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# Optimization of adiabatic buncher and phase rotator

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# R&D goal: “affordable” $\nu e, \nu \mu$ -Factory

- Improve from baseline:

- Collection

- Induction Linac → “high-frequency” buncher

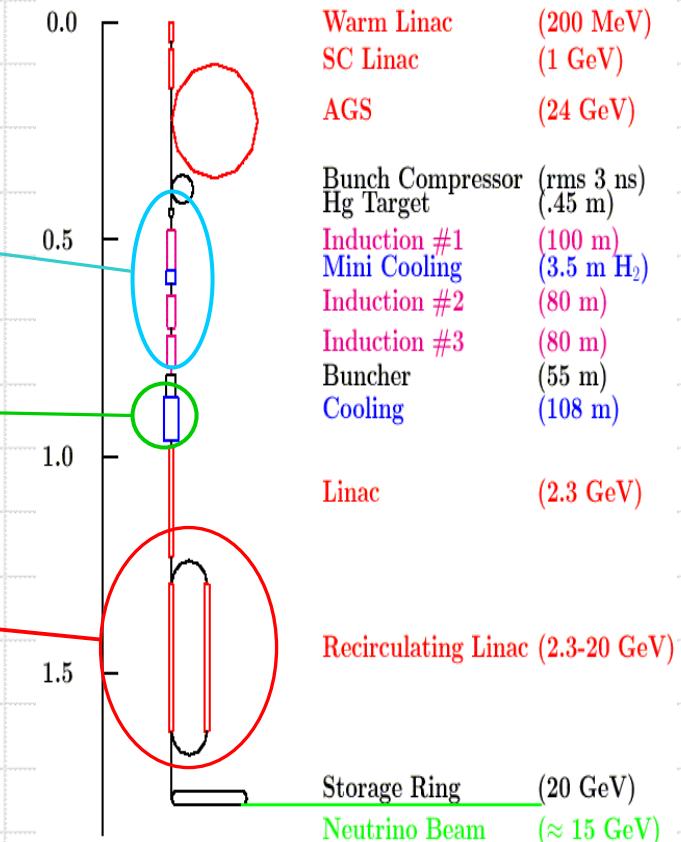
- Cooling

- Linear Cooling → Ring Coolers

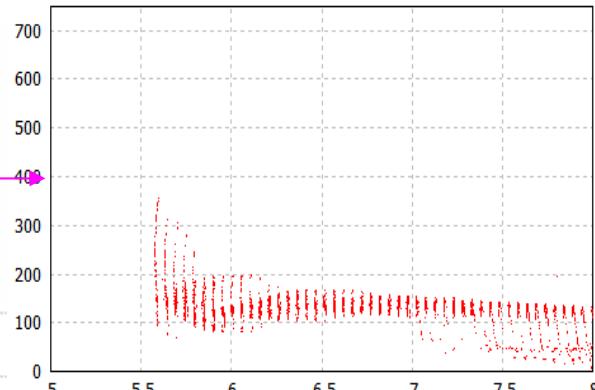
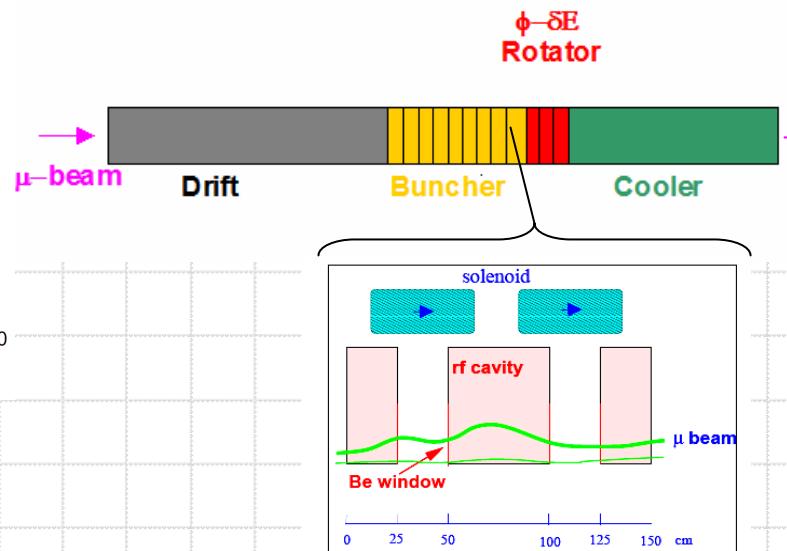
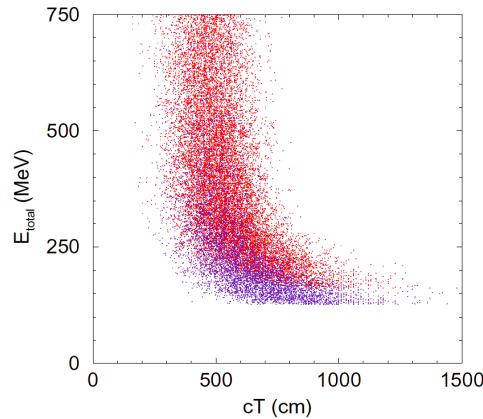
- Acceleration

- RLA → “non-scaling FFAG”

$$\mu^- \rightarrow e^- + \bullet_e + \nu_\mu \quad \text{and/or} \quad \mu^+ \rightarrow e^+ + \bullet_\mu + \nu_e$$

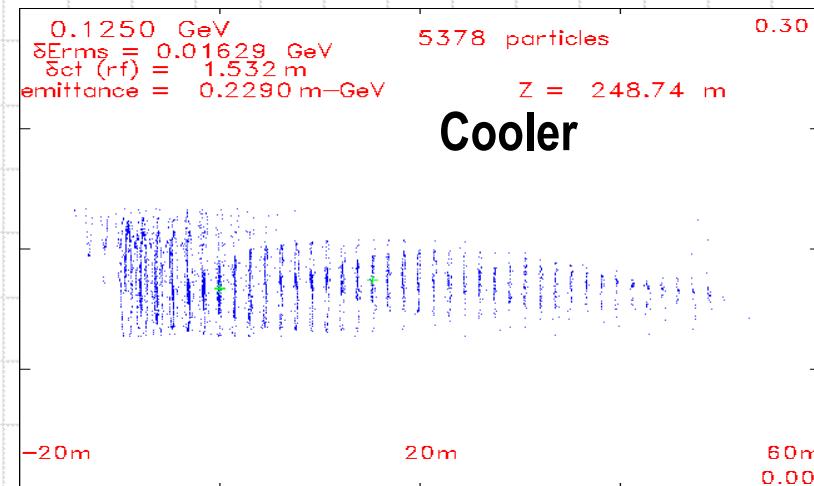
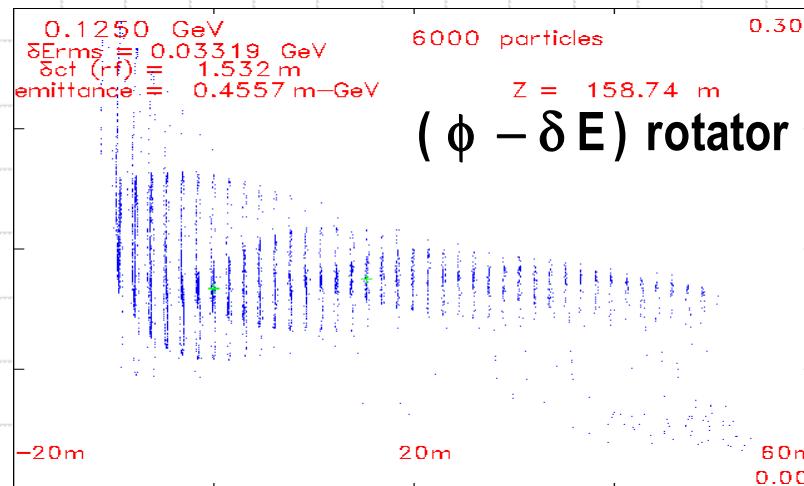
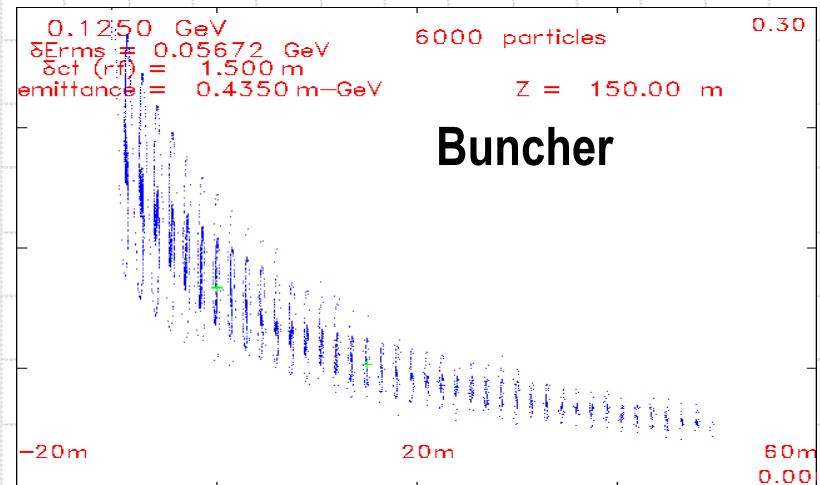
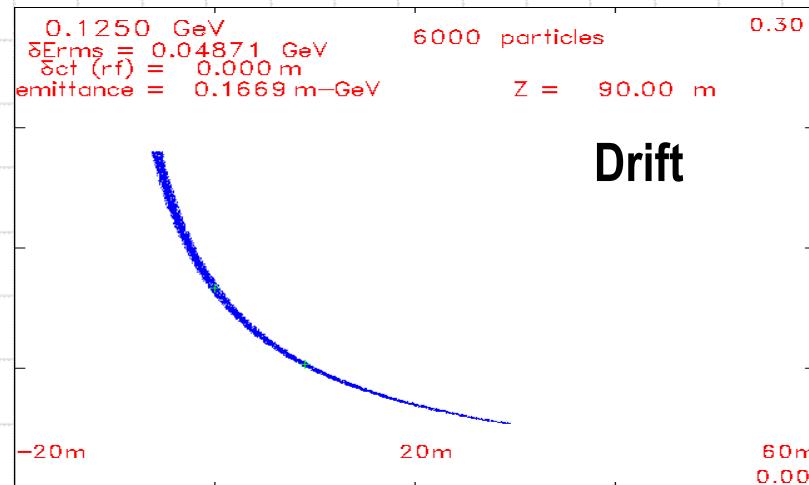


# Key Parameters of Buncher and Rotator



- Drift (Length  $L_D$ )
- Buncher (Length  $L_B$ , RF Gradients  $E_B$ , Final RF frequency  $\lambda_{RF}$ )
- Phase Rotator ( Length  $L_{\phi R}$ , Vernier offset, spacing  $N_{\phi R}$ ,  $\delta_V$ , RF gradients  $E_{\phi R}$  )

# Longitudinal Motion (2D simulations)



System would capture both signs ( $\mu^+$ ,  $\mu^-$ )

# Optimization approach formulation

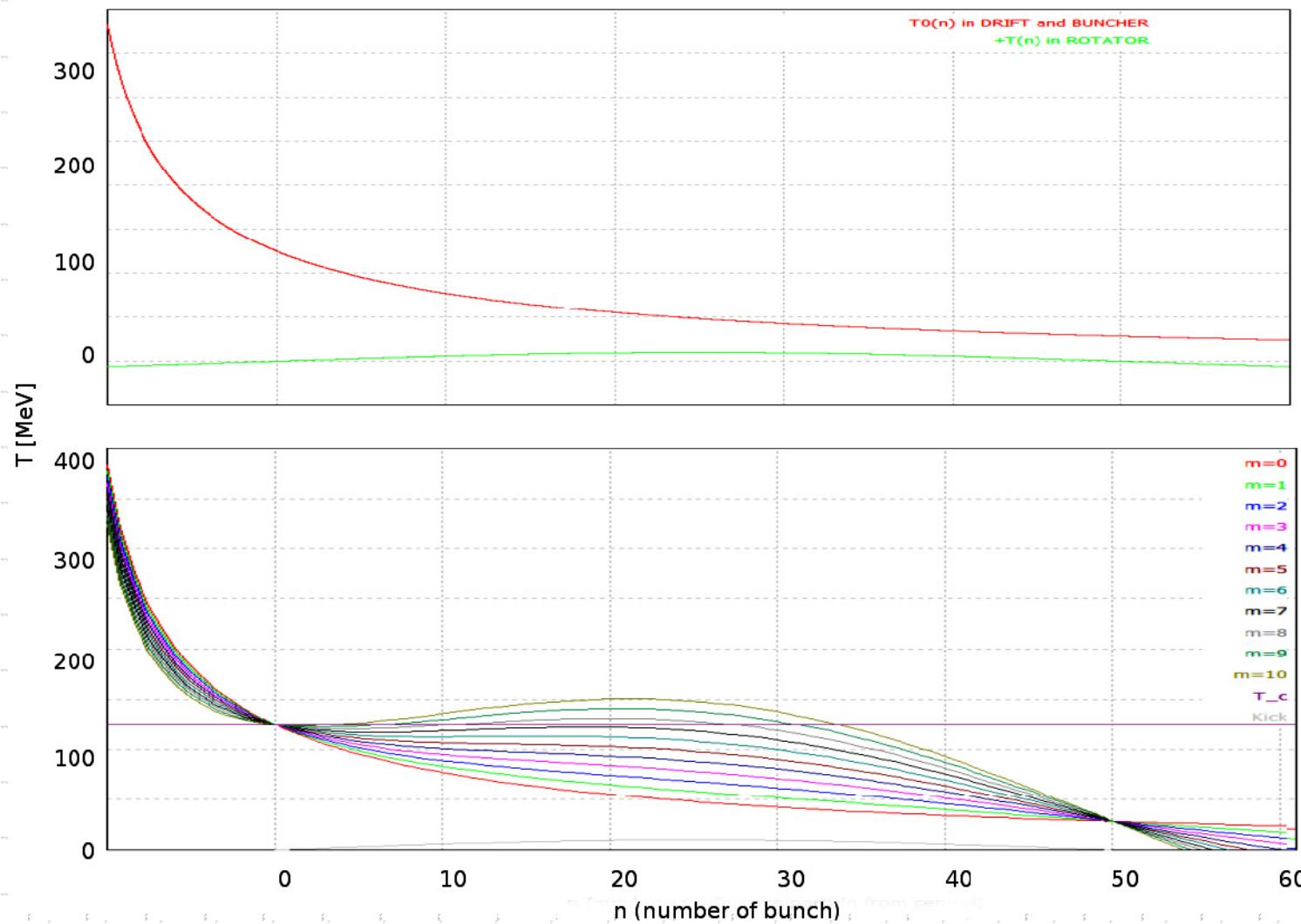
Moving to  $(T, n)$  phase space to study motion of the central particles from the buncher and rotator concept we derive formula describing dependence of the final energy of the central particle of each bunch on lattice parameters:

$$T(n, \dots) = T\left(n, \beta_c, \delta\left(\frac{1}{\beta_c}\right)\right) + m \cdot \Delta T(n, E_{RF}, \delta, n_1, n_2)$$

Problem separation:

- Central particles dynamics
- All beam particles dynamics

# Graphical Illustration



# Merit Functions

- Purpose of the structure is to reduce overall beam energy spread and to put particles energies close to some central energy. It seems natural to use objective function which has the form:

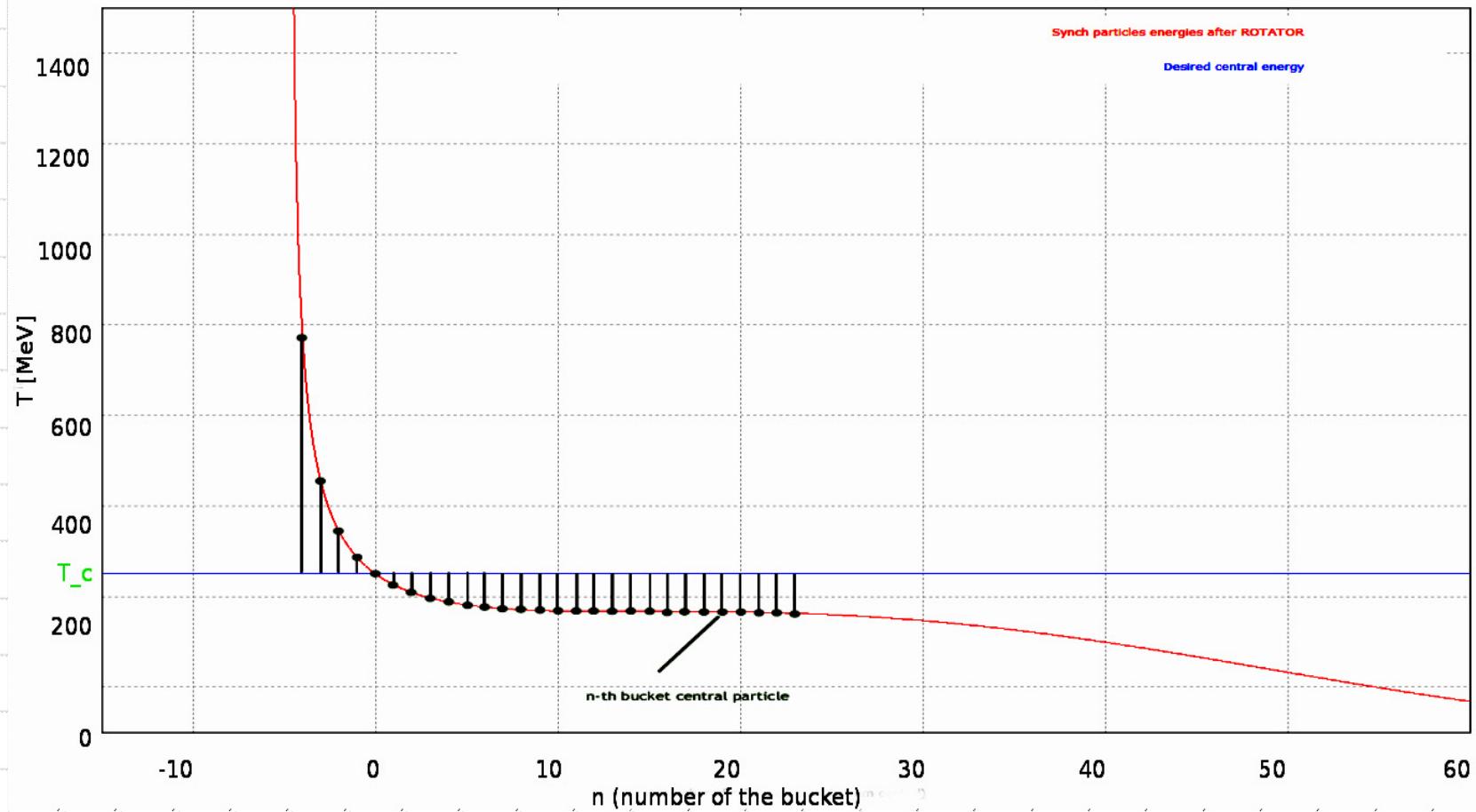
$$I = \sum c_n (T(n, m...) - T_c)^2$$

weight coefficients

measure of energy  
spread

# Merit function 1

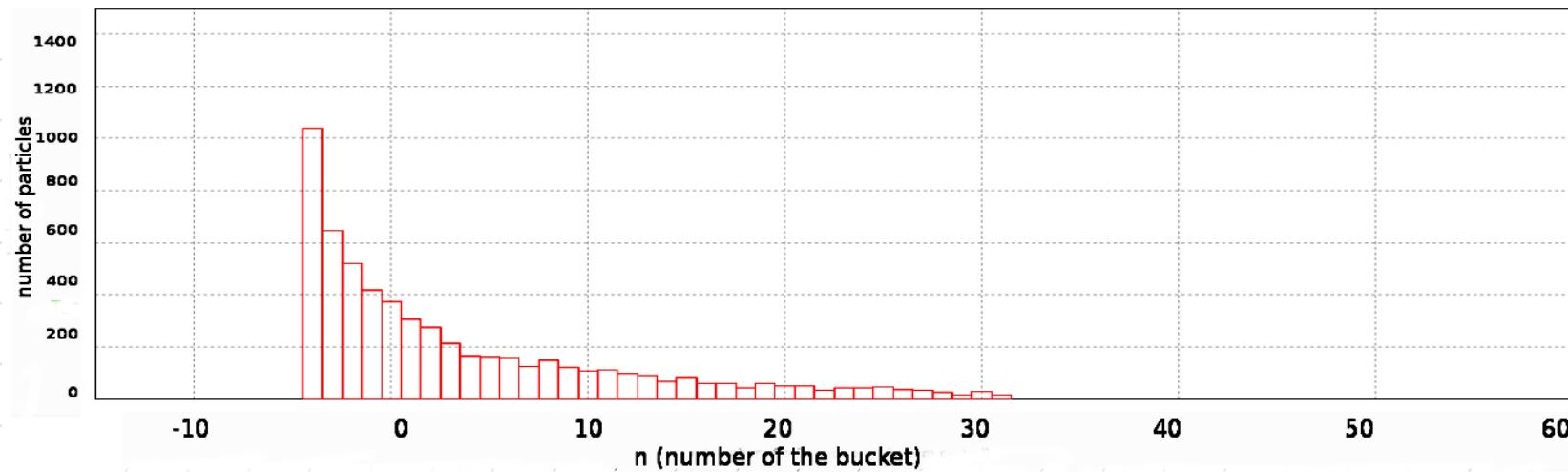
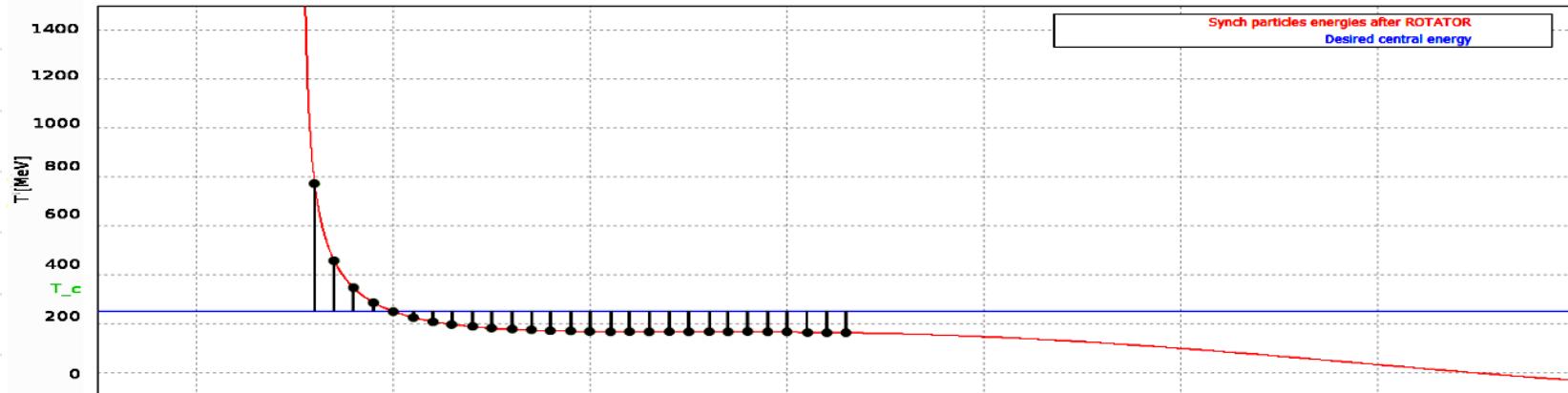
- First, we can set  $C_n = 1, \nabla n$  and get



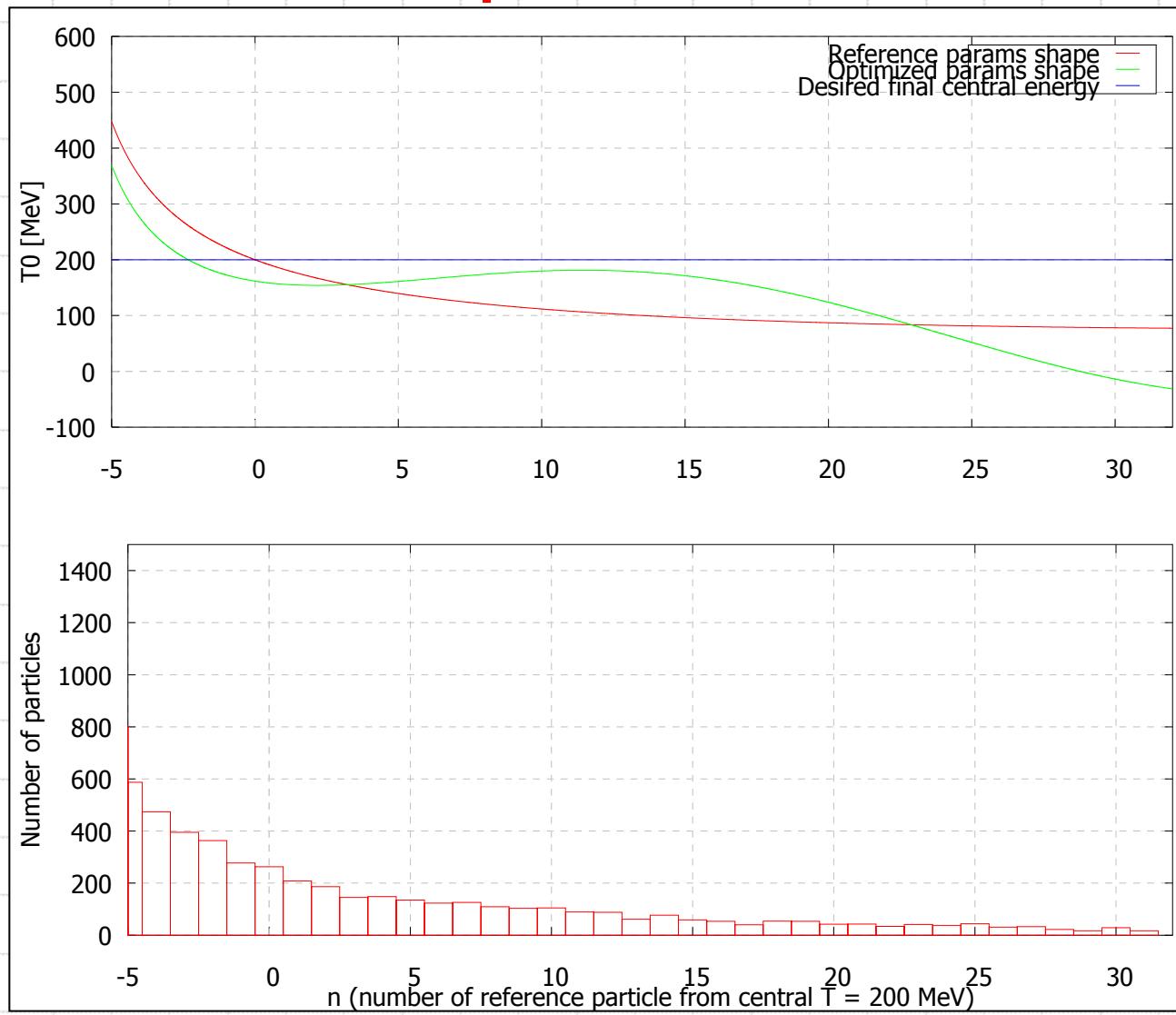
# Merit Function 2

We can use particle energies distribution to get weights

$$I_2 = \sum c_n (T(n, m) - T_c)^2$$



# Optimization with OBJ1

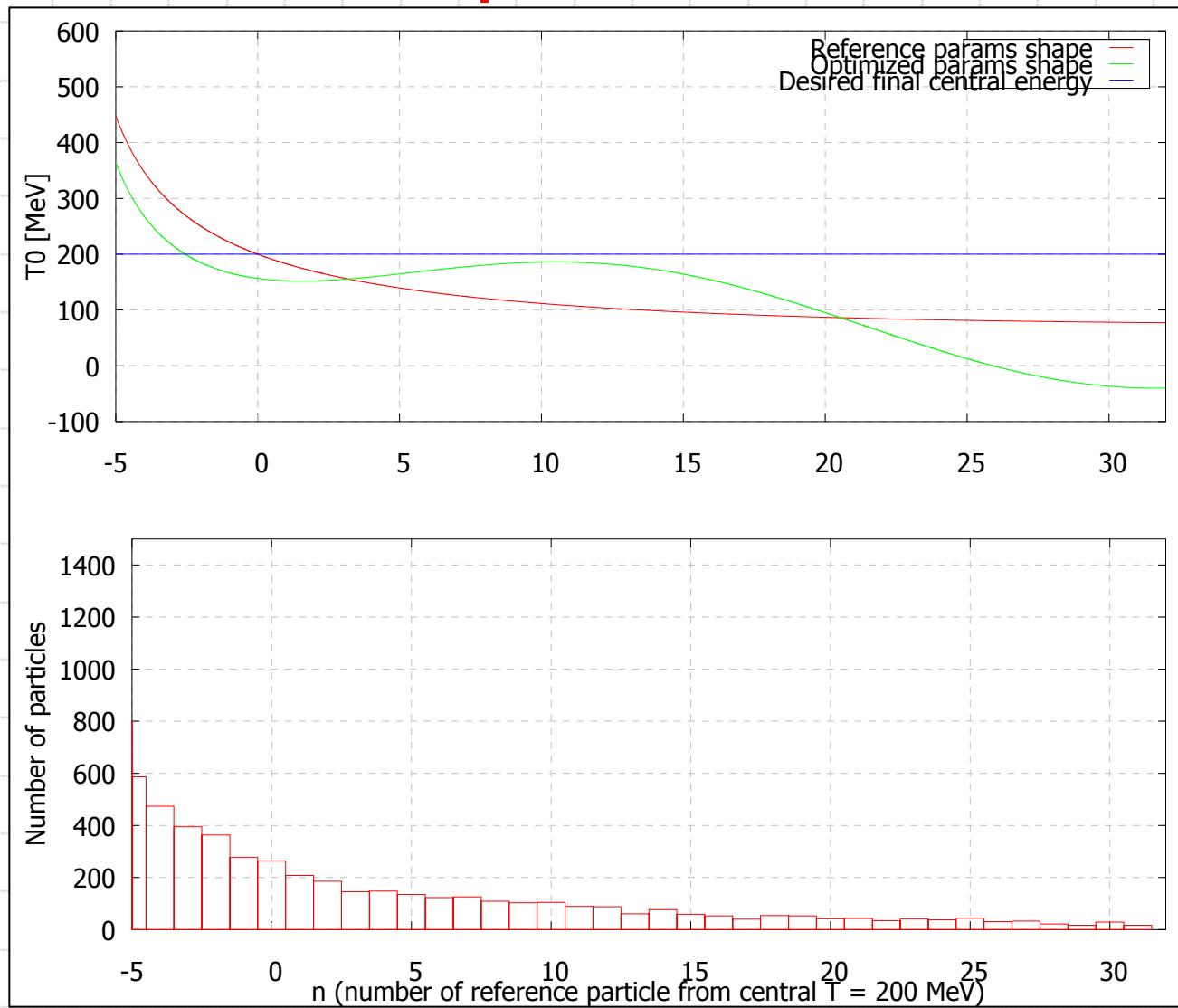


Fixed params:  
 Desired central kinetic energy ( $T_c$ ) = 200.00000000000000  
 $T_0$  in buncher ( $T_0$ ) = 200.00000000000000  
 Drift+Buncher length ( $L_{\text{buncher}}$ ) = 150.00000000000000  
 Final frequency (final\_freq) = 200000000.000000

Variied params:  
 1st lever particle ( $n_1$ ) : 0 ==> 3.00000000000000000000  
 2nd lever particle ( $n_2$ ) : 18 ==> 6.00000000000000000000  
 Vernier parameter (vernier) : 0.032 ==> 0.08  
 RF gradient ( $V_{RF}$ ) : 8 ==> 9.00000000000000000000  
 Number of RFs in rotator ( $m$ ) : 10 ==> 10.00000000000000000000

Objective functions:  
 619593.7642709546 ==>  
 522561.7532899606 = -  
 97032.01098099403  
 !! 750907264.4334378 ==>  
 615875434.3135488 = -  
 135031830.1198890  
 -1066.685941047459 ==>  
 869.7924497231580 =  
 196.8934913243012  
 749769445.5366095 ==>  
 615118895.4079534 = -  
 134650550.1286561

# Optimization with OBJ2

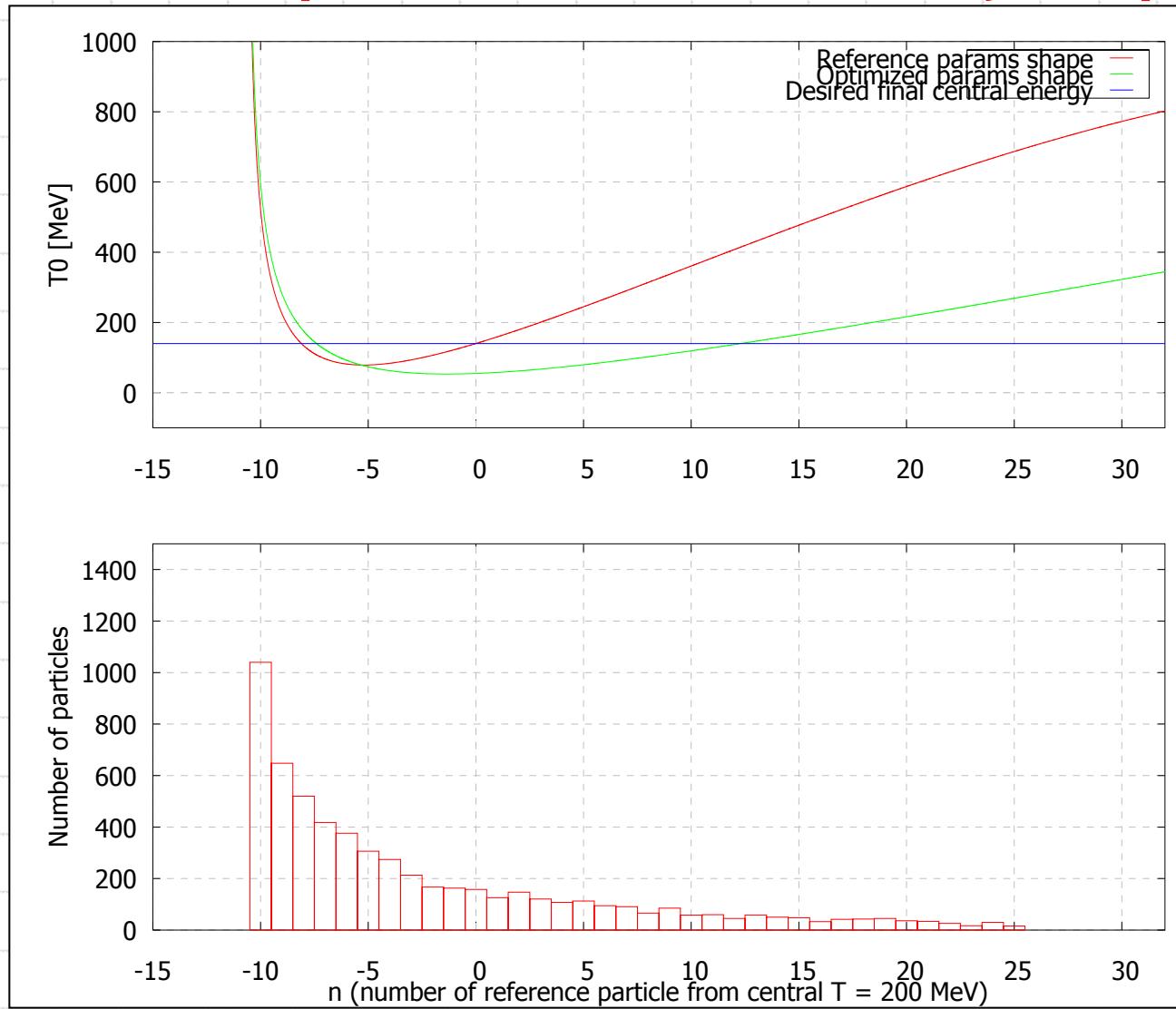


Fixed params:  
 Desired central kinetic energy ( $T_c$ ) = 200.00000000000000  
 $T_0$  in buncher ( $T_0$ ) = 200.00000000000000  
 Drift+Buncher length ( $L_{\text{buncher}}$ ) = 150.00000000000000  
 Final frequency (final\_freq) = 200000000.0000000

Variied params:  
 1st lever particle ( $n_1$ ) : 0 ==> 3.00000000000000000000  
 2nd lever particle ( $n_2$ ) : 18 ==> 6.00000000000000000000  
 Vernier parameter (vernier) : 0.032 ==> 0.07  
 RF gradient ( $V_{RF}$ ) : 8 ==> 9.00000000000000000000  
 Number of RFs in rotator (m) : 10 ==> 10.00000000000000000000

Objective functions:  
 619593.7642709546 ==>  
 522561.7532899606 = -  
 97032.01098099403  
 !! 750907264.4334378 ==>  
 615875434.3135488 = -  
 135031830.1198890  
 -1066.685941047459 ==>  
 869.7924497231580 =  
 196.8934913243012  
 749769445.5366095 ==>  
 615118895.4079534 = -  
 134650550.1286561

# Optimization with Study 2b params



**Fixed params:**

- Desired central kinetic energy ( $T_c$ ) = 140.00000000000000
- $T_0$  in buncher ( $T_0$ ) = 140.00000000000000
- Drift+Buncher length ( $L_{buncher}$ ) = 150.00000000000000
- Final frequency (final\_freq) = 200000000.000000

**Varied params:**

- 1st lever particle ( $n1$ ) : ==> 7.000000000000000
- 2nd lever particle ( $n2$ ) : ==> 14.000000000000000
- Vernier parameter (vernier) : ==> 0.100000000000000E-01
- RF gradient ( $V_{RF}$ ) : ==> 14.000000000000000
- Number of RFs in rotator ( $m$ ) : ==> 97.000000000000000

**Objective functions:**

- !! 485808.1248155629 ==> 316581.0365864867 = - 169227.0882290762 589204203.9100170 ==> 312400472.6813000 = - 276803731.2287170 -860.6933665745339 ==> 1008.554979752263 = - 147.8616131777286 588463410.8387516 ==> 311383289.5341169 = - 277080121.3046346

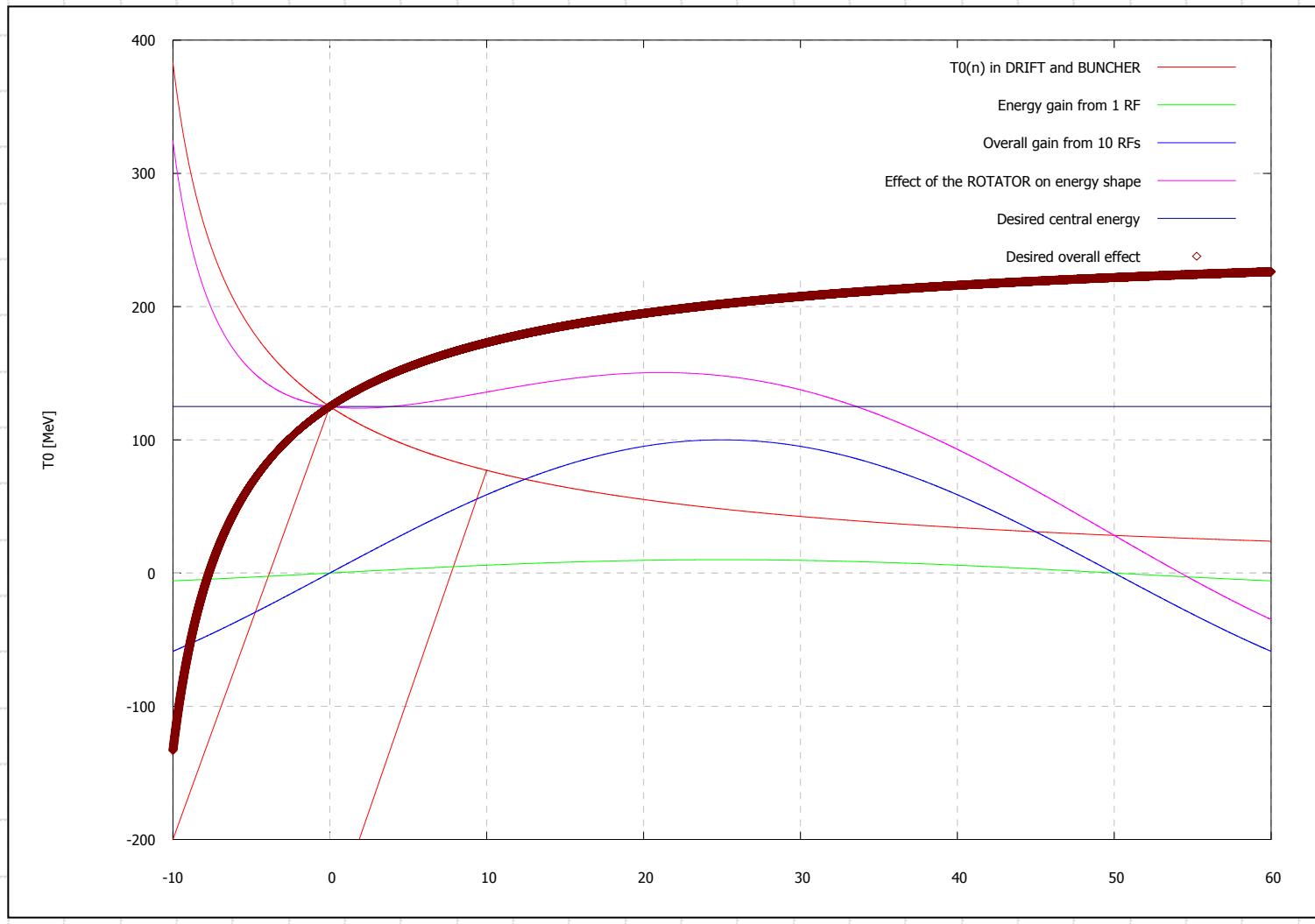
## To do

- Check optimized parameters for the whole beam distribution (COSY, ICOOL). Is it better for all particles? 6D-dynamics.
- Different merit functions? Other parameters to incorporate (bucket capture, cost, etc...). Playing with the model.
- Tuning overall rf field effect?
- Develop different optimal sets of parameters (on cost, number of muons survived, captured, final frequency, etc. or combinations). Verify by simulations.

# Summary

- ✓ An approach for optimization of bunch central energies in buncher and phase rotator is proposed.
- ✓ The algorithm is implemented in COSY Infinity. It enables optimization on any set of supported parameters (length of the buncher and rotator, final frequency, central energy, E field gradient, phases).
- ✓ Optimization runs are presented.
- ✓ Future plans are discussed.

# Overall RFs Effect



# Central Kinetic Energies after Buncher

- Moving to  $(T, n)$  phase space to study motion of the central particles from the buncher concept we derive following relation:

$$T(n, \beta_c, \delta(1/\beta_c)) = W_0 \left[ \frac{1}{\sqrt{1 - \left( \frac{\beta_c}{1 + n\beta_c \delta(1/\beta_c)} \right)^2}} - 1 \right]$$

Puts limits on  $n_{\min}$  and  $n_{\max} \Rightarrow n_{\text{bunches}}$ !

# Energy Gain in Rotator and Final Central Kinetic Energies

- From the rotator concept we derive amount of energy gained by n-th central particle in each RF (kept const in ROTATOR)

$$\Delta T(n, E_{RF}, \delta, n_1, n_2) = E_{RF} \sin\left(2\pi \cdot \delta \frac{n - n_1}{n_2 - n_1}\right)$$

- So final energy n-th central particle has after the BUNCHER+ROTATOR is a function of n,m,...

$$T(n, m, \dots) = T(n, \dots) + m \cdot \Delta T(n, \dots)$$