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FFAG lattice Without Opposite Bends with Distributed RF

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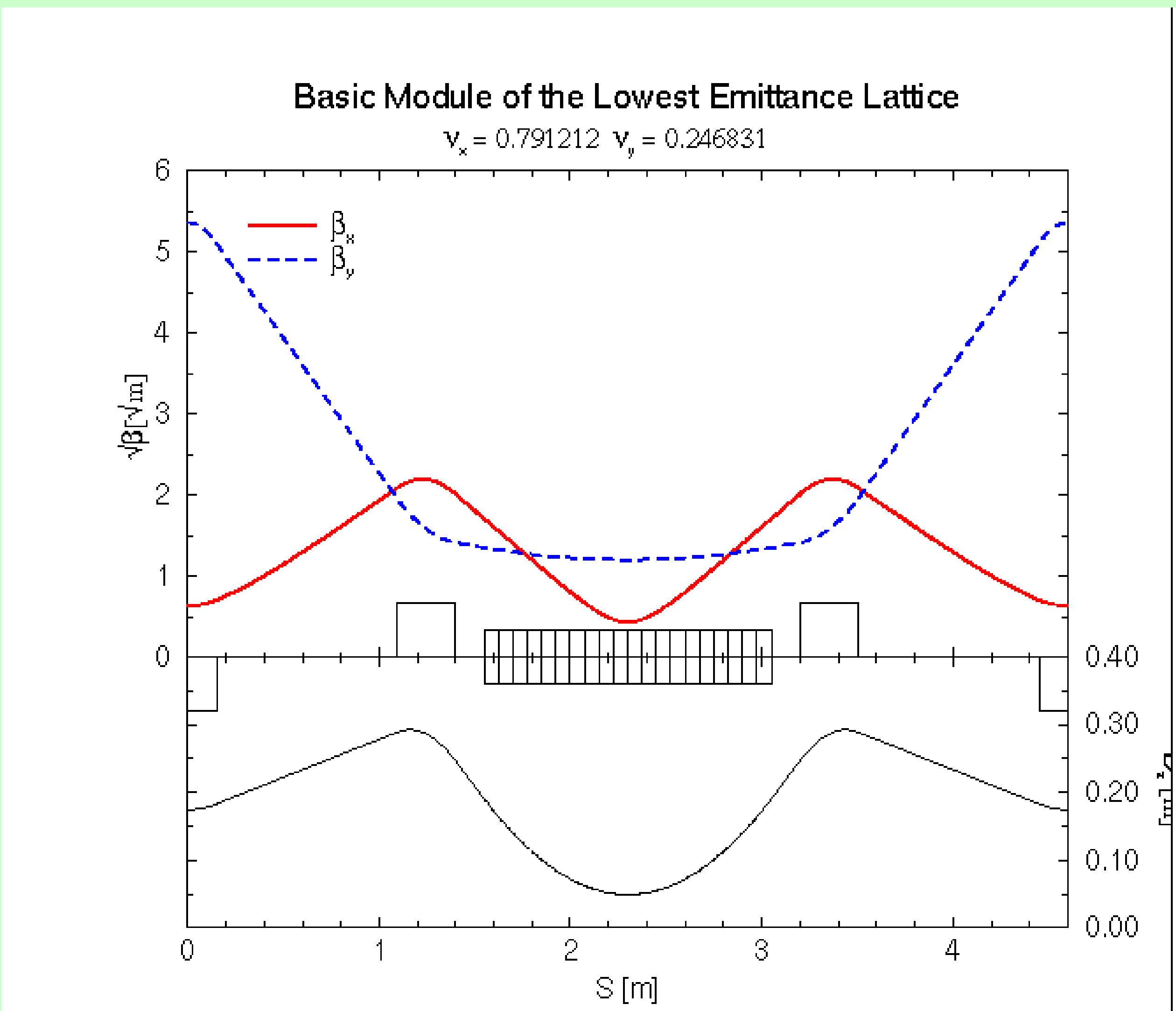
CONTENT:

Update with our lattice design:

- **A distributed RF looks very promising.**
- **Tools: SYNCH, COSY, MAD**
- **Lattice properties – a ring picture.**
- **Longitudinal simulation of the acceleration with the latest lattice solutions (Mike Blaskiewicz).**
- **Conclusions**

This is an old slide as a reminder of the the Montauk 99 presentation: a relevance to the minimum emittance lattice and muon acceleration lattice.

- The minimum emittance lattice requires reduction of the function H:
  - The normalized dispersion amplitude corresponds to the $\langle H \rangle^{1/2}$!!!



What are the basic parameters?

- Required Range of Energies (or dp/p)
  - the "central" energy or momentum p_o is in two examples presented later set to 10 GEV. The acceleration would be possible from 10 GeV up to 20 GeV.
  - Aperture limitation is defined by the maximum value of the DISPERSION function:
    - $\Delta x < +/- 30$ mm
    - if the $0.5 < dp/p < 1.5$ then:
      - **$D_x < 60$ mm**
- Why is the Minimum Emittance Lattice for the electronic Storage Rings Relevant?
  - The normalized dispersion amplitude Corresponds to the $\langle H \rangle^{1/2}$!!!

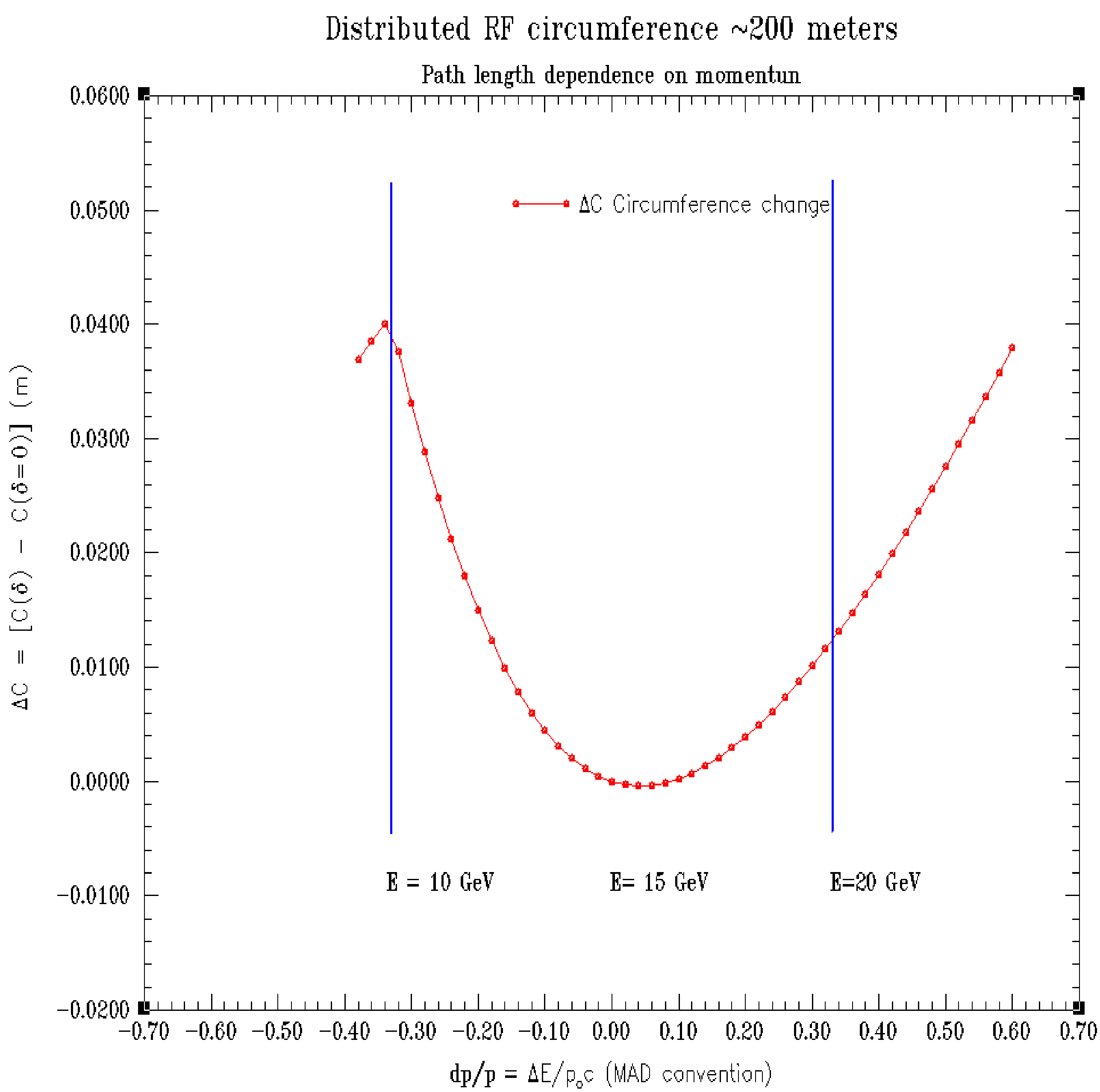
What was our promise given at the last meeting (BNL editors meeting):

- Construct a lattice where the dispersion will oscillate between positive and negative values but not exceeding 6 cm without opposite bending magnets.

  $\Delta x < D\ dp/p = 0.06 * (+-0.5) = +-0.03$ m

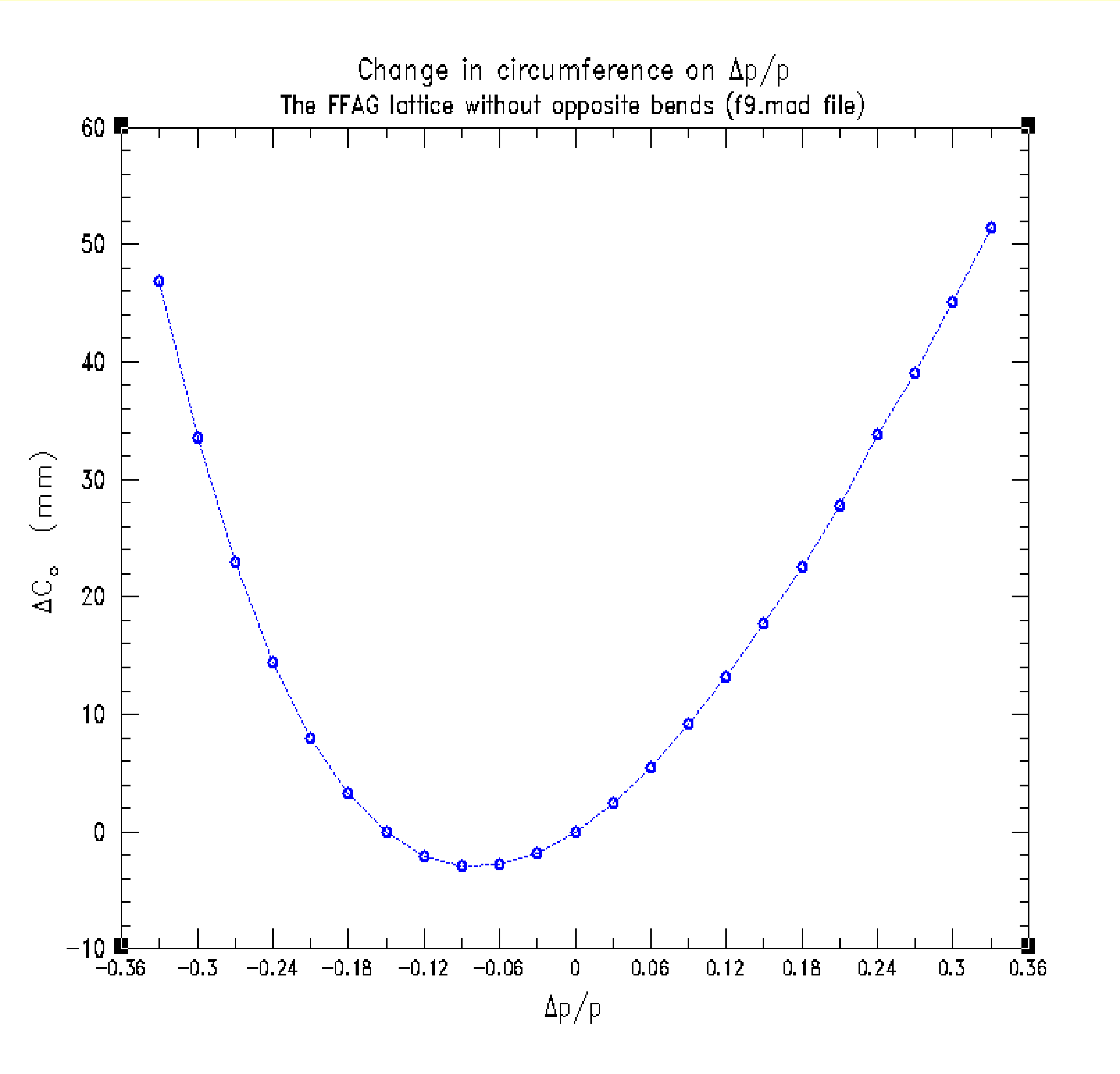
- Make a change in the circumference smaller to reduce the RF phase change.
- Try to combine the linac with a single arc.
- Or make enough room for the cavities within the ring.
- Longitudinal simulation of the multiple turns (10 – 20 turns)

The major result: reduced change of the circumference the 'SYNCH' result (with Ernie's combined function dipole subroutine correction)

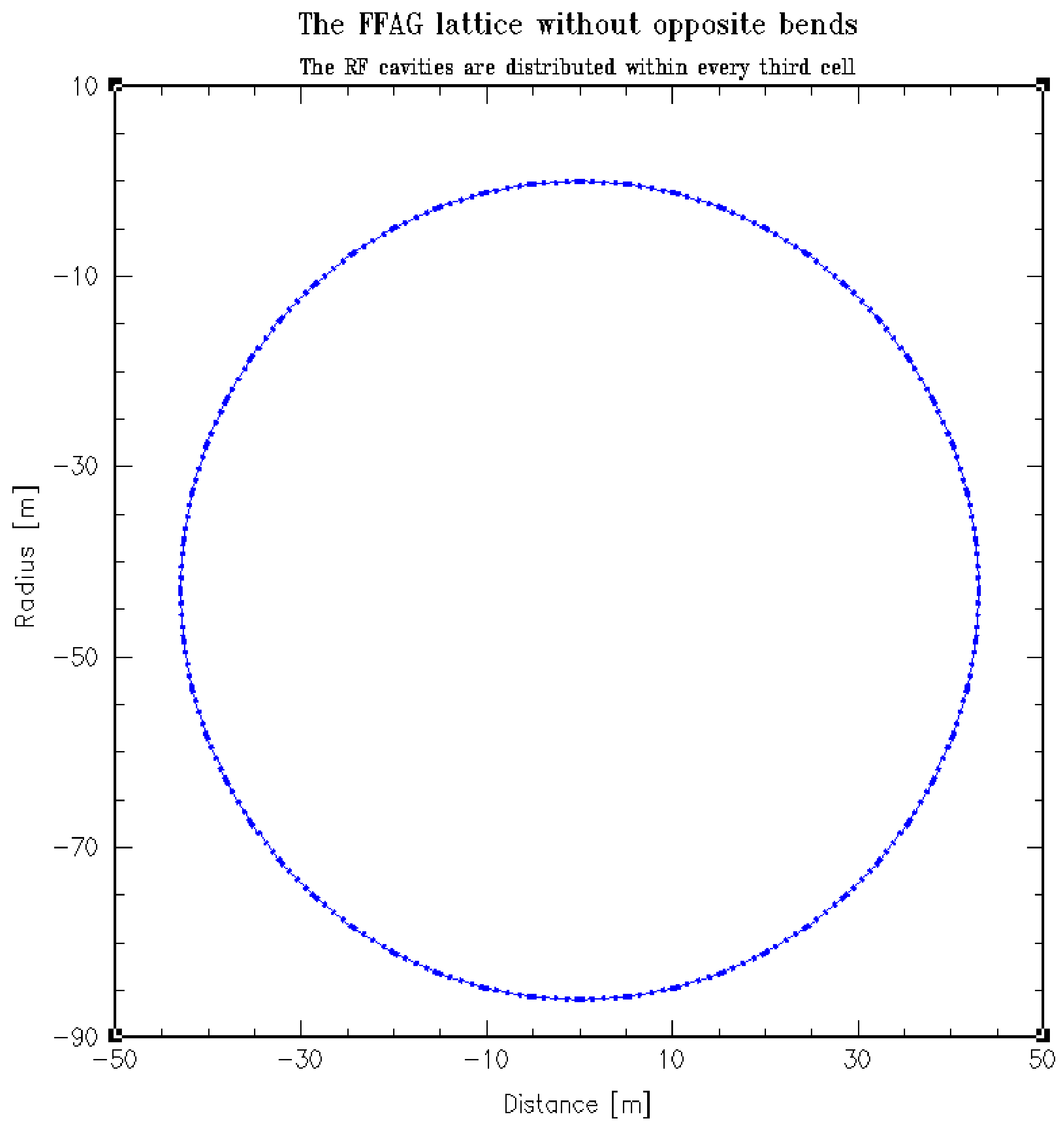


TEST data for different tools: SYNCHROTRON

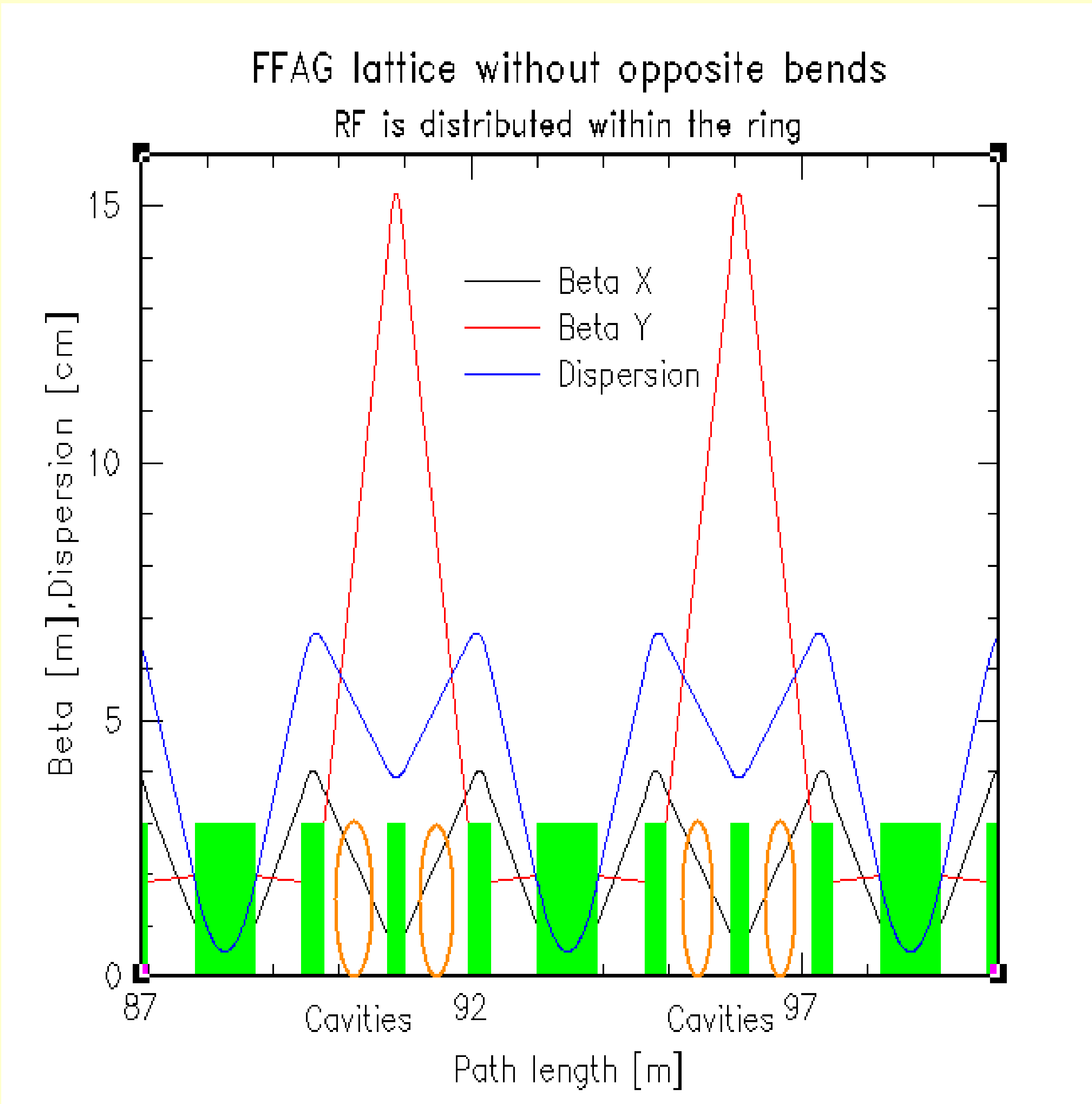
The major result: reduced change of the circumference the 'MAD' file result



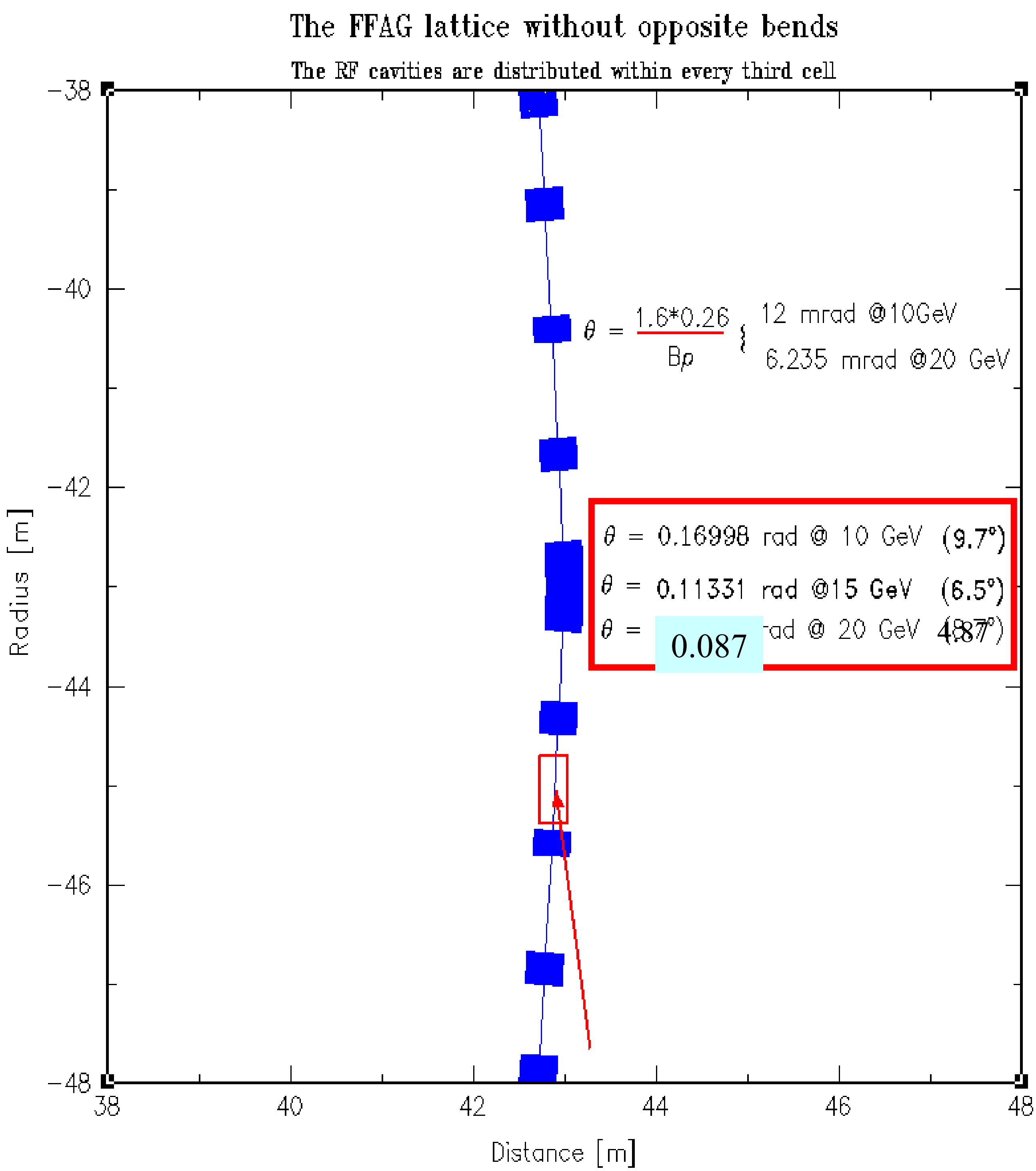
Picture of the 'MAD' ring



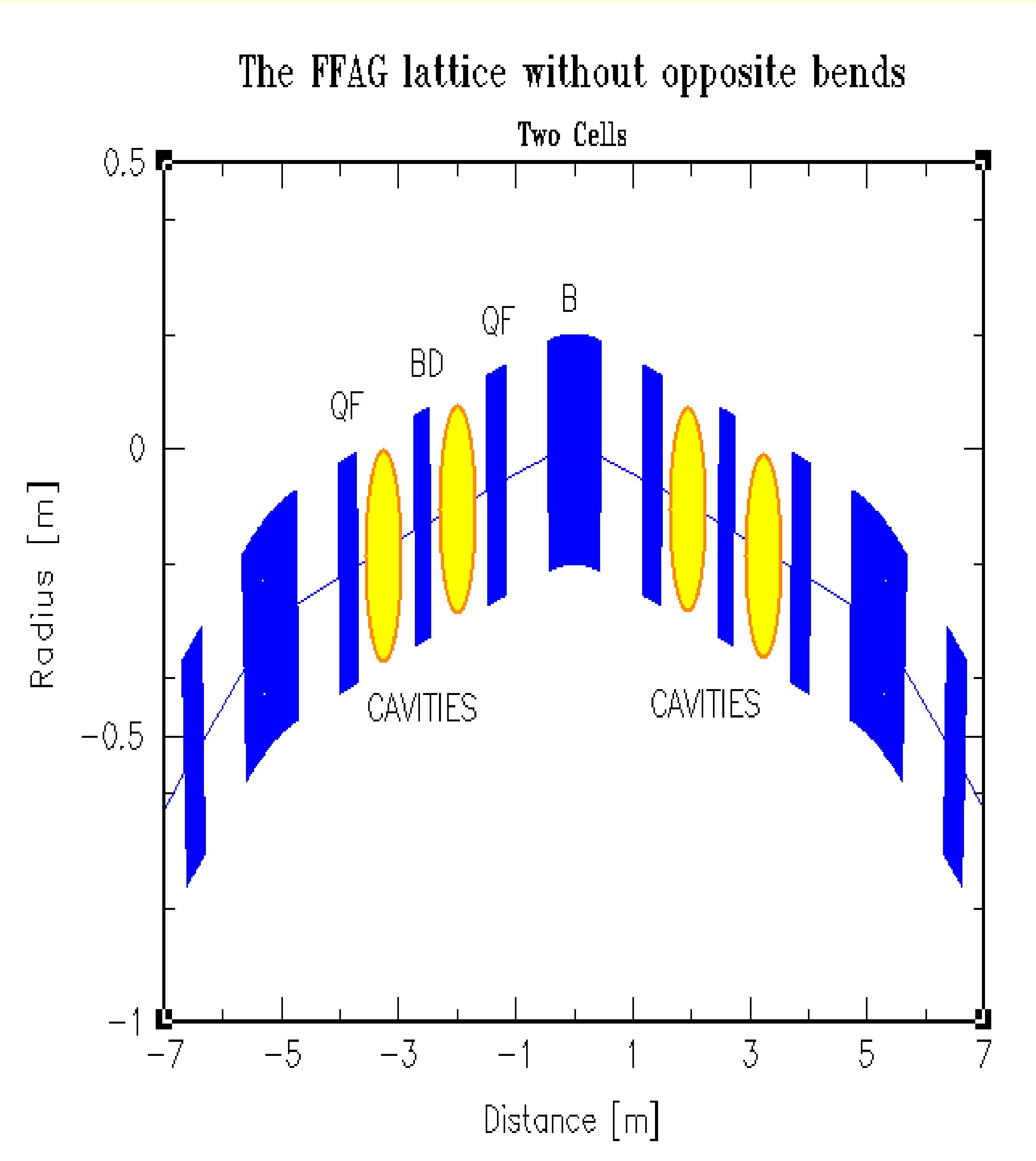
Betatron functions within the two cells



A part of the ring

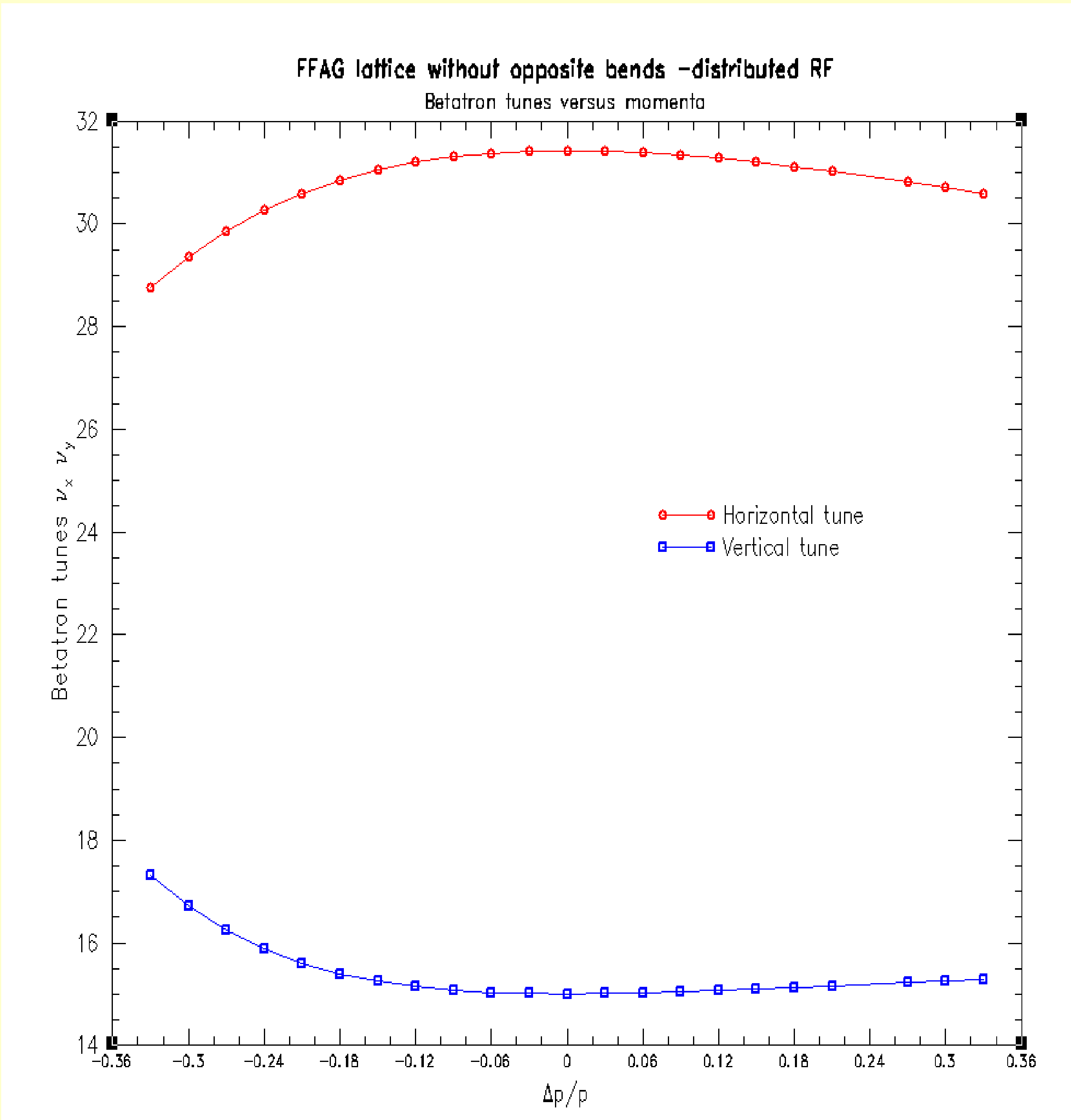


Two CELLS:

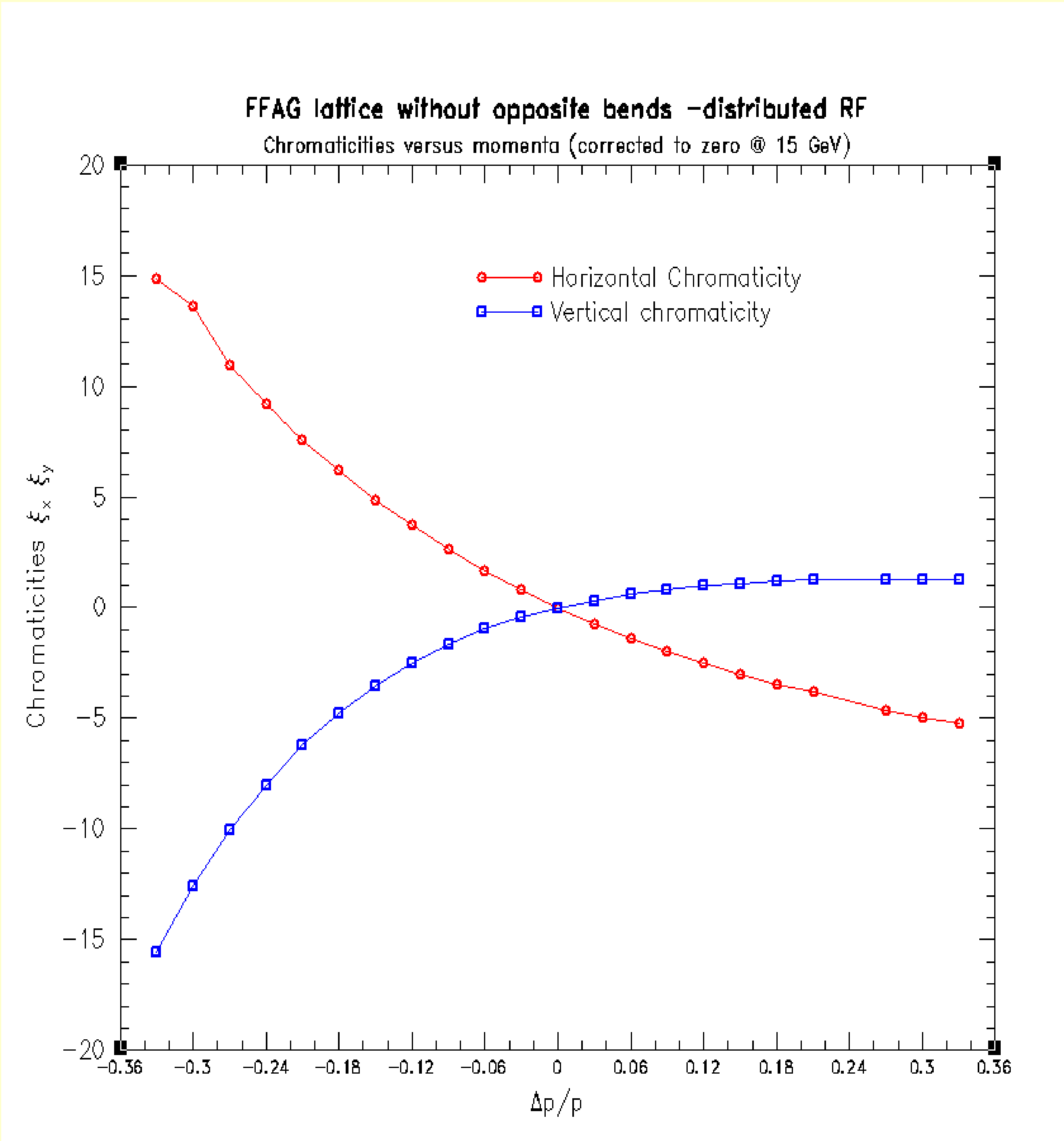


The FFAG lattice without opposite bends
Two Cells
QF
BD
QF
B
CAVITIES
CAVITIES
0.5
0
−0.5
−1
Radius [m]
−7
−5
−3
−1
1
3
5
7
Distance [m]

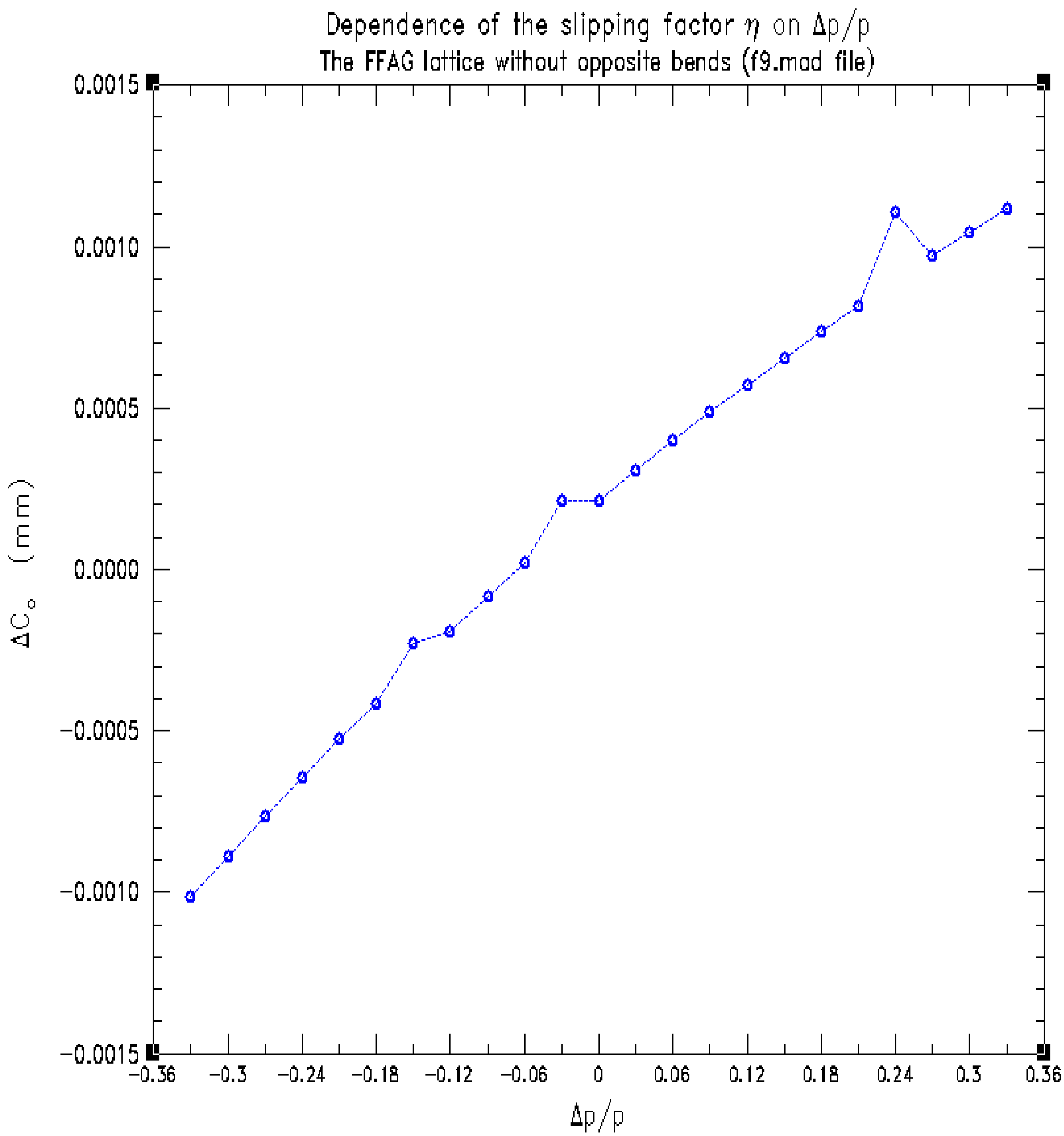
Betatron tunes during acceleration



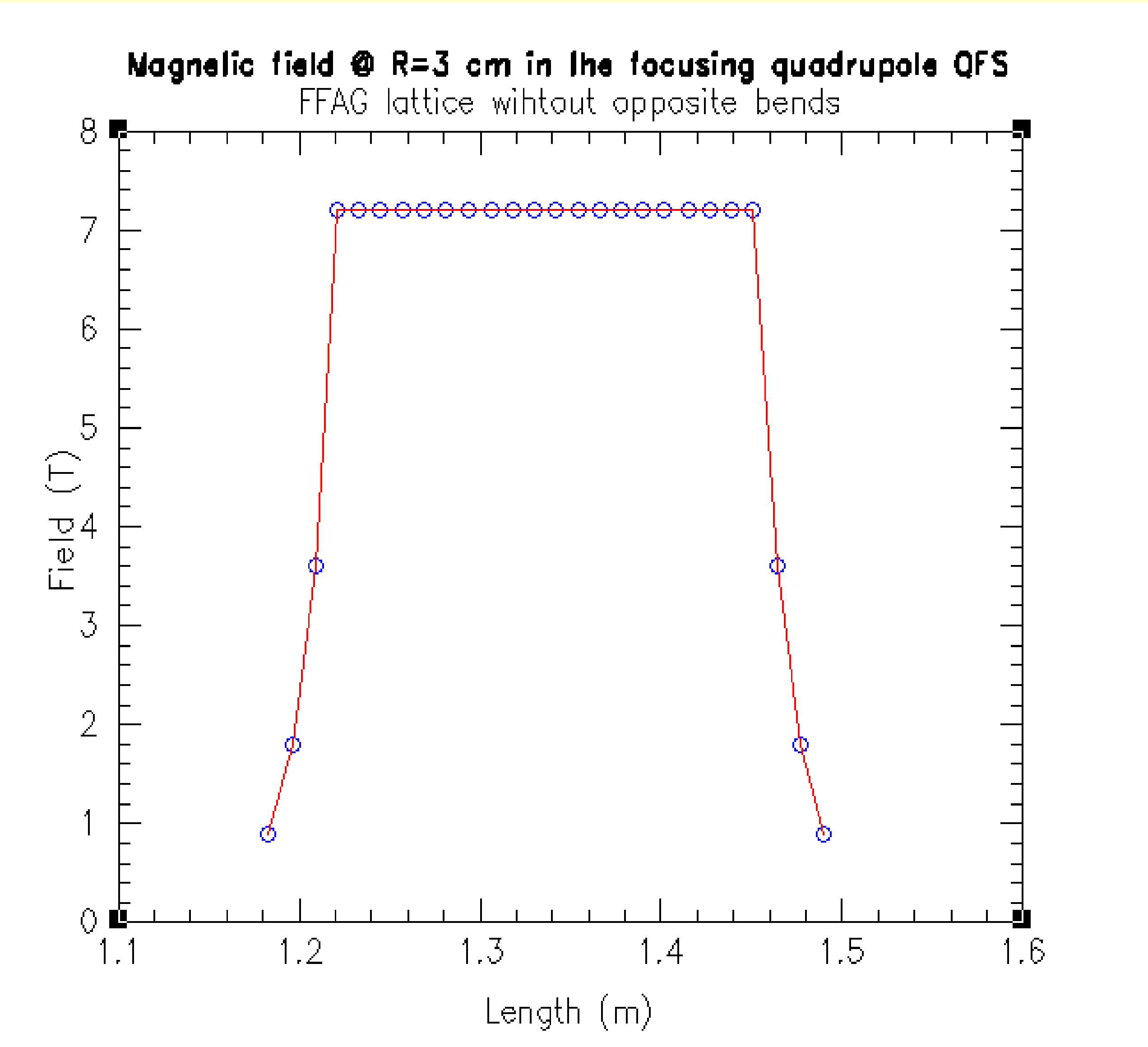
Chromaticities during acceleration – Corrected to zero at the central muon energy of 15 GeV



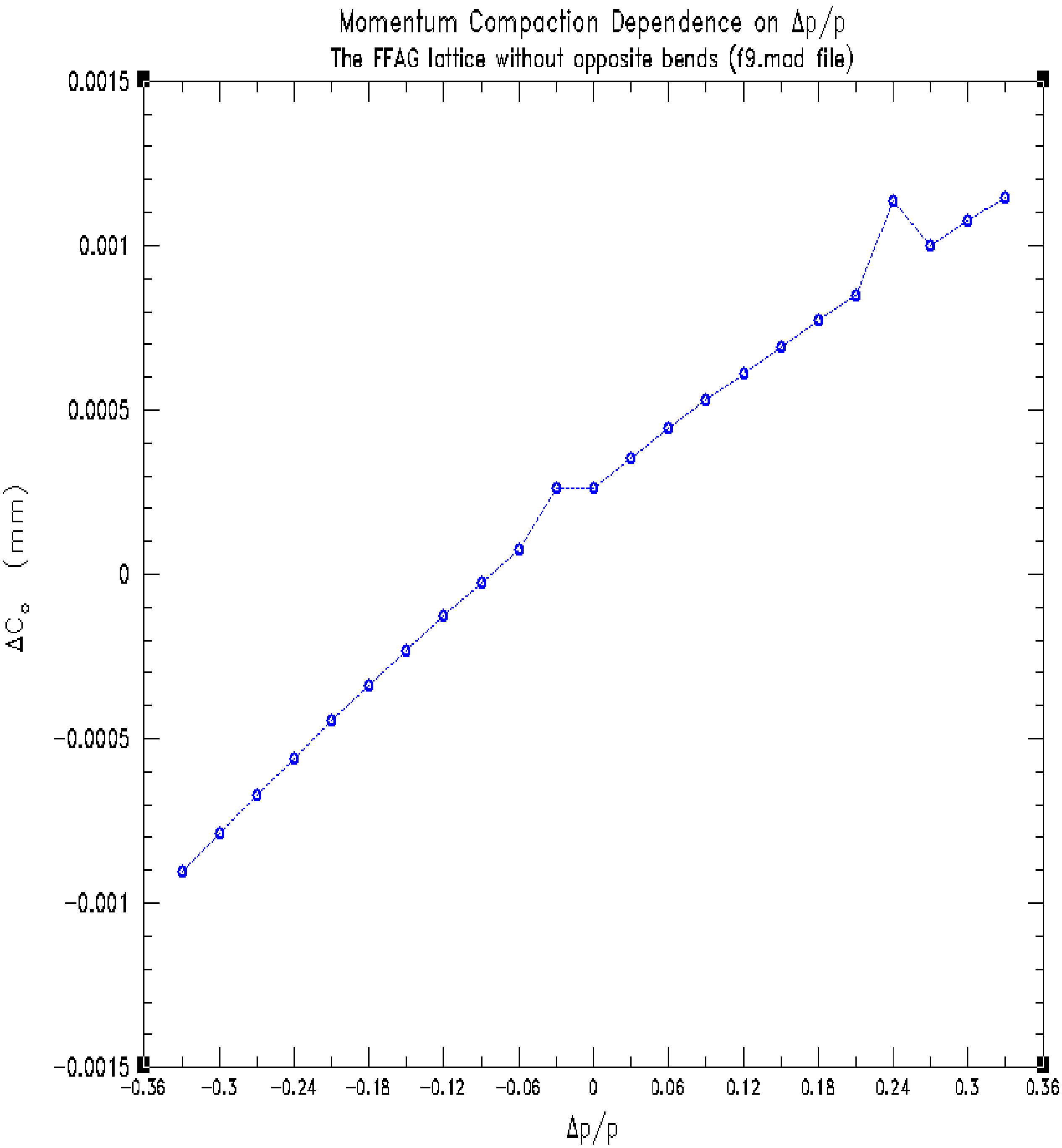
The slipping factor η during acceleration



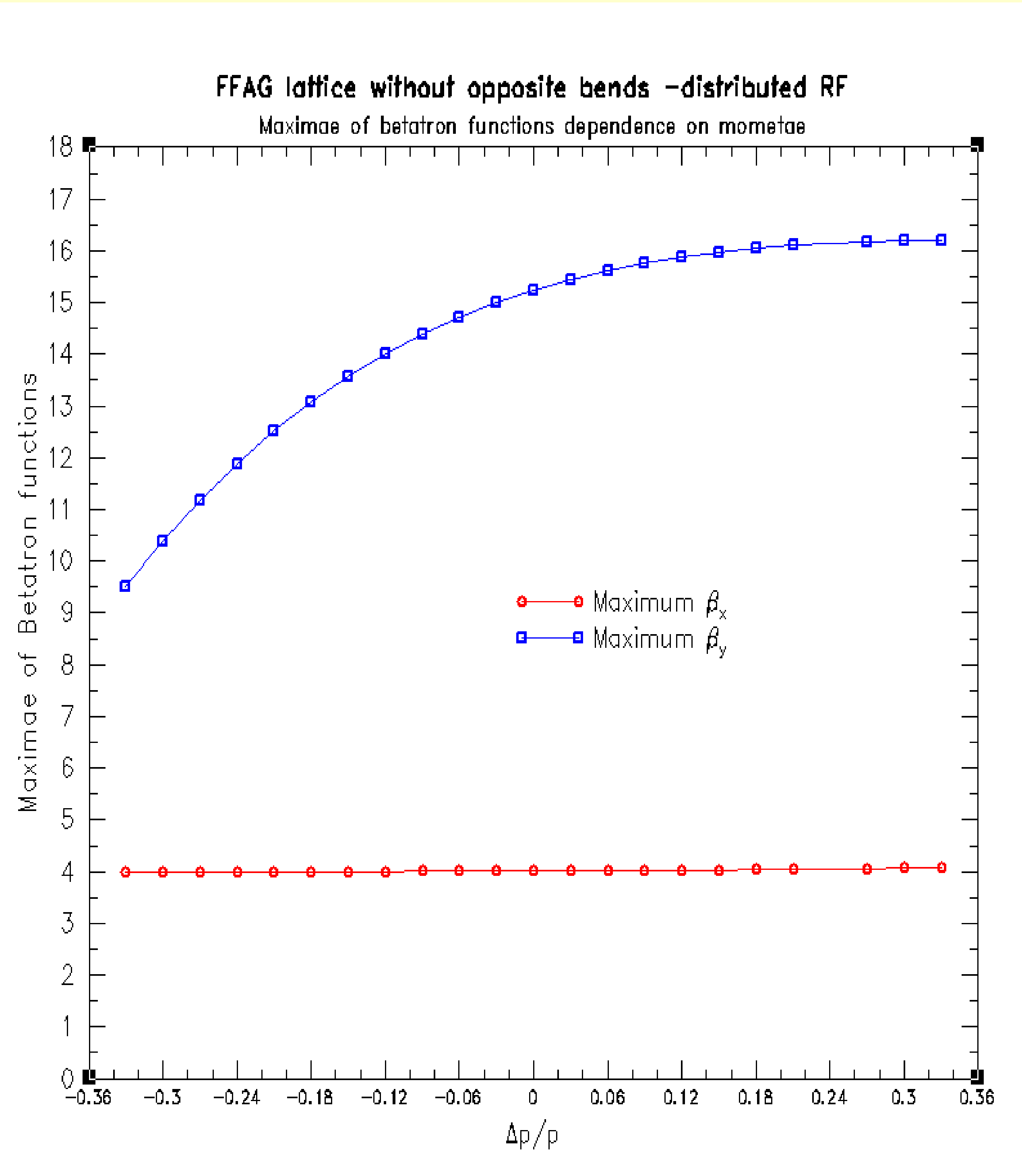
All previous results have a ~1m dipole divided into 100 pieces and quadrupoles divided into 26 and 46 pieces, as well they include the first attempt to include the end of the quadrupole field



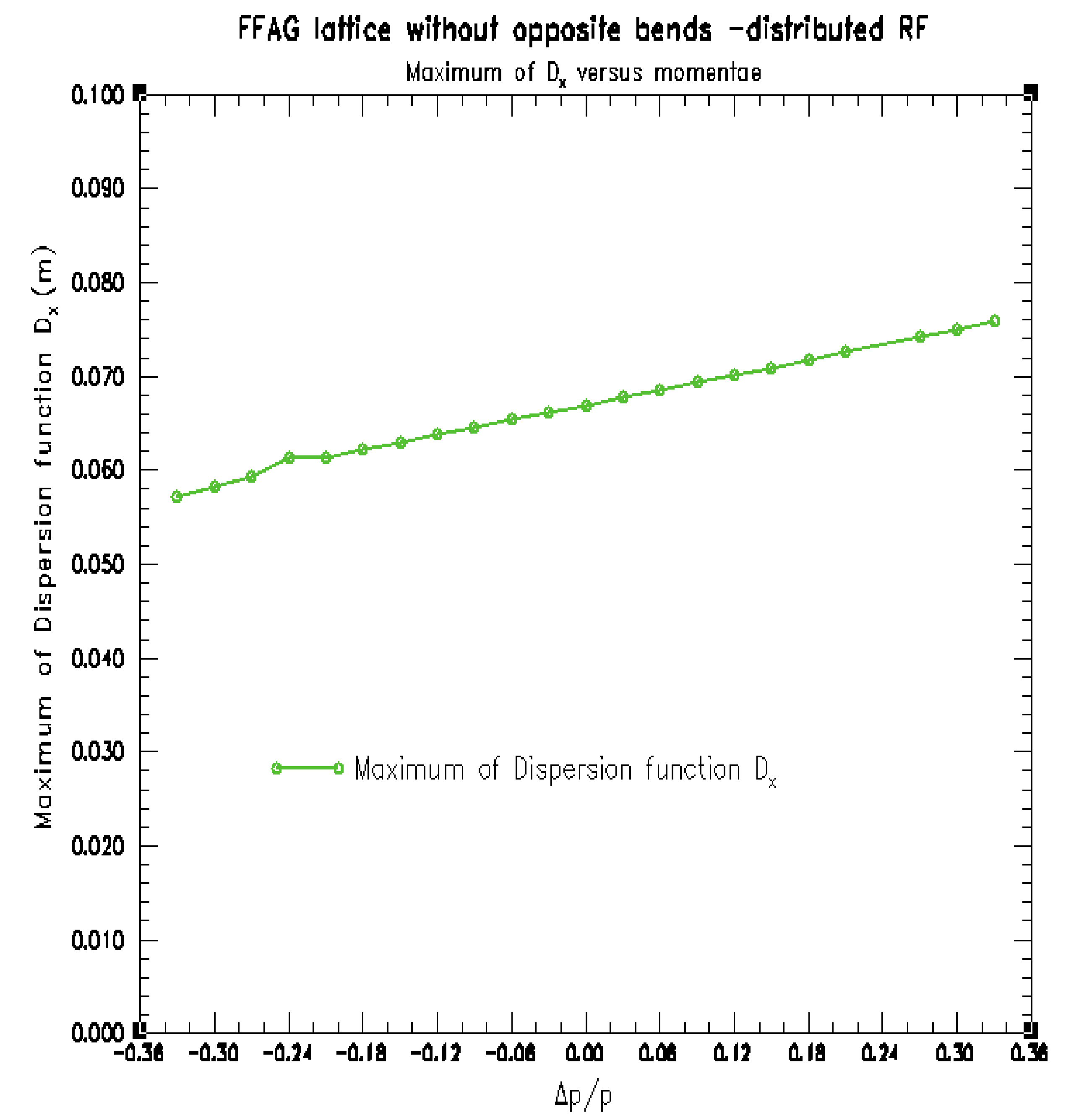
At negative Δp/p lattice is ‘imaginary γ_t’



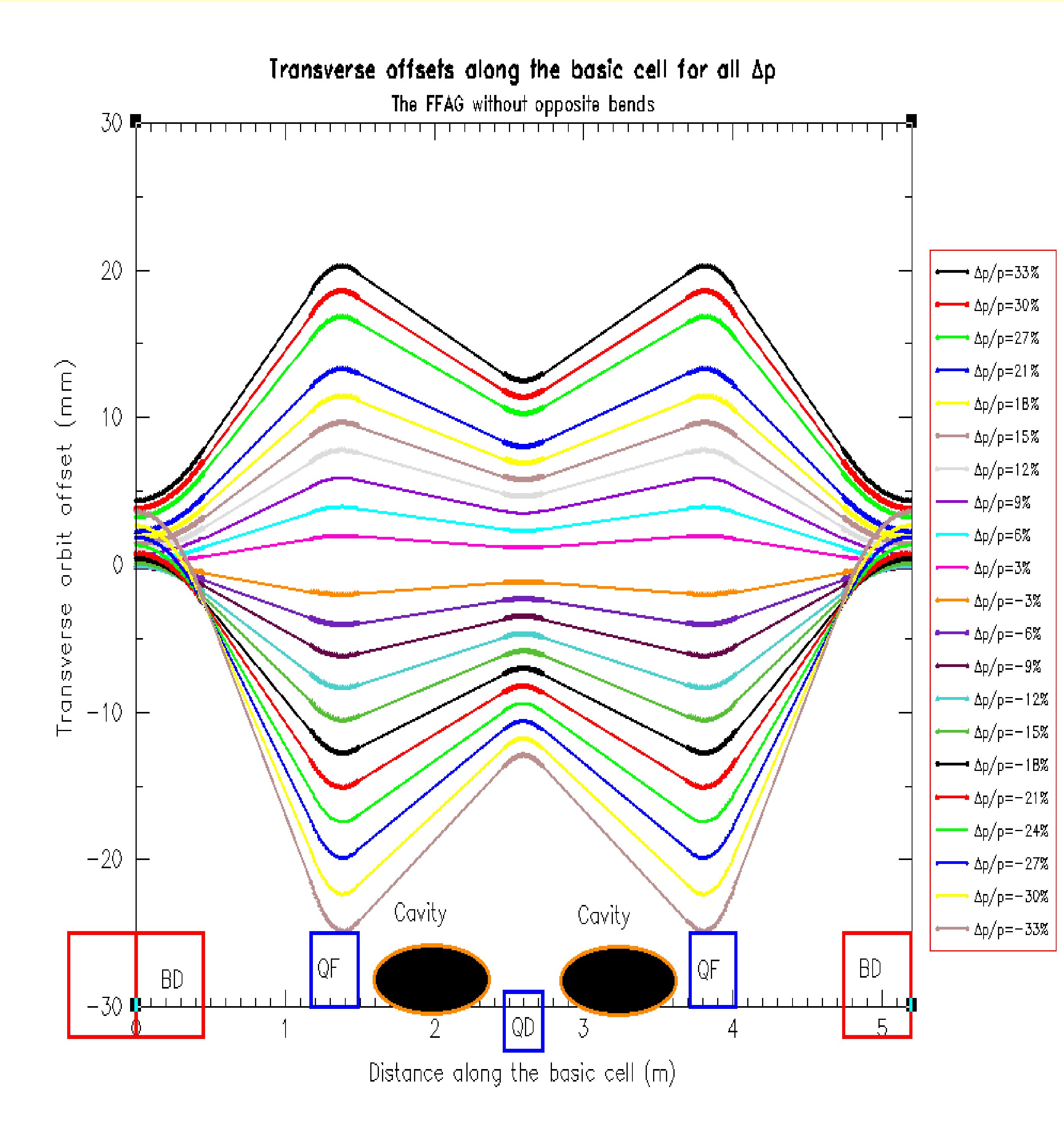
Maximae of the betatron functions during acceleration



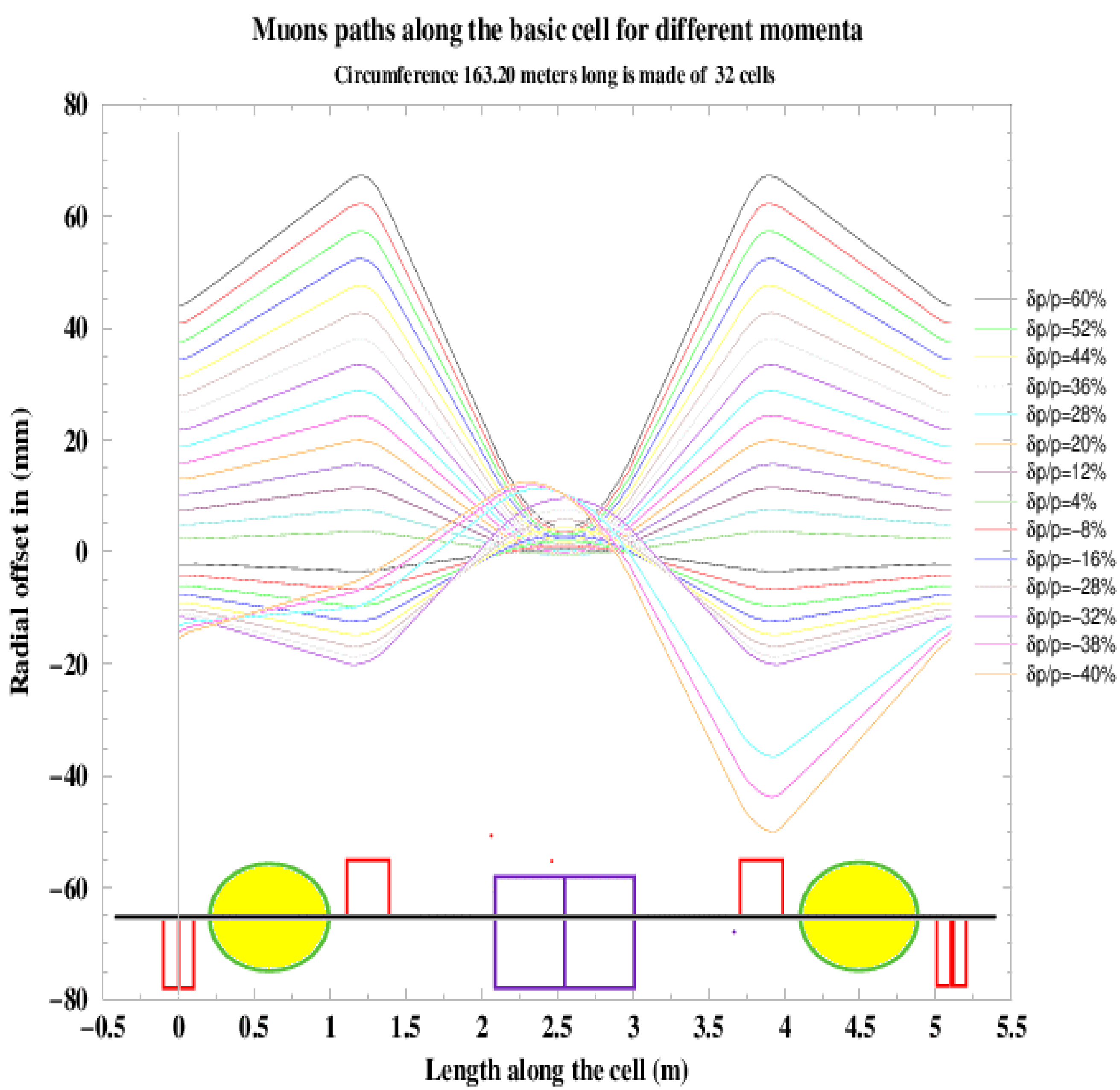
Maximum of the dispersion function during acceleration



A picture tells a story: particle path in the basic cell during acceleration

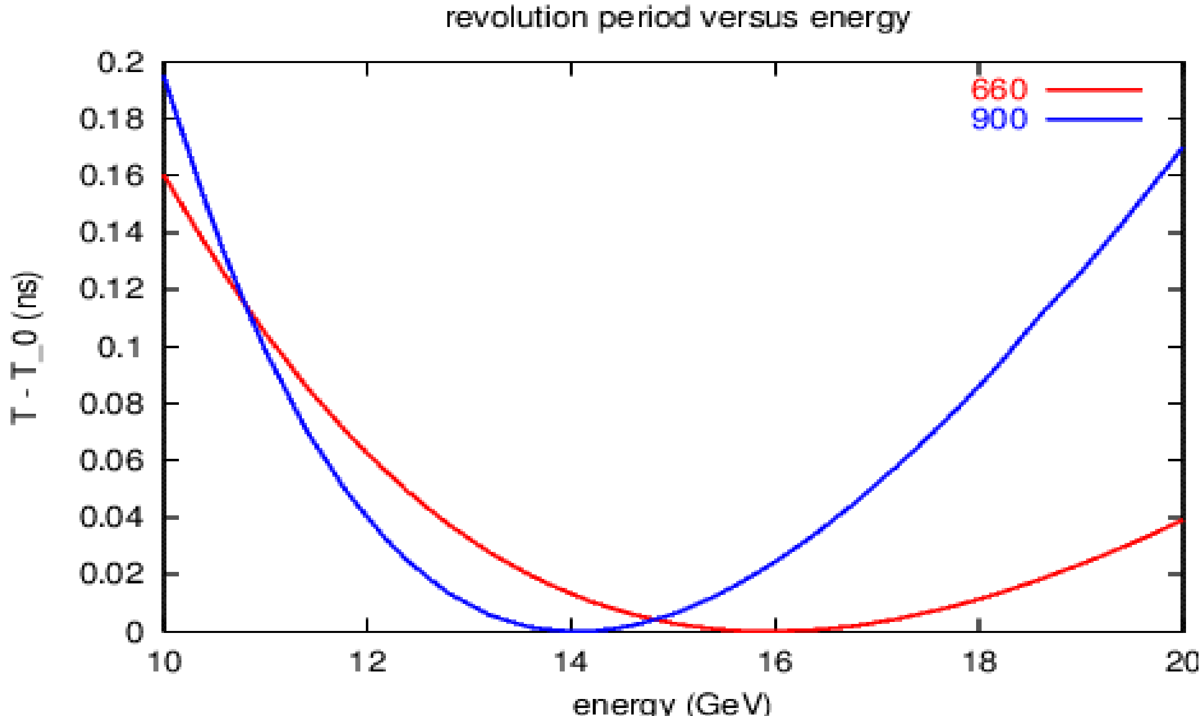


Particle path in one of the recent examples:



Simulation of the acceleration

RF considerations for FFAG rings
M. Blaskiewicz, BNL



660 ns lattice from D. Trbojevic and 900 ns lattice from E. Courant. Assume negligible energy input to the RF system during acceleration[1] 1-D update equations are

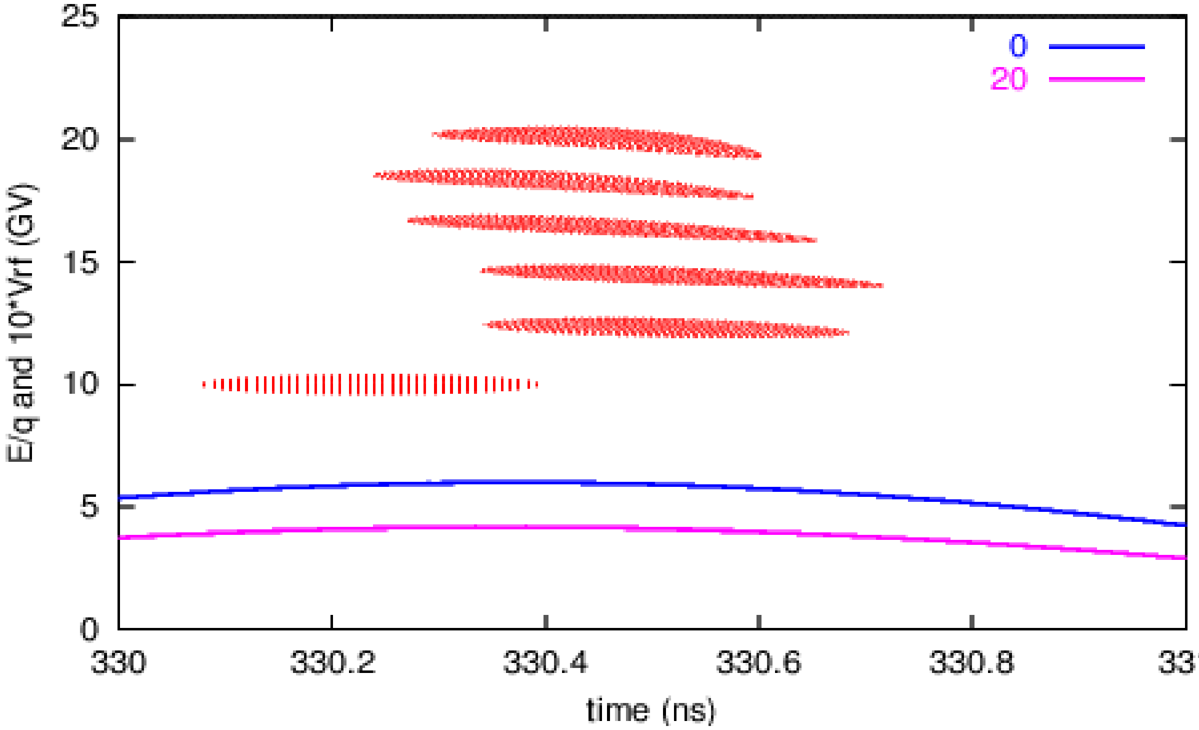
$$\tau_{n+1} = \tau_n + T(E_n) \quad (1)$$

$$\left(\frac{R}{Q}\right) I(t) = \frac{1}{\omega_{rf}} \frac{dV(t)}{dt} + \omega_{rf} \int_0^t dt_1 V(t_1) \quad (2)$$

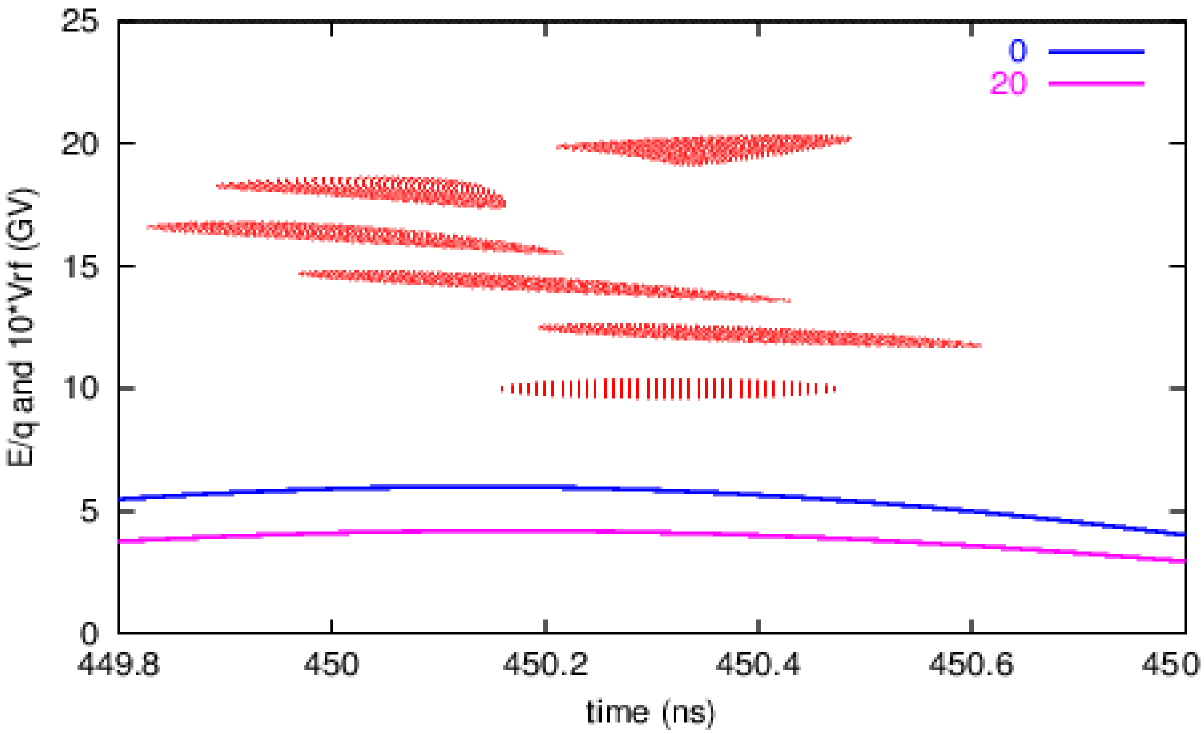
$$E_{n+1} = E_n + qV(\tau_{n+1}) \quad (3)$$

$I(t)$ smoothed by 0.5 ps. V(t) updated with $\Delta t = 0.15$ ps.

20 turn acceleration, 0.2 eV-s, 660 ns lattice



20 turn acceleration, 0.2 eV-s, 900 ns lattice



Energetics of the RF system

For 6.25×10^{12} muons the total charge is 1μC.
Assuming a factor of 2 voltage drop the initial stored energy in the RF cavities is

$$U = 10\text{GV} \times 1\mu\text{C} \times \frac{4}{3} = 13\text{kJ}$$

The stored energy is related to the voltage and impedance by

$$U = \frac{V^2}{2\omega_{rf}\left(\frac{R}{Q}\right)}$$

Taking a total voltage of 500 MV and $\omega_{rf} = 2\pi \times 200\text{MHz}$ one obtains $(R/Q) = 7.6$ kΩ.
The simulations used this impedance and $V = 600$ MV so the voltage dropped to 400 MV at the end of the cycle.
Taking 10 MV per cavity the requisite R/Q per cavity is 126Ω.
The stored energy per cavity is 300 J.
For $E = 10$ MV/m the volume is $0.7m^3$.
With 60 cavities some extra straight sections may be required but, since 10 GeV $\gg$ 106 MeV $= m_\mu c^2$, the straights will have a negligible effect on dT/dE.

References

[1] N. Holtkamp, D. Finley *eds.*, "A feasibility study of a neutrino source based on a muon storage ring", FNAL 2000.

Conclusions:

• The latest results in the FFAG lattice without opposite bends with distributed RF are very encouraging.

• Present codes MAD and SYNCH should be checked by either other codes or by an analytical calculation.

• If it is shown that the presented idea is really possible the whole muon acceleration should be redone.