Muon Acceleration in FFAG Rings

E. Keil and A.M. Sessler

FFAG Workshop, BNL

13 to 17 Oct 2003

My WWW home directory:

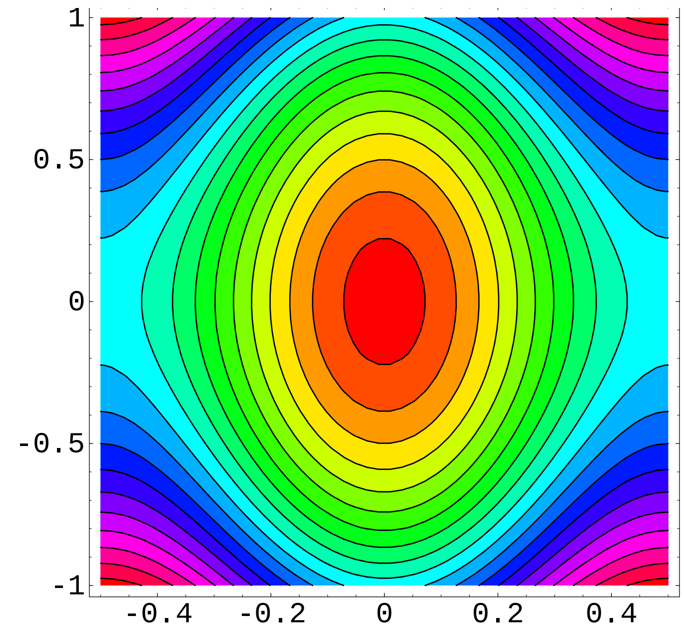
`http://keil.home.cern.ch/keil/MuMu/Doc/FFAG03/talk.pdf`

Scaled Longitudinal Hamiltonian

- Measure momentum offset y in units of half linear bucket height
- Measure phases φ in cycles with $-1/2 \leq \varphi \leq +1/2$
- For stationary buckets in FFAG rings
  - Stable fixed point at $\varphi = y = 0$
  - Unstable fixed points at $\varphi = \pm 1/2$ and $y = 0$
  - Hamiltonian

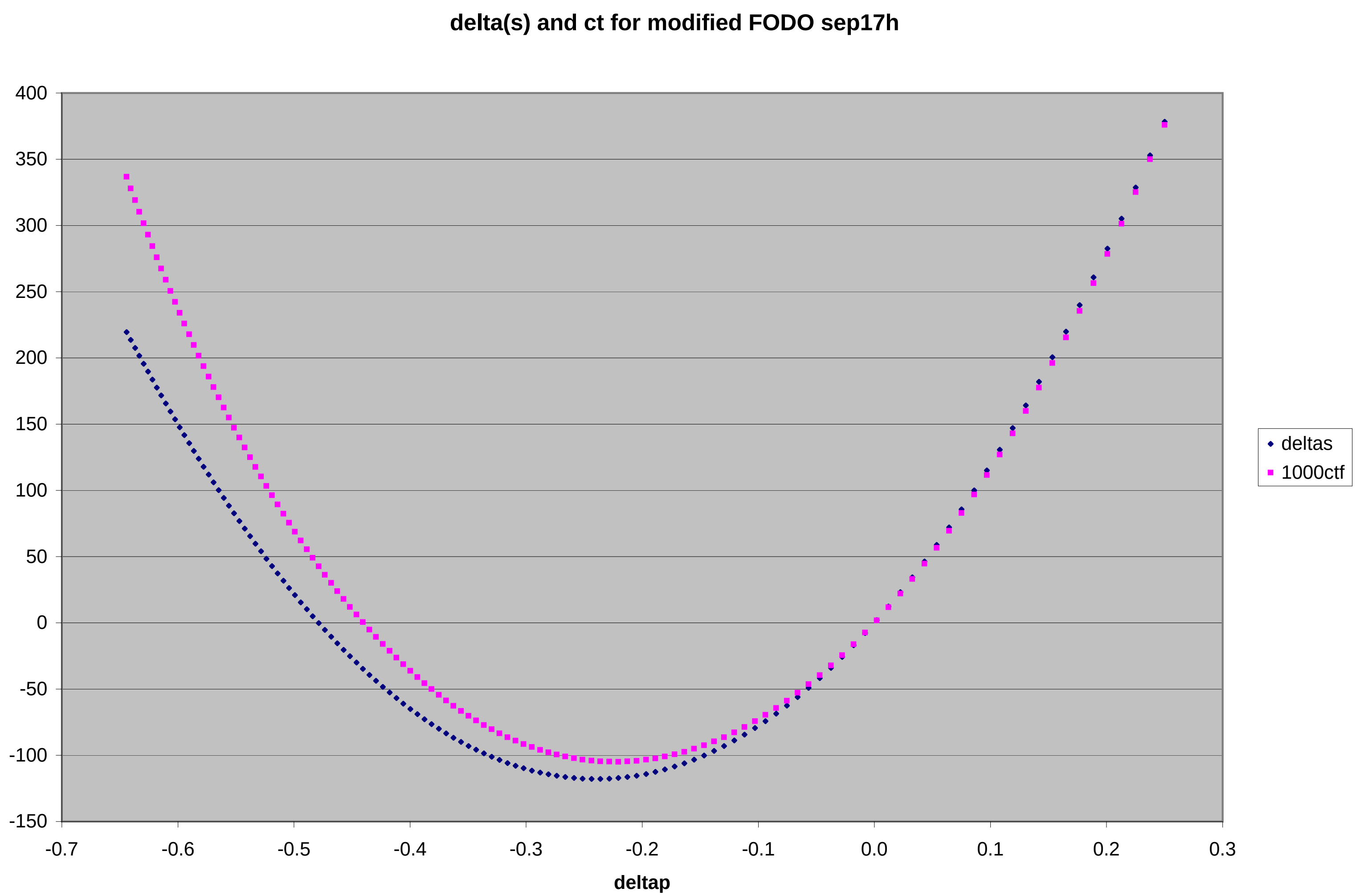
$$H(\varphi, y, a) = y^2 + \frac{2a}{3} y^3 + \sin^2 \pi\varphi$$

- Term in y^3 with coefficient a takes care of quadratic variation of travel time with relative momentum error δ

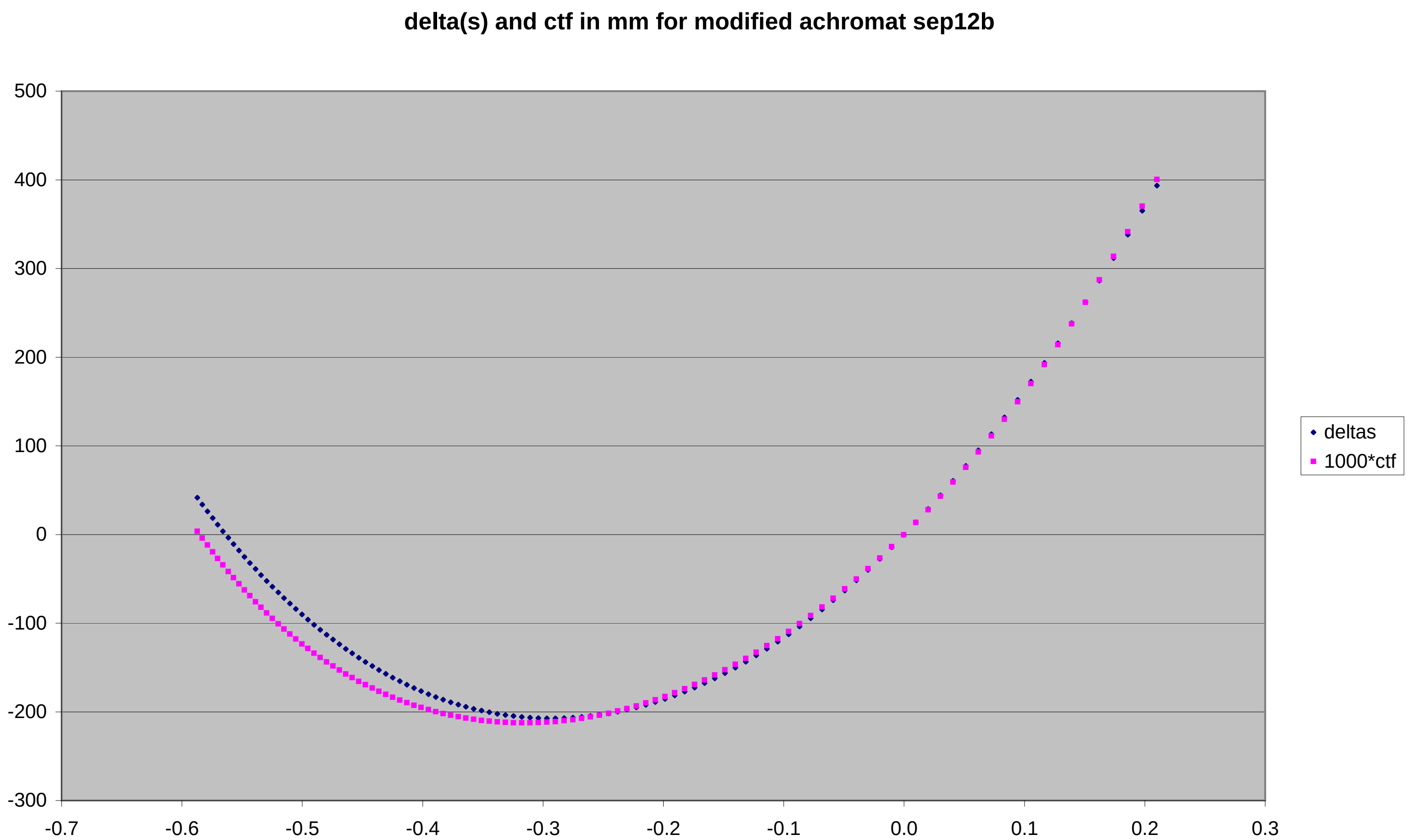


Contour plot of Hamiltonian for linear motion at $a = 0$. Muons move along level lines.

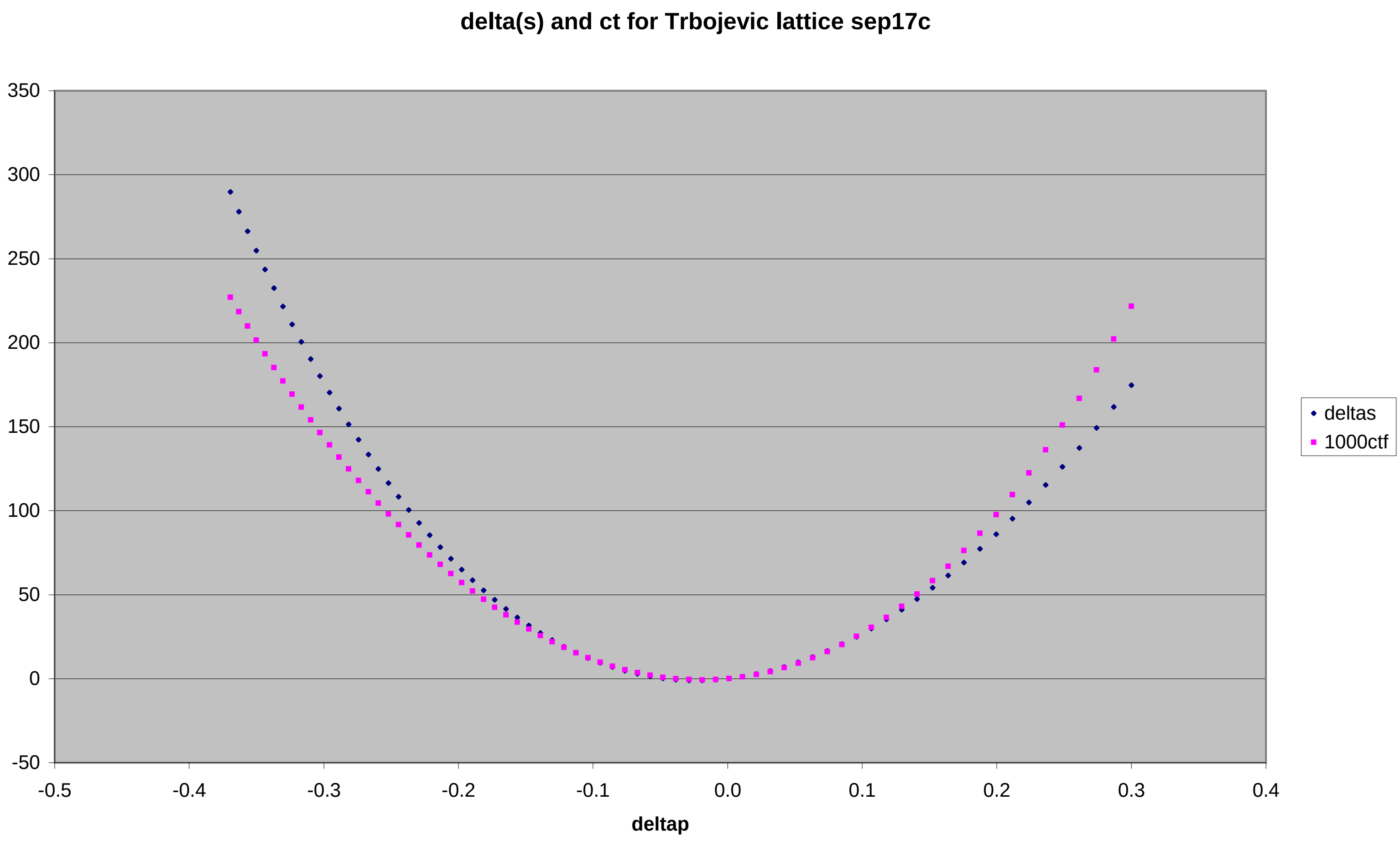
Path Length Variation in Modified FODO Lattice sep17h



Path Length Variation in Modified Achromat Lattice sep12b

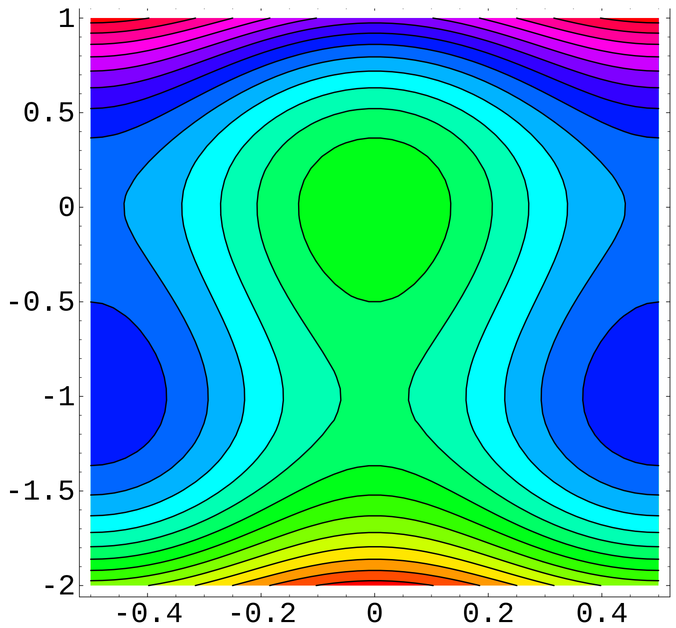


Path Length Variation in Trbojevic Lattice sep17c



Effect of $a \neq 0$ on Longitudinal Hamiltonian

- New stable fixed points at $\varphi = \pm 1/2$ and $y = -1/a$
- New unstable fixed point at $\varphi = 0$ and $y = -1/a$
- Ω-shaped trajectories start below fixed point at $\varphi = \pm 1/2$ and $y = -1/a$, circle around fixed point at $\varphi = 0$ and $y = 0$, and reach maximum y above it
- Acceleration in FFAG rings happens along these trajectories
- Find limit on a for Ω-shaped trajectories



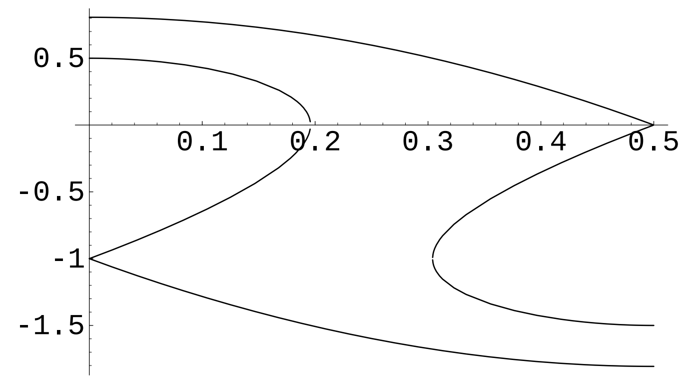
Contour plot of Hamiltonian at $a = 1$. Acceleration along light blue Ω-shaped trajectories.

Separatrices I

- Separatrices pass unstable fixed points
- 2 unstable fixed points and 2 separatrices when $a \neq 0$
- Find separatrices by solving for y:

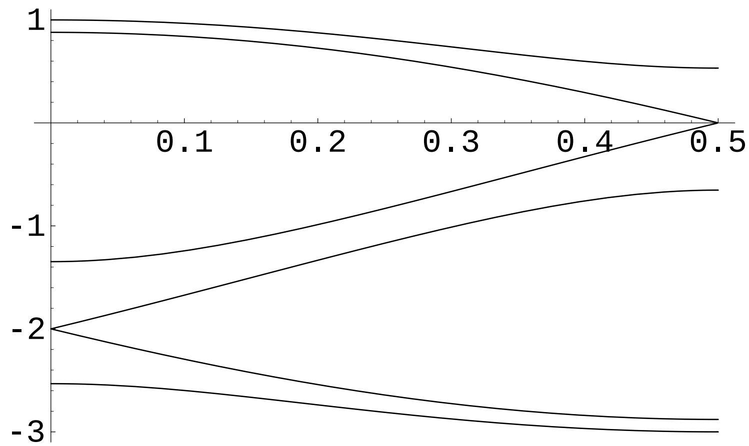
$$H(\varphi, y, a) = H(-1/2, 0, a)$$
$$H(\varphi, y, a) = H(0, -1/a, a)$$

- Use symmetry and plot for $0 \leq \varphi \leq 1/2$
- Acceleration along trajectories in S-shaped channel between islands starts between separatrices in lower right corner below $y = -3/2a$, and ends between separatrices in upper left corner above $y = 1/2a$

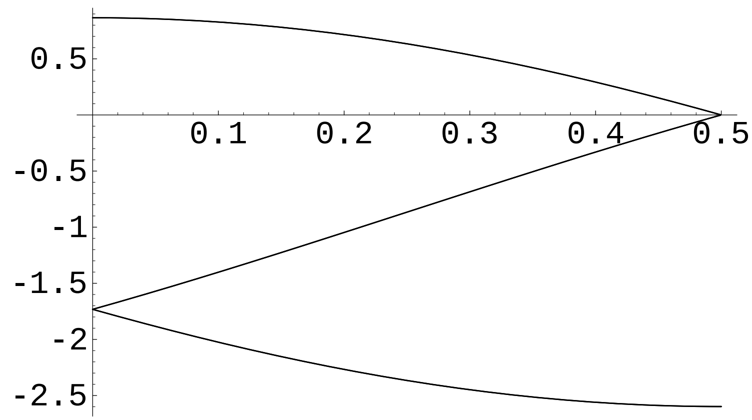


Separatrices at $a = 1$

Separatrices II



Separatrices at $a = 1/2$



Separatrices at $a = 1/\sqrt{3}$

- At $a = 1/2$ regular bucket centred at $\varphi = y = 0$ blocks acceleration across $y = 0$
- At $a = 1/\sqrt{3}$ buckets centred at $\varphi = y = 0$ and at $\varphi = 1/2$ and $y = -\sqrt{3}$ just touch, and channel of acceleration has width zero, agreeing with K.Y.Ng's result in Handbook, and possibly earlier writings

Momentum Scaling

- Used scaled momentum variable y with $y = 0$ at reference momentum p_r
- Obtained values y_i at injection and y_e at ejection
- Scale such that y_i coincides with assumed injection momentum p_i
- Scale such that y_e coincides with assumed ejection momentum p_e
- Get two relations of form

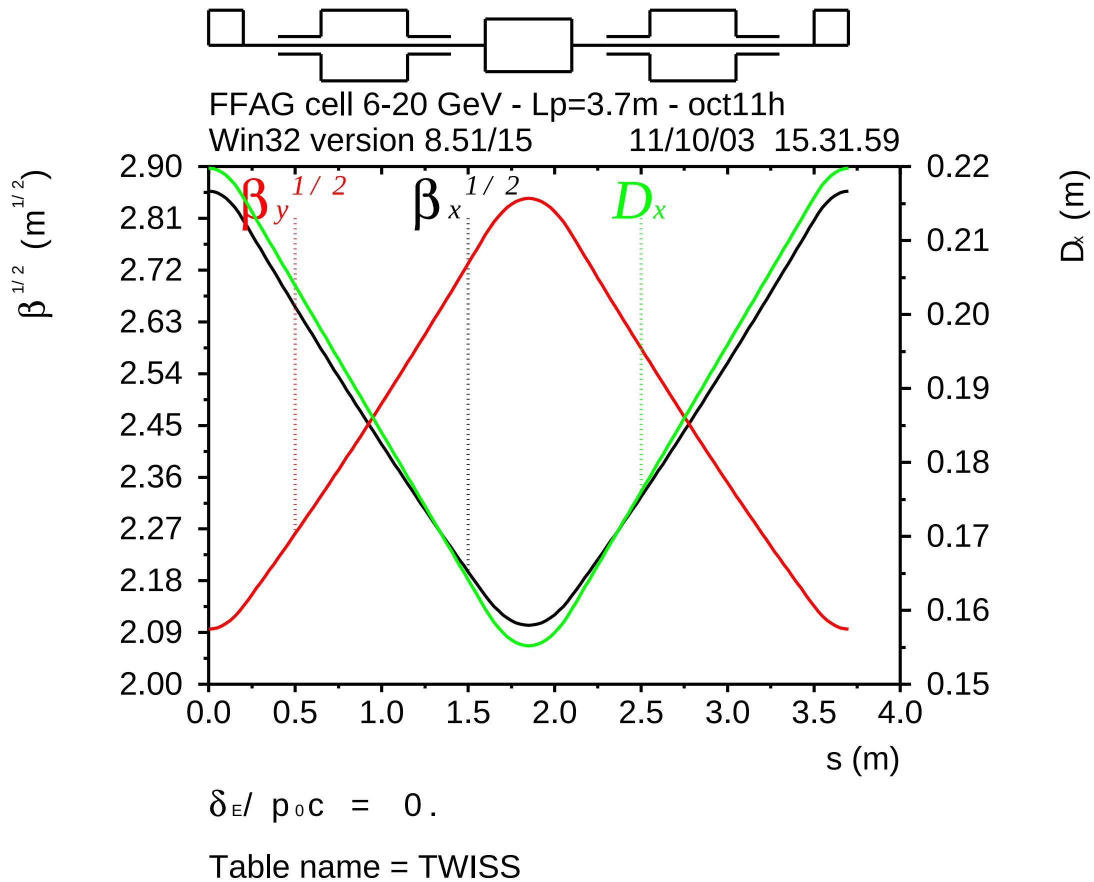
$$p = p_r + y\mathrm{d}p/\mathrm{d}y$$

- From orbit program get relative momentum offset δ_0 for second zero of travel time
- From longitudinal dynamics know $y_0 = -1/a$, and get third relation

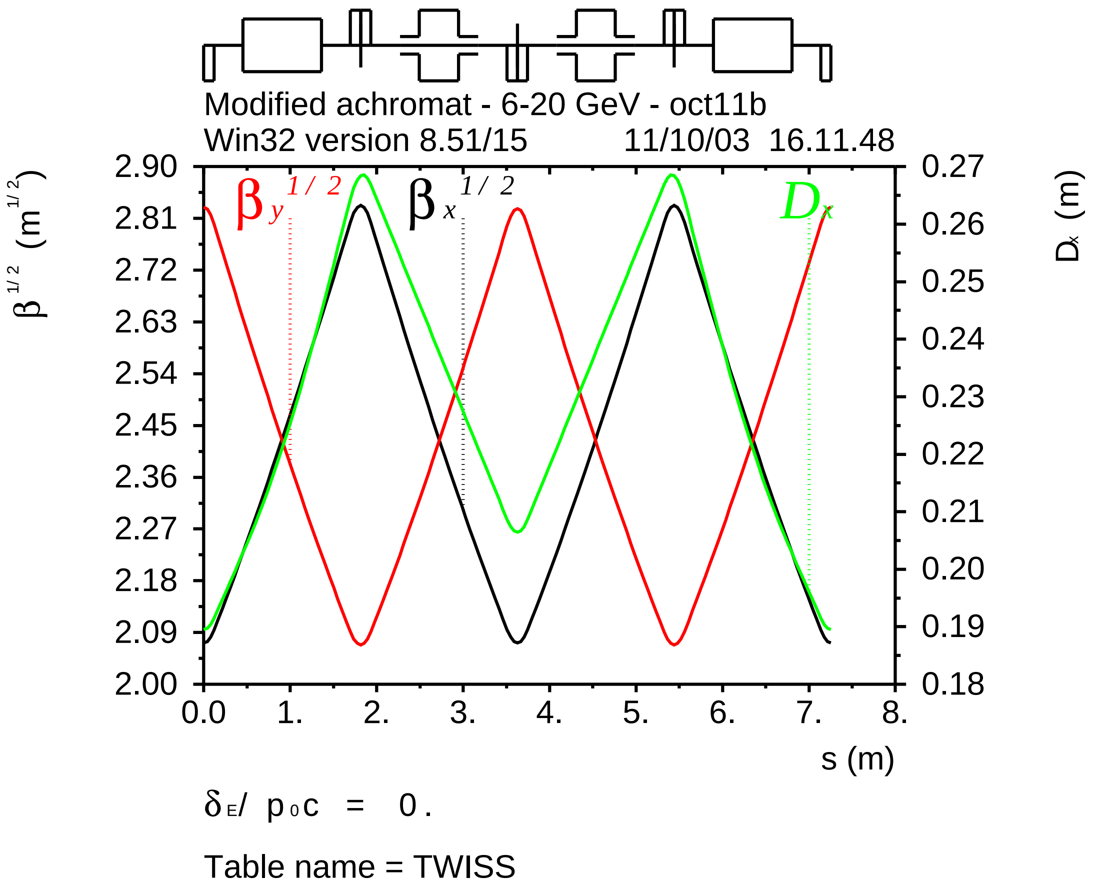
$$p_r\delta_0 = y_0\mathrm{d}p/\mathrm{d}y$$

- Have 3 equations in 3 unknowns p_r, $\mathrm{d}p/\mathrm{d}y$ and a

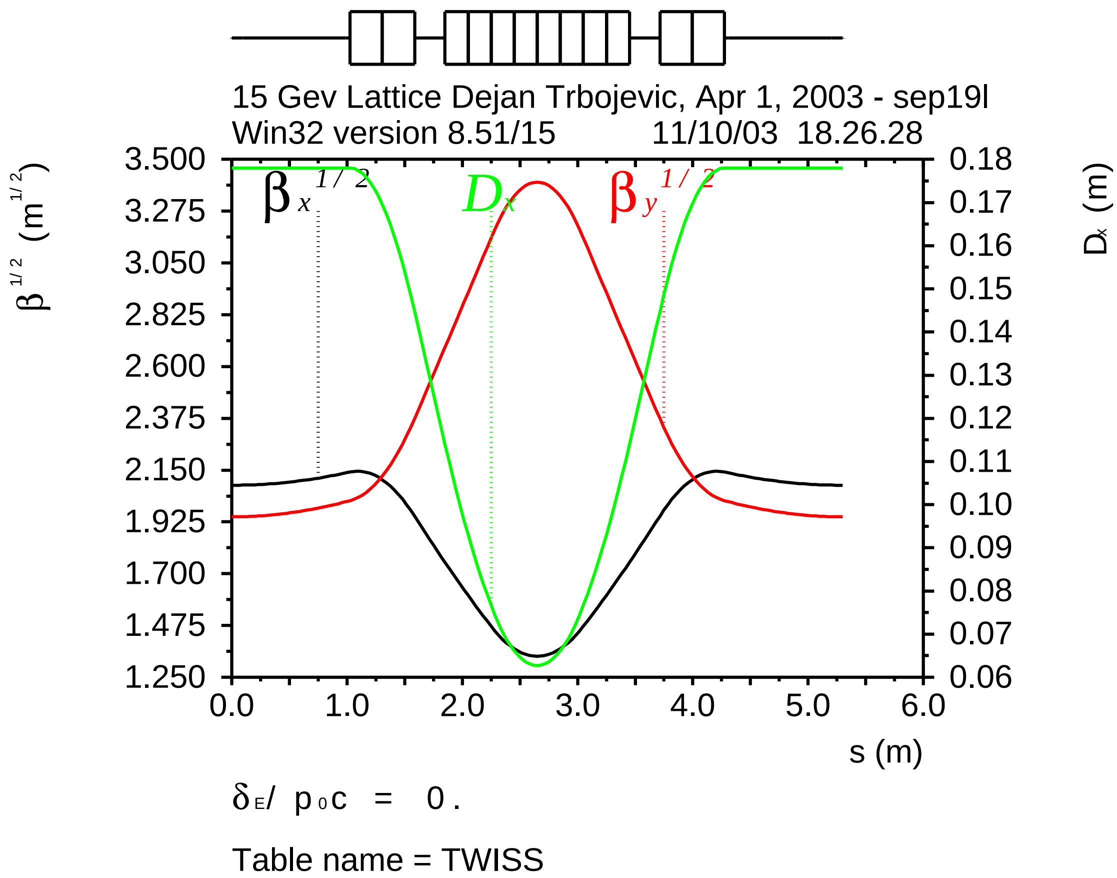
Modified FODO Lattice oct11h



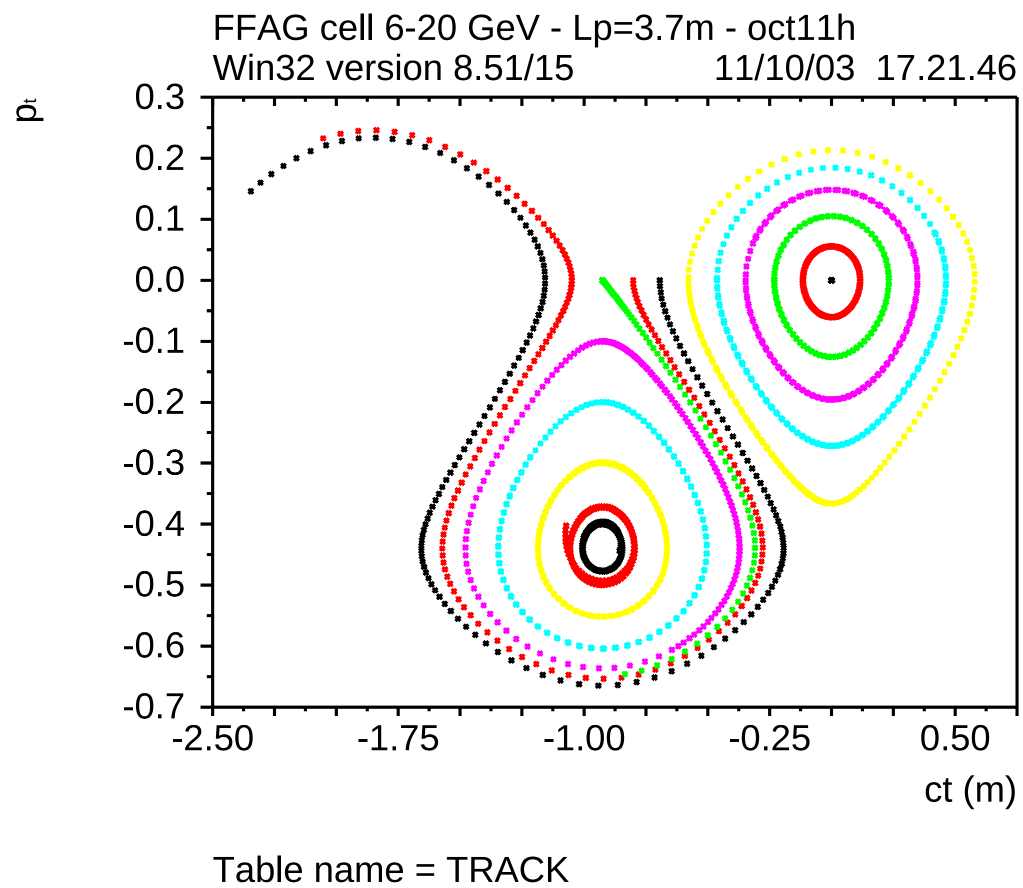
Modified Achromat Lattice oct11b



Trbojevic's Triplet Lattice oct11c

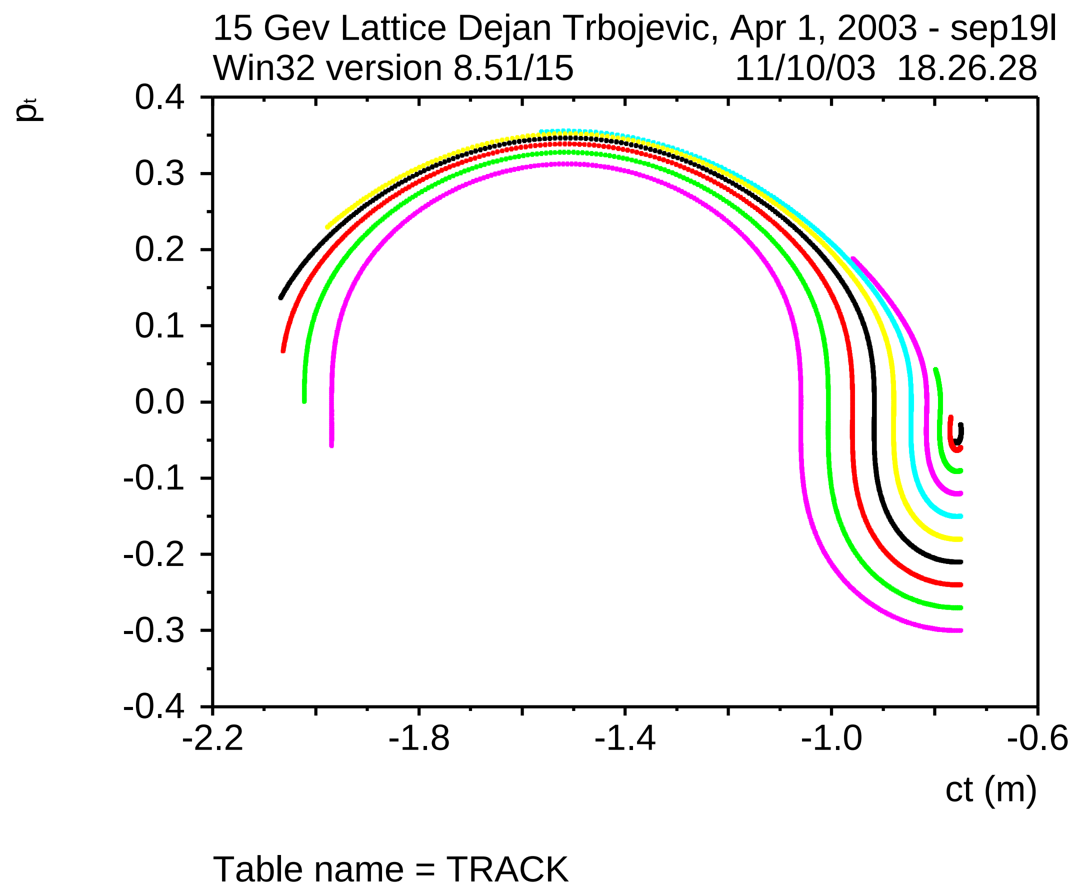


Modified FODO Lattice oct11h



2.5 MV/RF cavity, 30 turns, 5 dots/turn

Trbojevic's Triplet Lattice sep19l



10 MV/RF cavity, 15 turns, 30 dots/turn

Trbojevic's Triplet Lattice sep19l