Muon Acceleration in ‘Dogbone’ RLAs

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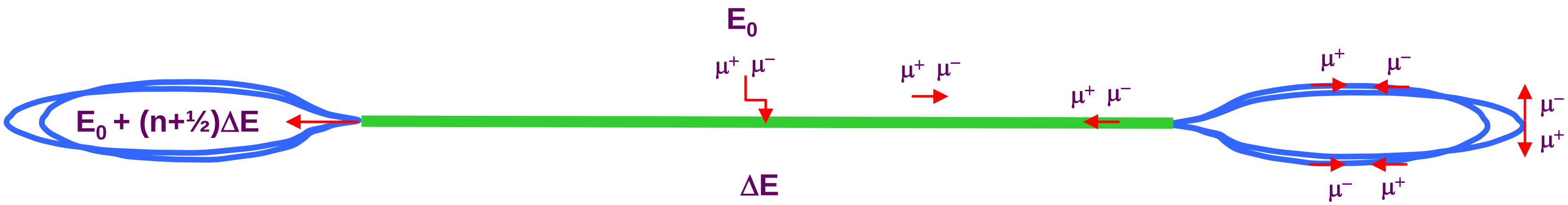
Overview

- Dogbone configuration
  - orbit separation
  - simultaneous acceleration of both μ^+ μ^- species
- **12.6 GeV Two-step-Dogbone RLA (4.5 pass) – ISS**
- Focusing scheme – Triplet vs FODO lattices
  - multi-pass linac optics
  - phase slippage in the linac
  - ‘droplette’ Arc lattice
- 15 GeV Dogbone RLA (6.5 pass)

Simultaneous acceleration of both μ^+ μ^- species



orbit separation at linac's end ~ energy difference between consecutive passes (2ΔE)

Baseline Acceleration Scenario (ISS)

- The scheme involves three superconducting linacs (200 MHz, 15 MeV/m):
  - a single pass linear Pre-accelerator
  - followed by a pair of multi-pass 'Dogbone' recirculating linacs (RLAs).
- Acceleration starts after ionization cooling at 273 MeV/c and proceeds to 12.6 GeV/c
- The beam may be injected into FFAG ring(s) for further acceleration

12.6 GeV Two-step-Dogbone RLA

Dogbone I (4.5 pass):

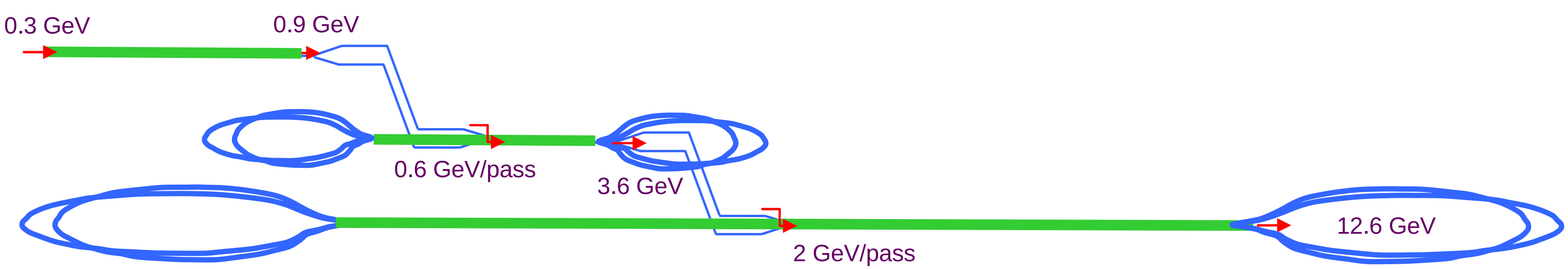
0.9 GeV to 3.6 GeV

$$\frac{E_f}{E_0} = 3.5$$

Dogbone II (4.5 pass):

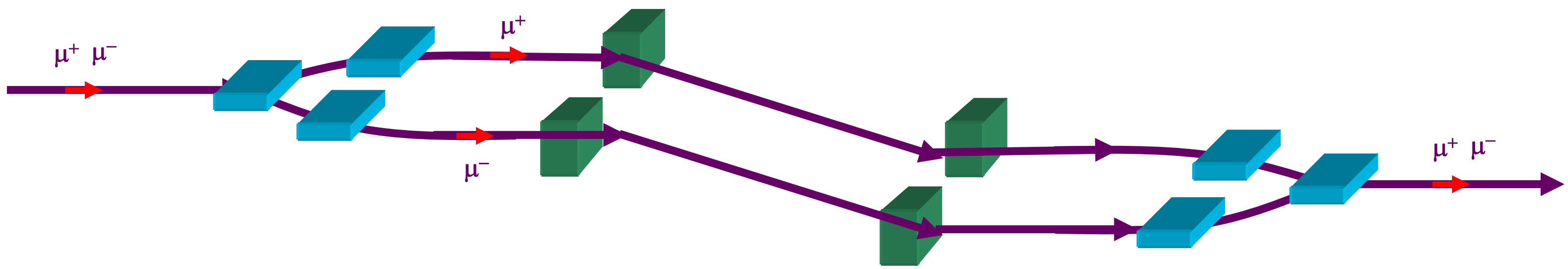
3.6 GeV to 12.6 GeV

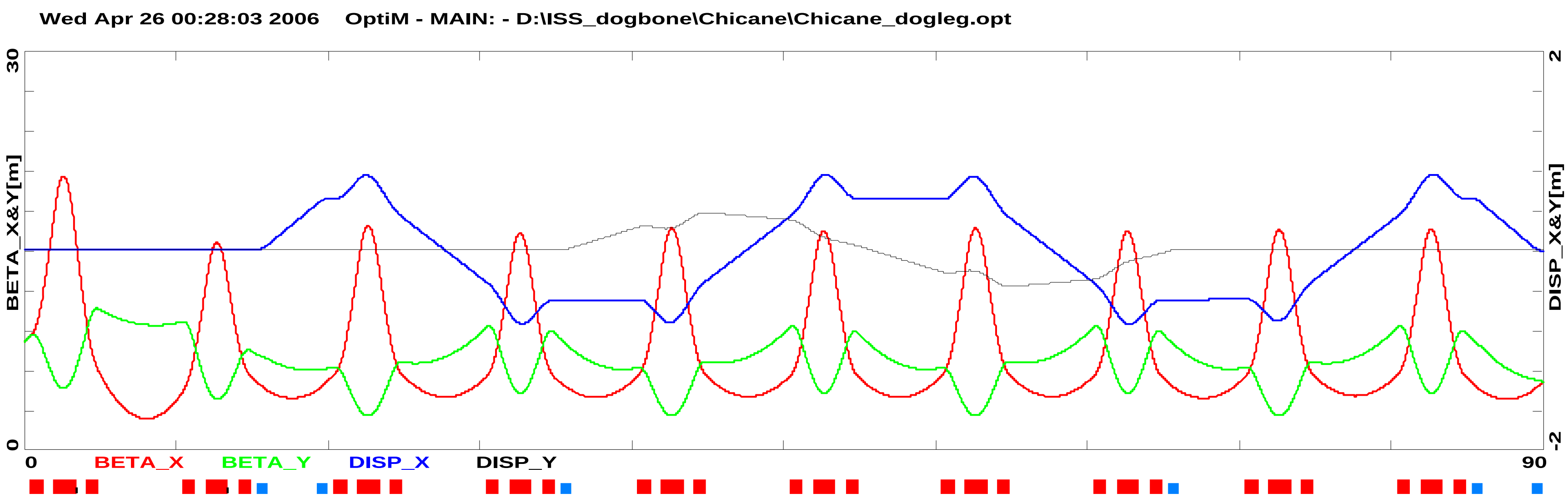
$$\frac{E_f}{E_0} = 4$$



For compactness all there components (Pre-accelerator, Dogbone I and Dogbone II) are stacked up vertically; $\mu^{\pm}$ beam transfer between the accelerator components is facilitated by the vertical double chicane

Injection double-chicane





Phase slippage in the linac

- The energy range for each 'Dogbone' RLA was chosen to give similar ratios of top-to-injection energies:
  - 3.5 for Dogbone I
  - 4.0 for Dogbone II
- The phase slippage of a muon injected with the initial energy E_0 and accelerated by ΔE in a linac of length, **L** , was calculated, where uniformly spaced RF cavities were phased for a speed-of-light particle
- The injection energies for both RLAs were chosen, so that a tolerable level of the RF phases slippage along a given length linac can be maintained.

Phase slippage in the linac

- A simple calculation of the phase slippage of a semi-relativistic muon accelerated in a linac, where uniformly spaced RF cavities are phased for the speed-of-light particle was carried out using the following cavity-to-cavity iterative algorithm for phase-energy vector

$$\begin{pmatrix} \phi b_{k,i+1} \\ Eb_{k,i+1} \end{pmatrix} := \begin{bmatrix} \phi b_{k,i} + \frac{h}{\lambda}\cdot 360 \cdot \left[\frac{1}{2}\cdot\left(\frac{m_\mu}{Eb_{k,i}}\right)^2\right] \\ Eb_{k,i} + h\cdot\Delta E_k \end{bmatrix}$$

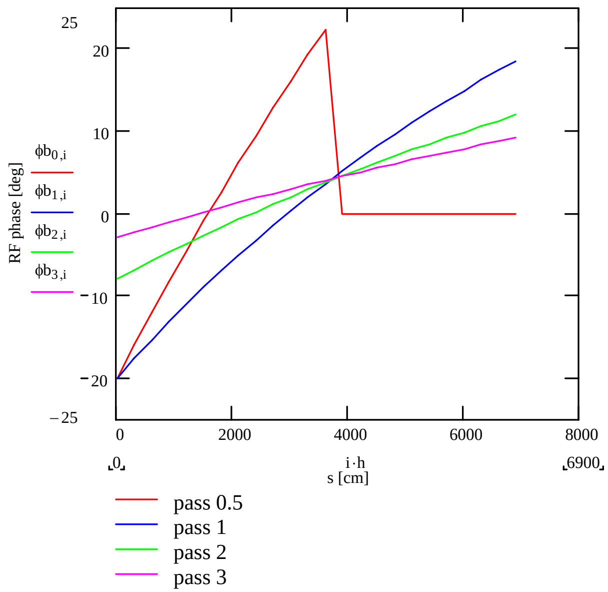
where

$$h := \frac{L_{linac}}{N_{cav}} \qquad \lambda := \frac{c}{f_0} \qquad k := 0..4 \qquad i := 0..1N_{cav} - 1$$

Phase slippage in the linacs

Dogbone I

$E_0 = 0.9GeV\ ,\ \Delta E = 0.6GeV\ ,\ L = 70m$

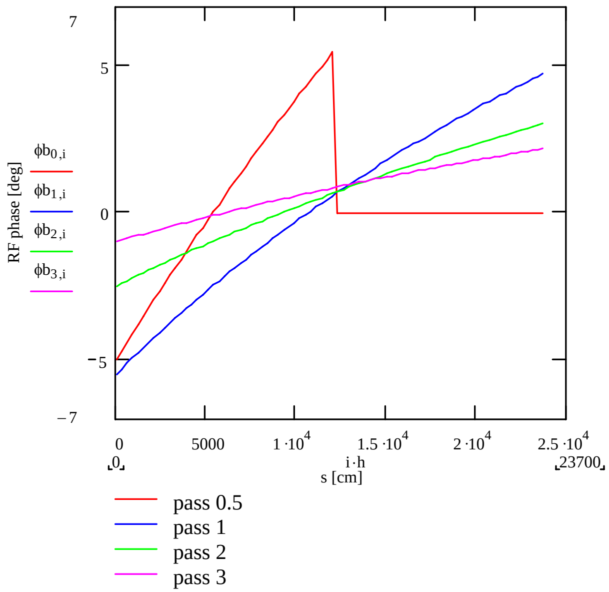


$\phi b_{0,0} := -20$ $\phi b_{1,0} := -20$

$\phi b_{2,0} := -8$ $\phi b_{3,0} := -3$

Dogbone II

$E_0 = 3.6GeV\ \ \Delta E = 2GeV\ \ L = 240m$



$\phi b_{0,0} := -5$ $\phi b_{1,0} := -5.5$

$\phi b_{2,0} := -2.5$ $\phi b_{3,0} := -1$

RF phase slippage along the multi-pass linacs; initial 'gang phases' for each pass were chosen for the optimum longitudinal bunch compression in each linac-Arc segment

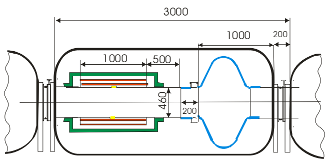
Longitudinal compression in the RLA

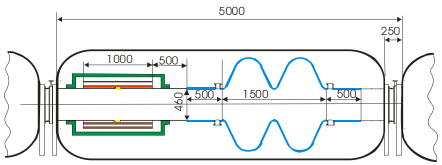
- Longitudinal bunch compression is required in the course of acceleration.
- To accomplish that, the beam is accelerated off-crest with non zero M_{56} ~ 6 m (momentum compaction) in the 'droplette' Arcs.
- This induces synchrotron motion, which suppresses the longitudinal emittance growth related to non-linearity of accelerating voltage.
- Without synchrotron motion the minimum beam energy spread would be determined by non-linearity of RF voltage across the bunch length, e.g. it would be equal to $1-\cos\phi \approx 9\%$ for bunch length $\phi = 30$ deg.
- The synchrotron motion within the bunch averages the total energy gain of tail's particle to the energy gain of particles in the core.

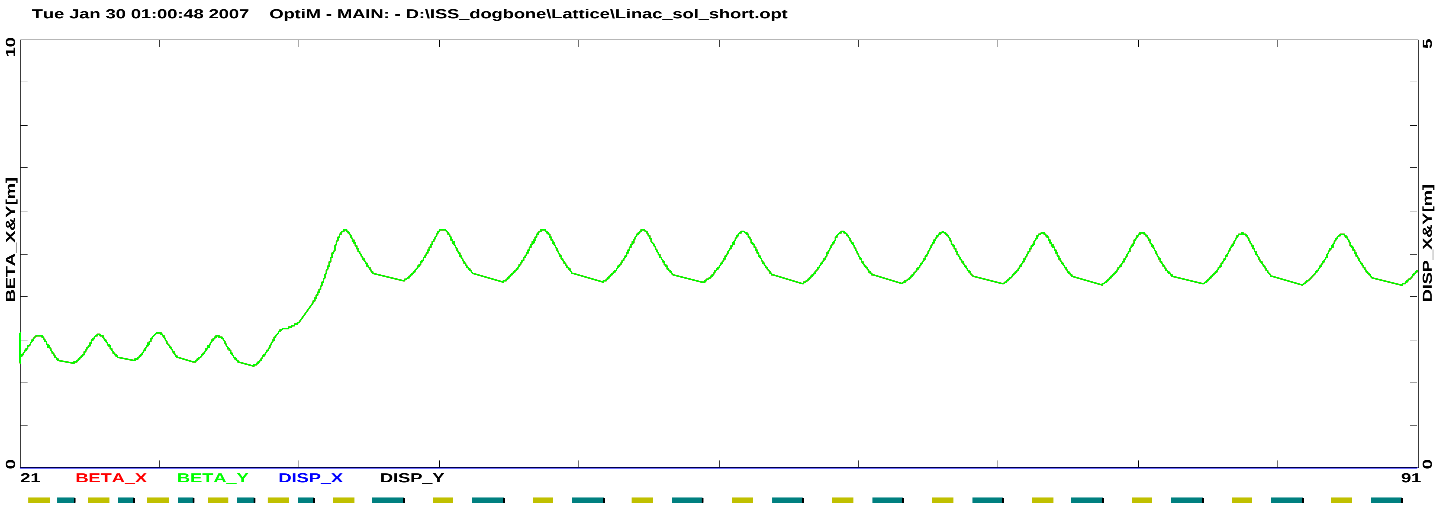
Initial beam emittance/acceptance after cooling at 273 MeV/c

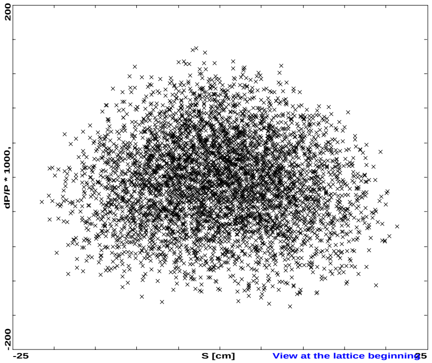
Normalized Emittances		ε_{rms}	**A = (2.5)^2 ε**
transverse emittance: $\varepsilon_x/\varepsilon_y$	mm·rad	**4.8**	**30**
longitudinal emittance: ε_l	mm	**27**	**150**
($\varepsilon_l = \sigma_{\Delta p}\ \sigma_z/m_\mu c$)			
momentum spread: $\sigma_{\Delta p/p}$		**0.07**	**±0.17**
bunch length: σ_z	mm	**176**	**±442**

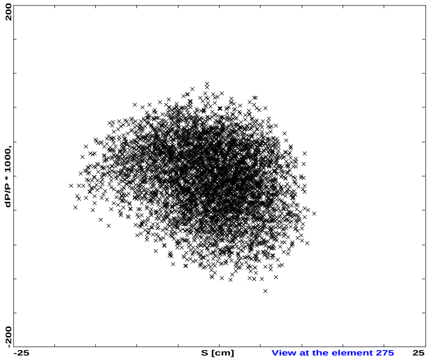
Linear Pre-accelerator – Longitudinal dynamics





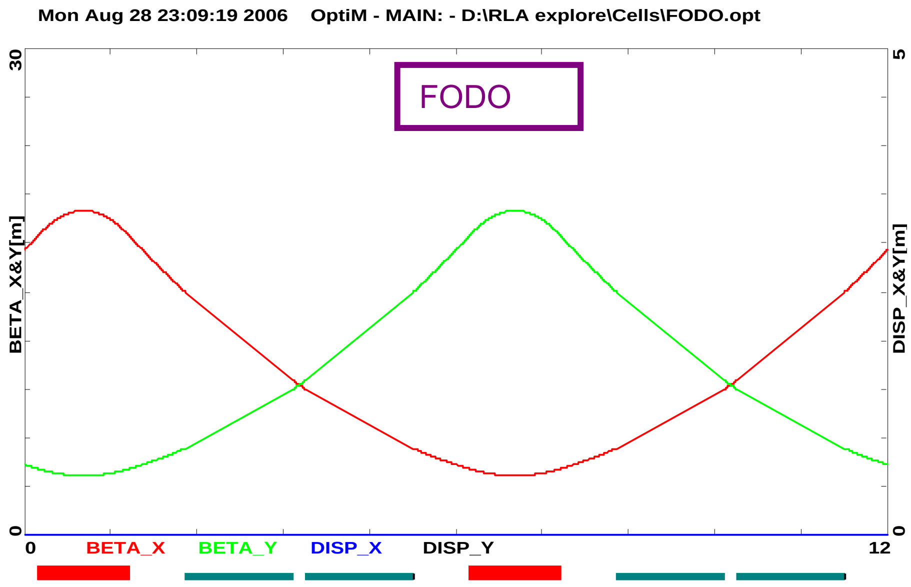


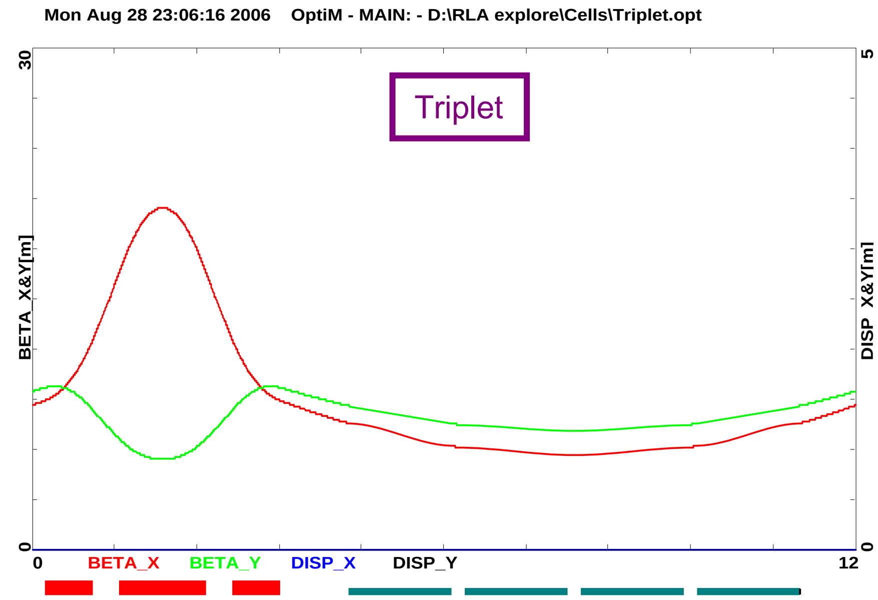




FODO vs Triplet focusing structure

- The same length
- The same phase advance per cell ($\Delta\phi_x = 90^0 = \Delta\phi_y$)





Advantages:

- much weaker quads (~3 times)
- shorter quads (total)
- easier chromaticity correction

Advantages:

- longer straight sections
- smaller vertical beta-function
- uniform variation of betas and dispersion

Multi-pass Linac Optics

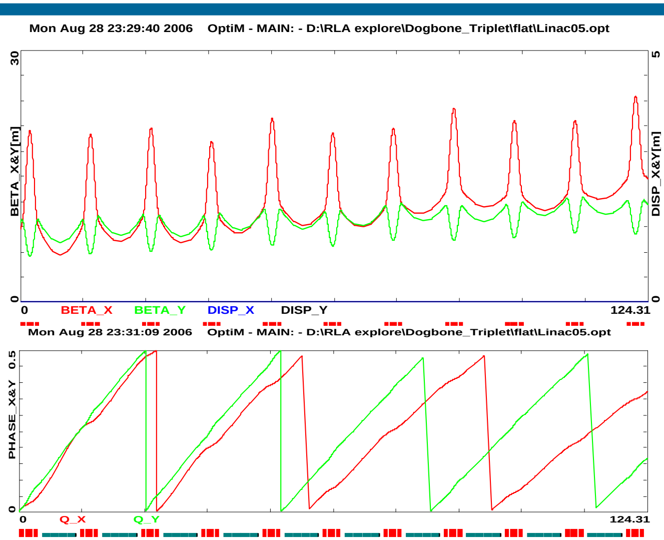
- The focusing profile along the linac (quadrupole gradients) need to be set so that one can transport multiple pass beams within a vast energy range (provide adequate transverse focusing for given aperture) .
- The beam is traversing the linac in both directions – one chooses a **'flat focusing profile'** (Bob Palmer) for the entire linac: e.g. the quads in all cells are set to the same gradient, corresponding to 90 deg. phase advance per cell determined for the lowest energy (injection) – no quad scaling with energy
- The requirement of simultaneous acceleration of both $\mu^{\pm}$ species imposes **mirror symmetry** of the 'droplette' Arcs optics (the two species move in the opposite directions through the Arcs). This in turn puts a constraint on the exit/entrance Twiss functions for the two consecutive linac passes:

$$\beta^{out}_{n} = \beta^{in}_{n+1} \quad \text{and} \quad \alpha^{out}_{n} = -\alpha^{in}_{n+1}$$

where n = 0, 1, 2.. is the pass index

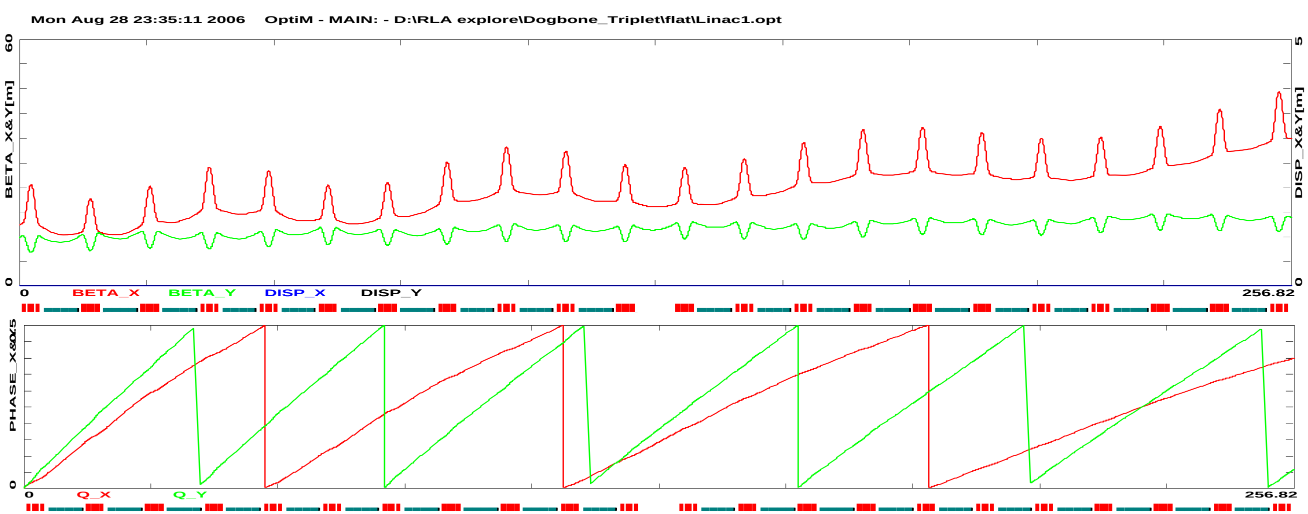
Triplet - ‘flat focusing' linac profile

mirror symmetry cond. ($\beta_{out}^{n} = \beta_{in}^{n+1}$, and $\alpha_{out}^{n} = -\alpha_{in}^{n+1}$, n - pass index)



‘half pass’ , 2-3 GeV

Initial phase adv/cell 90 deg – fixed quad gradients for the entire linac (no scaling with energy)

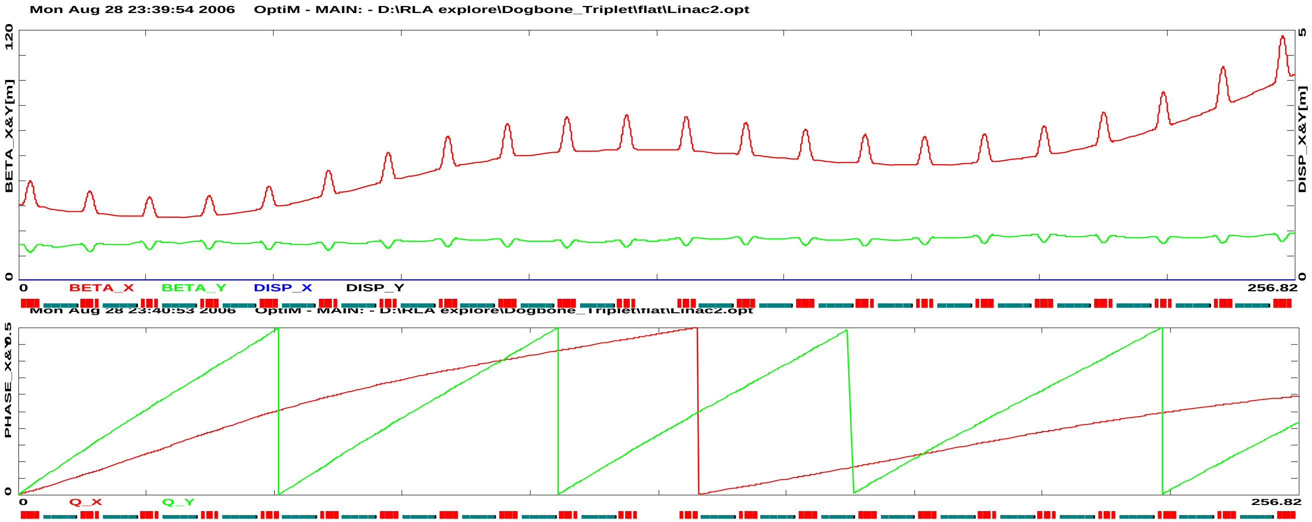


1-pass, 3-5 GeV

phase adv. drops much faster in the horizontal plane

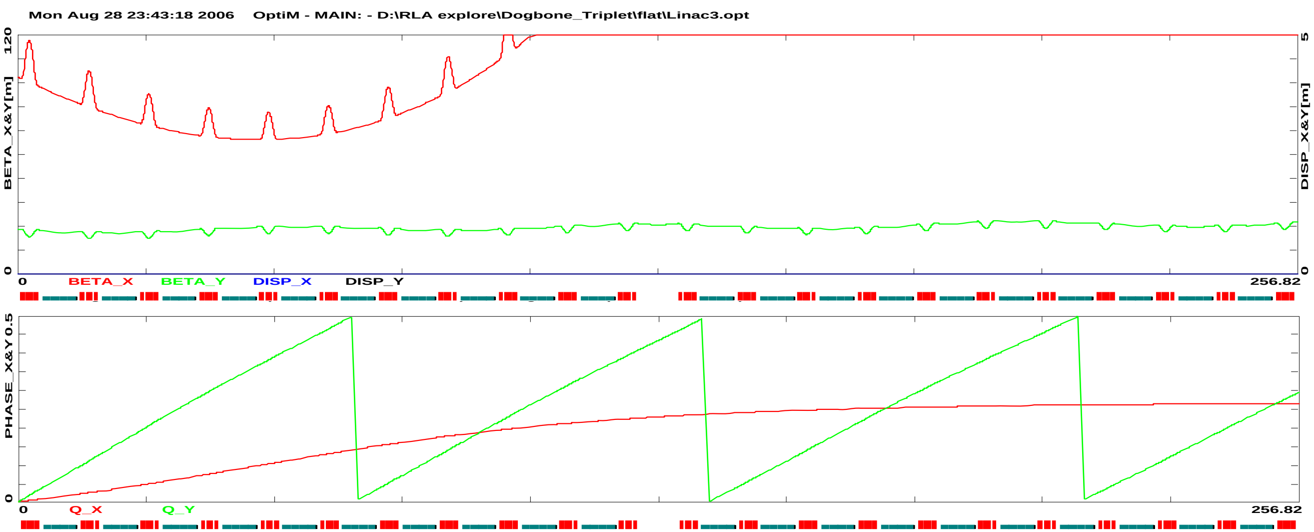
Triplet - 'flat focusing' linac profile

mirror symmetry cond. ($\beta_{out}^{n} = \beta_{in}^{n+1}$, and $\alpha_{out}^{n} = -\alpha_{in}^{n+1}$, n - pass index)



2-pass, 5-7 GeV

phase adv. drops much faster in the horizontal plane



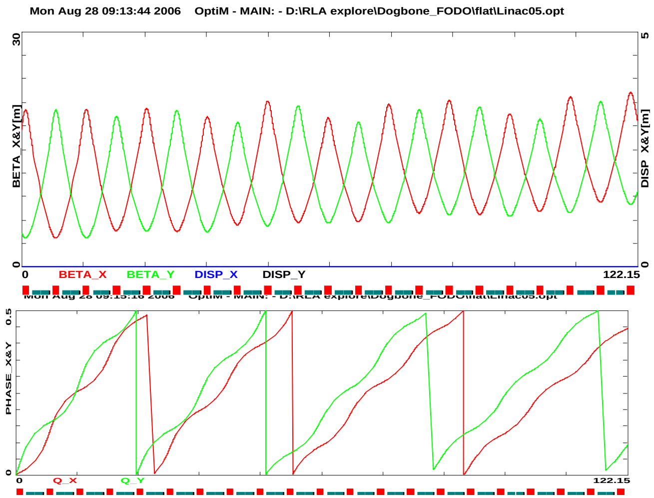
3-pass, 7-9 GeV

no phase adv. in the horizontal plane

FODO - ‘flat focusing' linac profile

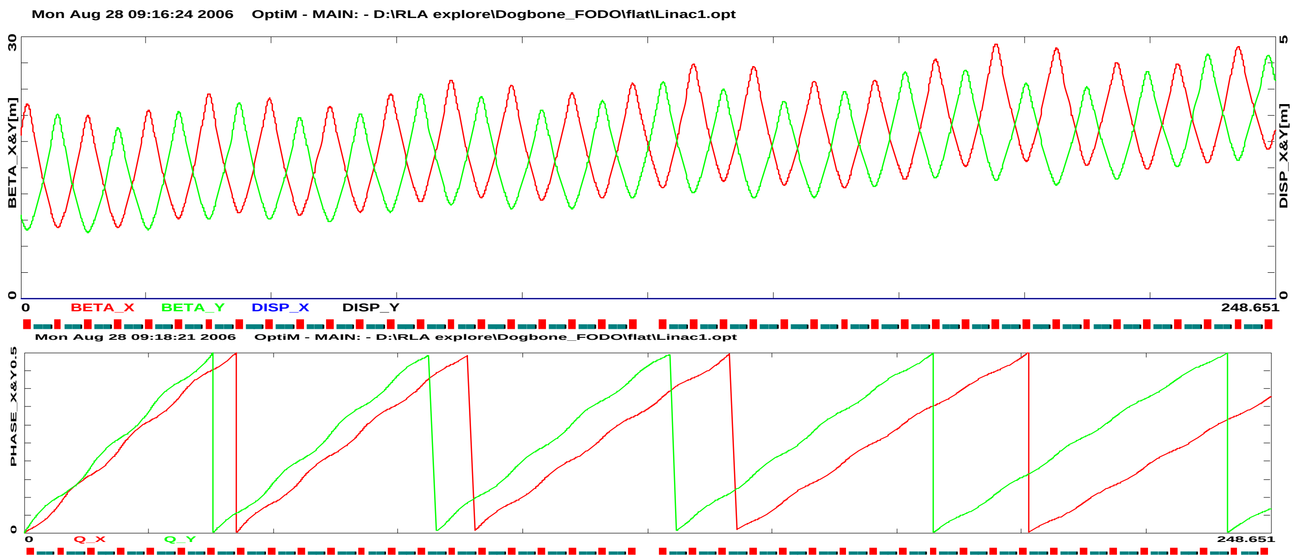
mirror symmetry cond. ($\beta_{out}^{n} = \beta_{in}^{n+1}$,and $\alpha_{out}^{n} = -\alpha_{in}^{n+1}$, n - pass index)

‘half pass’ , 2-3 GeV



initial phase adv/cell 90 deg – fixed gradient in all cells (no scaling with energy)

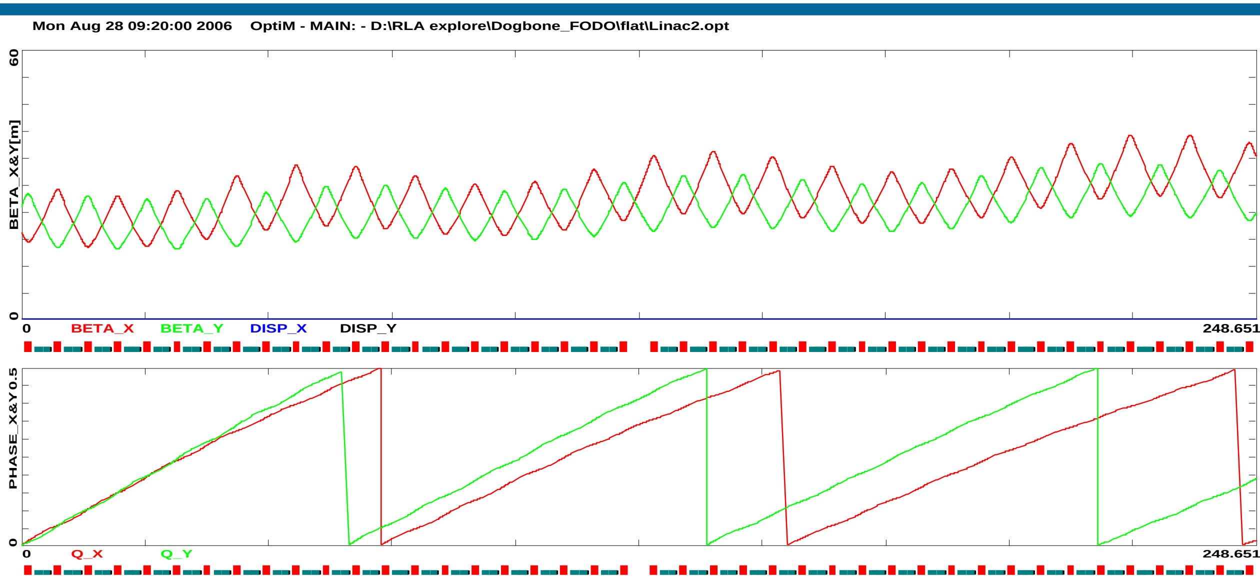
1-pass, 3-5 GeV



phase adv. diminish uniformly in both planes

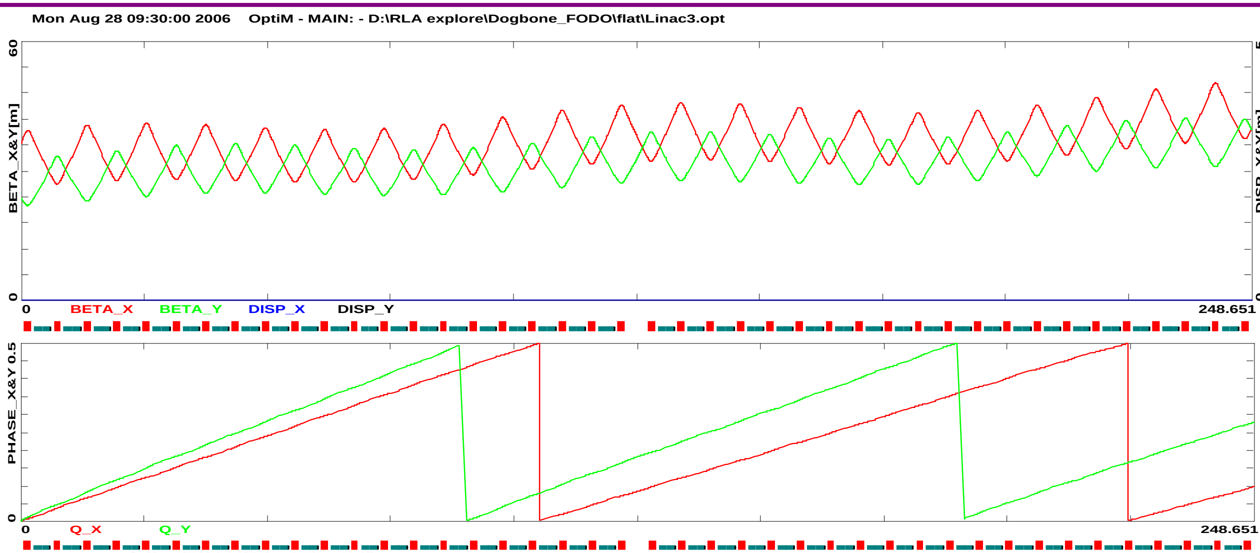
FODO - 'flat focusing' linac profile

mirror symmetry cond. ($\beta_{out}^{n} = \beta_{in}^{n+1}$, and $\alpha_{out}^{n} = -\alpha_{in}^{n+1}$, n - pass index)



2-pass, 5-7 GeV

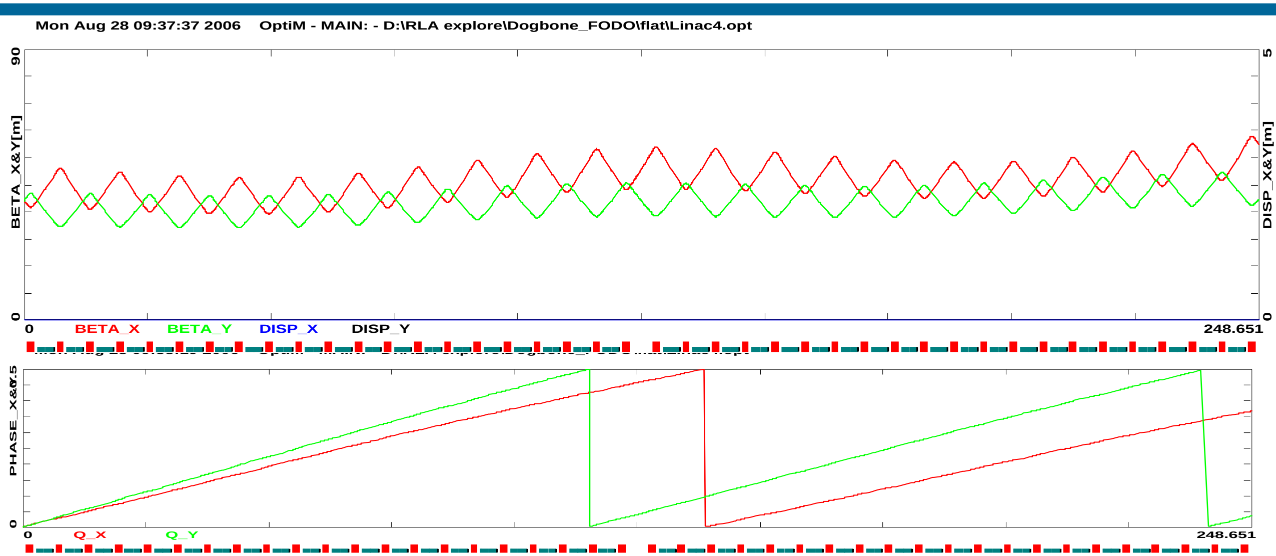
phase adv. diminish uniformly in both planes



3-pass, 7-9 GeV

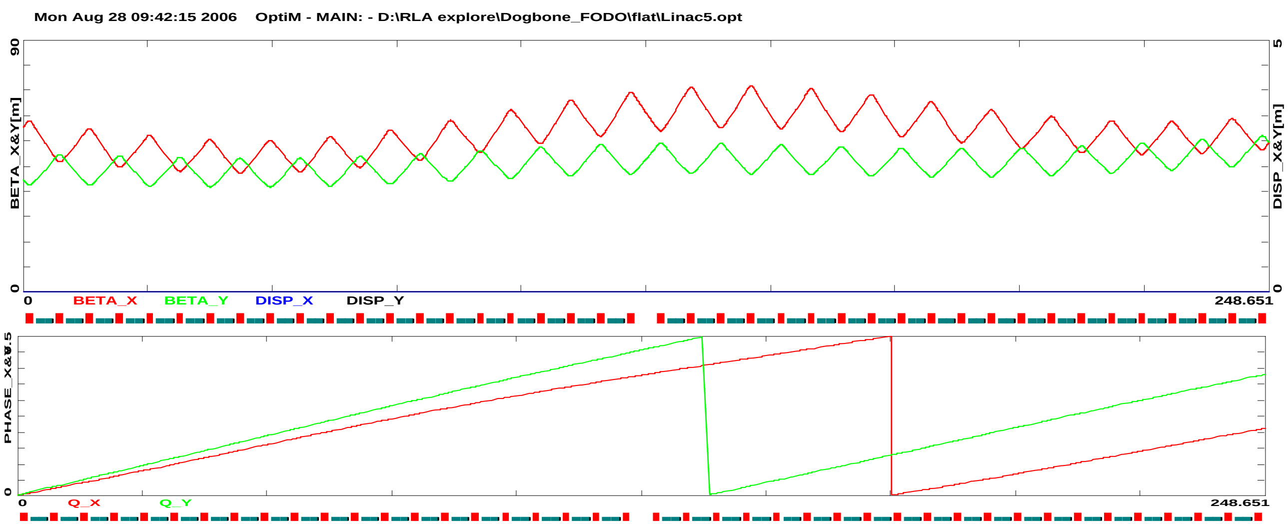
FODO - ‘flat focusing' linac profile

mirror symmetry cond. ($\beta_{out}^{n} = \beta_{in}^{n+1}$, and $\alpha_{out}^{n} = -\alpha_{in}^{n+1}$, n - pass index)



4-pass, 9-11 GeV

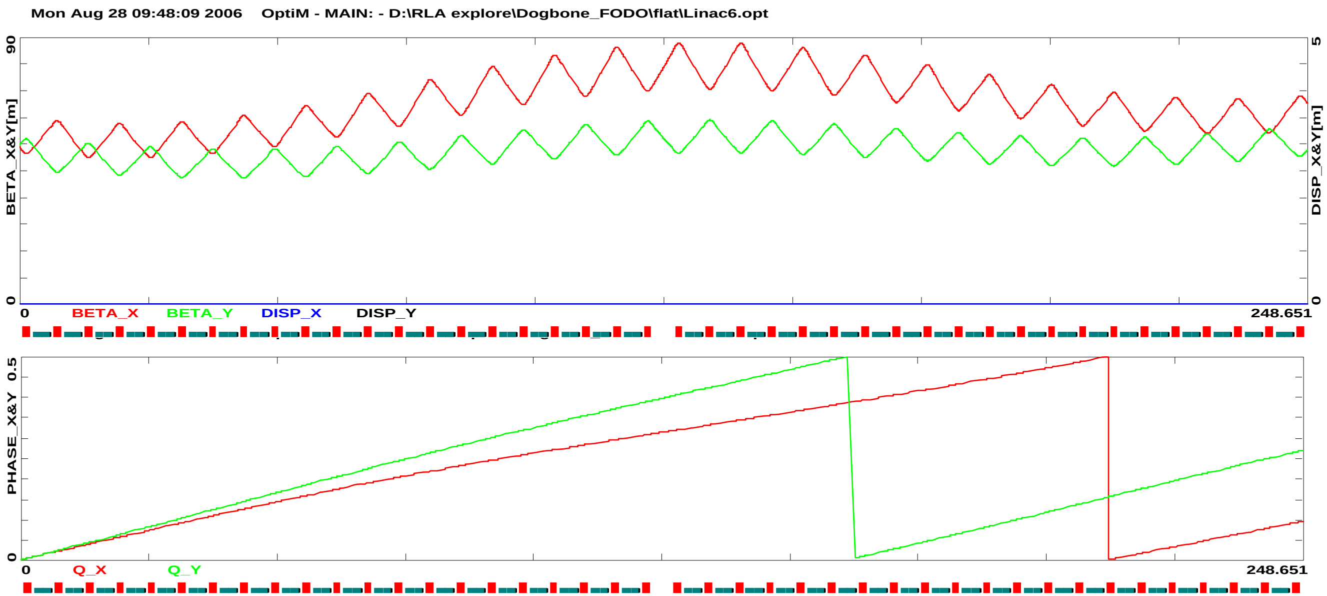
phase adv. diminish uniformly in both planes



5-pass, 11-13 GeV

‘flat focusing' linac profile

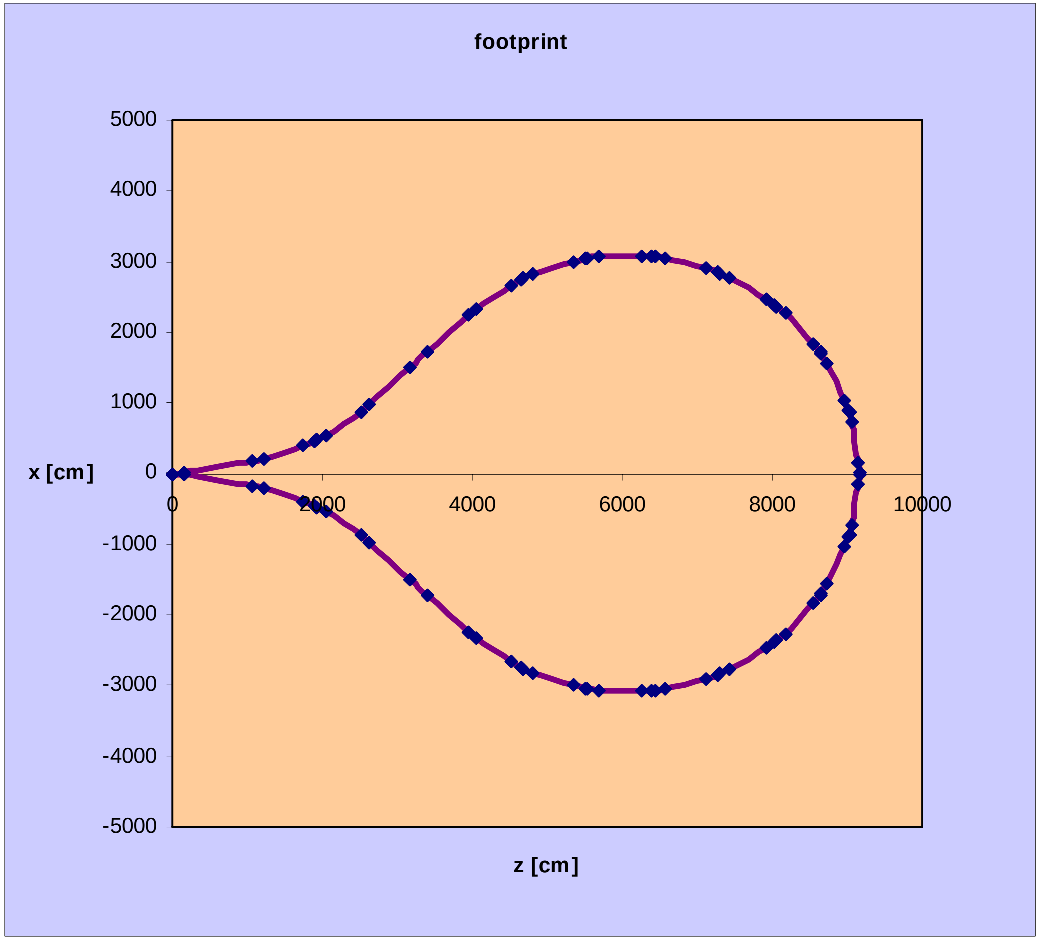
mirror symmetry cond. ($\beta_{out}^{n} = \beta_{in}^{n+1}$,and $\alpha_{out}^{n} = -\alpha_{in}^{n+1}$, n - pass index)



6-pass, 13-15 GeV

phase adv. diminish uniformly in both planes

'Droplette' Arc – Layout



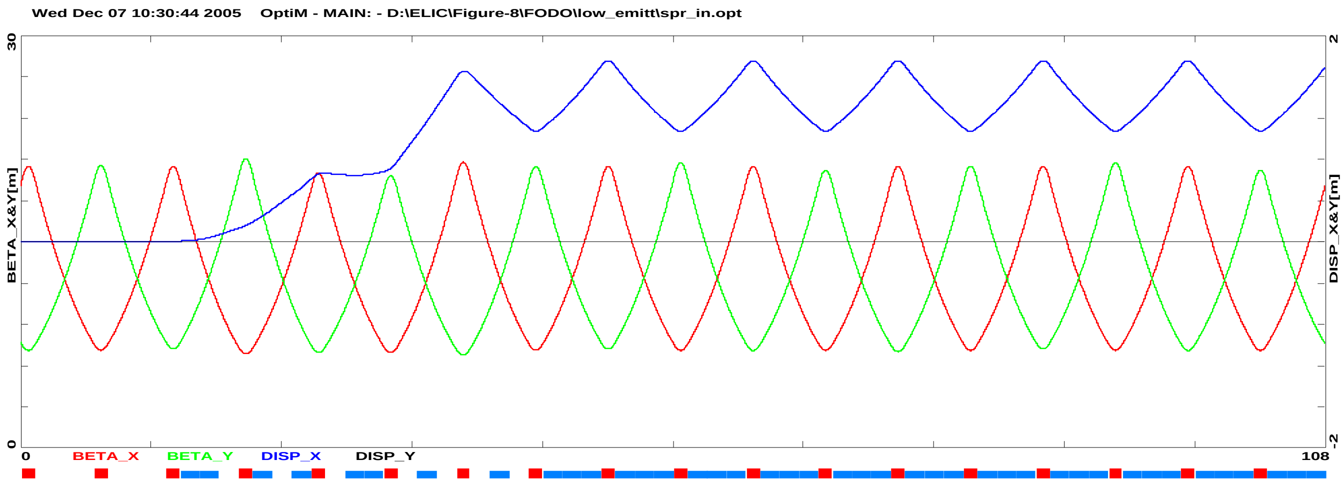
Arc dipoles

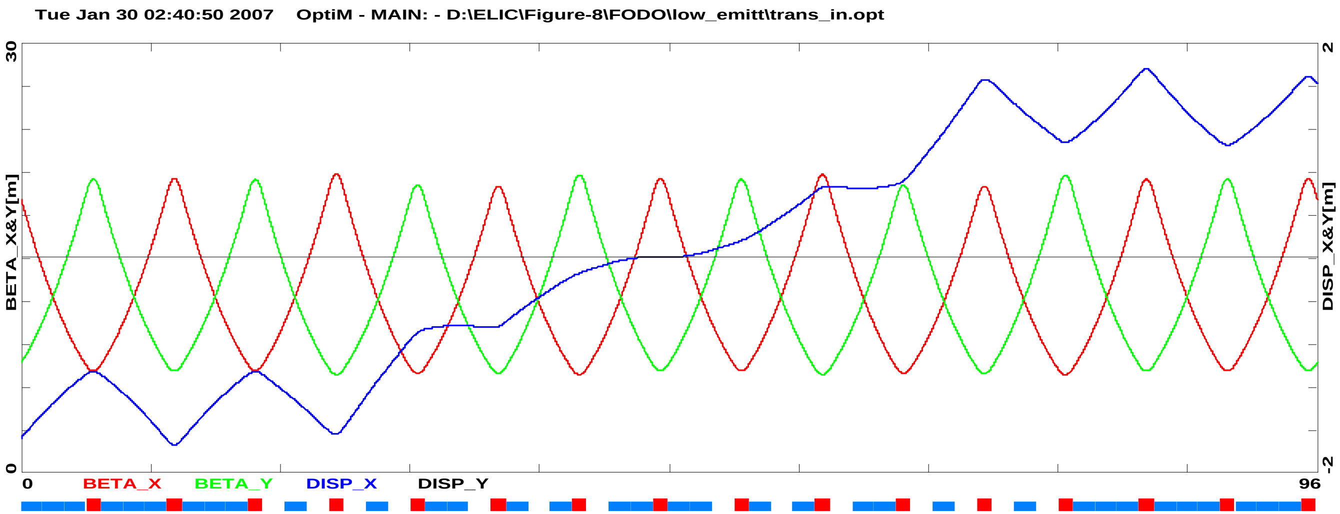
$Lb=150; => 150 cm
$ang0=10.3283; => 10.328 deg
$Nin=16; => 16
$Nout=2; => 2
$ang=(90+$ang0)/($Nin-2*$Nout); => 8.36 deg.
#
$Ang_out=$ang0+2*$Nout*$ang; => 43.77 deg.
$Ang_in=2*$Nin*$ang; => 267.54 deg.

$BP=$PI*$Hr*$ang/(180*$Lb); => 6.537 kGauss

$Lring=227.3 m

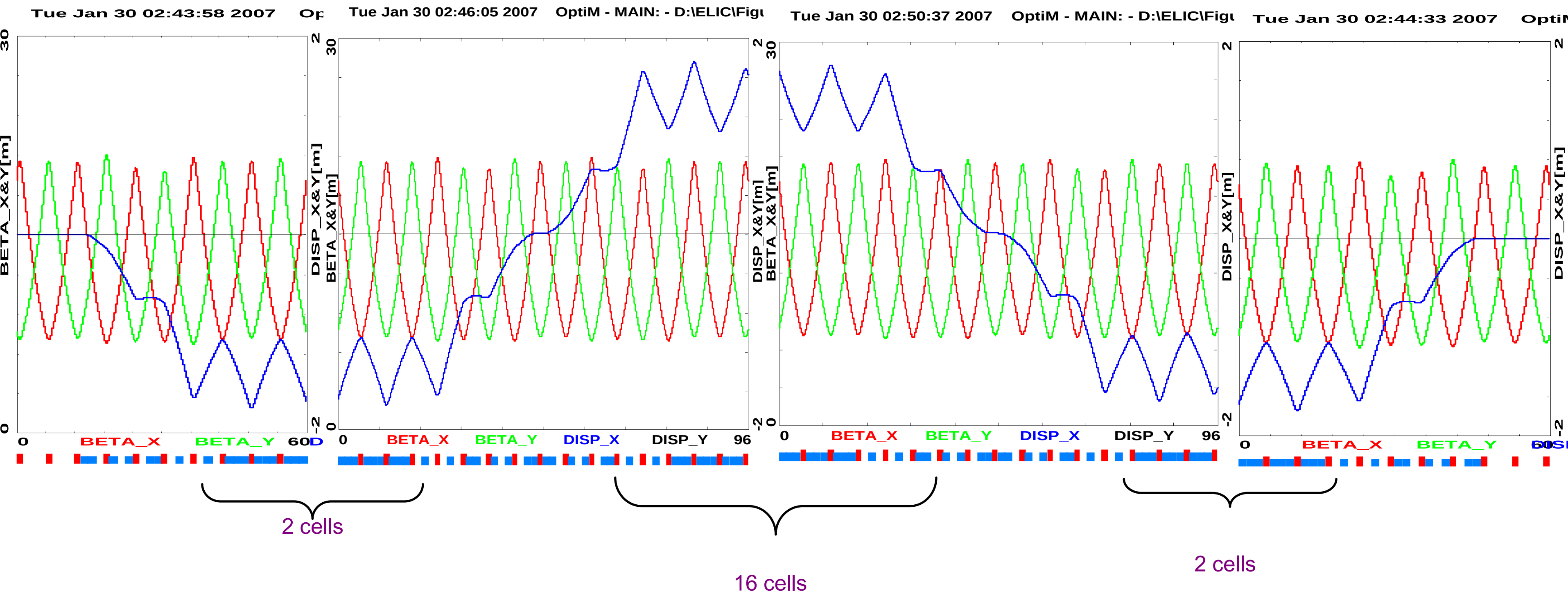
Spreader and 'Dispersion Flip' Lattices





‘Droplette’ Arc – Mirror-symmetric Optics

($\beta_{out} = \beta_{in}$,and $\alpha_{out} = -\alpha_{in}$, matched to the linacs)



$$M_{56} = -\int \frac{D_x}{\rho} ds = -D_x^{dip} \int d\left(\frac{s}{\rho}\right) = -D_x^{dip} \int d\theta_{rad} = -D_x^{dip} \times \theta_{rad}^{tot}$$

$$M_{56} = -\frac{5}{3}\pi \times 1.2\ m = -6.3\ m$$

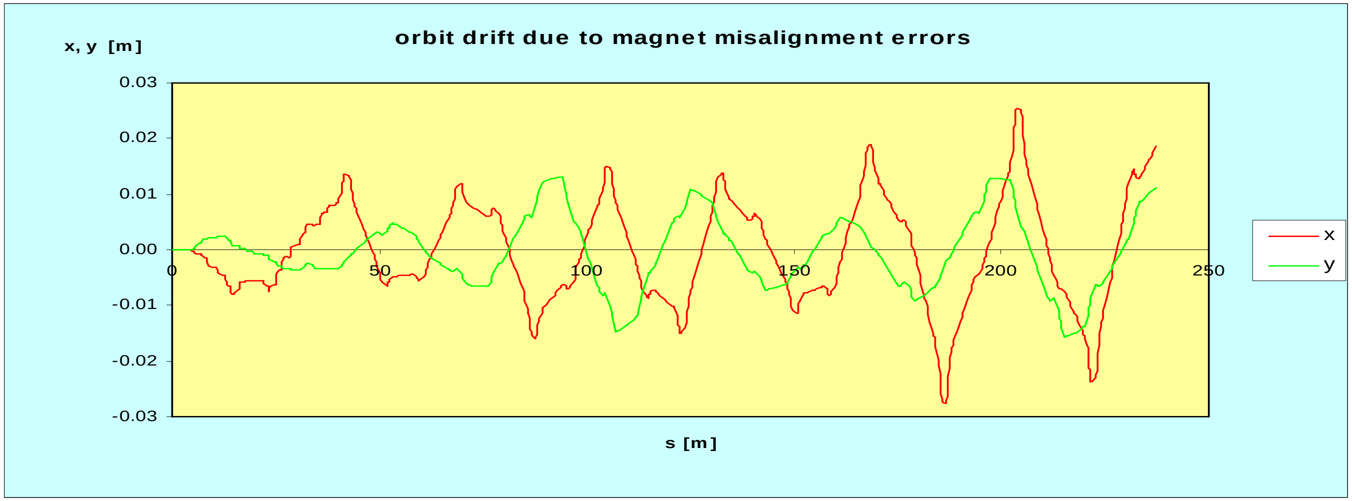
Magnet Misalignment Errors

- Lattice sensitivity to random misalignment errors was studied via DIMAD Monte-Carlo assuming:

quadrupole misalignment errors:

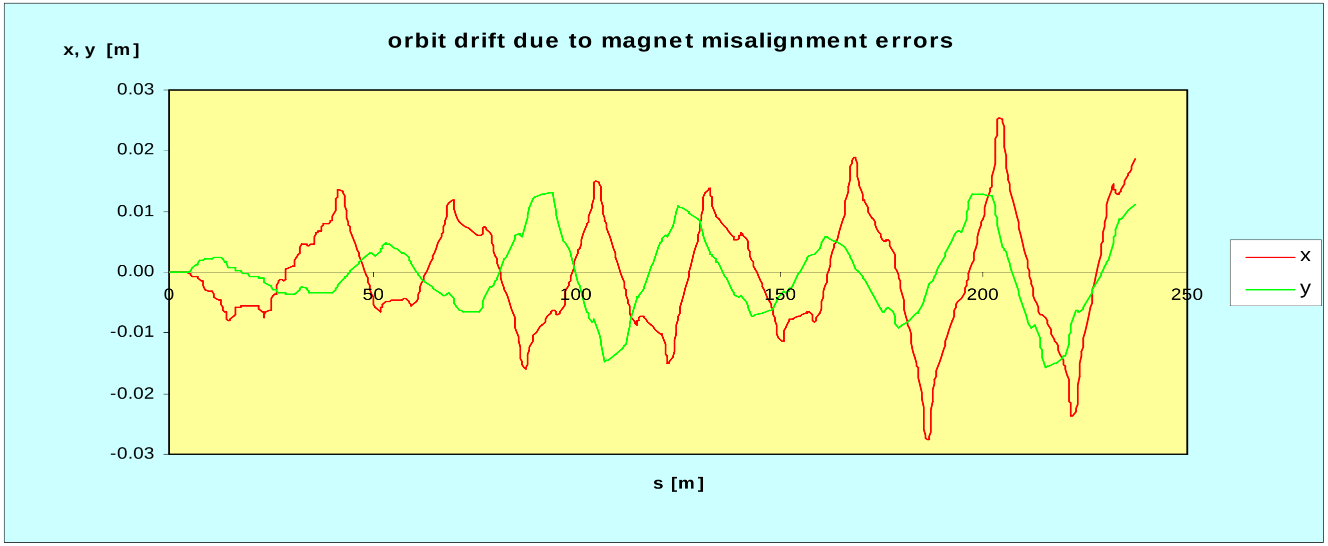
F:	$\sigma_x = \sigma_y = 1$ mm	$(\sigma_{x,y'} = \sigma_{x,y}/L)$	$\sigma_{x'} = \sigma_{y'} = 0.8 \times 10^{-3}$
D:	$\sigma_x = \sigma_y = 1$ mm		$\sigma_{x'} = \sigma_{y'} = 1.47 \times 10^{-3}$

- Gaussian distribution was chosen for individual quad misalignments
- Resulting reference orbit distortion (uncorrected) for Arc 2 is illustrated below



- Similar level of dipole misalignment errors had virtually no effect on random steering

Arc 2 – Magnet Misalignment Errors



RMS Orbit Displacement [m]:

X:	0.9486e-02
y:	0.7003e-02

Extr. Orbit Displacement [m]:

X_{max}:	0.2538E-01
X_{min}:	-0.2782E-01
y_{max}:	0.1434E-01
y_{min}:	-0.1697E-01

- Same level of orbit drifts due to quad misalignments for other 'Dogbone' segments (Arc 1, 3 and 4 and linacs)
- Orbit drifts at the level of ~3 cm can easily be corrected by pairs of hor/vert correctors (2000 Gauss cm each) placed at every triplet girder

Focusing Error Tolerances – Quadrupole Field Spec

- Cumulative Arc-to-Arc Optics mismatch as measured by Courant-Snyder invariant change:

$$\varepsilon' = \beta(\theta + \delta\theta)^2 + 2\alpha(\theta + \delta\theta)x + \gamma x^2$$
$$= \varepsilon\left(1 + \beta\Delta\Phi\sin(2\mu) + (\beta\Delta\Phi\cos\mu)^2\right),$$

$$\varepsilon_N = \varepsilon_0 \prod_{n=1}^{N}\left(1 + \beta_n\Delta\Phi_n\sin(2\mu_n) + (\beta_n\Delta\Phi_n\cos\mu_n)^2\right)$$
$$= \varepsilon_0 \prod_{n=1}^{N}\left(1 + \frac{1}{2}(\beta_n\Delta\Phi_n)^2 + \sqrt{(\beta_n\Delta\Phi_n)^2 + \left(\frac{\beta_n\Delta\Phi_n}{2}\right)^4}\sin(2\mu_n + \psi_n)\right),$$

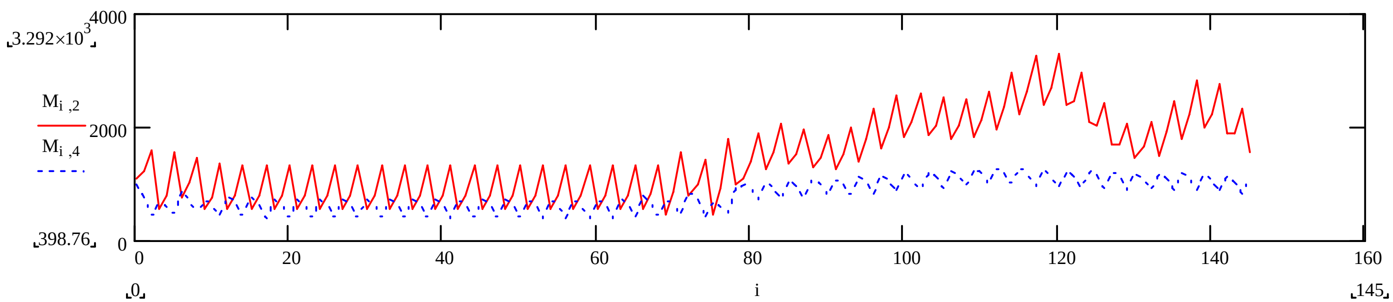
- Standard deviation of Courant-Snyder invariant:

$$\frac{\sigma_\varepsilon}{\varepsilon} = \frac{\sqrt{\overline{\Delta\varepsilon^2} - \overline{\Delta\varepsilon}^2}}{\varepsilon} \approx \sqrt{\frac{1}{2}\sum_{n=1}^{N}(\beta_n\Delta\Phi_n)^2} = \frac{\sqrt{\overline{\Delta\Phi^2}}}{\Phi_{\max}}\sqrt{\frac{1}{2F_{\min}^2}\sum_{n=1}^{N}\beta_n^2}$$

Focusing Error Tolerances – Quadrupole Field Spec

- By design, one can tolerate Arc-to-Arc mismatch at the level of 10% (to be compensated by the dedicated matching quads).

$$\left(\frac{\sigma_\varepsilon}{\varepsilon}\right)_{mis} = \sqrt{\frac{1}{2}\sum_{n=1}^{N}\left(\beta_n\delta\phi_1\right)^2} = \sqrt{\frac{1}{2}\Delta\phi_1^2\sum_{n=1}^{N}\left(\beta_n\right)^2_{quad} + \frac{1}{2}\delta\phi_1^2\sum_{n=1}^{N}\left(\beta_n\right)^2_{dipole}}$$



(GdL=65 kG)

$F_{min} = 1.5$ m

$$\sqrt{\frac{1}{2F_{min}^2}\sum_{n=1}^{N}\beta_n^2} \approx 190$$

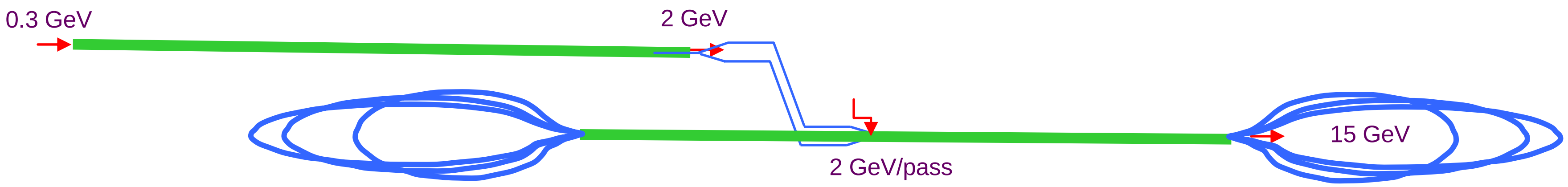
$$\frac{\Phi}{\phi_1^{max}} = 0.001$$

$$\Phi = \sqrt{\delta\phi_1^2 + \frac{3}{2}a^2\left(\delta\phi_2^2 + 2\delta\phi_1\delta\phi_3\right) + \frac{5}{2}a^4\left(\delta\phi_3^2 + 2\delta\phi_1\delta\phi_5 + 2\delta\phi_2\delta\phi_4\right) + \ldots}$$

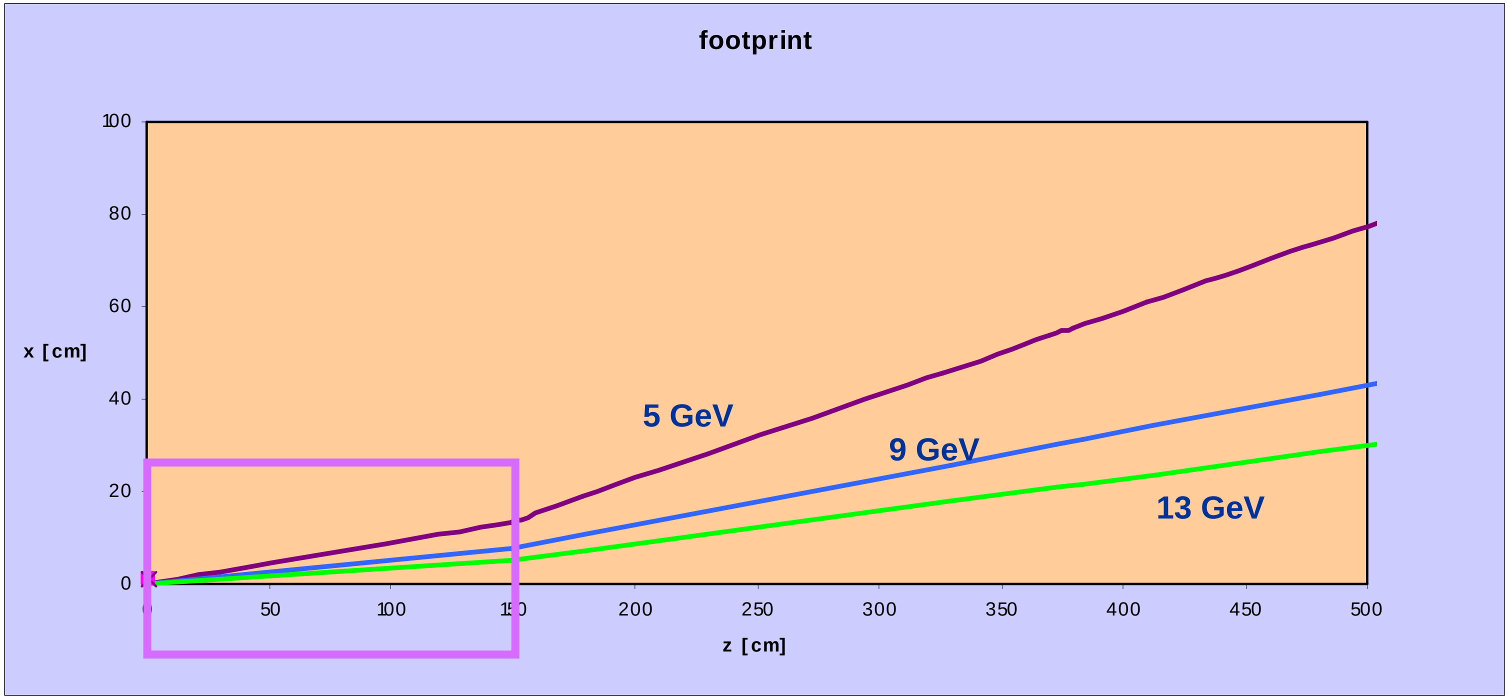
15 GeV Dogbone RLA (6.5 pass)

energy ratio:

$$\frac{E_f}{E_0} = 7.5$$



Beam Separation – Switchyard



Summary

- **'Dogbone'** RLA – preferred configuration
  - better orbir separation for higher passes
  - offers symmetric solution for simultaneous acceleration of μ^+ and μ^-
- **FODO** lattice more favorable (compared to the triplet) to accommodate large number of passes
  - uniform phase advance decrease in both planes
  - smaller variation of Twiss function – easier match to the Arcs
- Proposed **12.6 GeV two-step-dogbone RLA** (30 mm rad acceptance)
- **3.6 GeV Dogbone I** (4.5 pass) – error sensitivity studies
  - Magnet misalignment error analysis (DIMAD Monte Carlo on the above lattice) shows quite manageable level of orbit distortion for ~1 mm level of magnet misalignment error).
  - Great focusing errors tolerance for the presented lattice – 10% of Arc-to-Arc betatron mismatch limit sets the quadrupole field spec at 0.1%
- Aggressive **15 GeV dogbone RLA**