Question: isn't the neutrino cross-section very small?

$\sigma_{\nu N}^{total} \approx 7 \times 10^{-36}\, cm^2 \times E_\nu[TeV]$

The off-site dose limit is: $10\, mrem/year = 600\, TeV/kg/year$ (for min.ion.radiation) $\approx Energy\, of\, 10^3 \nu's$

The small cross-section is compensated by:

6×01^{26} N/kg

2×10^{21} ν/year