The Capture Solenoid as an Emittance-Reducing Element

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Why Do We Need "Cooling"?

Pions are produced at small/zero radius in a thin target, ⇒ Their initial emittance is nearly zero!

When the pions decay to muons their transverse momentum changes by ≈ 30 Mev/c, ⇒ Some increase in transverse emittance, particularly if the pions are well off axis when they decay.

But, the major issue is that the RMS emittance = size of phase-space ellipsoid that contains the pions, is large compared to the "true" emittance.

We make a great effort to reduce the RMS emittance by "ionization cooling".

In principle, other methods than ionization cooling can be used to shrink the apparent, RMS emittance to a value closer to the true, small emittance.

Such methods can be nondissipative.

In particular, magnetic fields alone can be used to decrease the apparent, RMS emittance (although this will not decrease the true emittance).

The production of pions in a strong magnetic field that later "tapers" down to ≈ 2 T in the decay and cooling channels can and does serve to reduce the apparent emittance.

However, the Collaboration has not made a thorough effort to optimize this process.

The Adiabatic Invariant of a Helical Orbit

If a particle is produced with transverse momentum $p_{0\perp}$ inside a solenoid of magnetic field strength B_0, then its trajectory is a helix of radius $R_0 = c\, p_{0\perp} / e\, B_0$ (in Gaussian units).

If the particle is produced close to the axis of the solenoid, then its maximum radius with respect to the magnetic axis is $2R_0$.

The magnetic flux through this helix is $\Phi_0 = \pi R_0^2 B_0 = \pi c^2 p_{0\perp}^2 / e^2 B_0 \propto 1 / B_0$ for a given transverse momentum.

If the solenoidal magnetic field is varied "slowly" with position, the motion of the particle has an "adiabatic invariant", $R\, p_\perp \propto \Phi_0$, where r = radius of helix (and not the radial coordinate of the particle with respect to the magnetic axis.

Pseudo-Emittance

Thus, use of a higher capture field implies a lower invariant quantity $R\, p_\perp$, which has the dimensions of a transverse emittance.

But $R\, p_\perp$ is not THE transverse emittance (and a magnetic field alone cannot reduce the true transverse emittance).

However, the quantity $r\, p_\perp$ is a kind of "pseudo-emittance" of practical relevance to the design of the magnetic transport system, such that a high field B_0 in the capture solenoid reduces the "pseudo-emittance" of the system.

Effect of Adiabatic Tapering of the Capture Field If No Pion Decay

If the field is reduced slowly from B_0 in the capture solenoid to B in the decay/cooling channel, and the pions didn't decay, the helical trajectory inside the cooling channel obeys
$e R^2 B / c = R p_\perp = R_0 p_{0\perp} = c^2 p_{0\perp}^2 / e^2 B_0$.

If the system is designed to accept particles up to a given $p_{0\perp}$, then the radius and magnetic field of the decay/cooling channel obey $r^2 B \propto 1 / B_0$ (recalling that $r = 2 R$).

Hence, use of a larger field B_0 in the capture solenoid permits reduction of either r or B (or both) in the decay/cooling channel (⇒ cost savings, and increased technical feasibility).

Effect of Pion Decay

When pions decay to muons, $\pi \to \mu\nu$, the muons have 30 MeV/c momentum in the rest frame of the pion. [$p^*_\mu = E^*_\nu = (m_\pi^2 - m_\mu^2) / 2 m_\pi$], and 110 MeV energy [$E^*_\mu = m_\pi - E^*_\nu$].

Roughly speaking, the transverse momentum of the muon just after its creation by pion decay can differ from the pion's transverse momentum by 30 MeV/c in any transverse direction.

In the worst cases, the helix of the muon extends out to larger distance from the magnetic axis than that of the parent pion, and so the radius of the decay/cooling channel must be larger to maintain good acceptance.

As noted by Bob Palmer, this effect is mitigated if the pions decay in a region of stronger magnetic field, since the adiabatic invariant can also be expressed as $p_\perp^2 / B = p_{0\perp}^2 / B_0$.

This suggests that we should consider operating the decay channel at a magnetic field intermediate between that of the capture solenoid and the cooling channel (whose field could well be lower than that considered in Study 2 if the capture solenoid field is higher).

Outlook

Use of a higher capture field, followed by a better optimized decay channel, could improve our muons beams 4 ways:

- Increased μ/p ratio.
- Lower RMS emittance at the entrance to the cooling channel.
- Lower magnetic field in the cooling channel.
- Smaller radius for the cooling channel.

One can choose to back off on some of these parameters to obtain even better performance in the others.

⇒ Opportunity for rather extensive simulations.

Work on this is just beginning.

30 T ON TARGET NEUTRINO FACTORY/MUON COLLIDER FRONT END *

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Abstract

Recent advances in magnet technology suggest that it is possible to get higher magnetic field on the target and capture section of a Neutrino Factory/Muon Collider. We have carried out a *naive* simulation of the study2a front end with 30 T instead of 20 T magnetic field on target.

INTRODUCTION

The study2a front end [1] (see Fig.1) is the reference performance for the National Muon Accelerator Program (MAP). The performance number we focus here is N_μ^{24}, the number of muons per incident proton on target within the acceptances: $A_T = 30$ mm-rad , $A_L = 150$ mm and $100 \le p_z \le 300$ MeV/c at the end of the pre-cooler. For 24 GeV protons on Hg, $N_\mu^{24} \approx 0.18$.

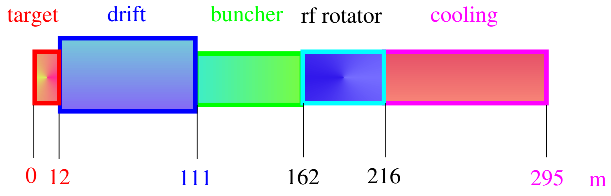

Figure 1: Layout of the Front-End.

NEW MARS RUNS

Recent careful studies of the proton beam energy and target geometry indicates the best proton energy is 8 GeV [2]. One would expect that the transmission for the 8 GeV case would be $N_\mu^8 = N_\mu^{24}\frac{8}{24} \approx 0.06$. It is shown in Fig. 2 that ICOOL give an even better transmission of $N_\mu^8 \approx 0.08$ which is $\approx 33\%$ better than expectations. Of course, this result reflect the optimization achieved with the geometry of the target system and the most efficient pion production at 8 GeV. This result was obtained with 20 T magnetic field on target.

CAPTURE REGION OPTIMIZATION AND RESULTS

Recently, it was suggested [3] that increasing the magnetic field on the target and re-optimizing the capture region we could improve even further the value of N_μ^8. We have taken a quick look at this and present here results that clearly encourage a systematic study of the target/apture region of a Neutrino Factory/Muon Collider, aiming to increase the performance of the front end or pre-cooler.

Using MARS15 [4] runs with 30 T on target we obtain $N_\mu^8 \approx 0.10$ which is an extremely interesting 66% larger transmission than we would have expected (see Fig. 3).

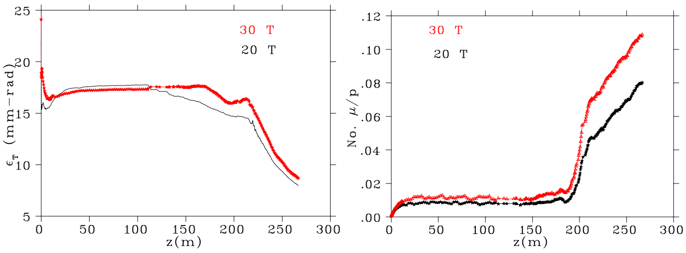

Figure 2: Comparison between 20 and 30 T examples: **left** transverse emittance vs z; **right** number of muons per incident proton on target vs z.

In this examples the constant magnetic field on both bunching and rotator sections was 2.6 T. If we reduce the field to the standard 1.75 T and disregard the lack of matching at the different magnetic field inter-phases, then

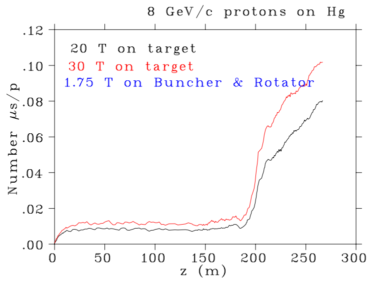

Figure 3: Comparison between 20 and 30 T examples: number of μs per incident proton on target vs z.

An important piece of information to be able to optimize the capture channel, we need to know the phase space of the initial pions as generated by MARS and collected at the end of the target. The following two Figs. 4 show the transverse and longitudinal phase space of all pions that decay into muons that are detected at the end of the channel (in black) superimpose with the same phase space of pions that decay

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* Work supported by ...

into muons that are inside the transverse and longitudinal acceptance defined above (in red).

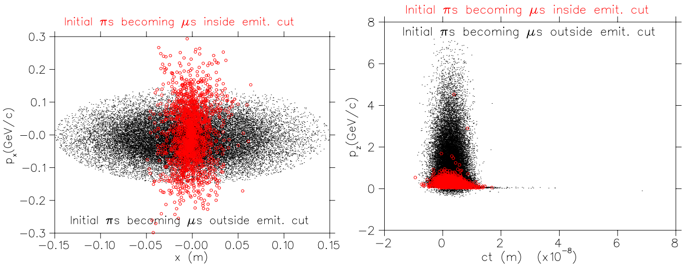

Figure 4: Initial pions: **left** Longitudinal phase space; **right** transverse phase space.

CONCLUSION

Although much work remains to be done, the scenario outlined here appears to lead to pre-cooler with enhanced performance.

REFERENCES

[1] J.S. Berg, *Cost-effective design for a neutrino factory*, Phys. Rev. ST Accel. Beams **9**, (2006) 011001.

[2] H. Kirk, MARS15 runs

[3] K. McDonald, presentation Dec, 2009

[4] .N. Mokhov, *MARS15*

A Geometry for a Rotating Solid Target for a Neutrino Factory

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Total length of W target, *A + B*, should be so long that unspent proton beam hitting the magnet has flux comparable to that of the secondary pions.

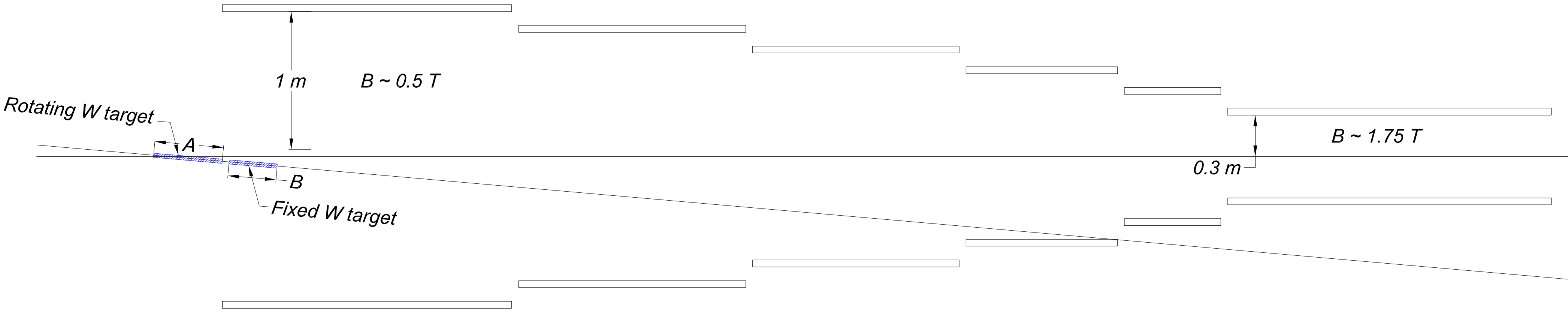
If need, say, *N* = 200 targets on the rotating wheel (axis parallel to the magnetic axis) to limit the radiation damage, then $A \approx 10\,\mathrm{cm}\cdot\ln(N) \approx 50\,\mathrm{cm}$.

Proton beam coaxial with target, and both tilted so the proton beam does not hit downstream beam window.

Target should be in magnetic field for good capture, but the field can be weak, say 0.5 T.

- Taper magnetic field from 0.5 T to nominal 1.75 T of pion transport solenoid.
- Rotating target can be upstream of first magnet
- Low field ⇒ long period for pion helices ⇒ reabsorption a minor issue.
- Target diameter can perhaps be larger than 2 cm

Serious flaw: For a pion transport channel of given *B* and *r*, the longitudinal-transverse momentum exchange due to the adiabatic invariant $r\,p_\perp$ implies that $p_\perp/p_{\perp 0} = (B/B_0)^{1/2}$ strongly favors use of B_0 much larger (not smaller) than *B* for maximal capture of pions.



Rotating Target Wheel Should Have an Air Bearing

CNGS rotating target failed due to radiation damage to lubrication of the bearings.

⇒ Use an air bearing for the rotating target wheel.