### FFAGs: Design and Performance for Rapid Acceleration

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### **Outline of Talk**

### 1). Types of FFAGs

- Scaling
  - a. Spiral Sector: complicated magnet design, will not be considered
  - b. Radial Sector: FODO or Triplet
- Nonscaling
  - a. Linear: quadrupoles + dipole fields only
  - b. Isoschronous: add a strong sextupole + small radius to minimize spread in orbits as a function of momentum

### Field Dependence in a Racial sector scaling FFAG

Radial dependence of magnetic field is:

$$B(r) = B_0 \left(\frac{r}{r_0}\right)^k;$$

where k is a const.

• Scaling optics require reverse bends

• geometric closure requires F and D magnet lengths to be unequal implying different tunes and betatron functions between the two transverse planes

• k=1 is a linear scaling FFAG; but dynamic range scales with k

#### Example: Scaling 0.3 –1 GeV KEK FFAG

- Force linearity:  $k=15 \rightarrow 1$
- Momentum compaction (as will be seen later) reduces by a factor of 8
- Since



The acceptance reduces from 0.3-1 GeV to 0.6-0.7 GeV for the same magnet aperture

 Clearly the relative amount of H.O. nonlinearities are a strong factor in determining the magnetic aperture

### Aperture of FFAG: Is it large with large k?

*larger ring --> large k --> large non-linear field?* 



Dynamic aperture depends mostly on phase advance/cell!

### What determines the strength of H.O. fields?

 If one looks at the field expansion it starts with the choice of "linear" optics, where the quadrupole strength is given by:

$$\kappa = \frac{0.3}{p}g = \frac{0.3}{p_0}\frac{k}{r_0}$$

where  $\kappa$  is the quad strength in m<sup>-2</sup>, g the gradient in T-m, and k and r<sub>0</sub>, the field index and radius of curvature which correspond to the chosen momentum, p<sub>0</sub>.

### But we know in linear optics:

Quad strength is related to focal length

$$\kappa l_{(1/2quad)} = \frac{1}{f_{(1/2quad)}}$$
 where  $f > L_{(1/2cell)}$ 

giving the stability condition.

$$\frac{0.3}{p_0} \frac{k}{r_0} l_{\{1/2\,quad\,\}} < \frac{1}{L_{(1/2\,cell\,)}}$$

 Since focal length and half cell length are related to phase advance:

$$\sin \frac{\varphi}{2} = \frac{L_{(1/2 \, cell\,)}}{f_{(1/2 \, cell\,)}} = \frac{0.3}{p_0} \frac{k}{r_0} l_{(1/2 \, cell\,)} L_{(1/2 \, cell\,)}$$

 Therefore, once the linear optics are specified, the higher-order components, expressed in powers of k/r<sub>0</sub>, are completely determined.

 And, clearly, larger phase advance means larger k/r<sub>0</sub> means larger relative nonlinearities so DA decreases with increasing phase advance/cell.

### Aperture of FFAG: Is it large with large k?

*larger ring --> large k --> large non-linear field?* 



Dynamic aperture depends mostly on phase advance/cell!

# **FFAG Parameters**

momentum(GeV/c)	0.3~1	1~3	3~10	10~20
number of sector	16	32	64	120
k number	15	63	220	280
average radius(m)	10	30	90	200
max. B field(T)	2.8	3.6	5.4	6.0
tune	5.826	13.704	27.911	22.333
	4.590	4.048	4.089	6.333
drift length(m)	2.120	3.299	5.046	5.668
BF length(m)	1.065	1.575	2.169	2.685
BD length(m)	0.367	0.544	0.813	1.062
orbit excursion(m)	0.77	0.52	0.813	0.49
transition γ	4	8	14.9	16.8

### **Relative Strengths of H.O. field components**

0.3-1 1-3 3-10 10-20

k/r <sub>0</sub>	1.5	2.1	2.4	1.4
$\frac{1}{2}(k/r_0)^2$	1.1	2.2	2.9	1.0
$(k/r_0)^3$	0.6	1.5	2.3	0.5

 Since magnet apertures are ~0.5 m or less, strongest nonlinearity is unquestionably the sextupole term

## **Simple Conclusions**

Magnet apertures ∝ 1/k
 (from momentum compaction dependence)

• Strength of nonlinearities or  $DA \propto k/r_0$ 

A high k value with appropriately enhanced radius of curvature results in a conservative magnetic aperture but retains large DA.

# **Nonscaling FFAGs**

- Most general application involves removal of reverse bends and low-order truncation of field expansion
- Two nonscaling designs for the Neutrino Factory:
  - Linear nonscaling FFAG
  - Isoschronous FFAG

# Linear Nonscaling FFAG

Uses only quad and dipole fields or linear optics

 "Minimum circumference" configuration of magnetic fields is current design



 Consequences: Completely linear equations of motion so DA is not an issue, reduced circumference

## Isochronous FFAG

- Adds a strong sextupole term which causes the optics to approach scaling conditions despite the elimination of the reverse bend dipole term
- With a high "k" value, α→0 and offmomentum orbits collapse about a central orbit, dramatically reducing magnet aperture and making the ring ~ isochronous since β~1.
- Consequences: ∆C →0, phase slip→0 and on-crest acceleration can be supported. DA is an issue, however.

# **Transverse Cooling Scenario/per plane**

 $\epsilon_{n}$  (rms)



# Mulit-GeV Nonscaling FFAGs for a Neutrino Factory or Muon Collider

- Lattices have been developed which, practically, support up to a factor of 4 change in energy, or
  - <u>almost unlimited momentum-spread acceptance</u>, which has immediate consequences on the degree of ionisation cooling required
- For example, the storage ring can accept approximately  $\pm 4\% \delta p/p$  @20 GeV (depending on the ring lattice design). If acceleration is completely linear, the absolute momentum spread is preserved, so at the exit of cooling (@~400 MeV) this translates into a  $\delta p/p$  of  $\pm 200\%$ implying little or no longitudinal cooling.
- THERE is a STRONG argument to let acceleration do the bulk of the LONGITUDINAL AND TRANSVERSE COOLING. The Linac/RLA has been the showstopper in this argument
  - (Upstream Cooling channels currently accept a maximum of ±22% for solenoidal-based and -22% to +50% for quadrupole based.)

### **Criteria for a competitive FFAG lattice**

#### Linearity in Optics

#### use of linear elements only

*nonscaling* FFAG: transverse DA=aperture of components.

Magnet apertures are reduced by the inclusion of nonlinear B field (*scaling* FFAG) at an expense in DA or increased circumference.

#### Number and Cost of Components:

#### Given: 1 vs. 4 arcs

single arc must transport a large energy increase

#### Aperture

- Comparable to RLA components ( $\approx 0.25$  m)
- Normal-conducting version?

# First challenge is to optimize the ring design(s) over the acceleration range

### According to:

- magnet design (aperture regulation, length vs. aperture)
- consistent performance--overridingly rf-phase-slip

The main concern in magnet design here are the large transverse (horizontal) orbit excursions and the correspondingly large magnet apertures.

### If the design is mindless, then

Horizontal apertures are typically >1/4 m for a factor of 3-4 gain in energy in a *nonscaling* FFAG. (For a *scaling* FFAG, apertures decrease as the radial nonlinearity of the field increases)

# Optimizing(minimizing) Magnet Design(apertures) for a 3-20 GeV acceleration

- Magnet aperture can be fixed and minimized in two sequential nonscaling FFAGs if the acceleration range is divided between the two according to approximate scaling laws:
- the magnet aperture scales roughly as the range in 1/p
- Δθ, which is the difference in the dipole bend from the central energy to the momentum limits; is closely given by the inverse of the momentum divided by the half cell length:

 $L_{1/2cell} \Delta \theta \cong .3 B_D(1/p-1/p_0)$ 

where  $B_D$  is the dipole field, p the upper (or lower) momentum bound for the cell,  $p_0$  the central energy, and  $L_{1/2cell}$  the length of the half cell.

one then solves for the momentum and angular acceptance for a specific magnet aperture and field which is equal between two consecutive accelerating rings.

# **3-20 GeV Acceleration Rings**

If one applies the previous scaling laws and solves for two rings in the range 3-20 GeV, then acceleration is optimized for a ring which is 3-6 GeV, followed by a ring from 6-20 GeV with the minimum horizontal aperture\*. More importantly, one achieves identical magnet parameters in both rings:

This table gives superconducting (SC) and normal (NC) magnet parameters applicable to both rings

	Length	Full Orbit swing	Horiz. Aperture	Max** Vert. Aperture
	(111)	(CIII)	(iuii,ciii)	(iuii,ciii)
"F" quad				
SC	0.15	15	~22	10
NC	0.45	15	~22	10
"D"+dipole	9			
SC	0.35	8	~15	10
NC	1.05	8	~15	10

\*\*the vertical aperture can be decreased with ring energy.

\*imposing the restriction that the magnet aperture is not significantly larger than the magnet length and that 6T/2T is the maximum poletip field for SC/NC.

### **General Ring Parameters**

Using these magnet parameters the following ring lattices apply:

	Cell	Drift/	Bend/	Total	Circum.
	Length	Cell	Cell	# cells	
	(m)	(m)	(rad)		(m)
3-6 GeV					
SC	4.5	4	0.87	72	324
NC	5.5	4	0.87	72	396
6-20 GeV					
SC	6.5	6	0.02	314	2041
NC	7.5	6	0.02	314	2355

The above table along with the pathlength dependencies shown previously are used for the rf simulations which follow.

# **Summary of Ring Design**

- Component apertures comparable to RLA designs
- Standard magnet strengths
- Normal conducting version completely equivalent to superconducting
- Lengths and apertures comparable: the optics are not fringe-field dominated
- Reduction of total number of magnetic components by at least a factor of 2

# **RF in a FFAG for rapid acceleration**

#### RF Voltage

#### Reduced rf voltage requirments:

- primarily through increased number of turns
- secondarily through near-crest operation--analogous to a cyclotron rather than a synchrotorn

#### Conventional rf gradients

### Pathlength Dependencies or RF Phase-slips in FFAGs for Rapid Acceleration

- Problematic for both Scaling and Nonscaling FFAGs--on the order of 0.5-1 m total pathlength or circumference change over the acceleration cycle
- Parabolic shape for Nonscaling linear FFAGs and a linear shape for Scaling FFAGs as a function of momentum
- High-Q rf cannot respond in the microsecond beam circulation time to the pathlength or time-of-arrival-changes (<u>hence the RLA</u> solution)



## **Proposed Solutions for RF Phase-Slip in FFAGS**

- Chicanes which change pathlength as a function of momentum--successfully applied in scaling FFAGs but are not applicable to nonscaling FFAGs.
- Broadband rf which can be phased quickly but has the disadvantage of low acceleration voltages (1 MeV/m or less) and large power consumption for equivalent acceleration.
- Lower frequency rf (~25 MHz) until the effect of the phase-slip is not as significant
- This work, however, investigates the simplest approach: the application and optimization of a single high-frequency, high-Q rf system.
- Further-only nonscaling FFAGs will be considered because of the energy regime (multi-GeV) combined with the need to support an unusually-large transverse dynamic aperture requiring linear optics.

# Fixed RF system parameters (based on existing systems):

Based on the 200 MHz (NC) and 400 MHz (SC) cavities to be used in the CERN LHC

- Assume: 360 MW wall power available and a 50% conversion efficiency.
- Using 300 of the 314 cells in the ring, and 6 cavities installed in the 6m of drift space available per cell (1800 cavities total), then the allowed power consumption is 100 kW per cavity.
- With a gap voltage of 1.7 MV, the shunt resistance is then 14MΩ and the acceleration gradient is ≤3MV/m using 50-70 cm long cavities with 20-30 cm diameter bores
- Using the R/Q of 200 for the CERN cavities, the quality factor must be at least 7x10<sup>4</sup>
- The filling time for these cavities is 350µsec, which is to be compared to the 6.7 µsec circulation time for light-speed particles and a 2 km ring.

Vector feedback of the gap voltage was considered which could in principle reduce the filling time by a factor of 20, but waveform fidelity is insufficient and peak power rises--pure sinusoid is the only mode of operation possible.

### **General Considerations**

Because of the large momentum acceptance, the notions of synchronous phase and rf bucket cannot be applied for rapid acceleration combined with high-frequency rf. In effect, there is a lower limit to ∆E/E due to the optics (no lattice solution because FODO cell phase advance ≥180°), but the upper limit, in principle, is well beyond the extraction energy. If you inject a 20 GeV muon for the 6-20 GeV ring, it will accelerate and will not be lost due to the optics.

# Therefore, one has to define very carefully the performance goals of this machine and how to achieve them.

The nonscaling machine, in particular, can be made to run in a variety of input/output configurations with extreme changes in transverse and longitudinal beam dynamics..

### **RF Optimization:**

There are many optimization strategies, but we started with one in which the reference bunch receives the maximum possible acceleration on each turn. Various rf parameters are then changed and input/output acceptances and emittances are evaluated for performance.

Later we termed this mode, near-crest operation

Given the extreme amount of rf required, this was felt to be the most economically-feasible approach.

## **Optimization Strategy**

Using a **single frequency rf system**, the following parameters can be chosen:

- a. the single fixed frequency:
- b. The initial individual cavity phases
- c. the addition of a 2nd harmonic (to impose a flat-top on the waveform).
- d. during the course of the studies, overvoltages\*\* were also found to be important

\*\*overvoltage merely represents the % increase in rf voltage required with relative to pure on-crest acceleration, or the minimum acceleration voltage.

The resulting performance needs to be **benchmarked** against **standard acceleration**; ie. Imposing the correct phases on the rf cavities on a turn-by-turn basis in the simulation.

## **RF Parameter/Optimization Definitions**

### **RF parameters and terms**:

- <u>Ideal phases</u>: A set of ideal phases are calculated for a single reference particle cavity by cavity and turn by turn. This is the "standard" acceleration benchmark
- Fixed Frequency and best phases: Assuming initial phases of the cavities can be individually chosen, a mean square deviation of the actual phases of the reference particle from the ideal phases above is calculated for a starting value of the frequency. This calculation is summed over all rf stations and turns. A search is then performed on both the frequency and the initial phases of all cavities to minimize this deviation. The results are a set of "best initial cavity phases" for the reference bunch and these phases are little resemblence to the ideal ones.
- <u>Over-voltages</u>: Optimization was also carried out on overvoltages, in this case chosen so as to minimize the variation of the extraction energy for a reference particle, bunch to bunch.

# **Details of the Simulation**

- Complete decoupling from transverse motion
- <u>Independently-settable initial cavity phases.</u> One rf station comprises one cell or 6 cavities and the starting phase of each station is a free paramenter; ie. 300 initial phases
- <u>Pathlength is taken from the curve</u>. Gap crossing times for the reference particle are calculated from this curve based on the 2 km circumference
- <u>The machine acceptance is -10% at injection and +10% at extraction</u>.
  The lattice limit is -10% for injection (physical aperture limit and no closed orbit), and the corresponding upper limit (20 GeV) is +10% at extraction, again due to physical apertures, but again no corresponding lower limit (6 GeV).\*\*

### **Fidelity of the Acceleration**

Output cuts on the extracted emittance. With such a huge machine acceptance, orders of magnitude emittance blowup can be tolerated in longitudinal phase space. A cut in momentum spread must be applied to the final longitudinal phase space, in this case 10% of 20 GeV was applied.\*\*

\*\*This 10% cut can be viewed as a limit on emittance blow-up and later will be observed to restrict solutions to a conserved system

## **Conditions of the Simulation**

- Initially the longitudinal phase space is flooded with trial particles and tracked to 20 GeV. <u>A 20 GeV ±10% cut is</u> <u>applied at extraction</u> and surviving particles are used to map both input admittance and output emittance.
- 2. The input admittance is saved and used to populate ensembles for final results for increased accuracy.

# **RF Single Frequency Choices**

Harmonic Numbers for 3-6 and 6-20 GeV normal conducting (NC) and superconducting (SC) rings.

RF Frequency (MHz)	3-6 GeV Ring 0.3-0.4 km (SC – NC)	6-20 GeV Ring 2.0-2.4 km (SC – NC)
25	27 - 33	170 – 200
50	54 - 66	340 - 400
100	108 – 132	680 – 800
200	216 - 264	1360 - 1600

- U.S. design for a Neutrino Factory currently produces a 200 MHz train of 100 bunches after ionization cooling. Even with 100 bunches the lower ring is only half full and the higher energy ring 1/14 to 1/16th full.
- There is also an open question of how to accelerate from ~400 MeV to 2-3 GeV where the beam sizes are so large (>10cm diameter) ring injection/extraction become a problem.

## **Simulation Results**

In the following, 100 bunches with roughly 1600 particles per bunch were tracked.

#### Five-turn, 200 MHz acceleration: 9.33MV/cell\*

Description	Over- Voltage	Input Phase Space (eV-s)	Output Phase Space	Comments
Ideal Phases	none	1.21	1.18	Not linear
With Dual Harmonic	none	2.22	2.22	No tlinear
Best Phases**	none	-	-	<18 GeV final energy
With Dual Harmonic	none	0.15	0.15	0,
	40%	1.37	1.375	Not linear
With Dual Harmonic	1.66 x nom	1.94	1.91	Not linear

- \* 200 MHz FFAG acceleration with ≥4turns provides a potential replacement for the RLAs used in the U.S. Neutrino Factory Feasibility Studies
- \*\* does not imply no net acceleration--it implies particles did not reach 18 GeV.

### 5-turn, 200 MHz Acceleration--Output Longitudinal Phase Space



using Ideal Phases



Output phase space with Best Phases and 40% overvoltage (left) and with dual harmonic (right)

### **More Results:**

#### Ten-turn, 100 MHz Acceleration: 4.7 MV/cell

Description	n	Over- Voltage	Input Phase Space	Output Phase Space	Comments
Ideal Phases*					Not Considered
Best Phases		4%	-	-	<18 GeV
		30%	1.8	1.8	inter cricigy
7 turns		23%	3.7	3.7	6.7 MV/cell
With Dual Harn (10 turns)	nonic	27%	4.0	3.9	

\*For this study 200 MHz was emphasized

It is interesting to note that transmission doubles reducing the number of turns from 10 to 7. The U.S. Neutrino Factory only requires about 0.5 eV-s, so 10 turn operation is acceptable.

# 10-turn, 100 MHz Acceleration--Output Longitudinal Phase Space





Input phase space with +/- 10% band (left) and output phase space for Best Phases and 30% overvoltage (right)

# **General Conclusions:**

- Using single-frequency, but different initial phases for the cavities, and
- imposing a conserved output phase space

one can expect to transmit 1-2 eV-s for 20-40% overvoltages, with the approximate turn dependence given below:

RF freq	# turns
25 MHz	40? (extrapolation may not extend this far)
50 MHz	20
100 MHz	10
200 MHz	5

 Further studies also indicated that only 100 cells were required to achieve these transmissions; ie more cells do not improve machine dynamics. (multiple-frequency beating was investigated, but dismissed because of the bunch train.

#### Lower Frequencies, No Independent Phasing E. Forest and C. Johnstone

- --Clearly the longer the wavelength the less important the relative phases of the individual particles, and hence the longer the bunch length that can be accelerated.
- A recent study was performed on the 6-20 GeV ring for 5-turn acceleration only, but determining the final acceleration energy of a particle relative to the crest of the waveform at injection. For this study the rf frequency was varied from 25-200 MHz and:
  - The rf frequency was chosen to be a harmonic of the pathlength, 2041.1 m which represents a "central" value of the pathlength vs. momentum curve.
  - Keeping the ±10% cut, estimates can be made of the bunch length and longitudinal emittance transported.
  - To match to the storage ring, the bunch length would have to be doubled and the momentum spread halved.

# **Results, No Initial Phasing of cavities**

Approximate longitudinal phase space transmitted for 5 turns assuming ±10% momentum cut at 20 GeV (1.705 MV/ cavity)

RF Frequency	Over Voltage	$\Delta L_{bunch}$	ε <sub>L</sub>
(MHz)	%	(m)	(eV-sec)
25	-	-0.70 – 1.25 1.95	13.0
50	3	-0.20 — 0.65 0.85	5.7
100	8	0.06 – 0.42 0.26	1.7 (3.6*)
200	40	0.16 – 0.23 0.07	0.46

\*3.6 eV-sec is corresponding output phase space using Best Phases indicating importance of cavity phasing even at 5 turns and 100 MHz.

## **Summary of Results Based on Both Studies**

Subsequent studies of the maximum number of turns achievable with the same initial phases for all cavities were performed as a function of rf frequency. These yielded the following table when compared with the 100 and 200 MHz Ideal Phase and Dual Harmonic Studies.

Estimates of maximum number of turns which can successfully transport 1-2 eVsec within a ±10% momentum bite at 20 GeV. Significant (>10%) overvoltages are generally required for Best Phases and Dual Harmonic.

RF	# turns	# turns	Dual
Frequency	Same	Best	Harmonic
(MHz)	Phases	Phases	
25	15	30*	36*
50	10	20*	24*
100	5	10	12
200	2-3	5	6

\*Extrapolated from 100 and 200 MHz cases

# **General Conclusions**

- Setting the initial cavity phases can either approximately double the number of turns for the same (useful) output phase space, or double the transported bunch length, keeping within the defined momentum cuts.
- 2. <u>Overvoltages are required for 100-200 MHz operation</u>; raising the power requirements unless more turns are implemented. In that case there is little difference in power requirements for 5-turn 200 MHz and 10-turn 100 MHz operation.
- 3. <u>Acceleration works for the lower frequencies</u> with little or no overvoltage, but at a greatly reduced number of turns.
- 4. Dual harmonic promotes a large increase in the output phase space without increasing the momentum spread; i.e. it seems to decrease emittance blowup in  $\Delta p$ , implying more conserving dynamics.
- <u>Dual harmonic appears to increase the number of turns</u> for a given output useful output phase space, but only by about 20%.

Specific RF	Solutions for Rapid Ad	cceleration in a FFAG
200 MHz #/turns	rf voltage	conserved
5 turns	4 GeV/turn	<b>phase space</b> 1.4 eV-sec
<b>100 MHz</b> 10 turns	1.8 GeV/turn	1.8 eV-sec

#### AND:

- Based on existing SC cavity designs
- Number of turns varies inversely with frequency
- RF voltage decreases inversely with frequency
- Dual harmonic doubles conserved phase space, but does not appear needed to be compatible with upstream systems.

### Match to bunch train from cooling

- CERN cooling uses 88 MHz, and the 10-turn, 100 MHz results show an advantage over the 4-turn RLAs, but
- Frequency has been fixed @200 MHz for the bunch train in the U.S. scenario and 5-turn acceleration is not as competitive, ignoring the apparent elimination of the need for emittance exchange

If the 200 MHz solution is forced, can we increase the number of turns? Increasing #turns @200 MHz

 Turn dependency on circumference and rf accelerating voltage

3-6 GeV	7	0.3 km	0.6 GeV/turn
	14	0.6 km	0.3 GeV/turn
	"	0.3 km	0.4 GeV/turn
	28	1.2 km	0.15 GeV/turn
	**	0.6 km	0.2 GeV/turn
	"	0.3 km	0.3 GeV/turn

Increasing #turns @200 MHz

Turn dependency on circumference and rf accelerating voltage

6-20 GeV	7	2 km	2.8 GeV/turn
	14	4 km	1.4 GeV/turn
	"	2 km	2.0 GeV/turn
	28	6 km	0.7 GeV/turn
	"	4 km	1.0 GeV/turn
	"	2 km	1.4 GeV/turn