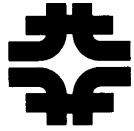


# Low-Energy Ionization “Cooling”

**David Neuffer**

Fermilab

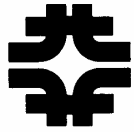


# Outline

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- **Low-energy cooling-protons**
- **ions and “beta-beams”**
- **Low-energy cooling – muons**
  - **emittance exchange**



# μ Cooling Regimes

- Efficient cooling requires:

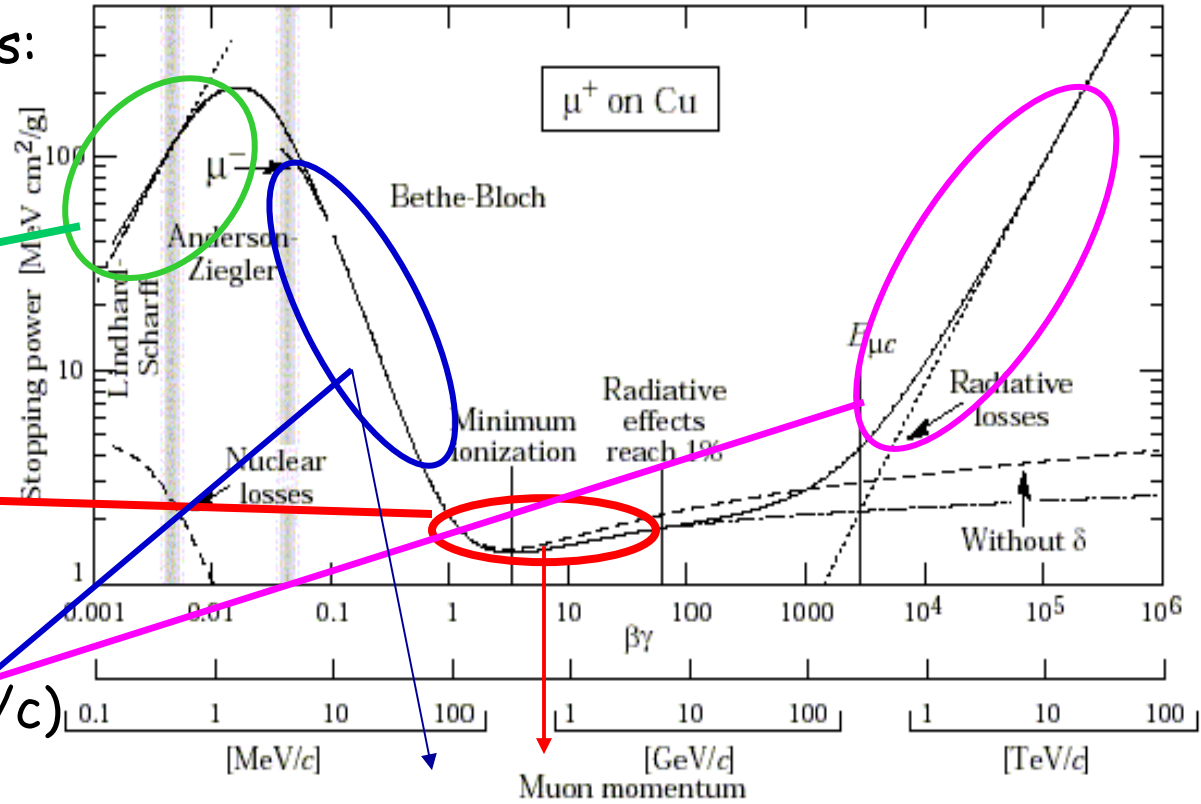
$$\frac{\partial \frac{dE}{dx}}{\partial E} > \sim 0$$

- Frictional Cooling (<1MeV/c)  $\Sigma_g \sim 3$

- Ionization Cooling ( $\sim 0.3\text{GeV}/c$ )  $\Sigma_g \sim 2$

- Radiative Cooling ( $>1\text{TeV}/c$ )  $\Sigma_g \sim 4$

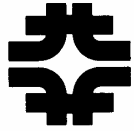
- Low- $\epsilon_+$  cooling  $\Sigma_g \sim 2\beta^2$   
(longitudinal heating)



$$\frac{dE}{ds} = 4\pi N_A \rho r_e^2 m_e c^2 \frac{Zz^2}{A} \left[ \frac{1}{\beta^2} \ln \left( \frac{2m_e c^2 \gamma^2 \beta^2}{I(Z)} \right) - 1 - \frac{\delta}{2\beta^2} \right]$$

$$\frac{d\epsilon_N}{ds} = -\frac{1}{\beta^2 E} \frac{dE}{ds} \epsilon_N + \frac{\beta\gamma \beta_{\perp}}{2} \frac{d\langle \theta_{rms}^2 \rangle}{ds}$$

$$\frac{d\langle \theta_{rms}^2 \rangle}{ds} = \frac{z^2 E_s^2}{3\beta^2 c^2 p_{\mu}^2 L_R}$$

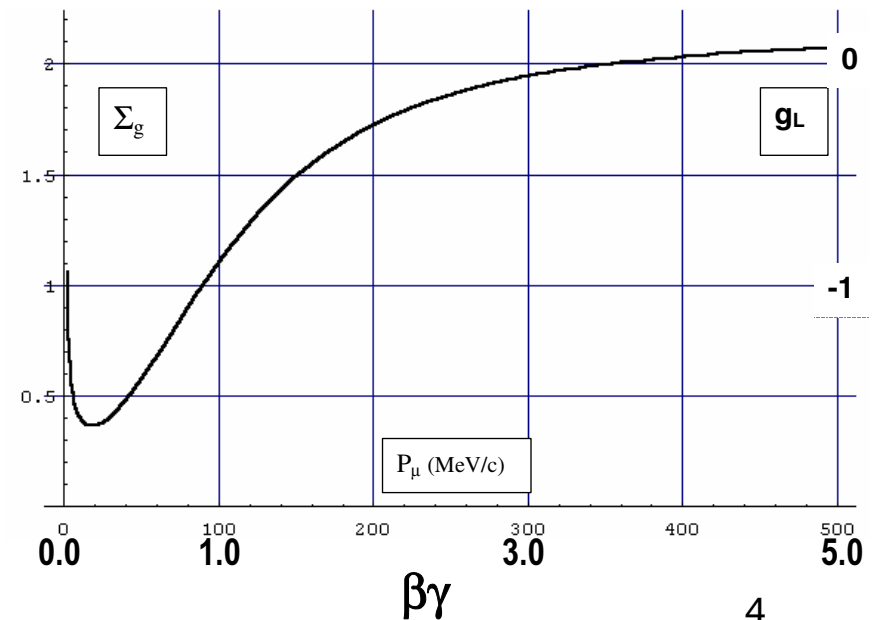


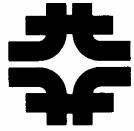
# Cooling/Heating equations

- Cooling equations are same as used for muons
  - mass =  $a m_p$ , charge =  $z e$
  - Some formulae may be inaccurate for small  $\beta=v/c$
  - Add heating through nuclear interactions
  - Ionization/recombination should be included
- For small  $\beta$ , longitudinal  $dE/dx$  heating is large
  - At  $\beta = 0.1$ ,  $g_L = -1.64$ ,  $\Sigma_g = 0.36$
  - Coupling only with  $x$  cannot obtain damping in both  $x$  and  $z$

$$g_L \cong -\frac{2}{\gamma^2} + \frac{2(1 - \frac{\beta^2}{\gamma^2})}{\left(\ln\left[\frac{2m_e c^2 \beta^2 \gamma^2}{I(Z)}\right] - \beta^2\right)}$$

$$\Sigma_g \cong 2\beta^2 + \frac{2(1 - \frac{\beta^2}{\gamma^2})}{\left(\ln\left[\frac{2m_e c^2 \beta^2 \gamma^2}{I(Z)}\right] - \beta^2\right)}$$



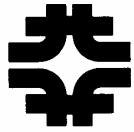


# Low-energy “cooling” of ions

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- **Ionization cooling of protons/ ions has been unattractive because nuclear reaction rate is competitive with energy-loss cooling rate**
  - **And other cooling methods are available**
- **But can have some value if the goal is beam storage to obtain nuclear reactions**
  - **Absorber is also nuclear interaction medium**
  - **Y. Mori – neutron beam source**
    - NIM paper
  - **C. Rubbia, Ferrari, Kadi, Vlachoudis – source of ions for  $\beta$ -beams**



# Miscellaneous Cooling equations

$$\frac{dE}{ds} = 4\pi N_A \rho r_e^2 m_e c^2 \frac{Z^2}{A} \left[ \frac{1}{\beta^2} \ln \left( \frac{2m_e c^2 \gamma^2 \beta^2}{I(Z)} \right) - 1 - \frac{\delta}{2\beta^2} \right]$$

$$\epsilon_{N,eq} = \frac{z^2 \beta_{\perp} E_s^2}{2g_x \beta a m_p c^2 L_R \frac{dE_{z,a}}{ds}}$$

$$\frac{d\sigma_E^2}{ds} = -2 \frac{g_L}{\beta^2 E} \frac{dE}{ds} \sigma_E^2 + 4\pi (r_e m_e c^2)^2 z^2 n_e \gamma^2 \left( 1 - \frac{\beta^2}{2} \right)$$

$$\sigma_{E,eq}^2 = \frac{(m_e c^2)(a m_p c^2) \beta^4 \gamma^3}{2 g_L \ln[]} \left( 1 - \frac{\beta^2}{2} \right)$$

$$\frac{d\epsilon_N}{ds} = -\frac{1}{P} \frac{dP}{ds} \epsilon_N + \frac{\beta_{\perp}}{2} \frac{z^2 E_s^2}{\beta^2 m c P L_R}$$

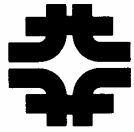
$$\ln[] \equiv \left[ \ln \left( \frac{2m_e c^2 \gamma^2 \beta^2}{I(Z)} \right) - \beta^2 \right]$$

For small  $\beta$ :

Better for larger mass?

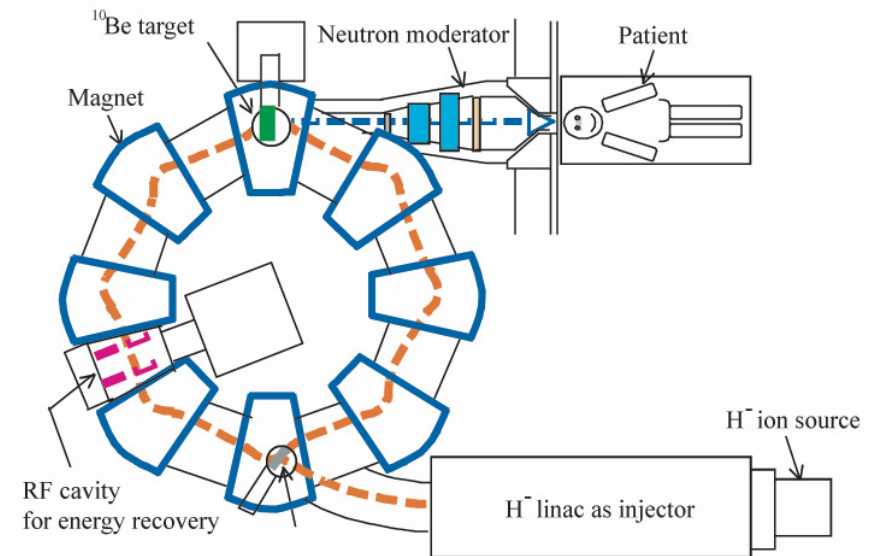
$$\frac{\sigma_{p,eq}^2}{P_{z,a}^2} \simeq \frac{m_e \gamma}{2 a m_p g_L \ln[]}$$

$$\frac{d\langle \theta_{rms}^2 \rangle}{ds} = \frac{z^2 E_s^2}{\beta^2 c^2 p_{\mu}^2 L_R}$$



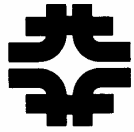
# Example

- **ERIT-P-storage ring to obtain directed neutron beam (Mori-Okabe, FFAG05)**
- **10 MeV protons**
  - $^9\text{Be}$  target for neutrons
  - $\sigma \approx 0.5$  barns
  - $\beta = v/c = 0.145$
  - Large  $\delta E$  heating
- **Baseline Absorber**
  - $5\mu$  Be absorber
  - $\delta E_p = \sim 36$  keV/turn
- **Design Intensity**
  - 1000Hz,  $6.5 \times 10^{10}$  p/cycle
  - 100W primary beam
  - < 1.5 kW on foil
    - 0.4 kW at  $n_{\text{turns}} = 1000$



**Table 1: Reference parameters of the ERIT Ring**

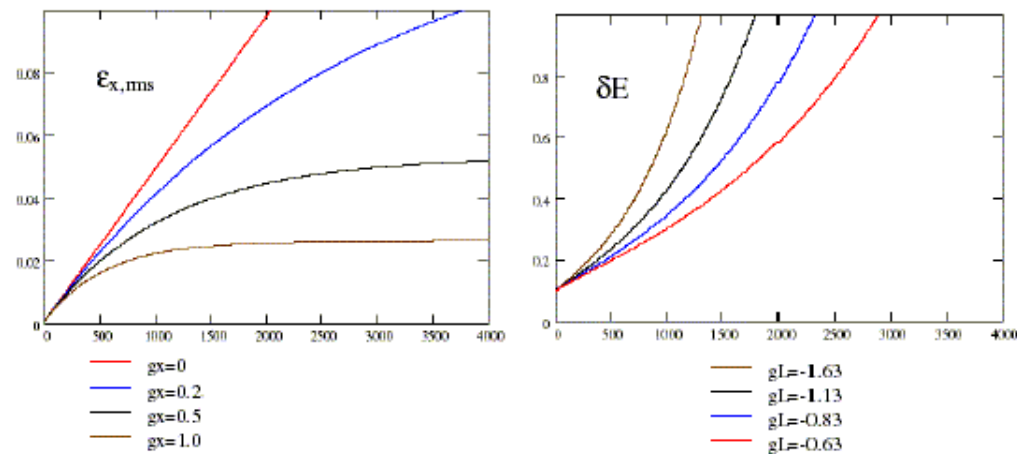
Parameter	Symbol	Ref. Value	Units
Beam Kinetic Energy	$E_p$	10	MeV
Beam Momentum	$P_p$	137.4	MeV/c
Beam velocity	$\beta=v/c$	0.145	
Beam current	$I_p$	40	mA
Ring Circumference	$C$	11.3	m
Ring tunes	$\nu_x, \nu_y$	1.89, 1.34	
Mean Betatron function	$\langle \beta_{\perp} \rangle$	0.95,	m
Maximum betatron functions	$\beta_{x,\text{max}}, \beta_{y,\text{max}}$	1.48, 2.03	m
Dispersion (at wedge)	$\eta_{\theta}$	0.6	m
Transition gamma	$\gamma_t$	1.7	
Energy loss (Be) at ref. energy	$dE/ds$	72	MeV/cm
Sum of partition numbers (at $E_p$ )	$\Sigma_p$	0.37	
Absorber central thickness	$\delta z$	5	$\mu$
Mean energy loss / turn	$\delta E_{\text{AVE}}$	36	keV
Rf voltage	$V_{\text{rf}}$	200	kV/turn
Rf harmonic	$h$	5	
Rf frequency	$f_{\text{rf}}$	19.25	MHz
Longitudinal focusing function	$\beta_{\theta}$	2.1	Radians/MeV



# ERIT results

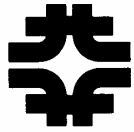


- With only production reaction, lifetime is 30000 turns
- With baseline parameters, cannot cool both x and E
  - Optimal x-E exchange increases storage time from 1000 to 3000 turns (3850 turns = 1ms)



- With x-y-E coupling, could cool 3-D with  $g_i = 0.12$ 
  - Cooling time would be  $\sim 5000$  turns
  - With  $\beta_{\perp} = 0.2m$ ,  $\delta E_{rms} = 0.4MeV$ ,  $\epsilon_{\perp,N} = 0.0004m$  ( $x_{rms} = 2.3cm$ )
  - $x_{rms} = 7.3cm$  at  $\beta_{\perp} = 2m$  (would need  $r = 20cm$  arc apertures)  
(but 1ms refill time would make this unnecessary)



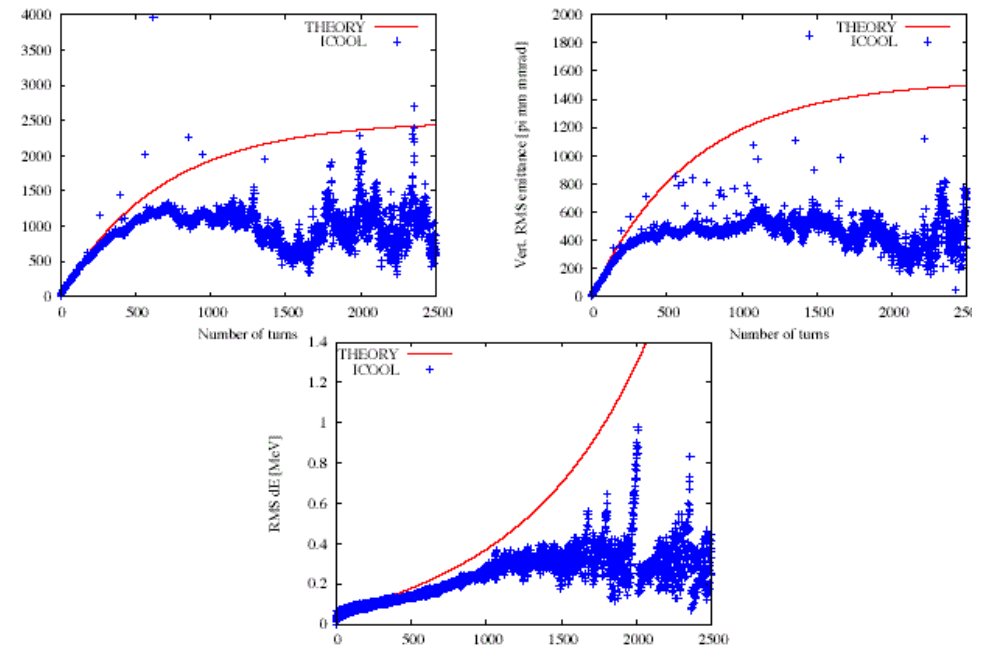
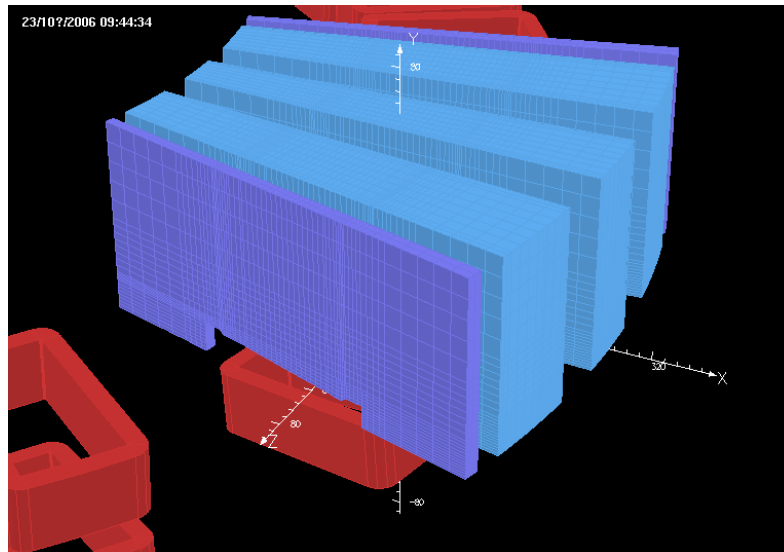
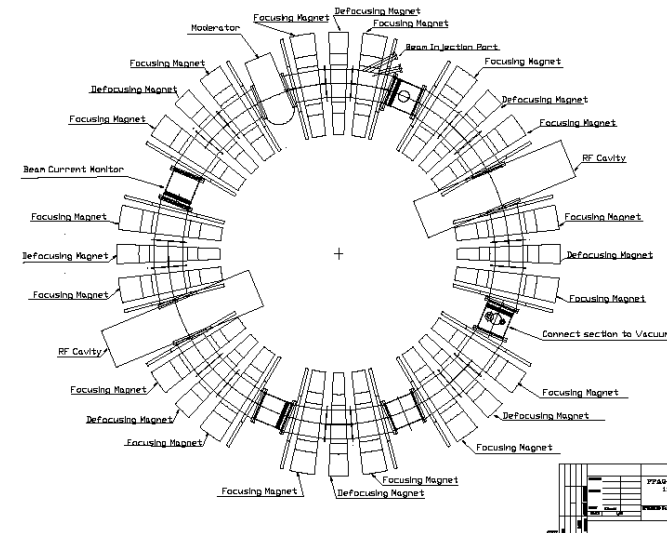


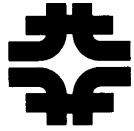
# ERIT-recent results



- Lattice changed from spiral to radial sector
  - spiral sector had too small vertical aperture
- With cooling effects, beam has ~1000 turn lifetime in ICOOL simulation

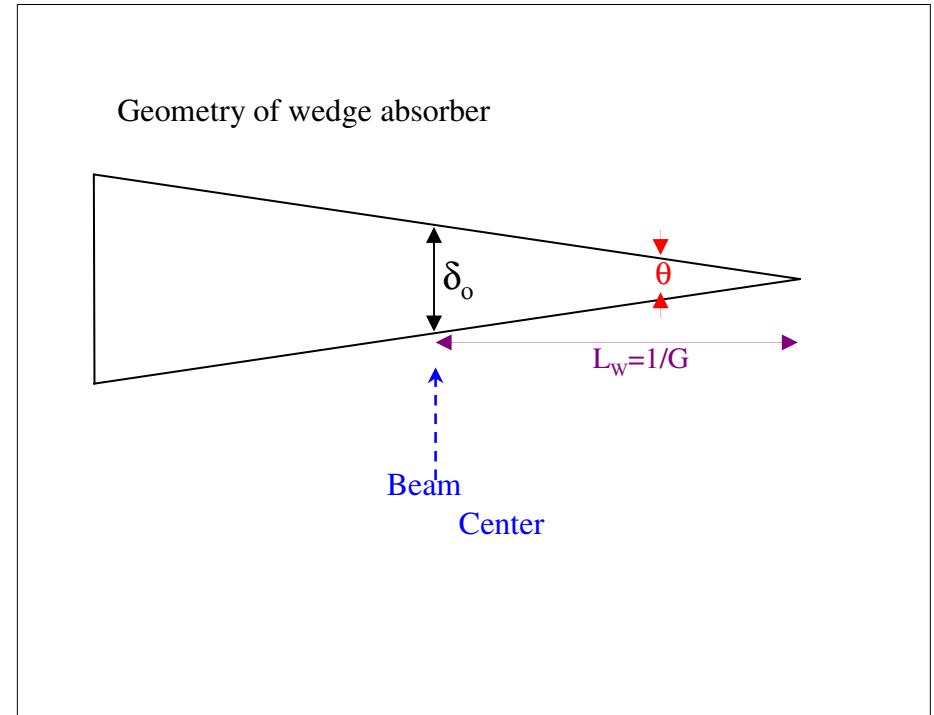
(Mori and Okabe FFAG06)



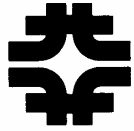


# Emittance exchange parameters

- with  $g_{L,0} = -1.63$ ,  $\eta = 0.5\text{m}$ ,
  - need  $G = 3.5\text{m}^{-1}$  to get  $g_L = 0.12$ 
    - ( $L_W = 0.3\text{m}$ )
- For 3-D cooling, need to mix with both x and y
  - Solenoid cooling rings
- Also “Moebius” lattice (R. Talman)
  - Single turn includes x-y exchange transport
    - solenoid(s) or skew quads
    - in zero dispersion region for simplicity
    - Solenoid:  $BL = \pi B\rho$
    - For 10 MeV p :  $BL = 1.44\text{T}\cdot\text{m}$
  - Complete period is 2 turns



$$g_L \rightarrow g_{L,0} + \frac{\eta\rho'}{\rho_0} = g_{L,0} + G\eta$$

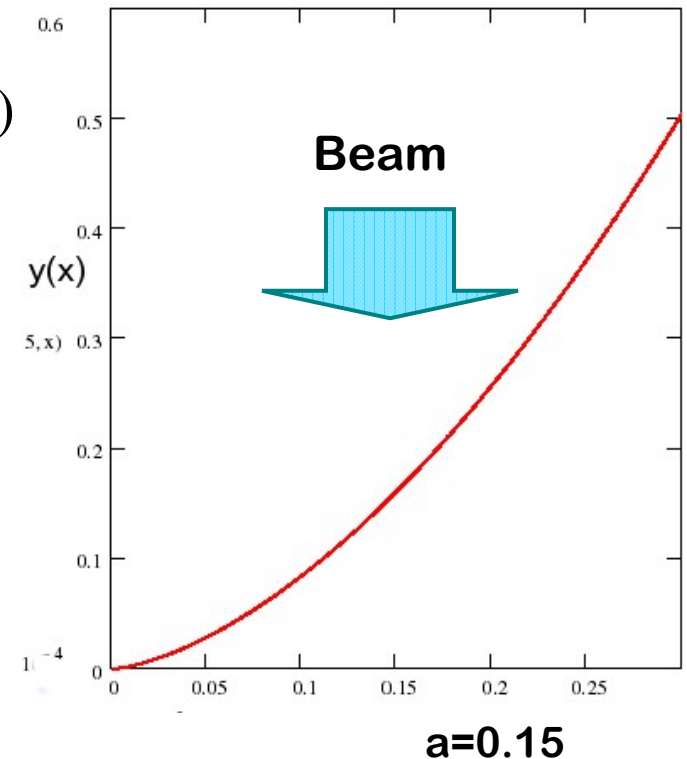


# “Wedge” for thin foil

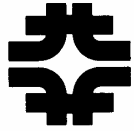
- Obtain variable thickness by bent foil (Mori et al.)

$$\delta(x) = \delta_R \sqrt{1 + y'(x)^2} = \delta_0 \left(1 + \frac{(x-x_0)}{L_W}\right) = \delta_R \left(1 + \frac{x}{a}\right)$$

- Choose  $x_0=0.15\text{m}$ ,  $L_W=0.3$ ,  $\delta_0=5\mu$ 
  - then  $a=0.15$ ,  $\delta_R=2.5$
  - for  $g_{L,0} = -1.63$ ,  $\eta = 0.5\text{m}$
- Barely Compatible with  $\beta^* = 0.2\text{m}$
- Beam energy loss not too large?
  - <1kW Power on foil



$$y = \int \sqrt{\left(\frac{x}{a}\right)^2 + 2\left(\frac{x}{a}\right)} \cdot dx$$
$$= \frac{(a+x)\sqrt{x^2 + 2ax}}{2a} - a \ln \left[ \sqrt{x} + \sqrt{a+x} \right]$$



# Other heating terms:

- **Mixing of transverse heating with longitudinal could be larger effect: (Wang & Kim)**

$$\frac{d\epsilon_z}{ds} = -g_L \frac{dP}{ds} \epsilon_z + \frac{1}{2} \beta_z \frac{d\delta_{rms}^2}{ds} + \frac{1}{2} \frac{\eta^2}{\beta_z} \frac{d\theta_{rms}^2}{ds}$$

$$\frac{d\epsilon_x}{ds} = -g_x \frac{dP}{ds} \epsilon_x + \frac{1}{2} \beta_x \frac{d\theta_{rms}^2}{ds} + \frac{1}{2} \frac{\eta^2}{\beta_x} \frac{d\delta_{rms}^2}{ds}$$

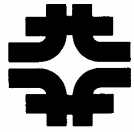
**At ERIT parameters:**  $\beta_x = 1.0m$ ,  $\beta_z = 16m$ ,  $\eta = 0.6m$ , Be absorber,  
 $d\delta^2/ds = 0.00032$ ,  $d\theta^2/ds = 0.0133$  **only 5%, 1.5% changes ...**

**At  $\beta_x = 0.2m$ , ~25% change ...**

$$\frac{d\langle \theta_{rms}^2 \rangle}{ds} = \frac{z^2 E_s^2}{\beta^2 c^2 P_a^2 L_R}$$

$$\frac{d(\delta_{rms}^2)}{ds} = 4\pi (r_e m_e c^2)^2 N_A \frac{Z}{A} \rho \frac{\gamma^2 z^2}{\beta^2 P_a^2} \left(1 - \frac{\beta^2}{2}\right)$$

$$\beta_L = \sqrt{\frac{\beta^2 P c C \lambda_{RF} \left(\frac{1}{\gamma^2} - \frac{1}{\gamma_T^2}\right)}{2\pi e V_{RF} \cos \phi_S}}$$



# $\beta$ -beam Scenario



## Ion production

### Ion Driver

25MeV Li ?

### Ion production:

Target - Ion ( $^8\text{B}$  or  $^8\text{Li}$  ?)  
Conversion Ring

### Beam preparation

ECR pulsed

### Ion acceleration

Linac

### Acceleration to medium energy

RCS

## Acceleration

### Acceleration to final energy

Fermilab Main Injector

8z  
GeV/c

## Neutrino source

Experiment  $\nu, \bar{\nu}$

### Neutrino Source

### Decay Ring

### Decay ring

$B\rho = 400 \text{ Tm}$

$B = 5 \text{ T}$

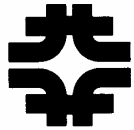
$C = 3300 \text{ m}$

$L_{ss} = 1100 \text{ m}$

$^8\text{B}: \gamma = 80$

$^8\text{Li}: \gamma = 50$

$\nu, \bar{\nu}$



# Conventional Beta beam ion source



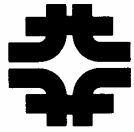
- **Want:**
  - lifetime  $\sim 1s$
  - large  $\nu$ -energy
  - $\nu$  and  $\nu^*$
  - easily extracted atoms
- **Number of possible ions is limited ( $\nu$  sources easier)**
- **Noble gases easier to extract**
  - ${}^6\text{He}_2$  “easiest”  $E_{\nu^*} = 1.94\text{MeV}$
  - ${}^{18}\text{Ne}_{10}$  for  $\nu$   $E_{\nu} = 1.52\text{MeV}$
- ( ${}^8\text{B}$ ,  ${}^8\text{Li}$ ) have  $E_{\nu, \nu^*} = \sim 7\text{MeV}$
- **Want  $10^{20}$   $\nu$  and  $\nu^*$  / year...**

$\nu^*$ -sources

Isotope	A/Z	T 1/2 (s)	$Q_{\beta}$ $\beta_s$ to $\beta_s$ (MeV)	$Q_{\beta}$ eff (MeV)	$E_{\beta}$ av (MeV)	$E_{\nu}$ av (MeV)	Ions/bunch	Decay rate ( $s^{-1}$ )	rate / $E_{\nu}$ av ( $s^{-1}$ )
${}^6\text{He}$	3.0	0.80	3.5	3.5	1.57	1.94	$5 \cdot 10^{12}$	$4 \cdot 10^{10}$	$2 \cdot 10^{10}$
${}^8\text{He}$	4.0	0.11	10.7	9.1	4.35	4.80	$5 \cdot 10^{12}$	$3 \cdot 10^{11}$	$6 \cdot 10^{10}$
${}^8\text{Li}$	2.7	0.83	16.0	13.0	6.24	6.72	$3 \cdot 10^{12}$	$3 \cdot 10^{11}$	$4 \cdot 10^9$
${}^9\text{Li}$	3.0	0.17	13.6	11.9	5.73	6.20	$3 \cdot 10^{12}$	$1 \cdot 10^{11}$	$2 \cdot 10^{10}$
${}^{11}\text{Be}$	2.8	13.8	11.5	9.8	4.65	5.11	$3 \cdot 10^{12}$	$1 \cdot 10^9$	$2 \cdot 10^8$
${}^{15}\text{C}$	2.5	2.44	9.8	6.4	2.87	3.55	$2 \cdot 10^{12}$	$5 \cdot 10^9$	$1 \cdot 10^9$
${}^{16}\text{C}$	2.7	0.74	8.0	4.5	2.05	2.46	$2 \cdot 10^{12}$	$2 \cdot 10^{10}$	$6 \cdot 10^9$
${}^{16}\text{N}$	2.3	7.13	10.4	5.9	4.59	1.33	$1 \cdot 10^{12}$	$1 \cdot 10^9$	$1 \cdot 10^9$
${}^{17}\text{N}$	2.4	4.17	8.7	3.8	1.71	2.10	$1 \cdot 10^{12}$	$2 \cdot 10^9$	$1 \cdot 10^9$
${}^{18}\text{N}$	2.6	0.64	13.9	8.0	5.33	2.67	$1 \cdot 10^{12}$	$2 \cdot 10^{10}$	$6 \cdot 10^9$

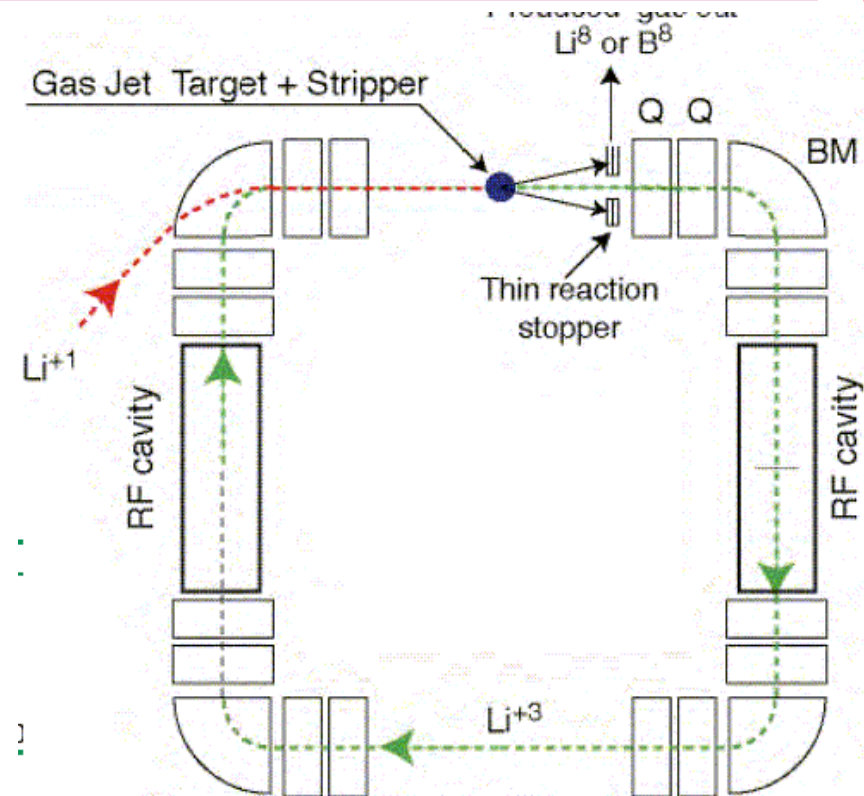
$\nu$ -sources

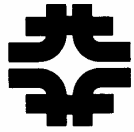
Isotope	A/Z	T 1/2 (s)	$Q_{\beta}$ $\beta_s$ to $\beta_s$ (MeV)	$Q_{\beta}$ eff (MeV)	$E_{\beta}$ av (MeV)	$E_{\nu}$ av (MeV)	Ions/bunch	Decay rate ( $s^{-1}$ )	rate / $E_{\nu}$ av ( $s^{-1}$ )
${}^8\text{B}$	1.6	0.77	17.0	13.9	6.55	7.37	$2 \cdot 10^{12}$	$2 \cdot 10^{10}$	$2 \cdot 10^9$
${}^{10}\text{C}$	1.7	19.3	2.6	1.9	0.81	1.08	$2 \cdot 10^{12}$	$6 \cdot 10^8$	$6 \cdot 10^8$
${}^{14}\text{O}$	1.8	70.6	4.1	1.8	0.78	1.05	$1 \cdot 10^{12}$	$1 \cdot 10^8$	$1 \cdot 10^8$
${}^{15}\text{O}$	1.9	122.	1.7	1.7	0.74	1.00	$1 \cdot 10^{12}$	$7 \cdot 10^7$	$7 \cdot 10^7$
${}^{18}\text{Ne}$	1.8	1.67	3.3	3.0	1.50	1.52	$1 \cdot 10^{12}$	$4 \cdot 10^9$	$3 \cdot 10^9$
${}^{19}\text{Ne}$	1.9	17.3	2.2	2.2	0.96	1.25	$1 \cdot 10^{12}$	$4 \cdot 10^8$	$3 \cdot 10^8$
${}^{21}\text{Na}$	1.9	22.4	2.5	2.5	1.10	1.41	$9 \cdot 10^{11}$	$3 \cdot 10^8$	$2 \cdot 10^8$



## $\beta$ -beam Scenario (Rubbia et al.)

- Produce Li and inject at 25 MeV
  - Charge exchange injection
- nuclear interaction at gas jet target produces  ${}^8\text{Li}$  or  ${}^8\text{B}$
- Multiturn with cooling maximizes ion production
- ${}^8\text{Li}$  or  ${}^8\text{B}$  is caught on stopper(W)
  - heated to reemit as gas
- ${}^8\text{Li}$  or  ${}^8\text{B}$  gas is ion source for  $\beta$ -beam accelerate
- Accelerate to  $B\rho = 400 \text{ T}\cdot\text{m}$ 
  - Fermilab main injector
- Stack in storage ring for:
- ${}^8\text{B} \rightarrow {}^8\text{Be} + e^+ + \nu$  or  ${}^8\text{Li} \rightarrow {}^8\text{Be} + e^- + \nu^*$  neutrino source



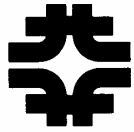


# Cooling for $\beta$ -beams (Rubbia et al.-NuFACT06)



- $\beta$ -beam requires ions with appropriate nuclear decay
  - ${}^8\text{B} \rightarrow {}^8\text{Be} + e^+ + \nu$
  - ${}^8\text{Li} \rightarrow {}^8\text{Be} + e^- + \nu^*$
- Ions are produced by nuclear interactions
  - ${}^6\text{Li} + {}^3\text{He} \rightarrow {}^8\text{B} + n$
  - ${}^7\text{Li} + {}^2\text{H} \rightarrow {}^8\text{Li} + {}^1\text{H}$
  - Secondary ions must be collected and reaccelerated
- Either heavy or light ion could be beam or target
  - Ref. 1 prefers heavy ion beam – ions are produced more forward (“reverse kinematics”)
  - He or  ${}^2\text{H}$  beam on Li has other advantages ....
- Parameters can be chosen such that target “cools” beam
  - (losses and heating from nuclear interactions, however...)





# $\beta$ -beams example: ${}^6\text{Li} + {}^3\text{He} \rightarrow {}^8\text{B} + n$



- **Beam: 25MeV  ${}^6\text{Li}^{+++}$**

- $P_{\text{Li}} = 529.9 \text{ MeV}/c$   $B\rho = 0.59 \text{ T}\cdot\text{m}$ ;  $v/c = 0.09415$

- **Absorber:  ${}^3\text{He}$**

- $Z=2$ ,  $A=3$ ,  $I=31\text{eV}$ ,  $z=3$ ,  $a=6$

- $dE/ds = 1180 \text{ MeV}/\text{gm}/\text{cm}^2$ ,  $L_R = 70.9 \text{ gm}/\text{cm}^2$

( $\rho_{\text{He-3}} = 0.09375 \text{ gm}/\text{cm}^3$ ) Liquid, ( $\rho_{\text{He-3}} = 0.134 \cdot 10^{-3} P \text{ gm}/\text{cm}^3/\text{atm}$  in gas)

- **If  $g_x = 0.123$  ( $\Sigma_g = 0.37$ ),  $\beta_{\perp} = 0.3\text{m}$  at absorber**

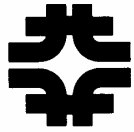
- $\epsilon_{N,\text{eq}} = \sim 0.000046 \text{ m}\cdot\text{rad}$
- $\sigma_{x,\text{rms}} = 1.2 \text{ cm}$  at  $\beta_{\perp} = 0.3\text{m}$ ,
- $\sigma_{x,\text{rms}} = 3.14 \text{ cm}$  at  $\beta_{\perp} = 2.0\text{m}$

$$\epsilon_{N,\text{eq}} \cong \frac{z^2 \beta_{\perp} E_s^2}{2g_x \beta a m_p c^2 L_R \frac{dE_{z,a}}{ds}}$$

- **$\sigma_{E,\text{eq}}$  is  $\sim 0.4 \text{ MeV}$**

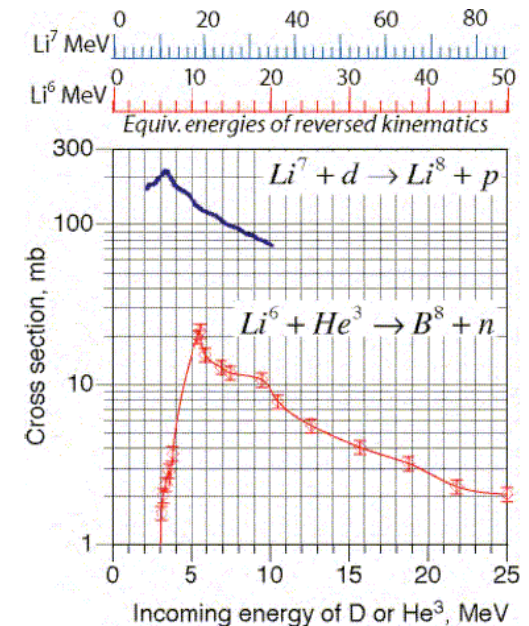
- $\ln[\ ] = 5.68$

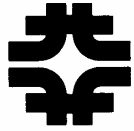
$$\sigma_{E,\text{eq}}^2 = \frac{(m_e c^2)(a m_p c^2) \beta^4 \gamma^3}{2g_L \ln[\ ]} \left(1 - \frac{\beta^2}{2}\right)$$



## Cooling time/power: ${}^6\text{Li} + {}^3\text{He} \rightarrow {}^8\text{B} + n$

- Nuclear cross section for **beam loss is 1 barn ( $10^{-24} \text{ cm}^2$ )** or more
- $\sigma = 10^{-24} \text{ cm}^2$  corresponds to  $\sim 5 \text{ gm/cm}^2$  of  ${}^3\text{He}$ 
  - $\sim 10$  3-D cooling e-foldings ...
- Cross-section for  ${}^8\text{B}$  production is  $\sim 10$  mbarn
  - At best,  $10^{-2}$  of  ${}^6\text{Li}$  is converted
- Goal is  **$10^{13}/\text{s}$  of  ${}^8\text{B}$  production**
  - then at least  $10^{15} \text{ Li}^6/\text{s}$  needed
- Space charge **limit is  $\sim 10^{12} \text{ }^6\text{Li}/\text{ring}$** 
  - Cycle time is  $< 10^{-3} \text{ s}$
  - If  $C=10\text{m}$ ,  $\tau=355 \text{ ns}$ ,  $2820 \text{ turns/ms}$
  - $5/2820=1.773 \cdot 10^{-3} \text{ gm/cm}^2$  ( $0.019 \text{ cm}$  @ liquid density ...)
  - $2.1 \text{ MeV/turn}$  energy loss and regain required ... ( $0.7\text{MV rf}$ )
  - **$0.944 \text{ MW}$  cooling rf power...**

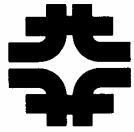




## Complementary case- ${}^7\text{Li} + {}^2\text{H} \rightarrow {}^8\text{Li} + {}^1\text{H}$



- Nuclear cross section for **beam loss is 1 barn ( $10^{-24} \text{ cm}^2$ )** or more
- $\sigma = 10^{-24} \text{ cm}^2$  corresponds to  $\sim 3.3 \text{ gm/cm}^2$  of  ${}^2\text{H}$ 
  - $\sim 9$  3-D cooling e-foldings ...
- Cross-section for  ${}^8\text{Li}$  production is  $\sim 100$  mbarn
  - $10^{-1}$  of  ${}^7\text{Li}$  is converted ??  **$10 \times$  better than  ${}^8\text{B}$  neutrinos**
- Goal is  **$10^{13}/\text{s}$  of  ${}^8\text{Li}$  production**
  - then at least  $10^{14}$   $\text{Li}^7/\text{s}$  needed
- Space charge **limit is  $\sim 10^{12}$   ${}^7\text{Li}/\text{ring}$** 
  - Cycle time can be up to  $10^{-2}$  s, but use  $10^{-3}$  s
  - If  $C=10\text{m}$ ,  $\tau=355$  ns, 2820 turns
  - $3.3/2820=1.2 \times 10^{-3}$  gm/cm $^2$  (0.007 cm @ liquid D density ...)
  - 1.3 MeV/turn energy loss and regain required ... (0.43MV rf)
  - **0.06 MW** cooling rf power...



# Space charge – Direct/inverse

?

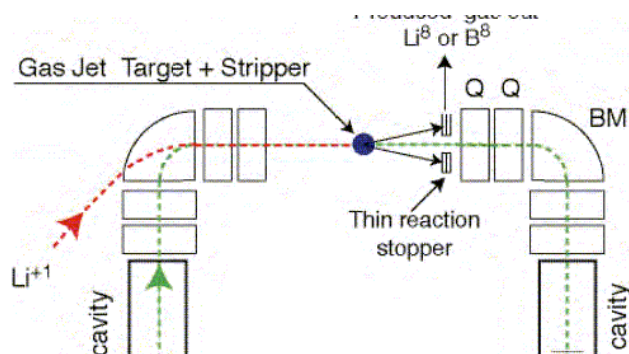
- At  $N = 10^{12}$ ,  $B_F = 0.2$ ,  $\beta = 0.094$ ,  $z=3$ ,  $a=6$ ,  $\epsilon_{N,rms} = 0.000046$ :

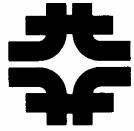
$$\delta v \approx 0.2$$

- tolerable ??
- Space charge sets limit on number of particles in beam and on transverse emittance
- Effect is reduced for “direct kinematics”
  - (D/He beam, Li target)

$$\delta v \cong \frac{z^2 r_p N_{tot}}{4\pi\beta\gamma^2 a B_F \epsilon_{N,rms}}$$

- Is “direct” source better than “inverse” source?
  - ${}^6\text{Li}_3$  beam +  ${}^3\text{He}_2$  target
  - or
  - ${}^3\text{He}_2$  beam +  ${}^6\text{Li}_3$  target
- Beam energy, power on target less (1/2 to 1/3)
- Li foil or gas-jet target?
  - Gas-jet nozzle for wedge effect
- > 0.1 MW power on target



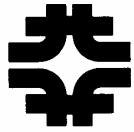


## Rubbia et al. not completely wrong

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- ...But contains mistakes
- Longitudinal emittance growth 2× larger
  - (than NuFACT06 presentation)
  - synchrotron oscillations reduce energy spread growth rate but not emittance growth rate
- Emittance exchange needs x-y coupling and balancing of cooling rates to get 3-D cooling
  - More complicated lattice
- **3-D cooling needed to get enough ions**
- Increases equilibrium emittance, beam size
  - increase needed for space charge, however
- Ion production to storage ring efficiency is not 100% ...



# $\mu$ Low-Energy “cooling”- emittance exchange



- $dP_\mu/ds$  varies as  $\sim 1/\beta^3$

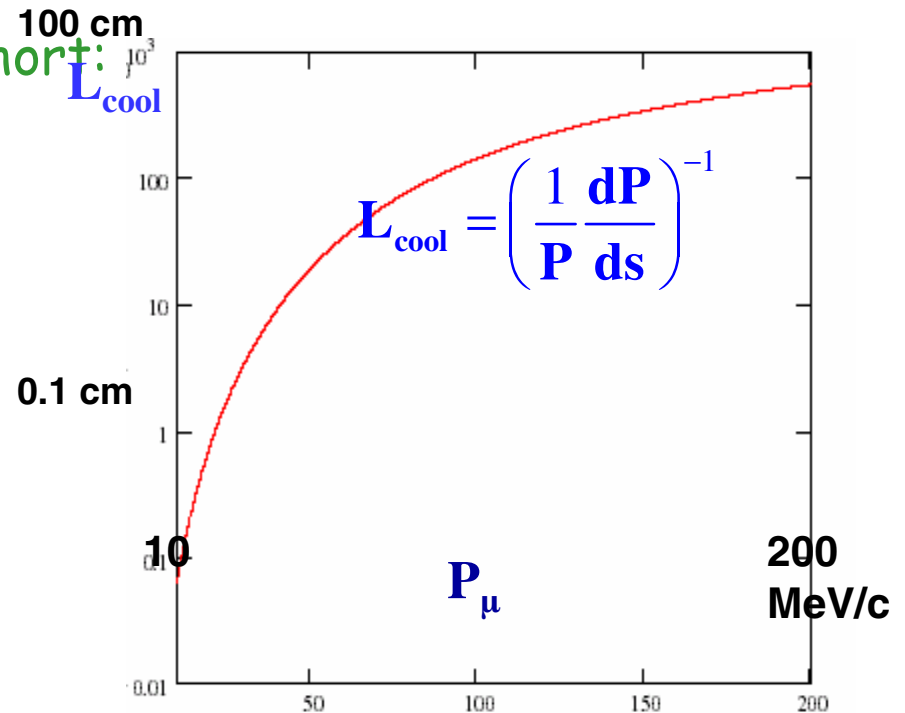
- “Cooling” distance becomes very short:

$$\frac{dP_\mu}{P_\mu ds} = 16 \text{ cm}^{-1} \quad \text{for liquid H at } P_\mu = 10 \text{ MeV}/c$$

- Focusing can get quite strong:
  - Solenoid:  $\beta_\perp \approx \frac{2B\rho}{B} = \frac{2P_\mu}{0.3B}$
  - $\beta_\perp = 0.002 \text{ m}$  at 30T, 10MeV/c

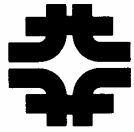
- $\epsilon_{N,eq} = 1.5 \times 10^{-4} \text{ cm}$  at 10MeV/c

- Small enough for “low-emittance” collider



$$\epsilon_{N,eq} = \frac{\beta_\perp E_s^2}{2g_t \beta m_p c^2 L_R \frac{dE}{ds}}$$

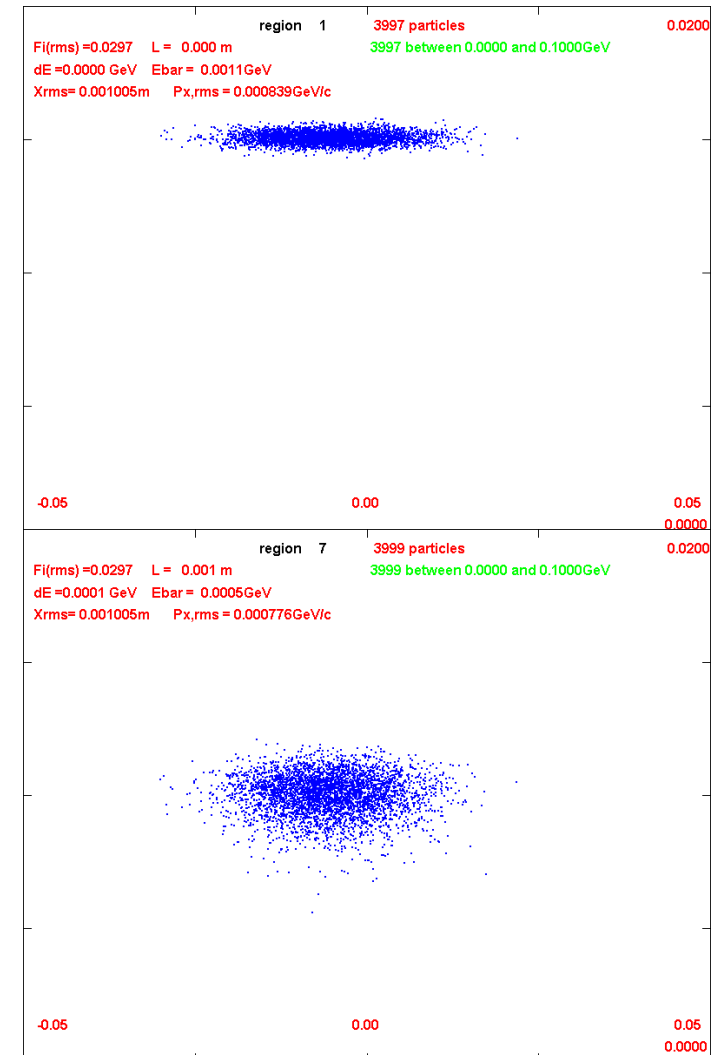
$$\epsilon_{N,eq} \propto \beta^2 \quad !!$$

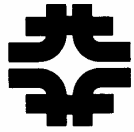


# ICool Simulation results



- **Low-Energy muons in H<sub>2</sub> absorber**
  - 50 MeV/c (4 cm H<sub>2</sub>)
  - 30 MeV/c (0.7cm H<sub>2</sub>)
  - 15 MeV/c (0.8mm H<sub>2</sub> or 80μ Be or... )
  - Could use gas absorbers/jets ?
  
- **Results follow rms eqns**
  - less multiple scattering ...
  
- **Typical section:**
  - reduces P by 1/3P
  - $\delta p$  increases by factor of 2
  - $\epsilon_x, \epsilon_y$  reduced by  $1/\sqrt{2}$





# ICool Multiple Scattering effects

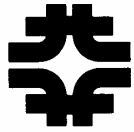


- **New Model 6 (Fano model) much less rms scattering than Model 4(Moliere/Bethe)**
  - At 200 MeV/c  $\mu$  on  $H_2$ , M4 scattering  $\sim 10\% >$  rms eq.
  - M6 scattering ( $\theta^2$ ) is  $\sim 30\%$  less than rms eq.
- **Low energy scattering less at low momentum**
  - At 15 MeV/c, M4 scattering  $\sim 40\% <$  than rms eq.
  - M6 scattering ( $\theta^2$ ) is  $\sim 60\%$  less than rms eq.
- **Which is more accurate? rms eq., model 4 or 6 or ??**

$$\frac{d\epsilon_N}{ds} = -\frac{1}{\beta^2 E} \frac{dE}{ds} \epsilon_N + \frac{\beta\gamma \beta_{\perp}}{2} \frac{d\langle \theta_{rms}^2 \rangle}{ds}$$

$$\frac{d\langle \theta_{rms}^2 \rangle}{ds} = \frac{z^2 E_s^2}{\beta^2 c^2 p_{\mu}^2 L_R} \left(1 + 0.038 \ln\left(\frac{ds}{L_R}\right)\right)^2 \quad \epsilon_{N,eq} = \frac{\beta_{\perp} E_s^2}{2g_t \beta m_{\mu} c^2 L_R \frac{dE}{ds}}$$



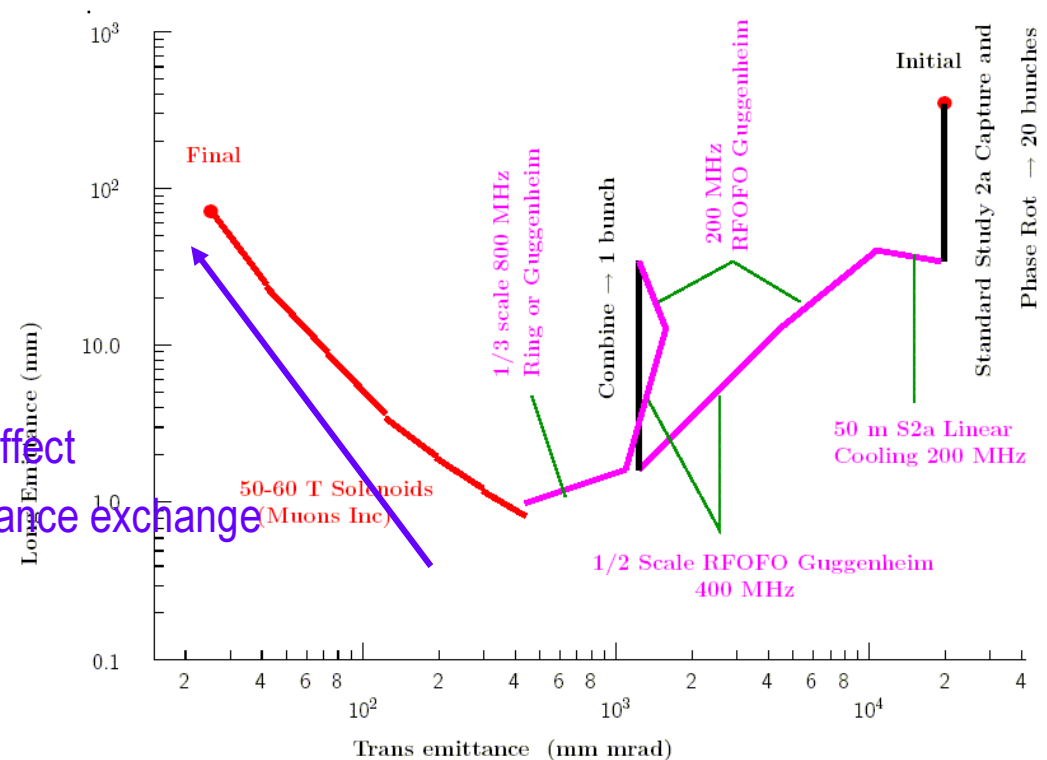


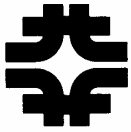
# Comments



- Can fit into end-stage cooling (with similar effects?)
- Can use gas jet absorbers to avoid having windows
  - $P_{\text{jet}} > 1 \text{ atm}$  possible
- Need “rf” to reduce dp/p (longer bunches for multistep )
  - 1mm bunch can grow to 1m bunch length
- Voltage is relatively small
- $L_{\mu} = 660\beta\gamma$
- Reacceleration of ~1m bunches

Approximate effect  
Of low-E emittance exchange



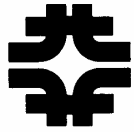


# Summary



- **Low energy ionization cooling has possible important applications**
  - Protons for neutron generation (Mori et al.)
  - $\beta$ -beam source production (Rubbia et al.)
  - Cooling of  $\mu$ 's to minimum transverse emittance
    - REMEX that might work ...
- **“Cooling” is predominantly emittance exchange**
  - X-y-z exchange needed for “real” cooling





# ILC Status



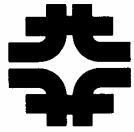
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# X-sections, kinematics

