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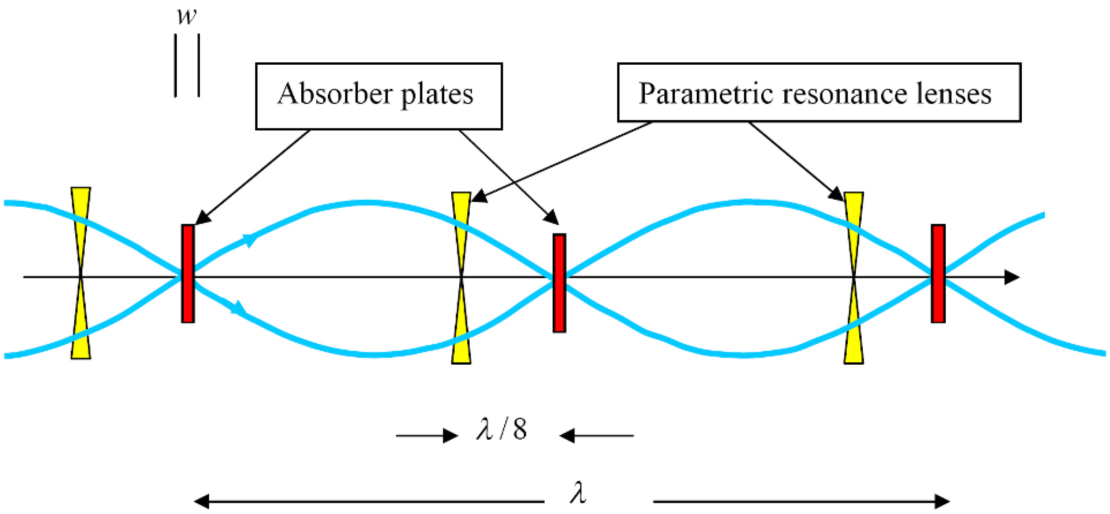
Status of Phase Ionization Cooling using Parametric Resonance

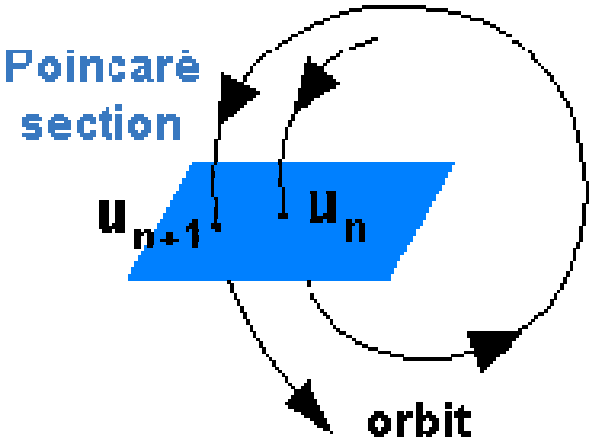
David Newsham

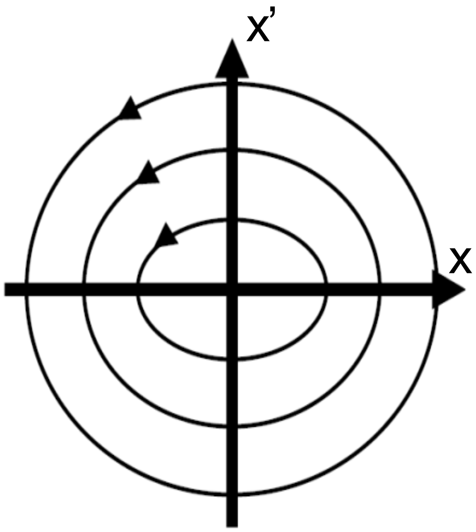
Muons, Inc.

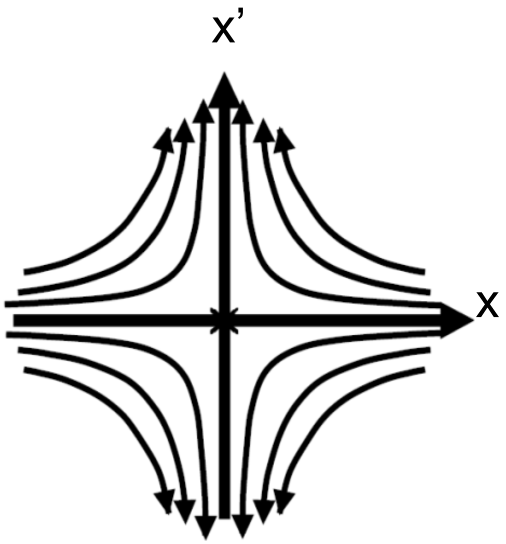
Muon Collider Design Workshop – BNL
3-7 December 2007

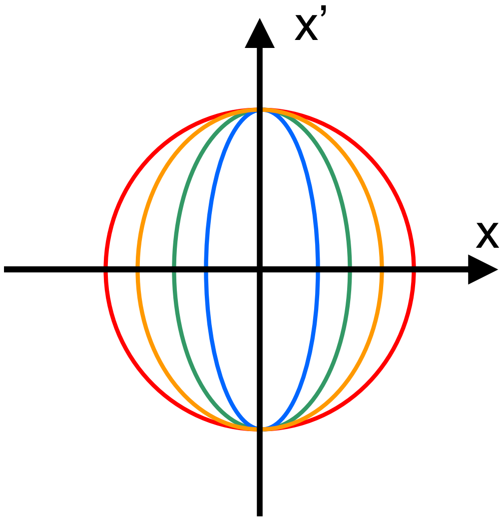
PIC Concept


w
Absorber plates
Parametric resonance lenses
λ/8
λ


Poincaré section
u_{n+1}
u_n
orbit

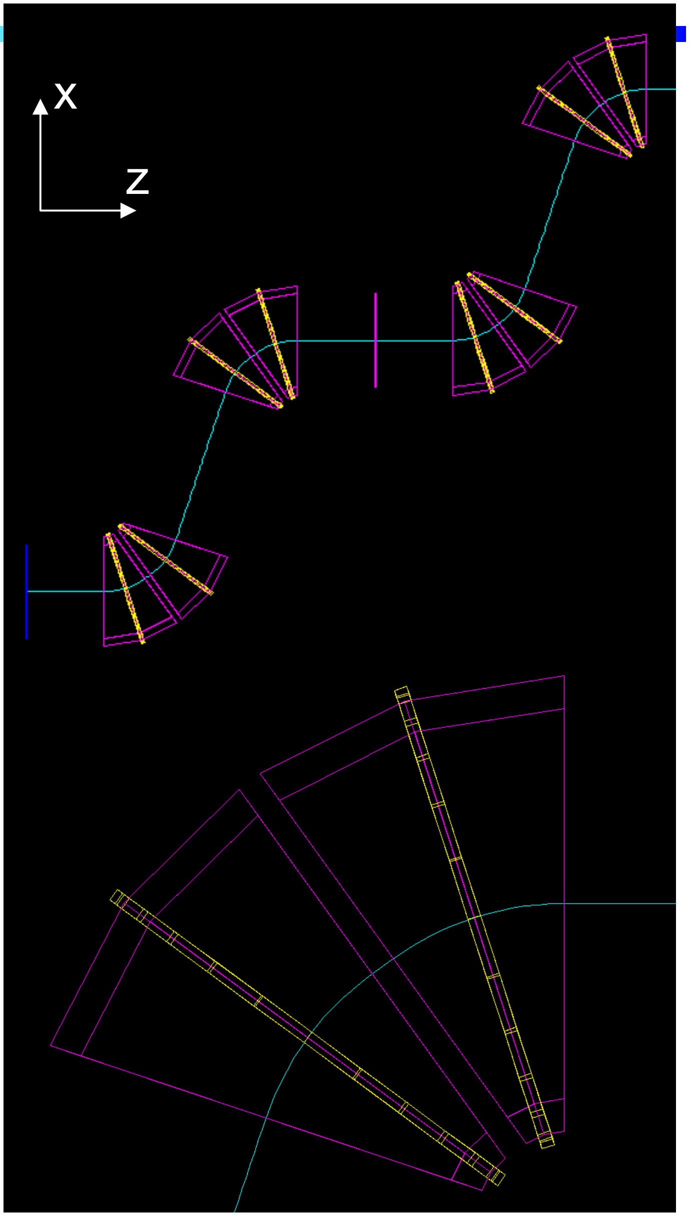

x'
x


x'
x

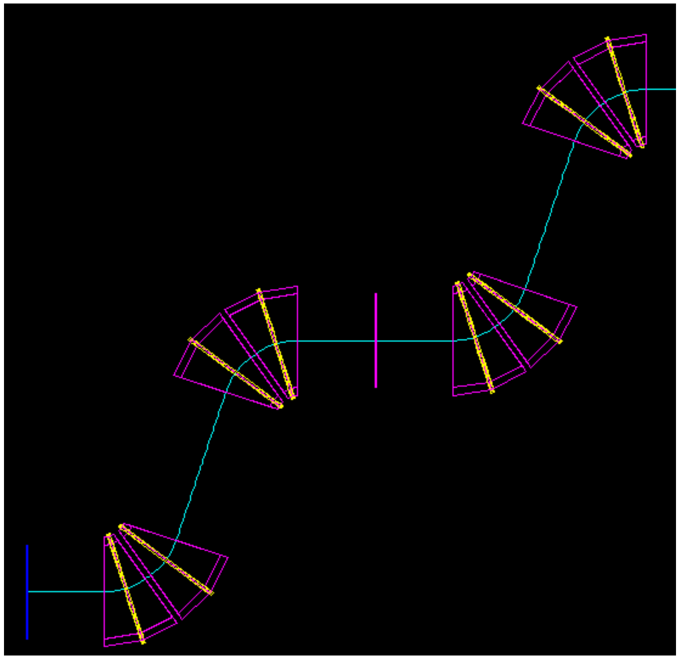

x'
x

September PIC Lattice Design

- Only uses dipoles and quadrupole magnets with no “fringe” fields.
- Each bend (½-cell) consists of 2 sector dipoles and 2 thin quads that act in unison.
  - All bends have the same angle (dipole field magnitude) only the bend direction changes.
  - All quads have the same unperturbed field.
  - Quads are thin to minimize sagitta effects.
  - Quad field literally encircle the dipoles.
- 2 independent quantities:
  - Bend angle – field automatically adjusted by G4Beamline
  - Quadrupole field gradient
- Y tune is only affected by quadrupole gradient
- X tune is affected by both quad gradients and bend angle.
- End effect is focusing in both planes.
- Fringe field effects will change parameter settings. Both tunes will be coupled but can be characterized by the same 2 independent parameters. Aberrations will be affected by fringes.

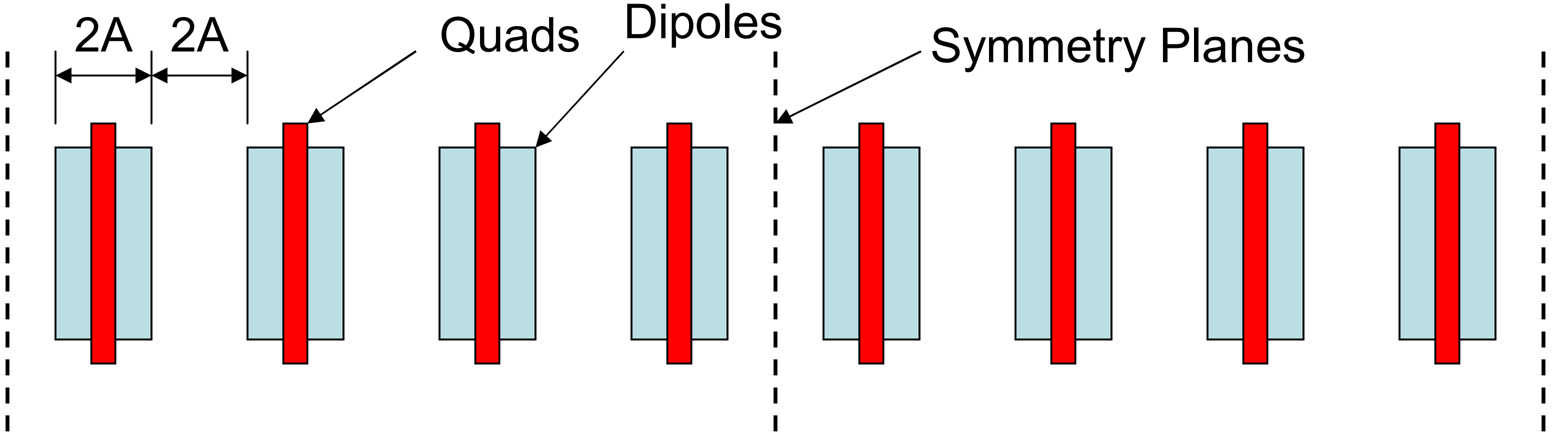


Post September Lattice Considerations



- Aberration (2nd order) compensation theory (Derbenev) requires that x and y have different phase advances per cell.
  - Doesn't require dramatically new lattice.
  - Solves the symmetric (x-y) perturbation problem.
- The same theory requires sextupole magnets with a spatial wavelength of ½ dispersion period.
  - Current dipole-pairs are not appropriately spaced.
  - Current dipole-pairs can be separated and the quads made into combined-function multipole magnets.
- Increased symmetry simplifies lattice (although "simple" and "PIC" should never be used together).

2-‘Cell’ Perturbation Layout



$+\theta$	$+\theta$	$-\theta$	$-\theta$	$+\theta$	$+\theta$	$-\theta$	$-\theta$	Bend
+Q	+Q	+Q	+Q	+Q	+Q	+Q	+Q	Quad
0	0	$\alpha\Delta Q$	ΔQ	$-\Delta Q$	$-\alpha\Delta Q$	0	0	Pert

$\alpha = -0.0504$

Based on Mathematica thick lens matrix analysis including perturbation

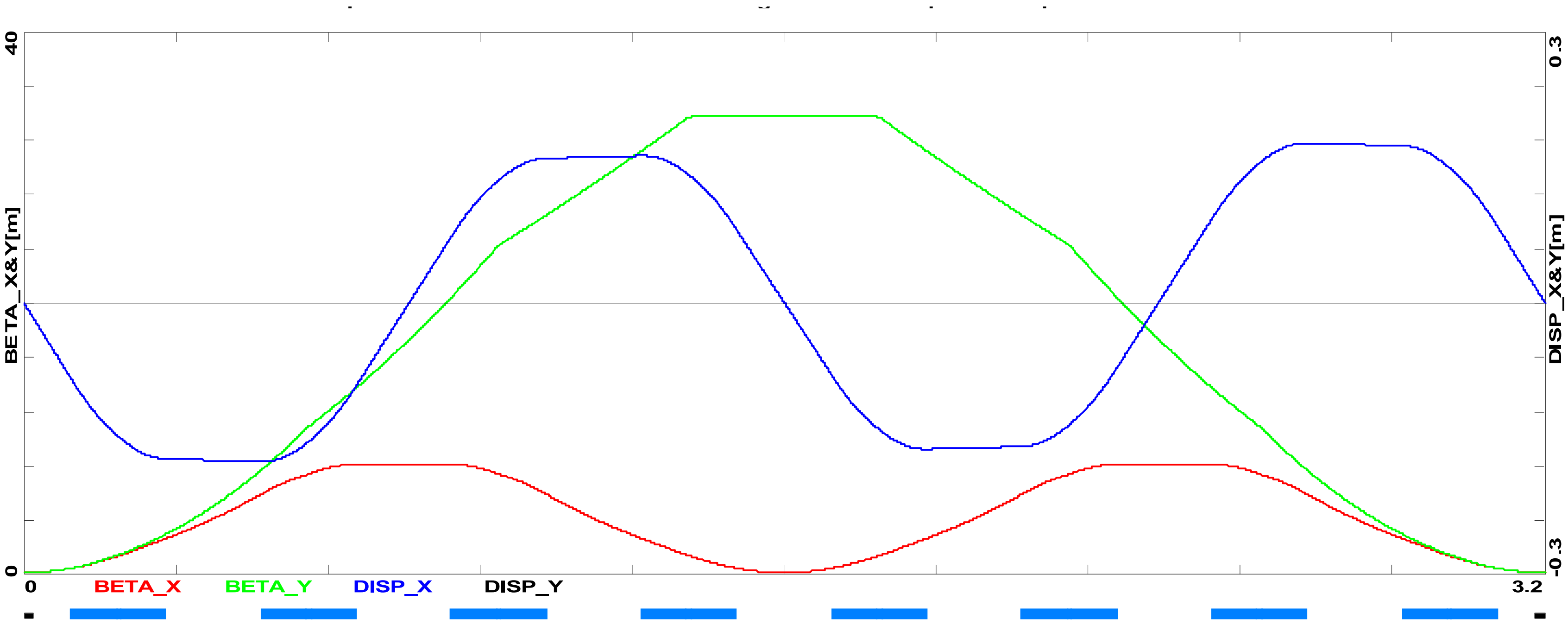
Thick Lens Matrix Results

Tune Condition	υ_x	υ_y	Dipole Bend Angle	Quad Gradient (1 cm)
$\lambda_x = \lambda_y = 2\lambda_D$	½	½	88.42°	-48.97 T/m
$\frac{1}{2}\lambda_x = \lambda_y = 2\lambda_D$	¼	½	69.23°	-48.97 T/m
$\lambda_x = \frac{1}{2}\lambda_y = 2\lambda_D$	½	¼	70.74°	-12.70 T/m

(Previous Lattice Design)

- OptiM and Mathematica give identical thick matrix results.
- G4Beamline results are nearly identical to the thick lens matrix analysis.
- Bottom condition chosen initially because of lower quadrupole gradient for same nominal bend angle
  - Same condition chosen studied by Derbenev.

September Lattice



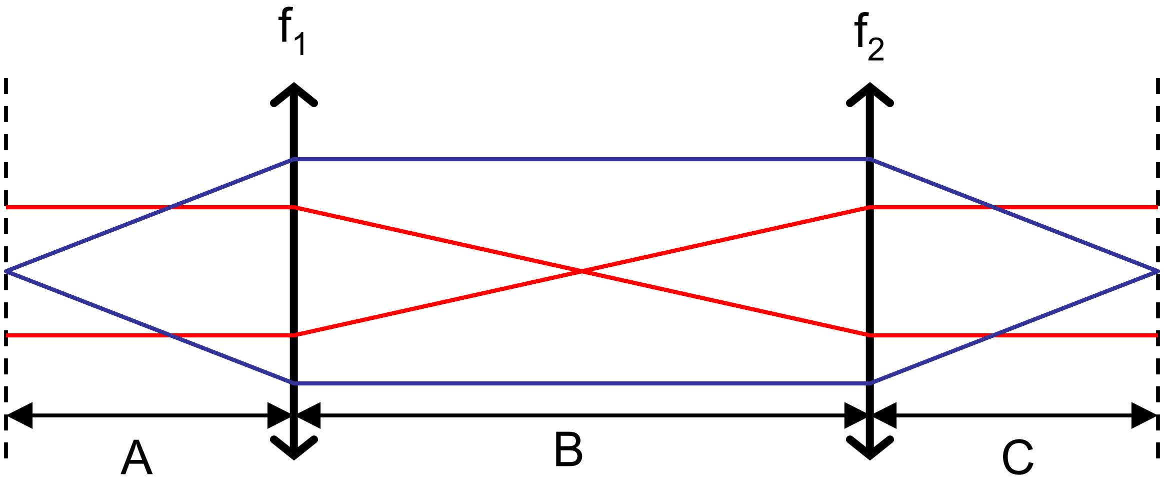
$\sigma_x = \sigma_y = 6$ mm

$\theta_x = \theta_y = 200$ mrad

$\beta_{Beam} = \sigma / \theta = 30$ mm

$\varepsilon_{rms} = \sigma\,\theta = 1.2$ mm

½-Integer (π) with 2 Lenses



$$\text{for } M = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$f_1 = f_2 = f$$

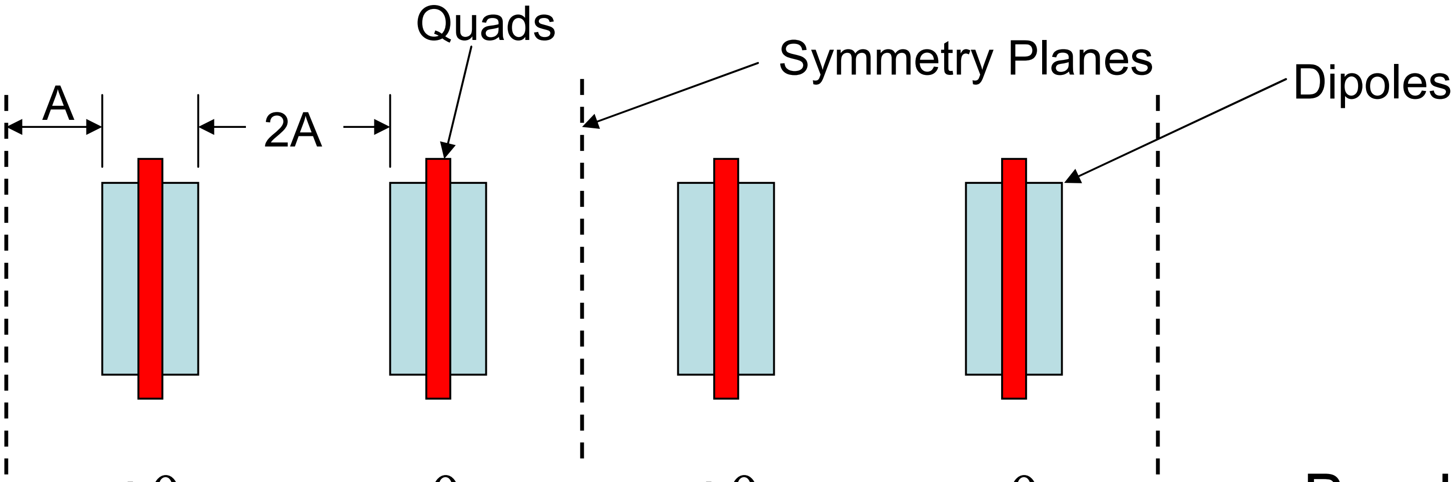
$$B = 2f$$

$$A + C = B = 2f$$

$$A = C \Rightarrow f = A = \tfrac{1}{2} B$$

Simplified 2-‘Cell’ Layout

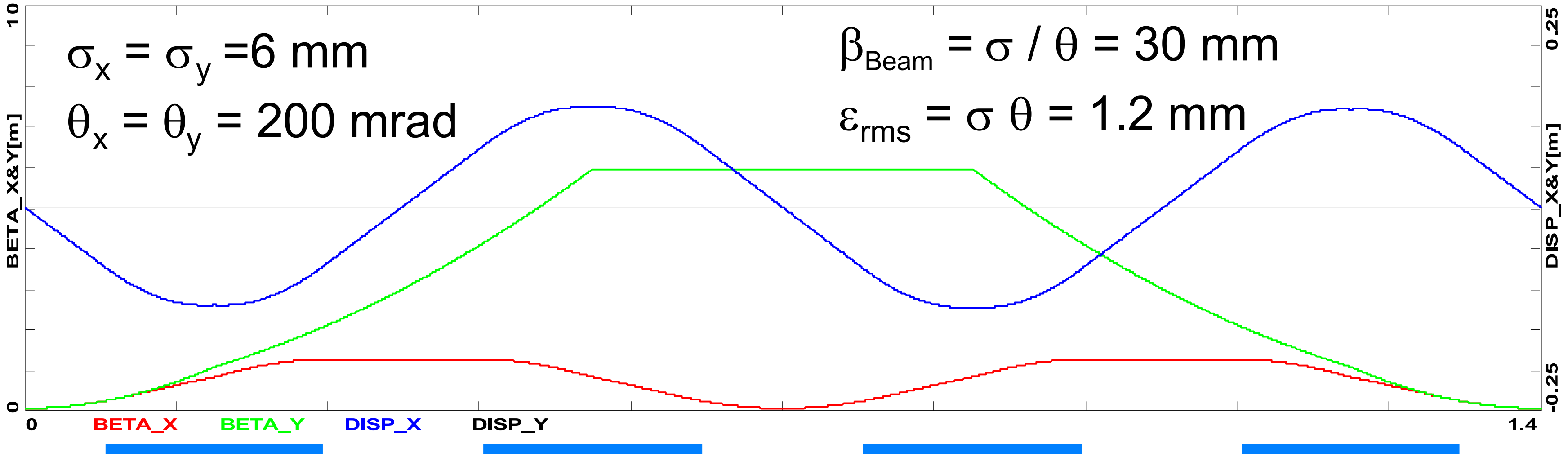
with Perturbation



$+\theta$	$-\theta$	$+\theta$	$-\theta$	Bend
$+Q$	$+Q$	$+Q$	$+Q$	Quad
$\rho\alpha Q$	αQ	$-\alpha Q$	$-\rho\alpha Q$	Pert

- Fewer dipole/quads:
  - Reduced (spherical-like) aberrations
  - Reduced base cell from 1.6 m to 0.7 m
- Absorbers every alternate cell ($\upsilon_x=2\pi$, $\upsilon_y=\pi$)
- From matrix analysis $\rho = -0.175$

Acceptance/Magnet Size



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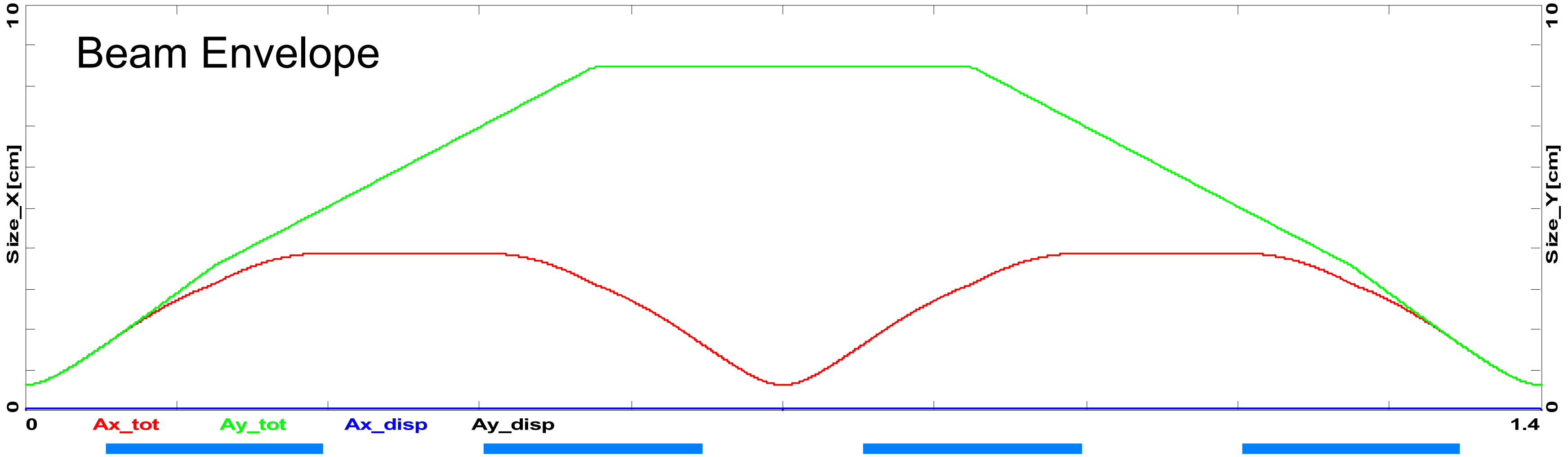
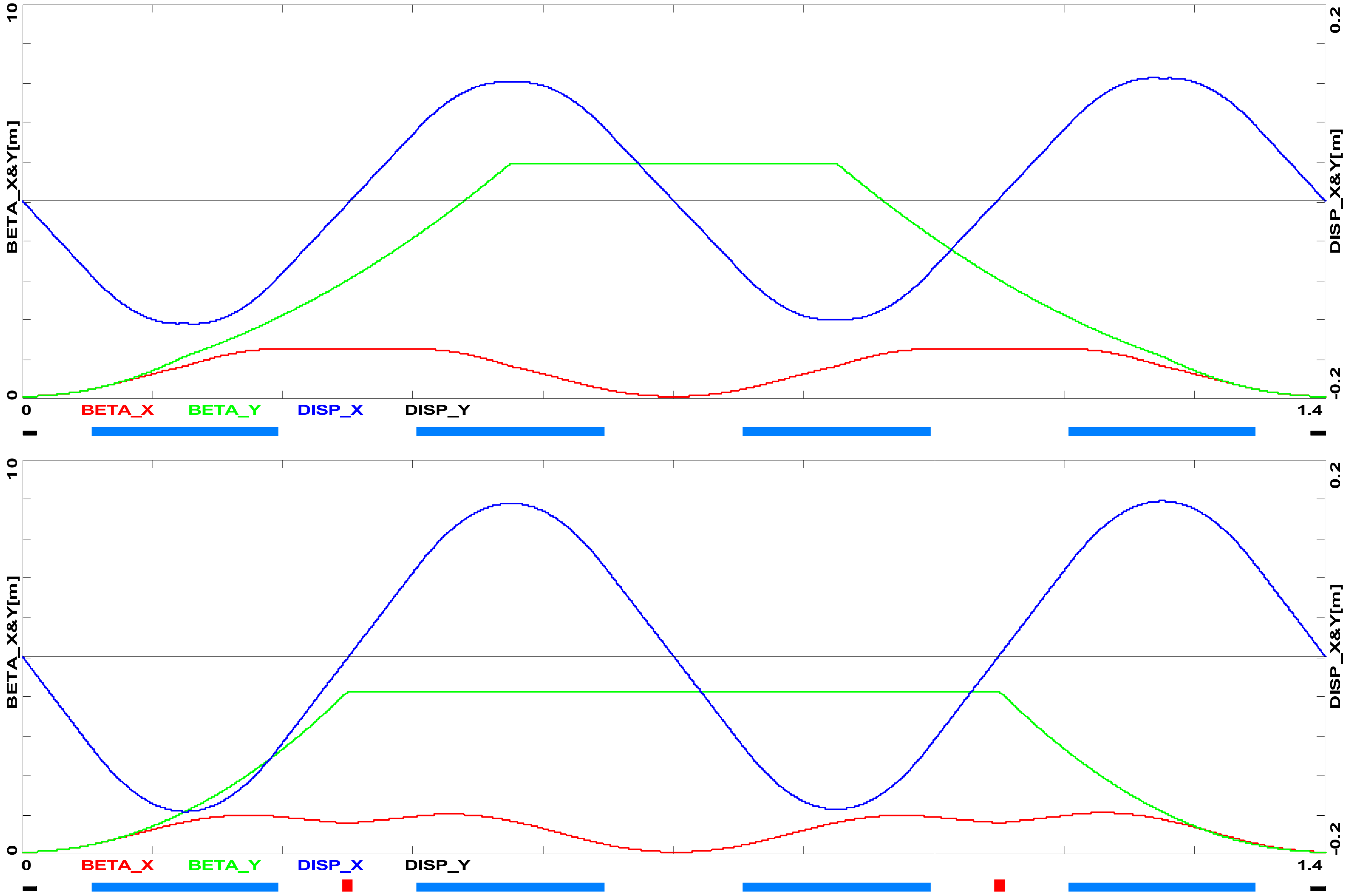


Beam Envelope

Separated Function Design



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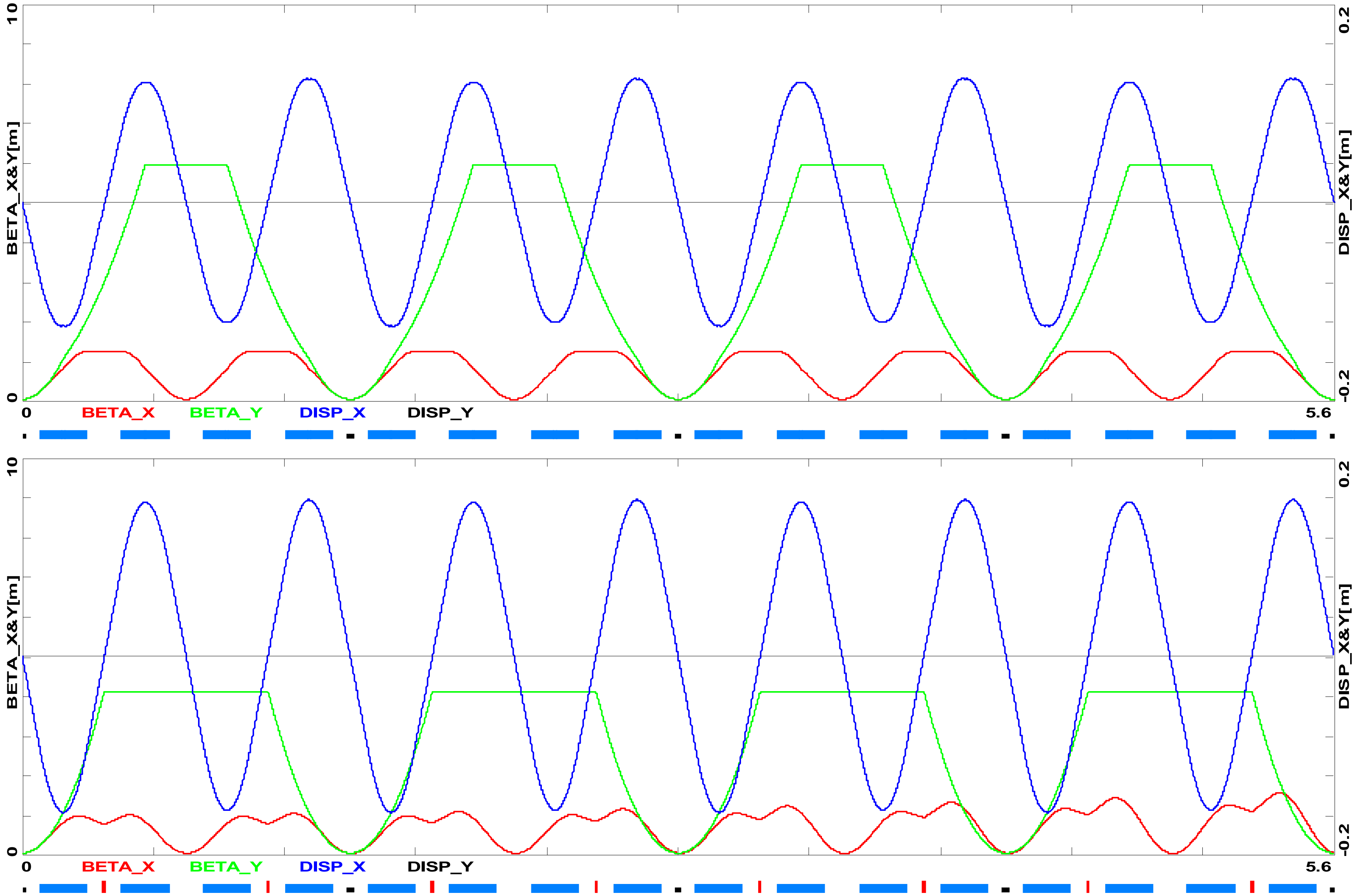
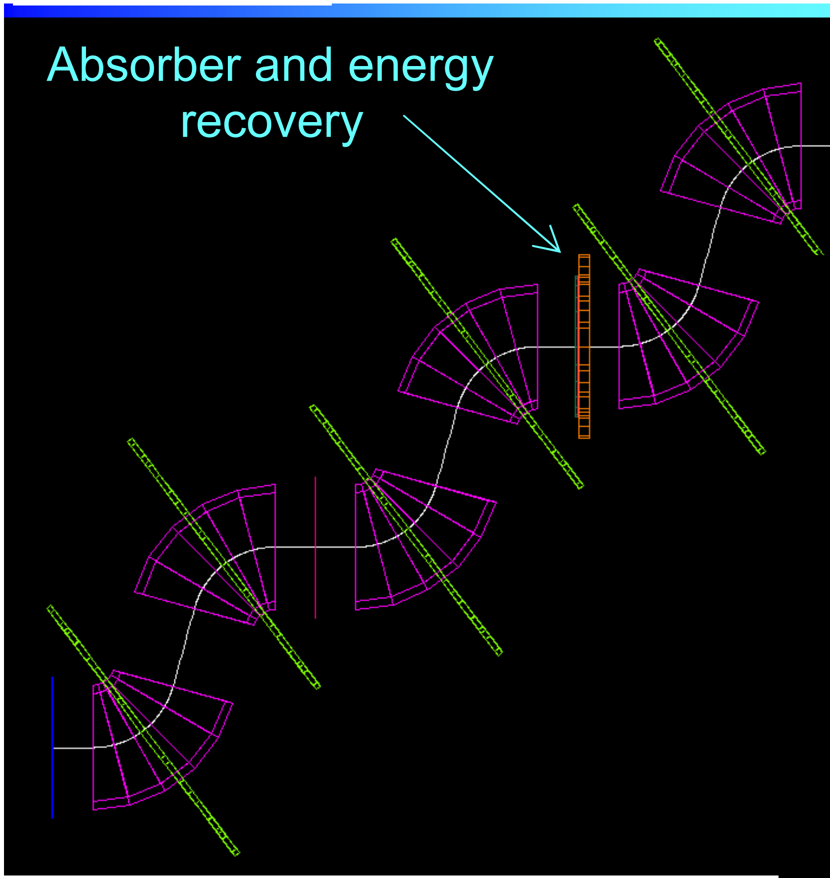
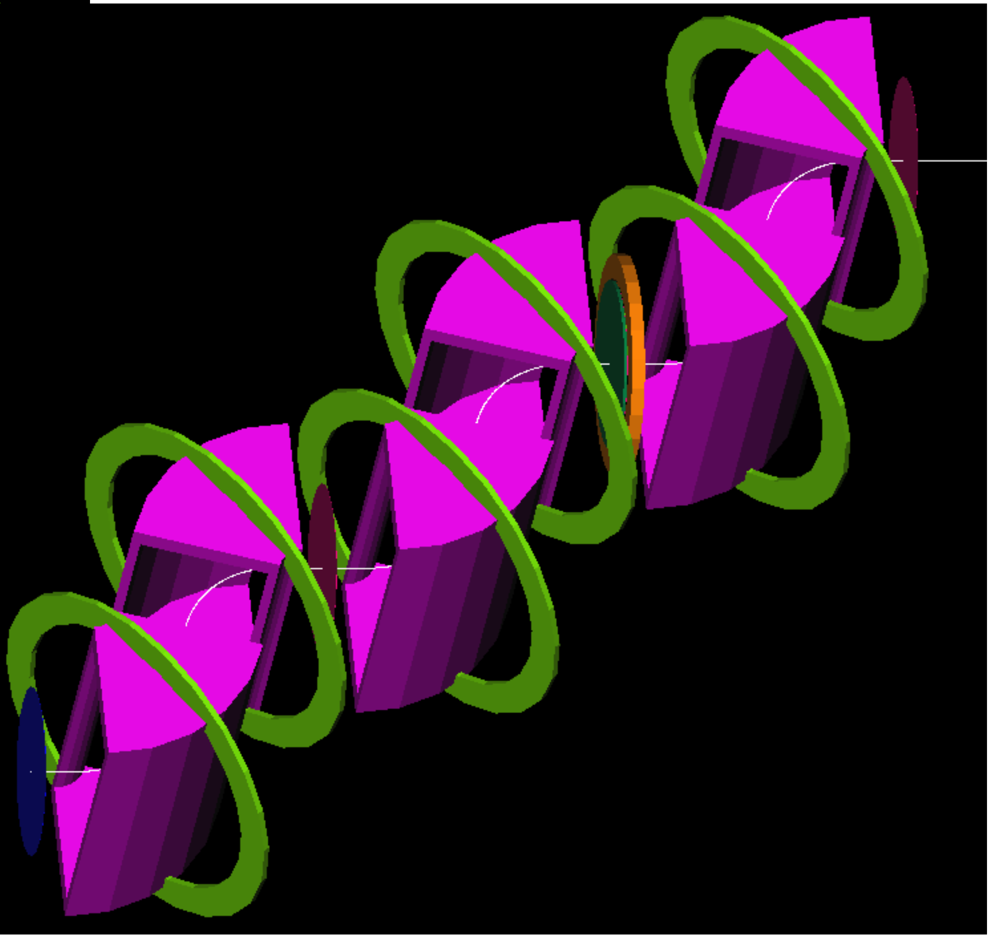
Separated Function Design Has Issues



PIC Lattice Mark-IV

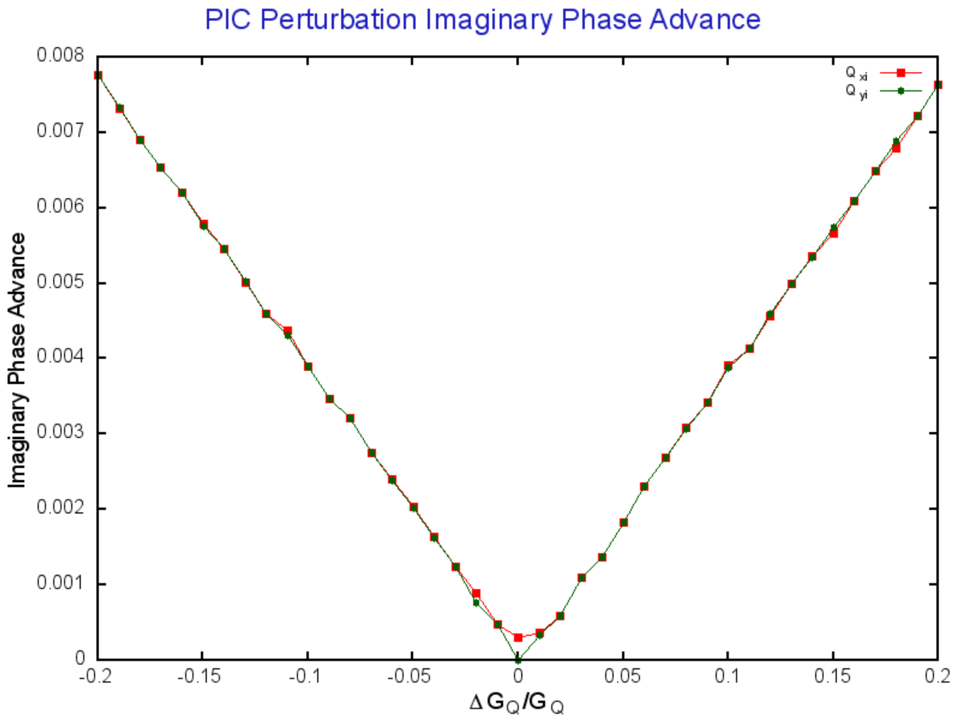


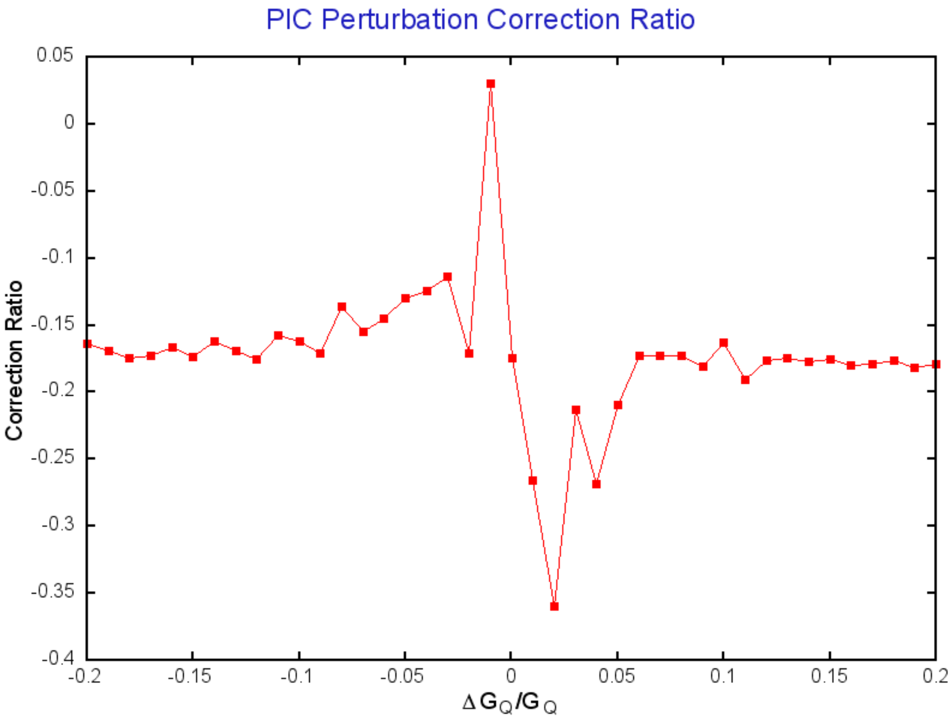
Absorber and energy recovery



θ = 74.6° ⇒ B = 2.17 T
Q ~ -56 T/m (1 cm long)
Base Cell = 0.7 m

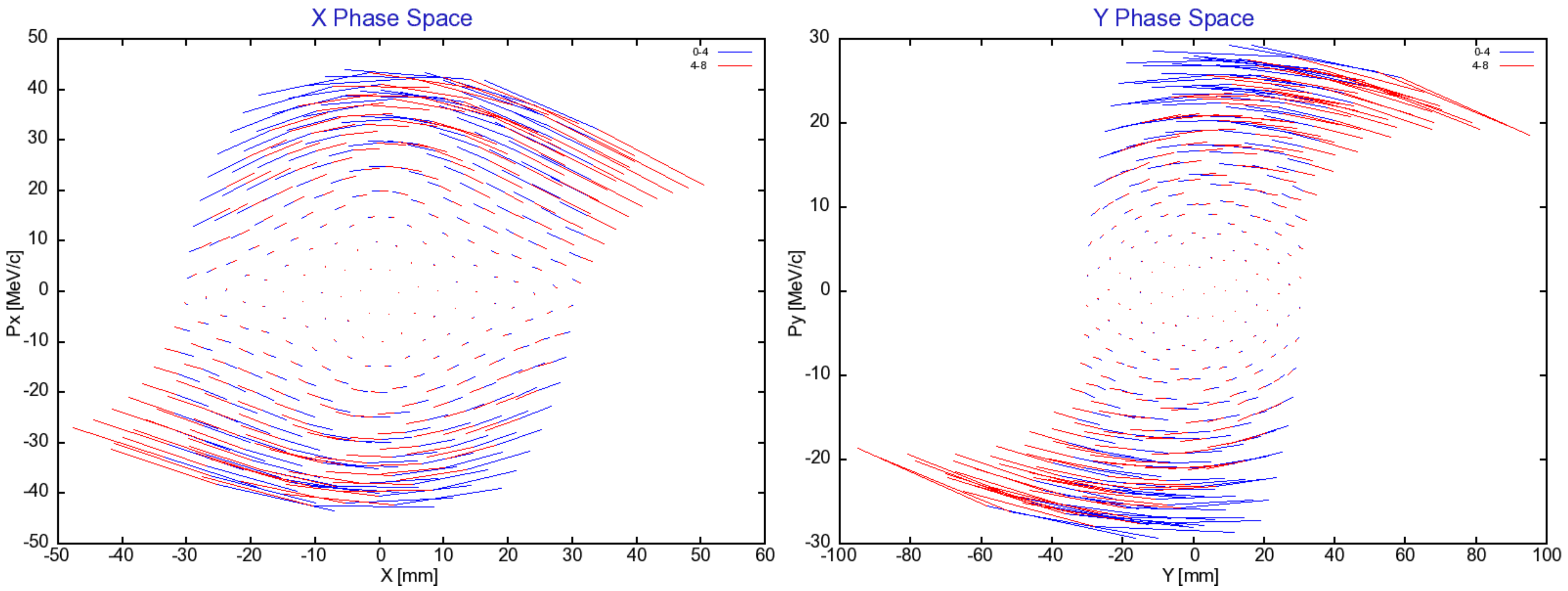
Equalize X-Y Growth


PIC Perturbation Imaginary Phase Advance


PIC Perturbation Correction Ratio

Phase Space

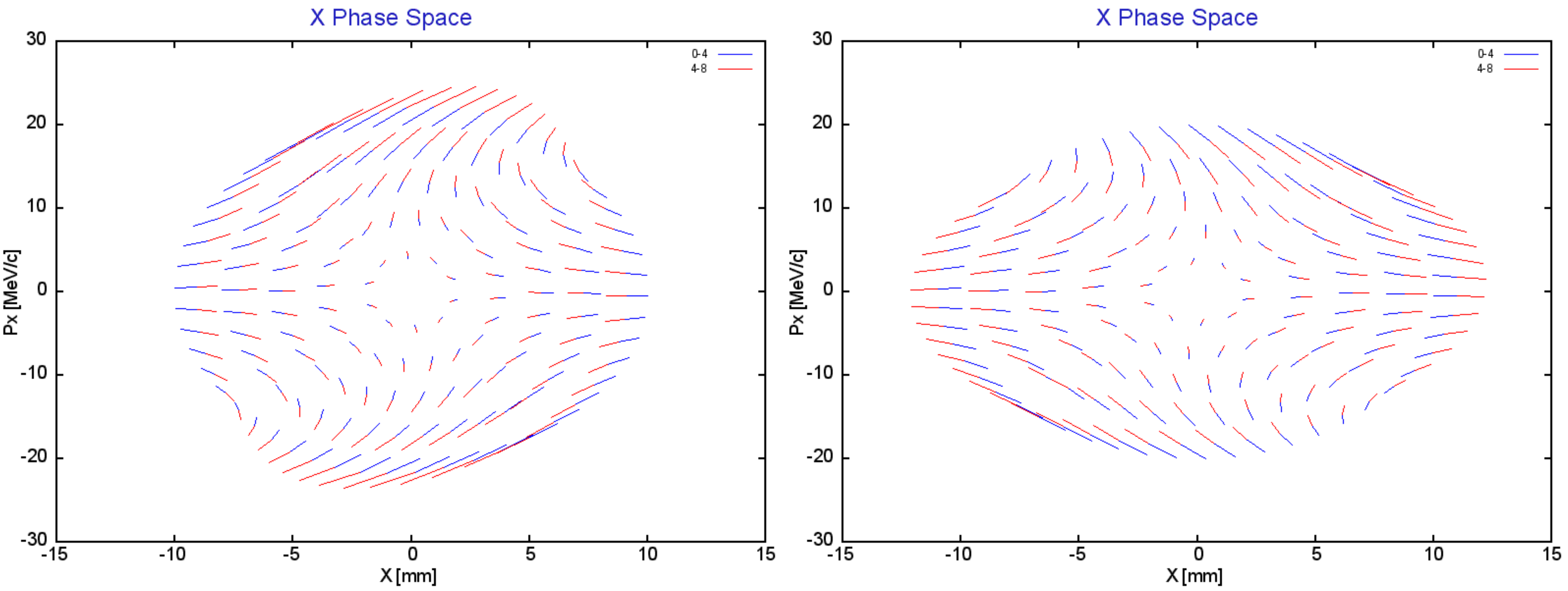
No Perturbation



Acceptance limited by physical apertures

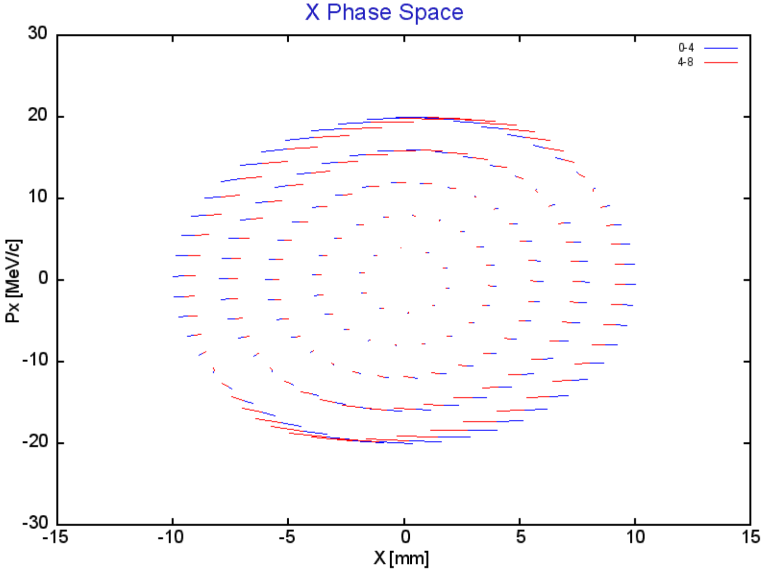
Add Perturbation

No Absorber

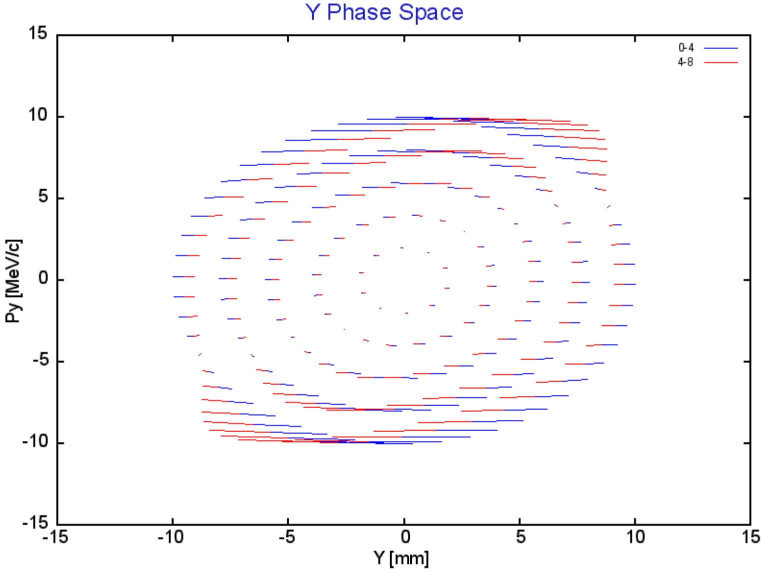

α = +10%

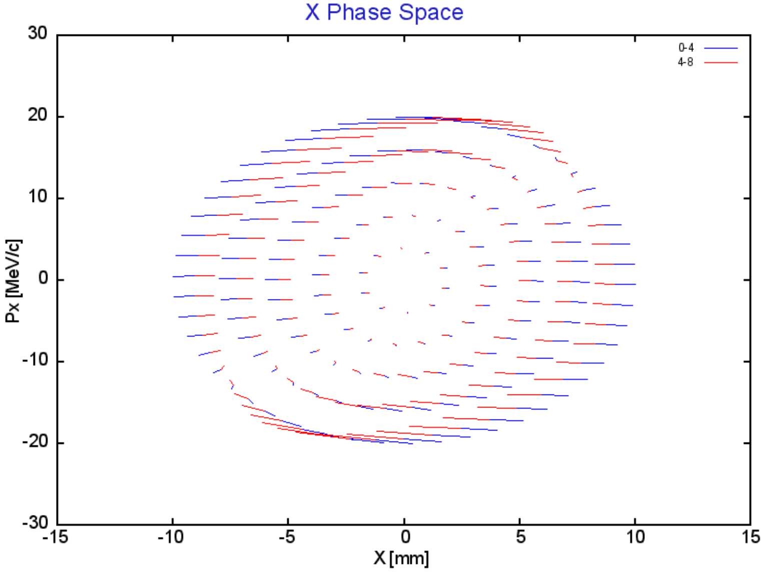
α = -10%

PIC Cooled Phase Space

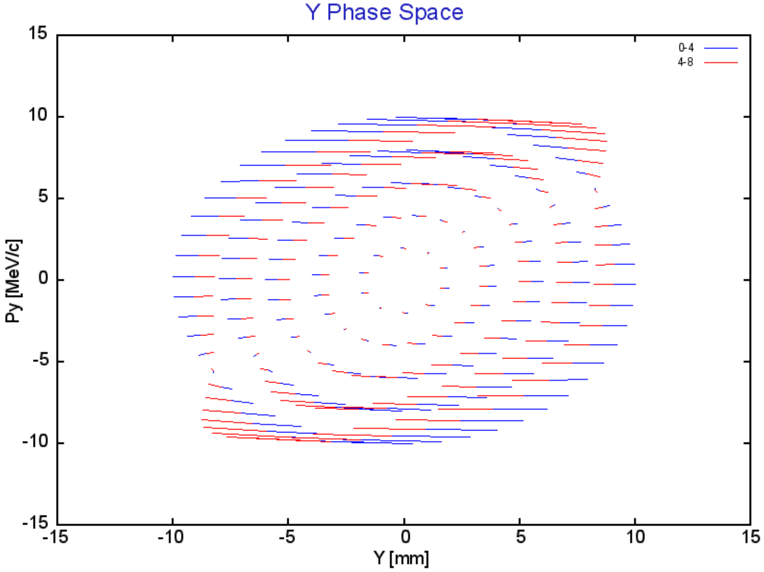

X Phase Space

α = 5%
3.5 mm Be

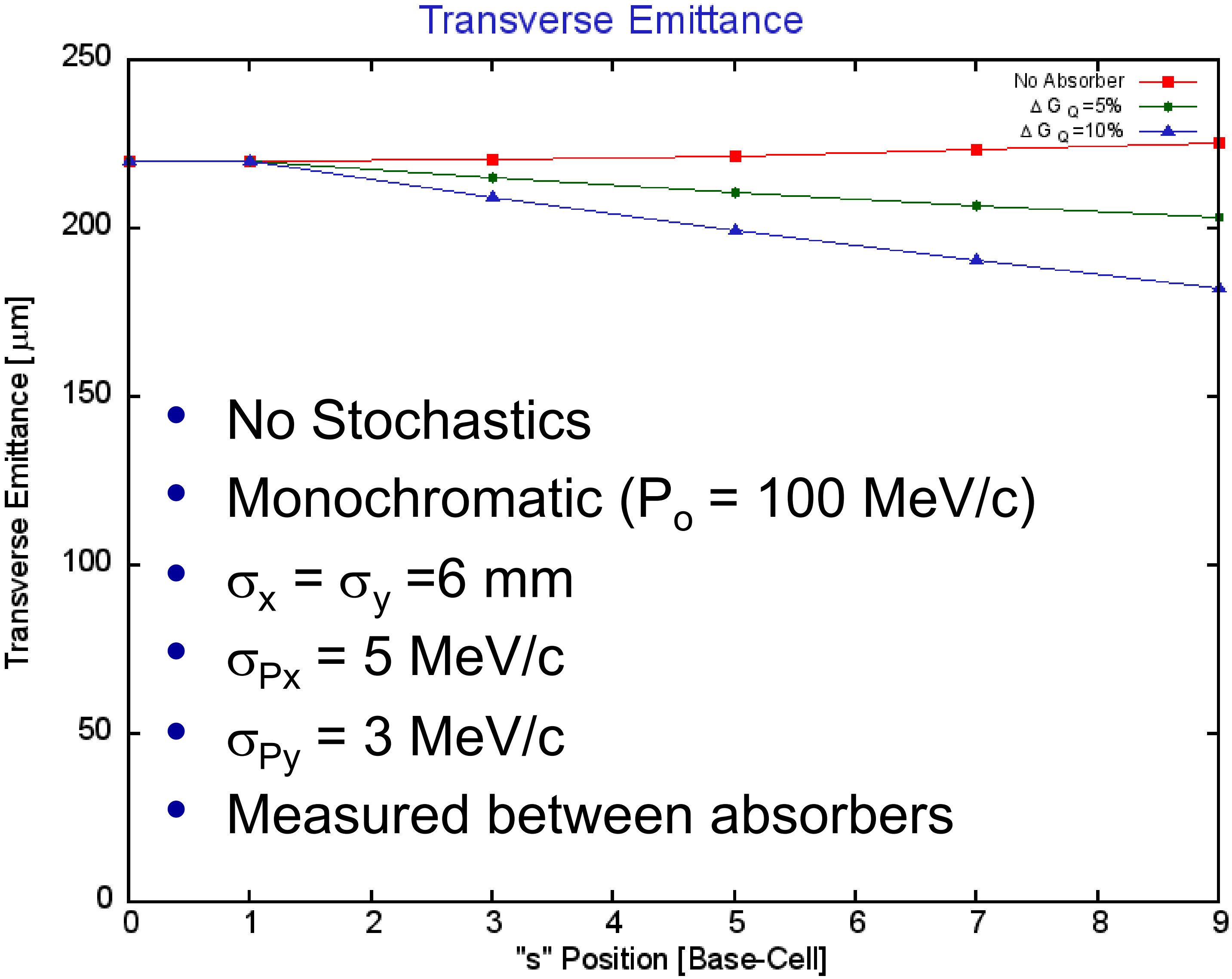

Y Phase Space


X Phase Space

α = 10%
7 mm Be

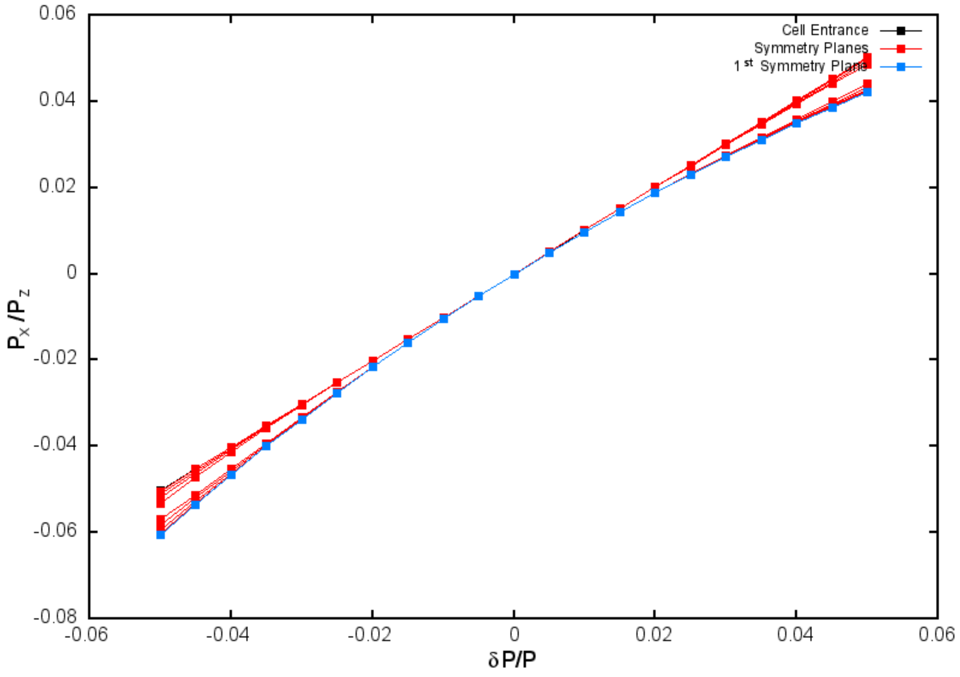

Y Phase Space

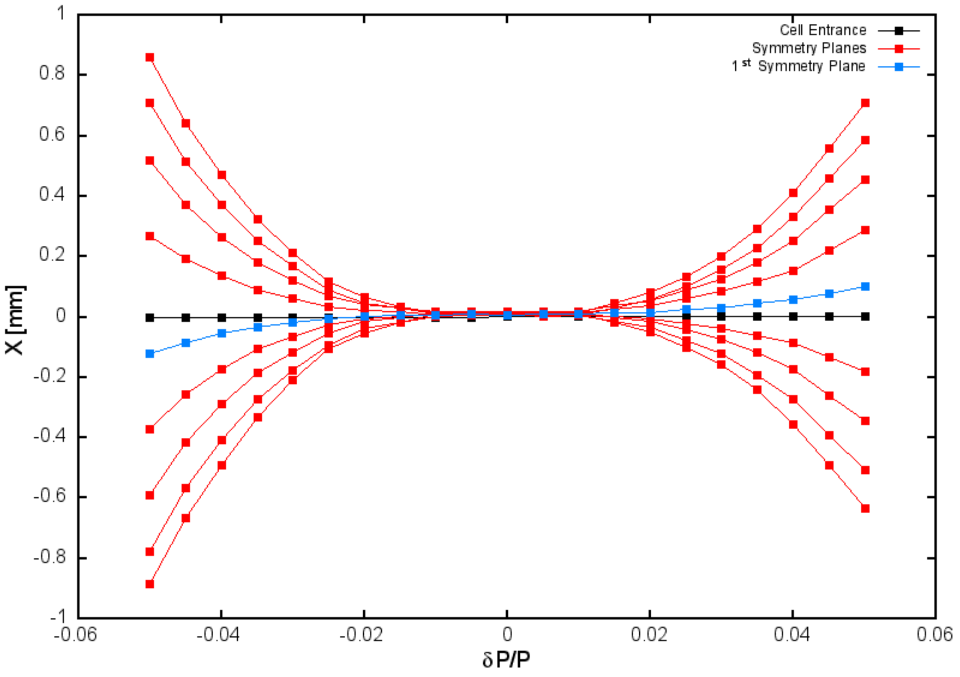
Emittance



- No Stochastics
- Monochromatic (P_o = 100 MeV/c)
- $\sigma_x = \sigma_y$ =6 mm
- σ_{Px} = 5 MeV/c
- σ_{Py} = 3 MeV/c
- Measured between absorbers

Dispersion Matching


Dispersion Prime in X


Dispersion in X

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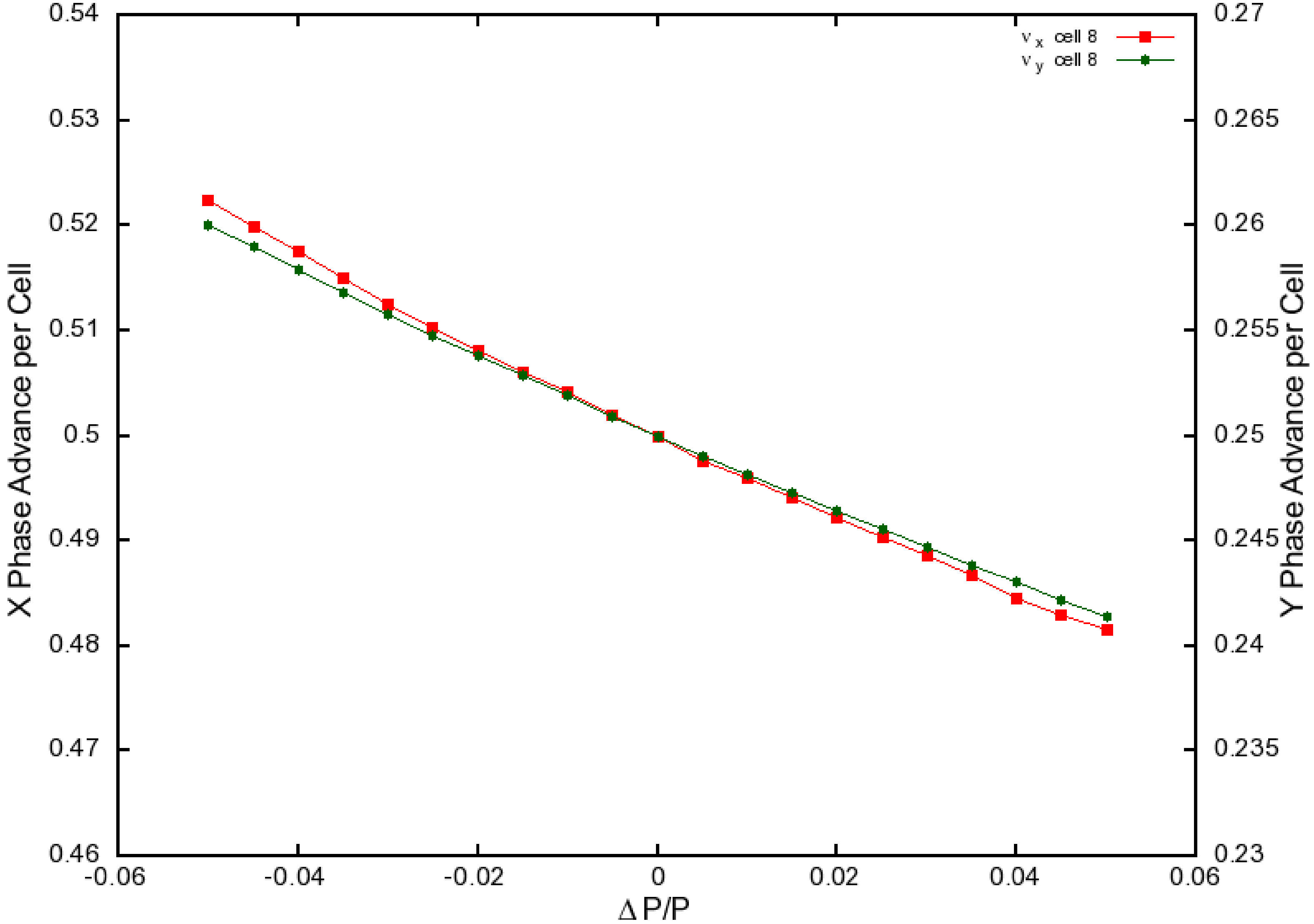
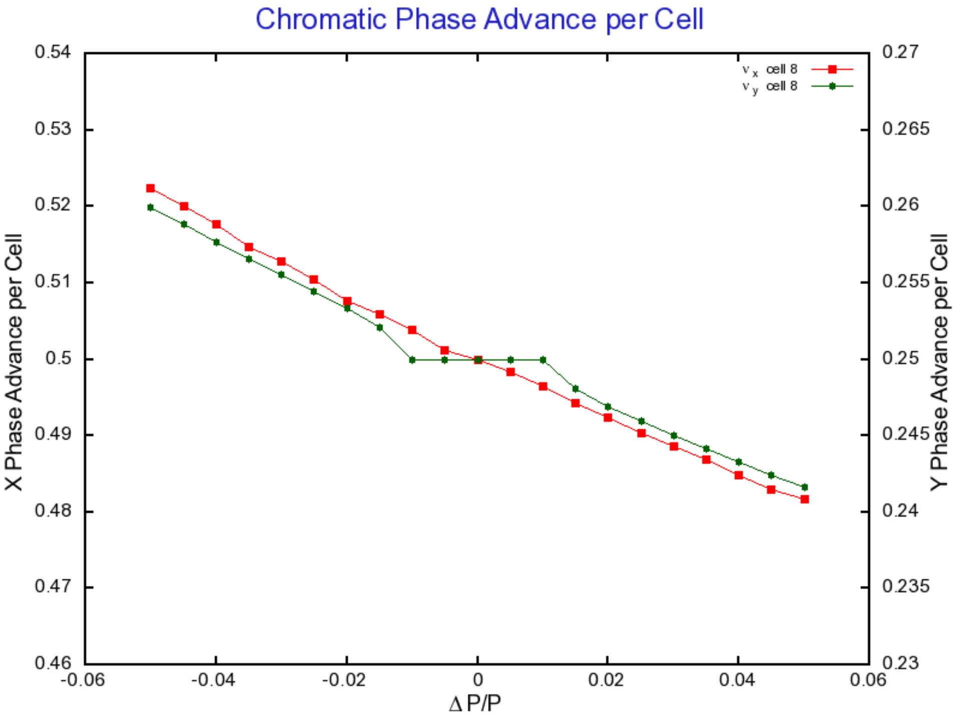
Longitudinal Issues

Chromatic Phase Advance per Cell

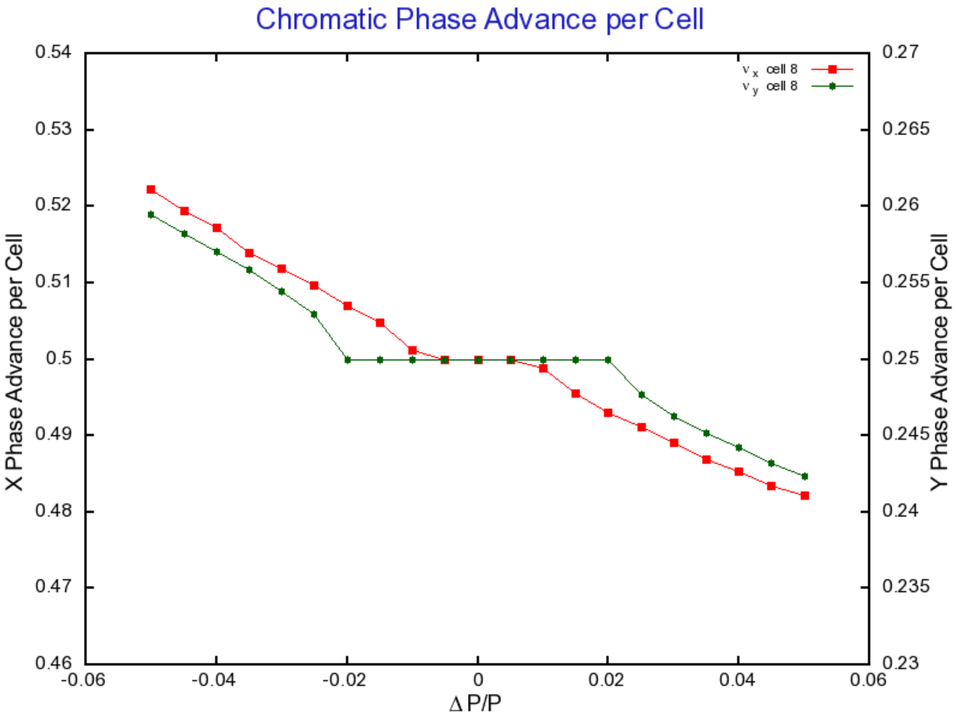


Perturbation Effect

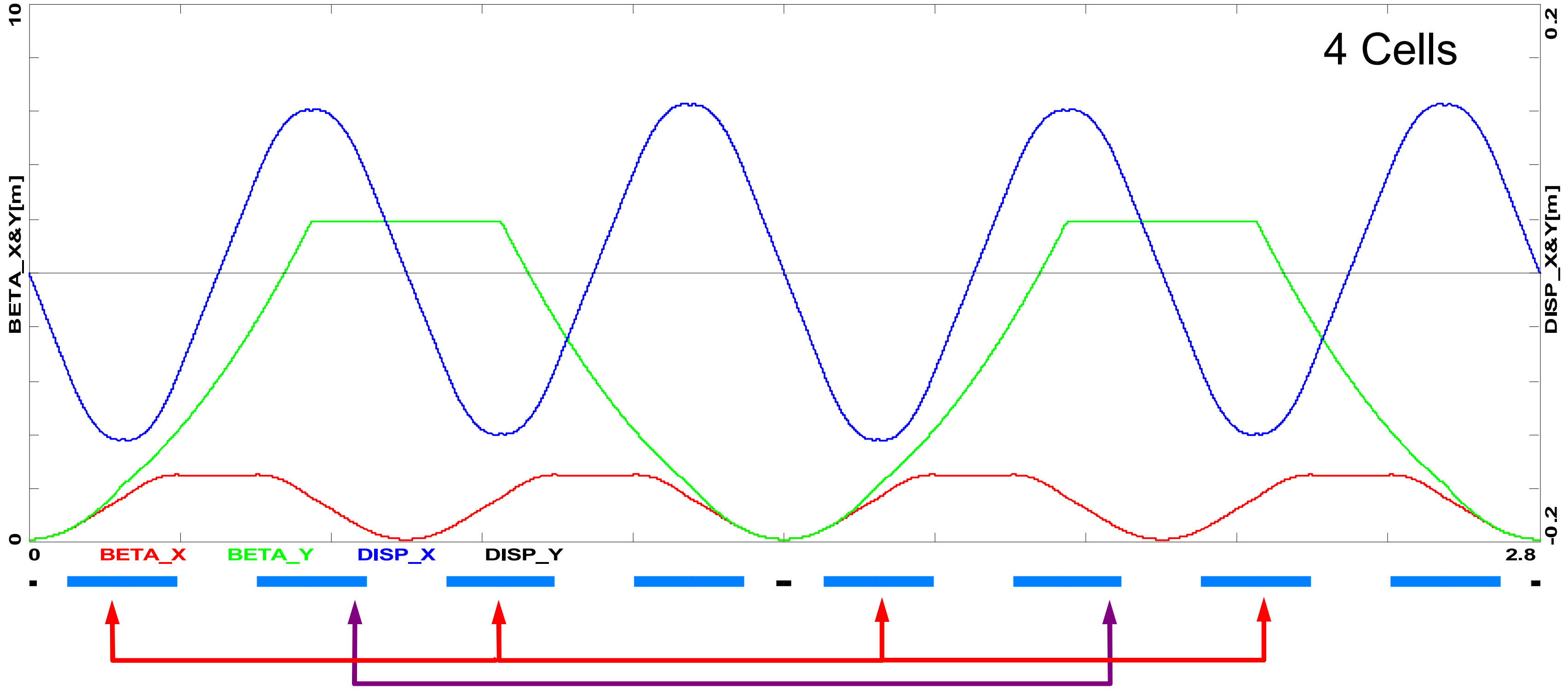
α = +5%



α = +10%



Chromatic Correction



- Standard method uses sextupole families spaced by π phase advance.
    - This method drives the ½ integer resonance unless balanced
    - Requires minimum 2π phase advance in both planes for implementation (4 cells = 2.8 m)

Conclusions

- Symmetric perturbation issue is solved by using different horizontal & vertical phase advances.
- Lattice design shows transverse cooling within aberration controlled region for monochromatic beam.
  - RF recovery not physically realistic
- Transverse acceptance is an issue due to large angular spread
  - Angle spread is fixed by the multiple scattering in the absorber.
  - Reducing lattice length would help
  - Baseline study using possibly unphysically large apertures will identify design parameters to optimize
- Next-order transverse aberration control theory in place, but needs to be implemented.
- Study longitudinal effects in terms of “flat” chromatic tune region with perturbation.
- Fundamental chromatic aberration control not effectively implemented.
- Isochronous lattice for initial PIC cooling without energy recovery needs to be designed.