VI. ACCELERATION

A. Introduction

Following cooling and initial bunch compression, the beams must be rapidly accelerated. In this section some of the options in accelerator design will be described and examples of acceleration scenarios presented.

Separate acceleration scenarios are given here for a low momentum spread 100 GeV First Muon Collider (Higgs factory), and for a high luminosity 3 TeV collider. Ideally, though more difficult, this accelerator designed for the low energy machine should be extendable to the $\approx$ 250 GeV beam energy and from there to the $\approx$ 2 TeV beam energy needed for a very high-energy collider.

While acceleration of muons to high energy is clearly possible, an optimal and cost-effective acceleration complex is needed. In the scenarios described below, a low-frequency linac would take the beam from the end of cooling to an energy of $\approx$1 GeV followed by recirculating-linac systems to take the beam to 50-70 GeV. The multi-TeV energy regime can be reached through a series of very rapid cycling synchrotrons. Variations on the acceleration model and potential difficulties are discussed, including the use of Fixed Field Alternating Gradient (FFAG) accelerators in place of, or together with, the recirculating-linacs. Finally, topics for further study and research are described.

B. Accelerator options

The acceleration time is limited by muon decay ($\tau_\mu = 2.2\mu\ s$ at rest) and requires that:

$$eV'_{rf} \;>>\; 0.16\mathrm{MeV/m}$$

where eV'_{rf} is the acceleration rate. An acceleration rate whose value of 0.16 MeV/m is low for a linac, but very high for a conventional synchrotron.

At the lowest energies ($<$ 700 MeV), the momentum spread and beam sizes are so large that only a linac is feasible, and acceleration to full energy in a single-pass linac would be good, but it would be very expensive.

Thus, following the initial linac, some form of recirculating acceleration is preferred. A synchrotron would be possible, in principle, but the acceleration must occur so rapidly that conventional magnet ramping is unlikely to be practical. Two alternative multi pass methods are being considered: recirculating linac accelerators similar to those used at TJNAF and Fixed Field Alternating Gradient (FFAG) accelerators.

In a recirculating linac accelerator, the beam is circulated through the same linac for several passes, with separate, energy matched, fixed-field return paths for each pass. Each return path is optically independent and can be separately designed. In the initial lattice design for the muon recirculating linac accelerator, the return arcs are similar alternating gradient (AG) systems with the same dipole layout, but with differing quadrupole strengths to allow separate tuning and chronicity matching in each arc. Multiple aperture superconducting magnets have also been designed which would reduce the diameter of the recirculating linac, lowering muon loss from decay and possibly being more economical (see Fig 7.12 [44]). In either case, both the linac and return transports must accommodate large transverse emittances (rms) of $\approx 300\ \pi$ mm-mrad. Strong focusing is required not only to keep apertures down, but also to minimize orbit deviations due to the large momentum spreads, which in the initial stages of acceleration, can be as large as 10% rms.

More recently, an adaptation of the FFAG (Fixed-Field Alternating Gradient) accelerator concept has been proposed for $\mu+$ $\mu-$ acceleration [174,175]. In this variation, return transports are designed with a very large (factor of 5-10) energy acceptance, so that separate energy turns can pass through the same fixed-field elements. More acceleration turns are possible than with a recirculating linac accelerator which reduces the rf requirements; but the orbits and focusing properties are now energy-dependent. Such FFAG configurations require strong superconducting magnets with large apertures to accommodate the energy-dependent spread in closed orbits. The extra cost associated with increased magnet apertures must be evaluated against potential savings in the number of magnets and reduced rf/turn requirements.

For the higher energy stages, the muon life time is greater and the needed rate of energy increase is less. Thus, above a few hundred GeV, rapid accelerating synchrotrons become possible. In a rapid accelerating synchrotron, the beam is also multi-pass accelerated through an rf system, but the beam returns in a single arc, as the magnetic field is ramped to match the increase in beam energy. As above, more acceleration turns are possible than with a recirculating accelerator, but we now have a single moderate aperture return transport. But rapid cycling synchrotrons have higher power costs and the technical challenges associated with the rapid acceleration time needed to conserve muons.

Thus a complete system would likely include an initial linac followed by a sequence of recirculating linac accelerators and/or Fixed Field Alternating Gradient machines. Depending on the final collider energy, one or more rapid acceleration synchrotrons would follow. Each system increases the beam energy by a factor of 5-10.

C. Scenario examples

Several scenarios have been discussed earlier [176,177], see for instance the parameters (table XI) used in a simulation of longitudinal motion discussed below. The ones given here are more recent, and more detailed, but they should not be taken to be definitive. They are examples that were derived to probe the design problems and to show that solutions should be possible.

1. Acceleration for Higgs collider

Table IX gives an example of a sequence of accelerators for a 100 GeV Higgs Factory, *i.e.* a machine with very low momentum spread (0.003%, see table I) and relatively large rms transverse emittance ($\approx$ 300 π mm-mrad).

Following initial linacs, recirculating accelerators are used. The number of arcs in each recirculating accelerator is about 10. In this example, conventional fixed field 2 T magnets are used, but the effective ramp frequencies that would be needed if pulsed magnets were used are given for reference.

TABLE IX. Accelerator parameters for a Higgs Factory (100 GeV)

Acc. type		linac	linac	recirc	recirc	recirc	sums
Magnet type				warm	warm	warm	
rf type		Cu	Cu	Cu	Cu	Cu	
E^{init}	(GeV)	0.10	0.20	0.70	2	7	
E^{final}	(GeV)	0.20	0.70	2	7	50	
Circ.	(km)	0.04	0.07	0.07	0.19	1.74	
Turns		1	1	8	10	11	
Loss	(%)	2.31	3.98	7.27	7.91	13.94	31.06
Decay heat	(W/m)	0.89	1.98	11.04	12.99	12.44	
B_{fixed}	(T)			2	2	2	
Ramp freq.	(kHz)			281	79.83	8.00	
Disp	(m)			1	1.50	3	
β_{max}	(m)	0.83	1.42	3.00	5.31	21.08	
σ_z^{init}	(cm)	2.71	2.22	1.42	1.64	0.90	
$\Delta p/p^{\mathrm{init}}$	(%)	3.58	2.80	1.64	0.56	0.32	
σ_y	(cm)	1.09	1.14	1.01	0.85	0.94	
σ_x	(cm)			1.93	1.19	1.34	
Pipe full height	(cm)	10.92	11.42	10.14	8.49	9.39	
Pipe full width	(cm)	10.92	11.42	19.28	11.94	13.35	
rf freq.	(MHz)	200	200	200	200	400	
Acc/turn	(GeV)	0.20	0.40	0.17	0.50	4	
Acc time	(μs)			1	6	62	
η	(%)	5.15	5.36	6.36	2.84	6.92	
Acc. Grad.	(MV/m)	8	8	8	10	10	
Synch. rot's		0.62	0.63	0.62	3.92	23.16	
Cavity rad.	(cm)	54.37	54.88	54.88	60.47	38.26	
Beam time	(ms)	0.00	0.00	0.00	0.01	0.06	
rf time	(ms)	0.17	0.17	0.17	0.18	0.13	
Tot. peak rf	(GW)	0.05	0.10	0.05	0.26	4.71	5.17
Ave. rf power	(MW)	0.14	0.24	0.13	0.68	9.54	10.73
rf wall	(MW)	0.64	1.16	0.46	2.42	28.06	32.75

In this example, all the accelerating cavities are room temperature copper structures, and the accelerating gradients are modest (< 10 MeV/m). Nevertheless, the acceleration is rapid enough that the total losses from decay are only 30%. The heating from these decays is also modest ($\approx$ 10 W/m) because of the small number of turns and relatively low energy. Since no superconducting magnets or rf are used in this example, this heating should cause no problem.

In this machine, the transverse emittances are large and strong focusing is thus required, but the maximum momentum spread is moderate (up to 1.37% rms in the first recirculator) and is thus not likely to be a problem.

If the same machine is to also run at a high luminosity, with larger momentum spread, then although the six dimensional emittance is the same as in the Higgs collider discussed above, the transverse emittance is smaller

($\approx 90\pi$ mm-mrad rms), and the longitudinal emittance larger (by about a factor of 4) and the momentum spread in the first recirculating accelerator would be nearly 6% rms; or about 50% full width. This is a very large momentum spread that could only be accepted in FFAG like lattices as discussed below.

Similarly, if the same acceleration is to be usable as the front end for a 250 + 250 GeV or higher energy machine then the transverse emittance will again be less, the longitudinal emittance even larger, and the problem of very large momentum spread will be worse. Clearly, although not absolutely needed for a Higgs Factory, it is desirable to solve this problem even in that First Collider.

A separate parameter set for the high luminosity 50 + 50 GeV collider and a 250 + 250 GeV collider could have been presented, but their parameters are very similar to those of the front end of the 3 TeV machine given below, and are thus omitted here.

2. Acceleration for 3 TeV collider

For a high energy machine, the muon accelerators are physically the largest component and are also probably the most expensive. More work is needed on its design. Table X gives an early example of a possible sequence of accelerators for a 3 TeV collider.

Linacs are used up to 700 MeV, followed by recirculating linac accelerators. In the first of these, because of the very large longitudinal emittance, the momentum spread as the beam enters the first recirculating linac accelerator is 8.5% rms, which is very large. The lattice must have very strong focusing, small dispersion and large aperture. If this is not possible, higher energy linacs or lower frequency rf could relieve the requirement.

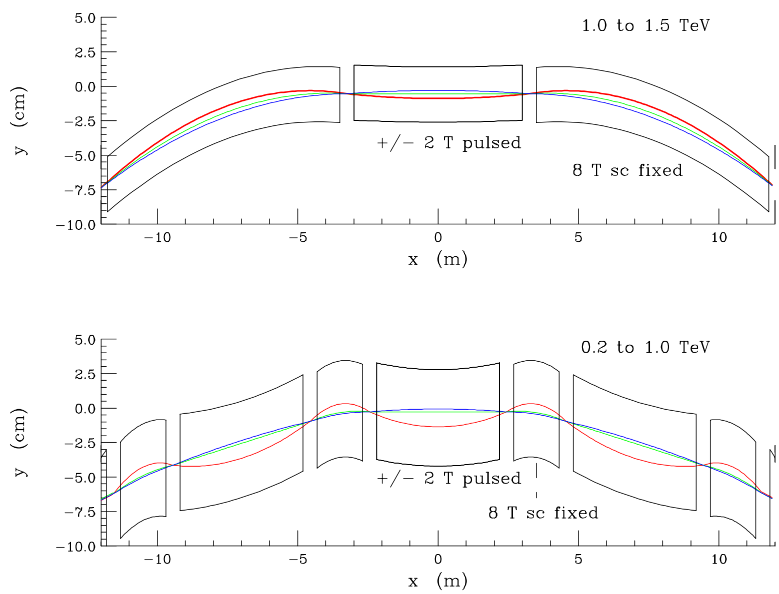

FIG. 38. Schematic of hybrid superconducting-pulse magnet accelerator ring

For the final three stages, pulsed magnet synchrotrons [178] are used. In the 200 GeV ring, all the magnets in the ring are pulsed, but in the last two rings a superconducting-pulsed hybrid solution is used. In these cases, if only pulsed magnets were used, then the power consumed would be too high, and because only low pulsed fields could be used, the circumferences would also be very large. It is thus proposed to use rings with alternating warm pulsed magnets and superconducting fixed magnets [179] (see figure 38). The fixed magnets are superconducting at 8 T; the pulsed magnets are warm with fields that swing from - 2 T to + 2 T. The effective ramp frequency is given in the table. Both of these rings are in the same tunnel, with the fraction of magnet length pulsed (vs. fixed) being different (73% and 43%).

In all the final three rings superconducting rf is employed to minimize the peak power requirements and to obtain high wall to beam efficiency and thus keep the wall power consumption reasonable. In the final two rings the frequency and cavity designs are chosen to be the same as that in the TESLA [180] proposal.

TABLE X. Parameters of Acceleration for 3 TeV Collider

Acc. type		linac	recirc	recirc	recirc	synch	synch	synch	sums
Magnet type			warm	warm	warm	warm	hybrid	hybrid	
rf type		Cu	Cu	Cu	SC Nb	SC Nb	SC Nb	SC Nb	
E^{init}	(GeV)	0.10	0.70	2	7	50	200	1000	
E^{final}	(GeV)	0.70	2	7	50	200	1000	1500	
Circ.	(km)	0.07	0.12	0.26	1.74	4.65	11.30	11.36	
Turns		2	8	10	11	15	27	17	
Loss	(%)	6.11	12.28	10.84	13.94	10.68	10.07	2.65	50.58
Decay heat	(W/m)	3.67	15.02	16.89	15.91	19.44	30.97	18.09	
B_{pulse}	(T)					2	2	2	
B_{fixed}	(T)		0.70	1.20	2		8	8	
frac pulsed	%						73	43	
Ramp freq.	(kHz)		162	57.34	8.00	2.15	0.50	0.79	
Disp.	(m)		0.40	0.60	0.80	1	2	4	
βmax	(m)	0.89	3.97	8.75	36.29	52.20	108	120	
Mom. compactn	%		1	-0.25	-0.50	-0.50	-0.50	-1	
σ_z^{init}	(cm)	16.34	8.53	5.29	3.57	1.59	0.96	0.78	
$\Delta p/p^{\text{init}}$	(%)	19.27	8.49	5.41	2.47	0.82	0.35	0.09	
σ_y	(cm)	0.45	0.45	0.42	0.48	0.22	0.16	0.08	
σ_x	(cm)		3.40	3.25	1.98	0.82	0.71	0.36	
Pipe full height	(cm)	4.46	4.52	4.22	4.77	2.20	1.62	0.78	
Pipe full width	(cm)	4.46	33.95	32.49	19.79	8.20	7.06	3.62	
rf Freq	(MHz)	200	100	200	200	800	1300	1300	
Acc./turn	(GeV)	0.40	0.17	0.50	4	10	30	30	
Acc. time	(μs)		3	8	62	232	1004	631	
η	(%)	3.82	0.96	1.97	1.11	10.15	14.37	12.92	
Acc. Grad.	(MV/m)	8	8	10	10	15	25	25	
Synch. rot's		0.81	0.76	1.02	5.82	19.14	54.29	31.30	
Cavity rad.	(cm)	54.88	110	60.47	76.52	19.13	11.77	11.77	
rf time	(ms)	0.04	0.12	0.05	0.56	0.40	1.25	0.96	
Tot. peak rf	(GW)	0.21	0.14	0.59	1.31	1.06	1.16	1.04	5.51
Ave. rf power	(MW)	0.14	0.25	0.45	11.04	6.32	21.91	15.07	55.18
rf wall	(MW)	0.64	0.88	1.62	32.47	18.59	44.72	30.76	130

D. Design issues

1. Recirculating linac accelerator lattice issues

Beam transport R&D for recirculating linac accelerators follows the model of the Thomas Jefferson National Accelerator Facility (TJNAF). The layout is a racetrack with linacs in the straight sections and multi-pass return arcs. At the ends of the linacs the multi-pass beam lines are recombined. A pulse magnet at each separation/recombination point is used to guide the beam into the energy matched return arc. Some initial lattice design concepts for recirculating linac accelerators are being developed. The basic return arc unit would be a FODO lattice, but with the quadrupole strengths varied in order to perturb the arc dispersion function and obtain nearly isochronous motion around the arcs. The arcs are dispersion matched by setting the arc phase advance to a multiple of 2π. Arc designs based upon the flexible momentum compaction (FMC) module can also be used.

In the special case of the very low momentum spread Higgs factory, the transverse emittances are very large ($\approx 300\ \pi$ mm-mrad rms), and will require strong focusing in the lattices. Momentum acceptance in the rings is, in this case, not a problem. But the longitudinal phase space of the muons in the other machines is much larger and requires, at low energies, either long bunches, or large momentum spreads. The requirement of high accelerating gradients argues for high frequencies, and thus short bunches. One therefore needs accelerators with large momentum acceptances.

In the 3 TeV example above, the acceptance at injection into the first recirculating accelerator is 8.5% rms. This is very large by conventional standards, but far less than that in the FFAG lattices being studied [174]. Thus the early return arcs of such a recirculating linac accelerator would have to have very strong focusing, and be FFAG-like. Of course, if a true FFAG accelerator were to be used for its avoidance of the switchyards and multi-aperture magnets, then the specified momentum spread would certainly not be a problem.

Permanent, ferric or superferric ($\approx$2 T), or high field superconducting magnets could be used for recirculating linac accelerators. The lower field magnets may be economic for initial turns, while high field magnets minimize particle travel times, and therefore decay losses. Designs for multi-aperture superconducting magnets suitable for recirculating linac accelerators have been developed [44], and superconducting magnets with as many as 18 apertures with 0.7–7 T fields have been designed. A variety of magnet configurations can be developed; cost/performance optimization will be needed in developing a final choice.

2. RF peak power requirements

Because of the need for rapid acceleration, the peak rf powers are high, and the resulting numbers of power sources large. For the linacs and early recirculating accelerators, the powers are high because of the high gradients and low frequencies needed to accelerate the long bunches. At these frequencies ($\approx$200 MHz), currently available sources (triodes and tetrodes) have relatively low maximum output powers and are expensive. Low temperature operation of the cavities, and superconducting or conventional SLED [181] systems, which would reduce the peak power requirements, are being considered.

Study of the example suggests that in the first two recirculators (up to 7 GeV) there is no hope for the rf to keep up with the beam loading. The cavity can only be filled in a suitable filling time (twice the time constant in these examples), and the rf voltage allowed to sag as the beam makes its multiple passes. If excessive sensitivity to beam current is to be avoided, then the stored energy must be large compared to that used, which is somewhat inefficient.

In the final recirculating accelerator, continuous filling (cw) is just possible, but requires yet higher peak power ($\approx$ 5 GW total at 400 MHz) because of the high acceleration rate. The use of superconducting cavities can reduce losses, and thus reduce this peak rf power source requirement, and was included in the above 3 TeV example. At this frequency (400 MHz), klystrons are available with greater power ($\approx$ 20 MW) than that of the sources at the lower frequencies, but a yet higher power klystron (50-100 MW) could probably be developed and would be desirable.

3. Pulsed magnet systems

A pulsed current 4 T magnet has been designed for acceleration to 250 GeV in 360 μs [182], but efficiency favors use of ferric materials in rapid acceleration magnets, although this would limit peak magnetic fields to $\approx$2 T. The average field can be increased by interleaving magnets swinging from -2 T to 2 T with fixed field 8 T superconducting magnets.

Faster pulsing magnets would require special materials to minimize energy losses from eddy currents. Options include silicon steel, metglass laminations, or finemet laminated tape or powdered solid. A 30μm metglass lamination suitable for several kHz cycling has been developed. A design of suitable pulsed magnets [179](see Fig. 39) has been shown to have sufficiently low losses for this application. The magnets employ cables made of many fine insulated strands (litz cable) and the yokes are made of very thin (0.28 mm), 3% Si-Fe laminations, possibly of metglas [183,184] for the higher rate cases. Detailed designs must be developed and prototypes constructed and the practical limits of recycling scenarios should be determined.

4. Superconducting linacs

While the gradients needed in the acceleration systems are not excessive, they are larger than previous experience at the lower frequencies. The high peak power pulsed operation poses power handling difficulties at lower energies and high peak current presents collective effect (wakefield) difficulties at higher energies. Higher gradients and efficiencies in all sections would improve performance.

The superconducting rf would operate in pulsed mode, matched to the acceleration time of up to a few ms. This pulse structure is similar to the multibunch acceleration mode planned for TESLA (25 MV/m at 1300 MHz designs), and studies indicate that this design could be adapted to $\mu^+ - \mu^-$ acceleration. At lower frequencies, structures such as the CERN 350 MHz superconducting rf cavities could be used. These cavities have been tested in pulsed mode operation, and tests indicate that pulsed acceleration fields $>$ 10 MV/m are possible [185].

The high single bunch intensities required for high intensities imply large higher order mode losses and large wakefield effects from the short, high intensity bunches. Higher order mode (HOM) load designs adapted from superconducting

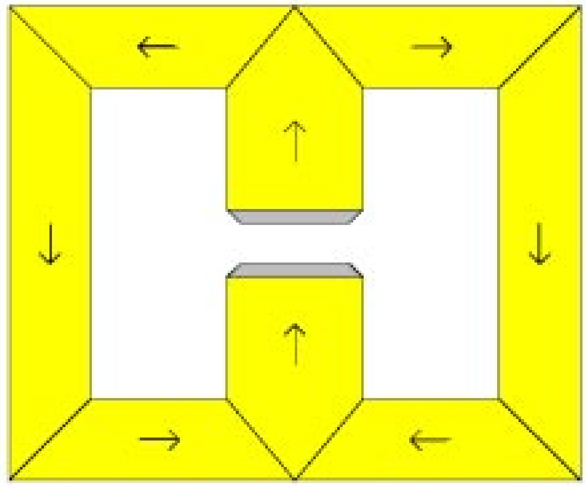

FIG. 39. A 2-D picture of an H frame magnet lamination with grain oriented 3% Si-Fe steel.The arrows show both the magnetic field and the grain direction.

rf experience could be used. HOM loads and wakefields are expected to vary as a^{-2} and λ^{-2} and $\sigma^{-1/2}$, where a is the cavity aperture, λ is the acceleration wavelength and σ is the bunch length [186,187]. Calculations indicate that the wakefields would limit bunch intensities to $\approx 2 \times 10^{12}$ with 1300 MHz superconducting rf in a recirculating linac accelerator scenario. The longitudinal dynamics is microtron-like or synchrotron-like and off-crest acceleration enables compensation of the linear part of the wakefields, with synchrotron-like phase stability [176].

E. Simulations

TABLE XI. Parameters of Acceleration for 4 TeV Collider

	Linac	RLA1	RLA2	RCS1	RCS2
E (GeV)	$0.1 \to 1.5$	$1.5 \to 10$	$10 \to 70$	$70 \to 250$	$250 \to 2000$
f_{rf} (MHz)	$30 \to 100$	200	400	800	1300
N_{turns}	1	9	11	33	45
V_{rf}(GV/turn)	1.5	1.0	6	6.5	42
C_{turn}(km)	0.3	0.16	1.1	2.0	11.5
Beam time (ms)	0.0013	0.005	0.04	0.22	1.73
$\sigma_{z,beam}$(cm)	$50 \to 8$	$4 \to 1.7$	$1.7 \to 0.5$	$0.5 \to 0.25$	$0.25 \to 0.12$
$\sigma_{E,beam}$(GeV)	$0.005 \to 0.033$	$0.067 \to 0.16$	$0.16 \to 0.58$	$0.58 \to 1.14$	$1.14 \to 2.3$
Loss (%)	5	7	6	7	10

A study [177] followed the longitudinal motion of particles through a similar sequence of recirculating accelerators (see table XI). Cavities similar to those proposed for TESLA [180] were assumed. Figure 40 shows, after optimization of parameters, the final longitudinal phase space distributions corresponding to wakefields estimated for four different bunch charges: a) very small, b) 0.83×10^{12} muons, c) 2.08×10^{12} muons, and d) 4.17×10^{12} muons.

For the design beam charge of 2×10^{12} muons (approximately as for Fig. 40(c)) the wakefield amplitude was estimated to be 2.5 MV/m, the accelerating phase was 35^o, and rf voltage depression 26%. The simulation used an initial longitudinal phase space of 20 eV-s. It gave negligible particle loss, a final longitudinal phase space of 21.6 eV-s, resulting in an increase of longitudinal emittance of only 8%.

F. Acceleration research needed

As discussed above, possible acceleration configurations have been developed, and critical longitudinal motion simulations have been performed. These calculations support the general feasibility of acceleration of muons from

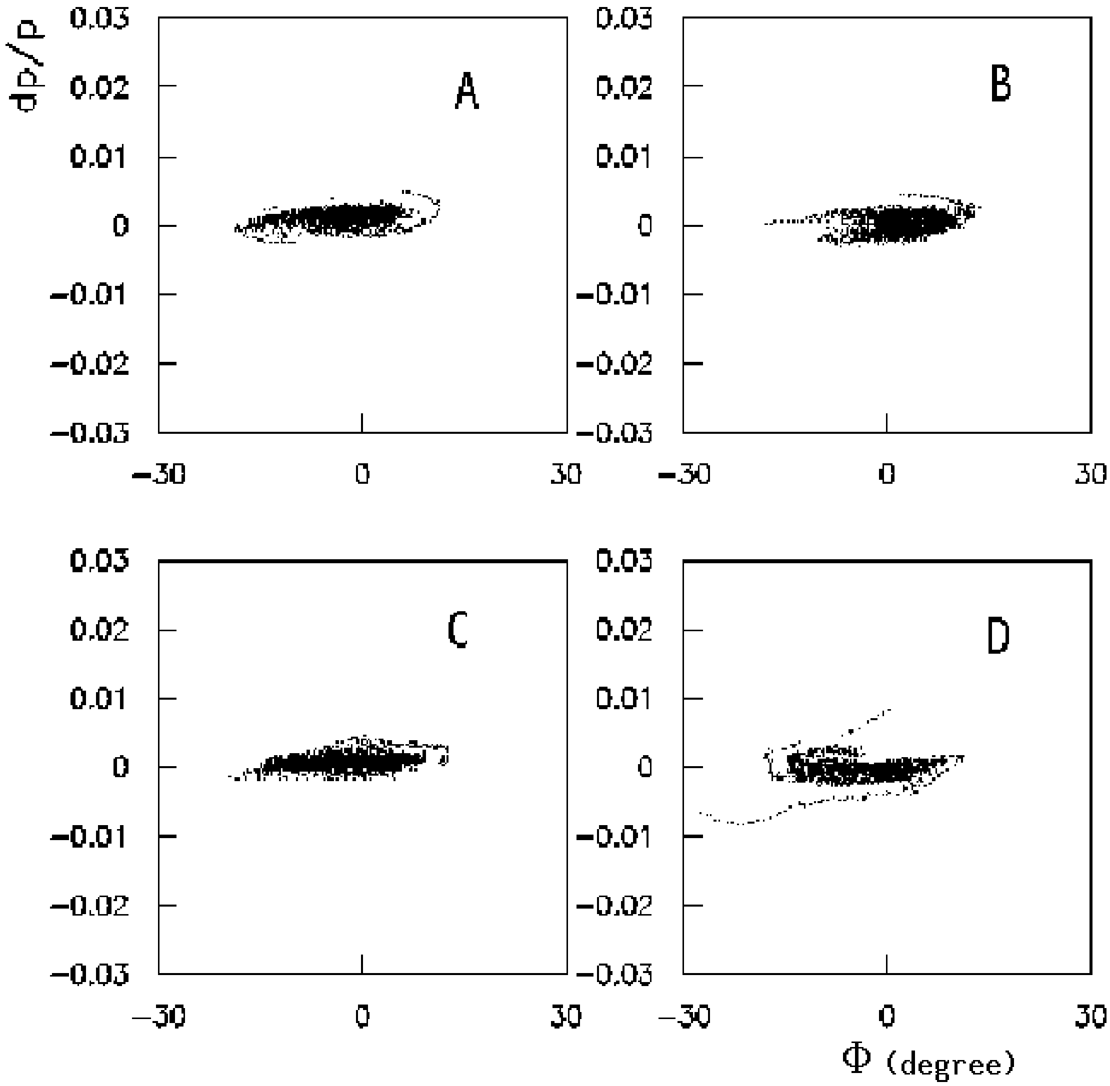

FIG. 40. Recirculating linac accelerator simulation results with wakefields, with beam accelerated from 200 to 2000 GeV in a 10-turn recirculating linac accelerator. Longitudinal phase space plots for different bunch charges: A) very small number; B) 0.83×10^{12}; C) 2.08×10^{12} and D) 4.17×10^{12} muons in a bunch.

cooling to collider energies. However the designs of acceleration systems have not been fully detailed and much work would be needed to obtain a buildable design. Complete transport lattices for linacs and return arcs have not yet been derived, and 6-D phase space tracking of beams through the accelerators has not been attempted. Also the geometry of combining and separating multi pass beams has not been worked out and optimized.

The rf requirements and systems have been specified at only the rudimentary requirements level, and have not been developed to a constructible level. Optimal configurations and choices of normal or superconducting rf must be developed, as well as more optimal choices in acceleration frequencies. The simple wakefield models used in the initial simulations should be expanded to obtain more realistic systems, and more precise calculations of wakefield effects must be developed.

Rapid accelerating systems have only been outlined at the simplest conceptual level. Prototype magnet design and testing are needed to test the limits of cycling rate and field strengths. Successful magnet concepts must then be specified in terms of stable beam transport configurations, including focusing and transport matching. While beam is stored for only a few turns, the individual bunch intensities are large enough that the possibility of single bunch instabilities must be considered and calculated. The larger number of passes in a recirculating linac accelerator places greater demands on the rf systems and higher order mode (HOM) loads, particularly for superconducting systems.