MHD Simulation for Free Surface Hg Jet Dispersal at Low Magnetic Reynolds Numbers

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Abstract

- **MHD system of equations and approximations**
- **Front tracking for free surface flows and EB elliptic solver**
- **Simulations of the MHD processes of mercury jet dispersal**
- **Conclusion and Future plan**

1. MHD system of equations

Full system of MHD equations

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u})$$

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\mathbf{u} = -\nabla P + \mu\Delta\mathbf{u} + \frac{1}{c}(\mathbf{J}\times\mathbf{B})$$

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)e = -P\nabla\cdot\mathbf{u} + \frac{1}{\sigma}\mathbf{J}^2$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) - \nabla\times\left(\frac{c^2}{4\pi\sigma}\nabla\times\mathbf{B}\right)$$

$$P = P(\rho, e), \qquad \nabla\cdot\mathbf{B} = 0$$

Low magnetic Re approximation & charge neutrality

$$\nabla\cdot J + \frac{\partial \rho_e}{\partial t} = 0$$

$$\mathbf{J} = \sigma\left(-\nabla\phi + \frac{1}{c}\mathbf{u}\times\mathbf{B}\right)$$

$$\Delta\phi = \frac{1}{c}\nabla\cdot(\mathbf{u}\times\mathbf{B}),$$

$$\text{with } \left.\frac{\partial\phi}{\partial\mathbf{n}}\right|_{\Gamma} = \frac{1}{c}(\mathbf{u}\times\mathbf{B})\cdot\mathbf{n}$$

$$\mathbf{B} = \mathbf{B}_{\text{ext}}(x,t), \quad \nabla\cdot\mathbf{B}_{\text{ext}} \equiv 0$$

2. Front tracking and EB method

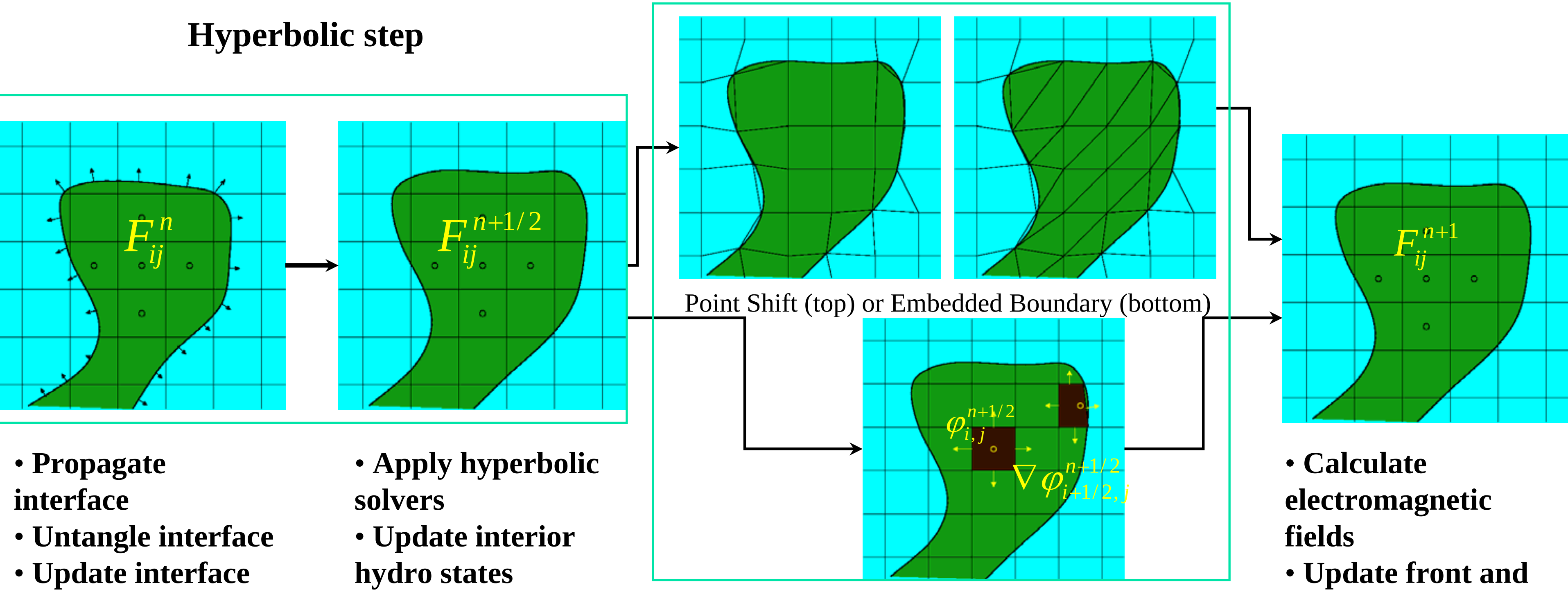
- The low magnetic Re MHD is a coupled hyperbolic/elliptic system. Operator splitting.

- The hyperbolic subsystem is solved on a finite difference grid in both domains separated by the free surface using front tracking numerical techniques.
  - Implemented in FronTier code
  - Riemann problem for interface propagation
  - Complex interfaces with topological changes in 2D and 3D
  - High resolution hyperbolic solvers
  - Realistic EOS models

- The elliptic subsystem is solved in geometrically complex domains
  - Embedded boundary finite volume discretization
  - Fast parallel linear solvers

Elliptic step

Hyperbolic step



- **Propagate interface**
- **Untangle interface**
- **Update interface states**

- **Apply hyperbolic solvers**
- **Update interior hydro states**

- **Generate finite element grid**
- **Perform mixed finite element discretization**
  or
- **Perform finite volume discretization**
- **Solve linear system using fast Poisson solvers**

- **Calculate electromagnetic fields**
- **Update front and interior states**

2. EB method

(1) Why EB

> **Point-shift grid generation and finite element discretization method**

- **Second order accurate for gradients**
- **Compatible with mixed finite element formulation**
- **Capable of generating grids for vector finite elements**
- **Not robust (especially in 3D)**

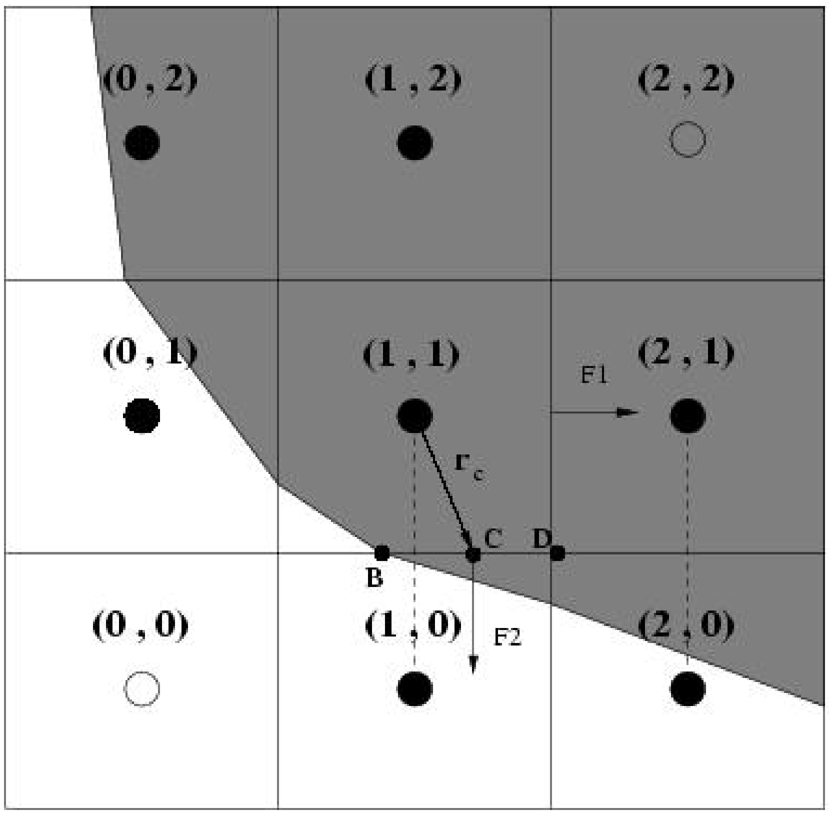
> **EB**

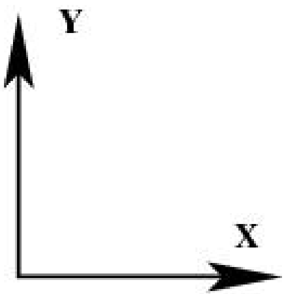
- **Advantages of dealing with complex geometric domains**
- **second-order accuracy of solution and robust**
- **Trivial work to implement the algorithm in parallel computing**

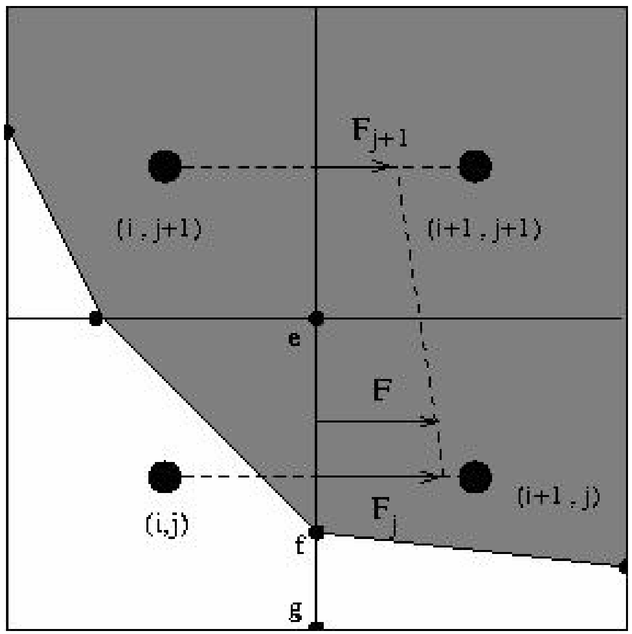
(2) Main Points

- Based on the finite volume discretizations
- Potential is treated as cell centered value, even if the center is outside the computational domain
- Domain boundary is embedded in the rectangular Cartesian grid, and solution is treated as a cell-centered quantity
- Using finite difference for full cell and linear interpolation for cut cell flux calculation

(3). Stencil Setting







$$\varphi = e^{k_1 X^2 + k_2 Y^2}$$

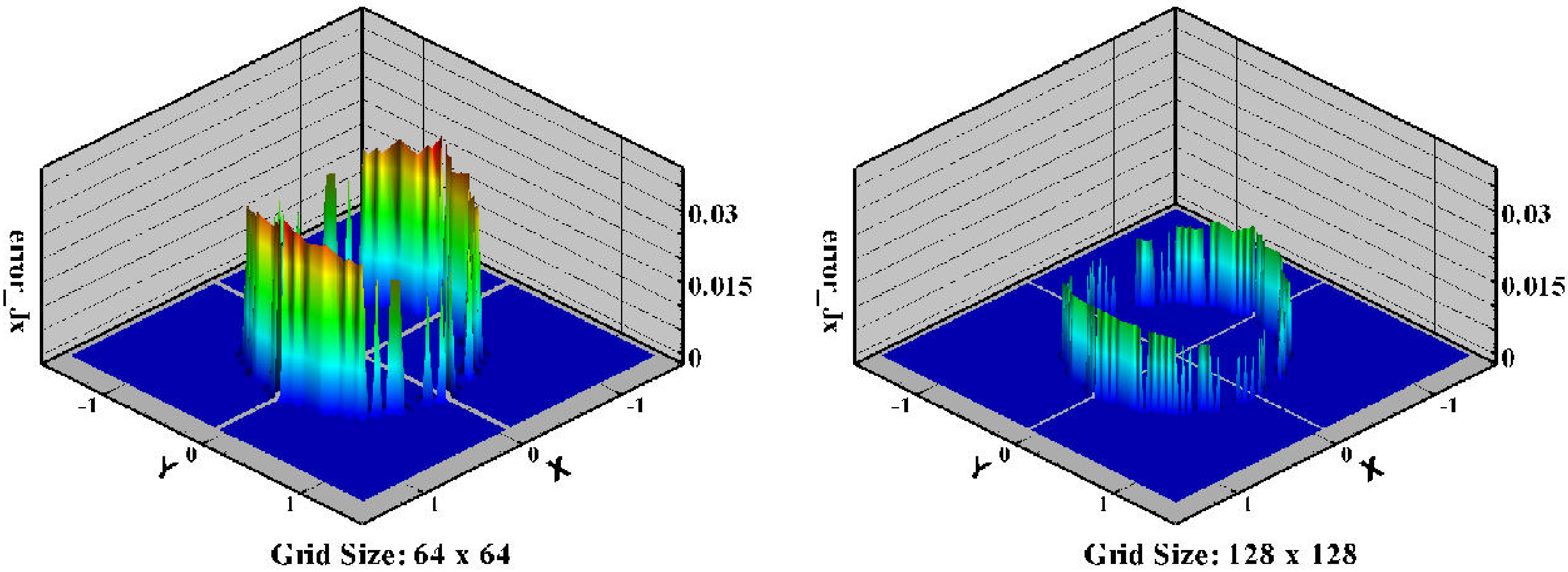
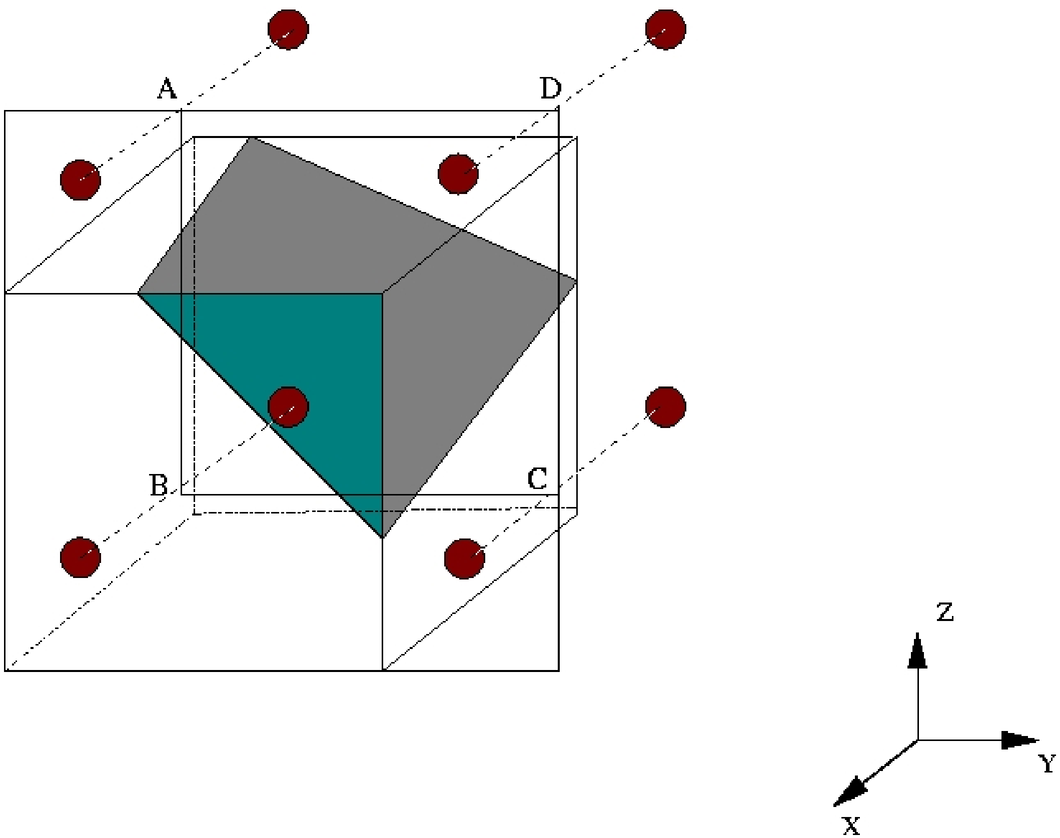


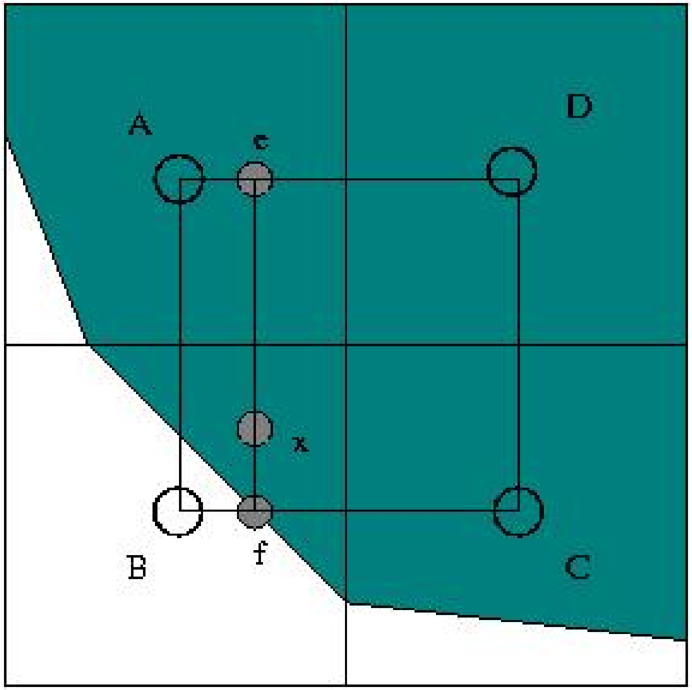
***Figure 5*. Illustration of flux error (X direction)**

(4). 3D implementation

* Same principle as 2D

* Bilinear interpolation of flux

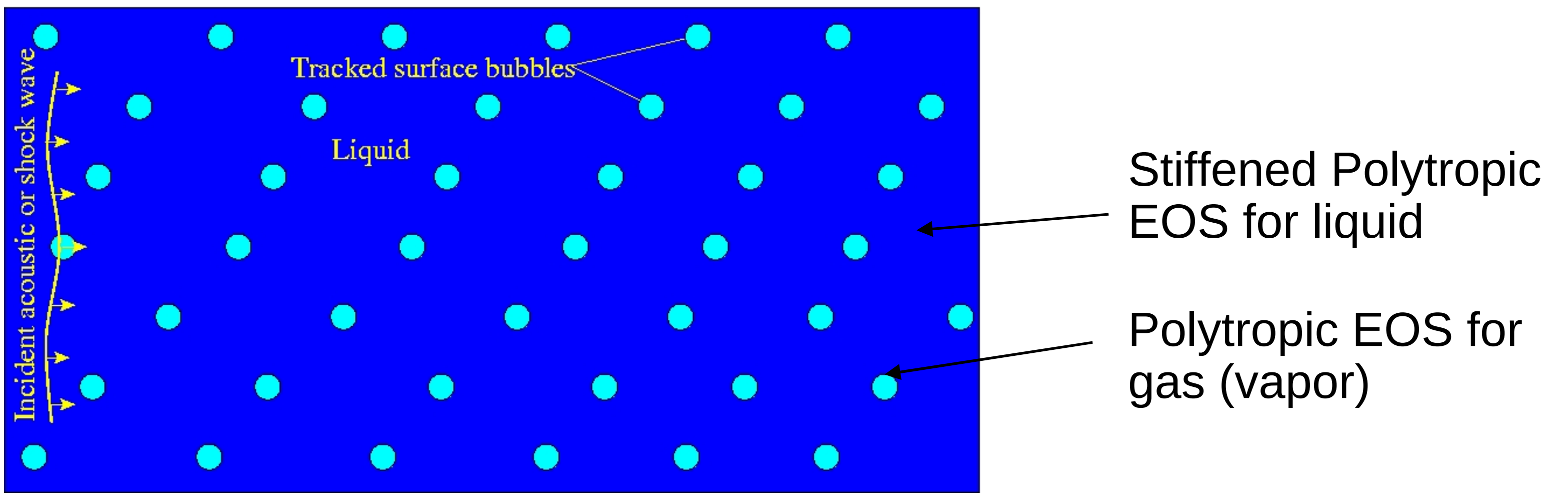




3. MHD Simulations

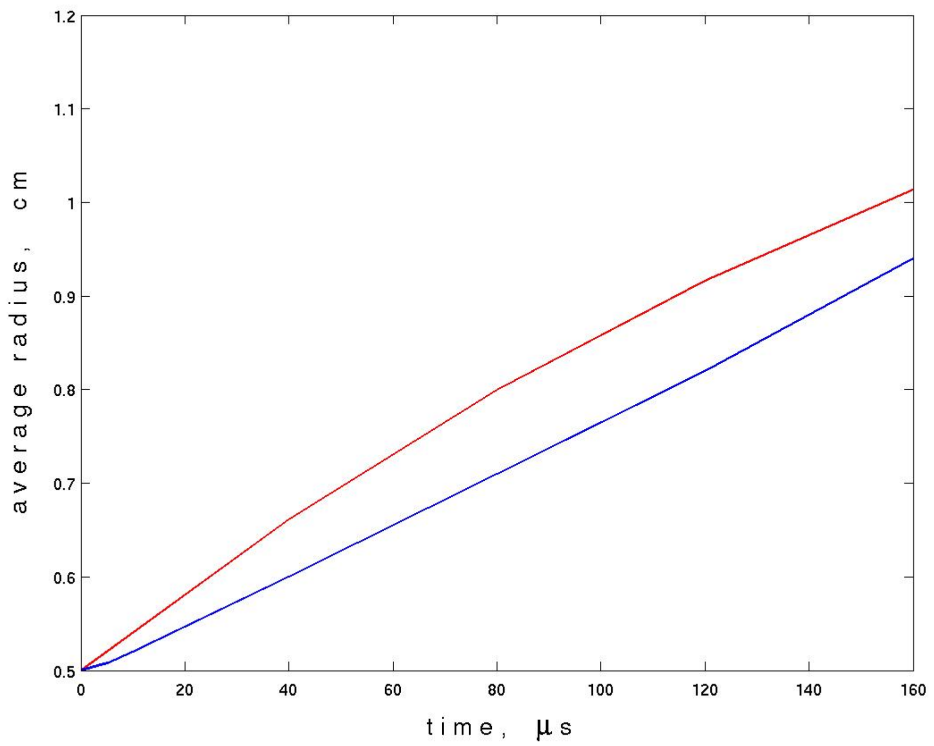
(1) Introduction of EOS models used

- **Heterogeneous method (Direct Numerical Simulation): Each individual bubble is explicitly resolved using FronTier interface tracking technique.**



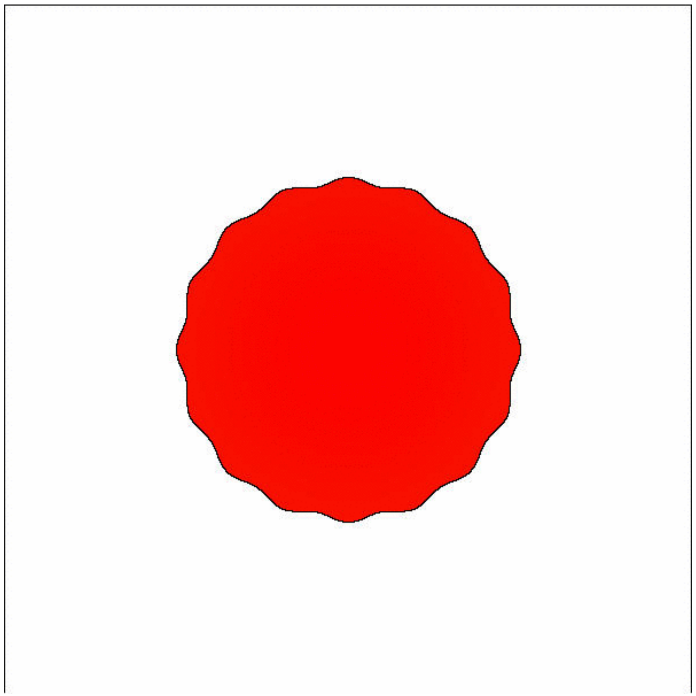
- **Homogeneous EOS model. Suitable average properties are determined and the mixture is treated as a pseudofluid that obeys an equation of single-component flow.**

(2) Comparison of Jet expansion with two EOS models (B = 0)

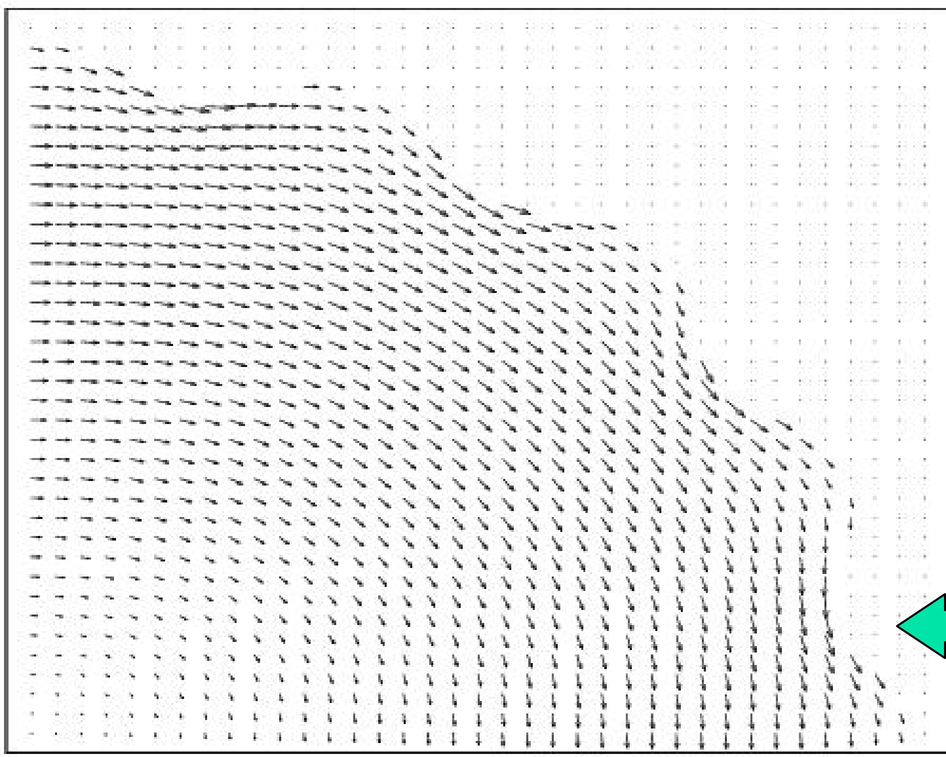


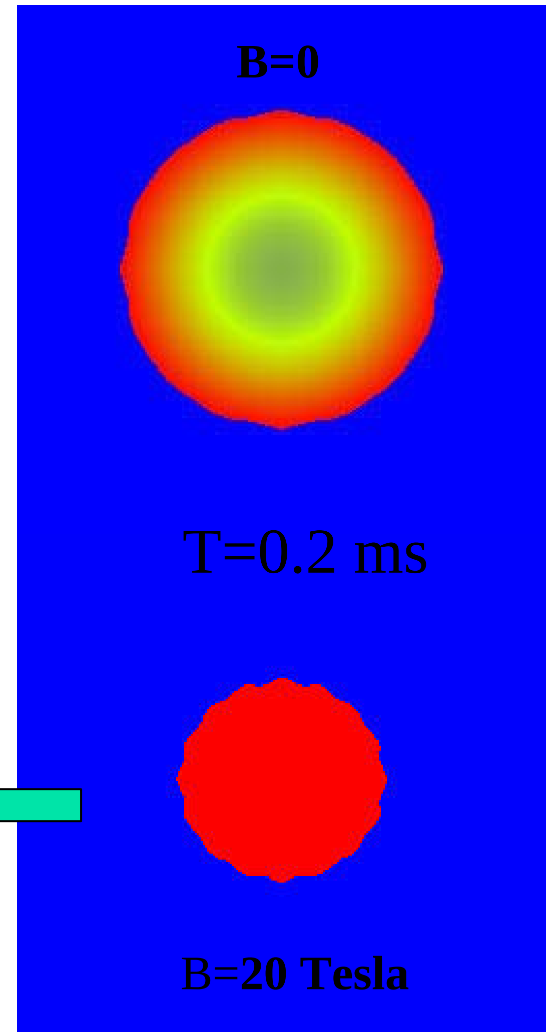
homogeneous

heterogeneous



(3) Mercury Jet simulation with S2phase EOS





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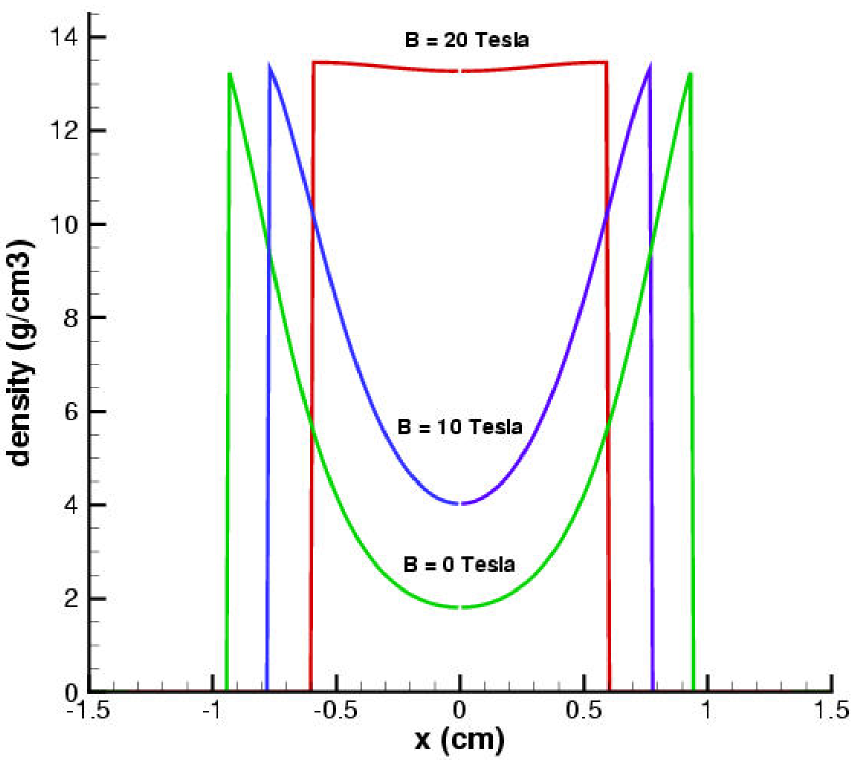
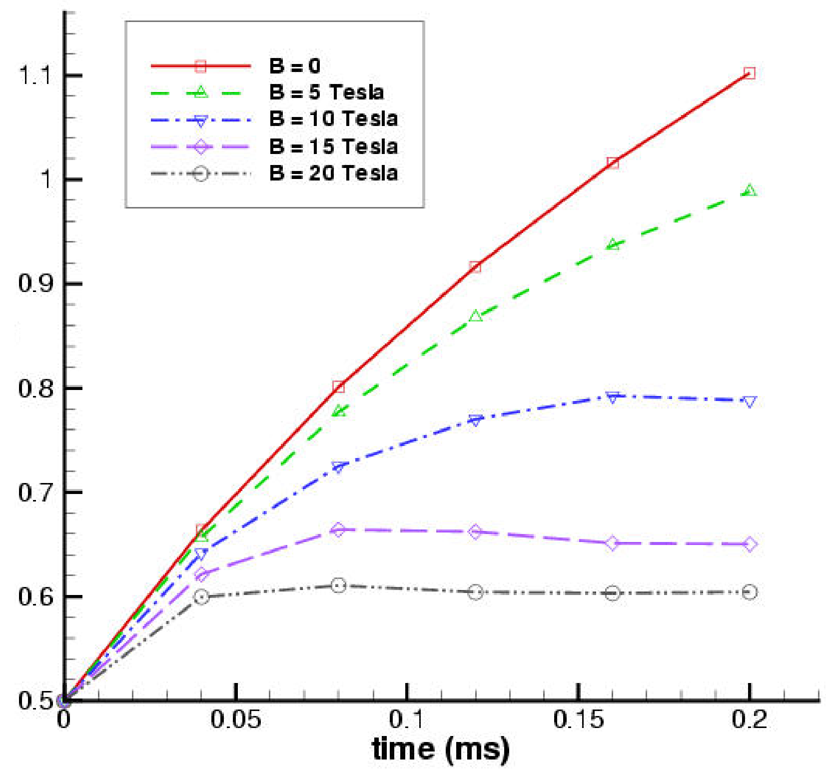
(4) Mercury Jet Evolution and Density Distribution





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4. Conclusions and Future plan

* Embedded Boundary Method for the 2D Neumann boundary elliptic equations are implemented into FronTier code and validated over geometrically complex domain. 3D implementation is finished and under testing.

* Without magnetic field, the two EOS model give similar jet expansion speed. With magnetic field , the growth of the two-phase domain and jet expansion for homogeneous model are strongly restricted by the magnetic field.

* The MHD running for 3D and heterogeneous model is under development.