Optimizing the Pion Capture and Decay Channel

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Overview:

- FS1 graphite target with 1 MW, 16 GeV proton source...

  ...80 cm long, 1.5 cm diameter, 100 mrad tilt angle

  ...Conservative approach

  ...Channel design applicable to other targets

  ...No optimization of target

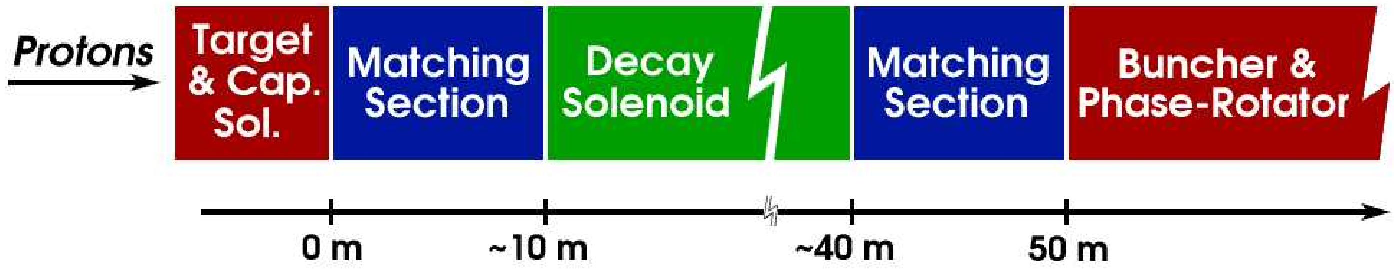
- Initial objective...

  ...Find properties that contribute to maximum yield

  ...Find best performance-to-cost ratio

  ...Determine the best *approach* to channel design

Principles of Channel Design:



- Pion capture and containment...

  ...Target region and initial matching section

  ...Focusing and transport with solenoids

  ...Take pions from high-field region at target (20 T) to low-field decay channel (1.25 T) *adiabatically*

- Muon capture and containment...

  ...Decay channel and final adiabatic section

  ..."All" muon beam by end of channel

  ...Final matching section may be unnecessary

Pion Capture & Containment:

- Capture solenoid...

  ...About 1 m long with $B_0 = 20$ T peak field on axis

  ...Captures pions in a $R_0 = 7.5$ cm radius beampipe with

$$p_T < e\,B_0\left(\frac{R_0}{2}\right) = 225 \text{ MeV/c}$$

- Adiabatic matching section...

  ...Tapered solenoid with constant magnetic flux

$$\Phi_0 \propto B_0\,R_0^2 = B(s)\,R(s)^2$$

  ...Design $R(s)$ and determine on-axis field

$$B(s) = B_0\left(\frac{R_0}{R(s)}\right)^2$$

Adiabaticity:

- Change field slowly...

...Minimize curvature of field lines, $|\kappa_B|$

...Minimize normalized field gradient, $|\frac{1}{B}\frac{\partial B}{\partial s}|$

...Conditions for adiabaticity:

$$a \ll \left(\frac{1}{B}\frac{\partial B}{\partial s}\right)^{-1} = \left(\frac{2}{R}\frac{\partial R}{\partial s}\right)^{-1}$$

and

$$a \ll |\kappa_B|^{-1} \equiv R_B$$

where a is the Larmor radius of the particle's orbit and R_B is the radius of curvature of the field lines

...Particle motion described by adiabatic invariants:

$$Ba^2 \quad \text{and} \quad \frac{p_T^2}{B}$$

Designing Matching Sections:

- Our demands...

...Initial and final radii fixed by conservation of flux:

$$R(s_1) = R_1 = 7.5 \text{ cm}$$

and

$$R(s_2) = R_2 = 30 \text{ cm}$$

...Initial and final direction of field fixed:

$$\frac{\partial R}{\partial s}(s_1) = \lambda \quad \text{and} \quad \frac{\partial R}{\partial s}(s_2) = 0$$

NOTE: $\lambda \neq 0$ is "unphysical" and *not* easy to fit!

...Radius function, $R(s)$, should be monotonic

Designing Matching Sections (continued):

- Previous designs...

  ...Simplistic choice for radius function:

$$R(s) = \sqrt{\alpha_0 + \alpha_1 s}$$

  where $\alpha_0 = R_1^2 - (R_2^2 - R_1^2)\left(\frac{s_1}{s_2 - s_1}\right)$ and $\alpha_1 = \left(\frac{R_2^2 - R_1^2}{s_2 - s_1}\right)$

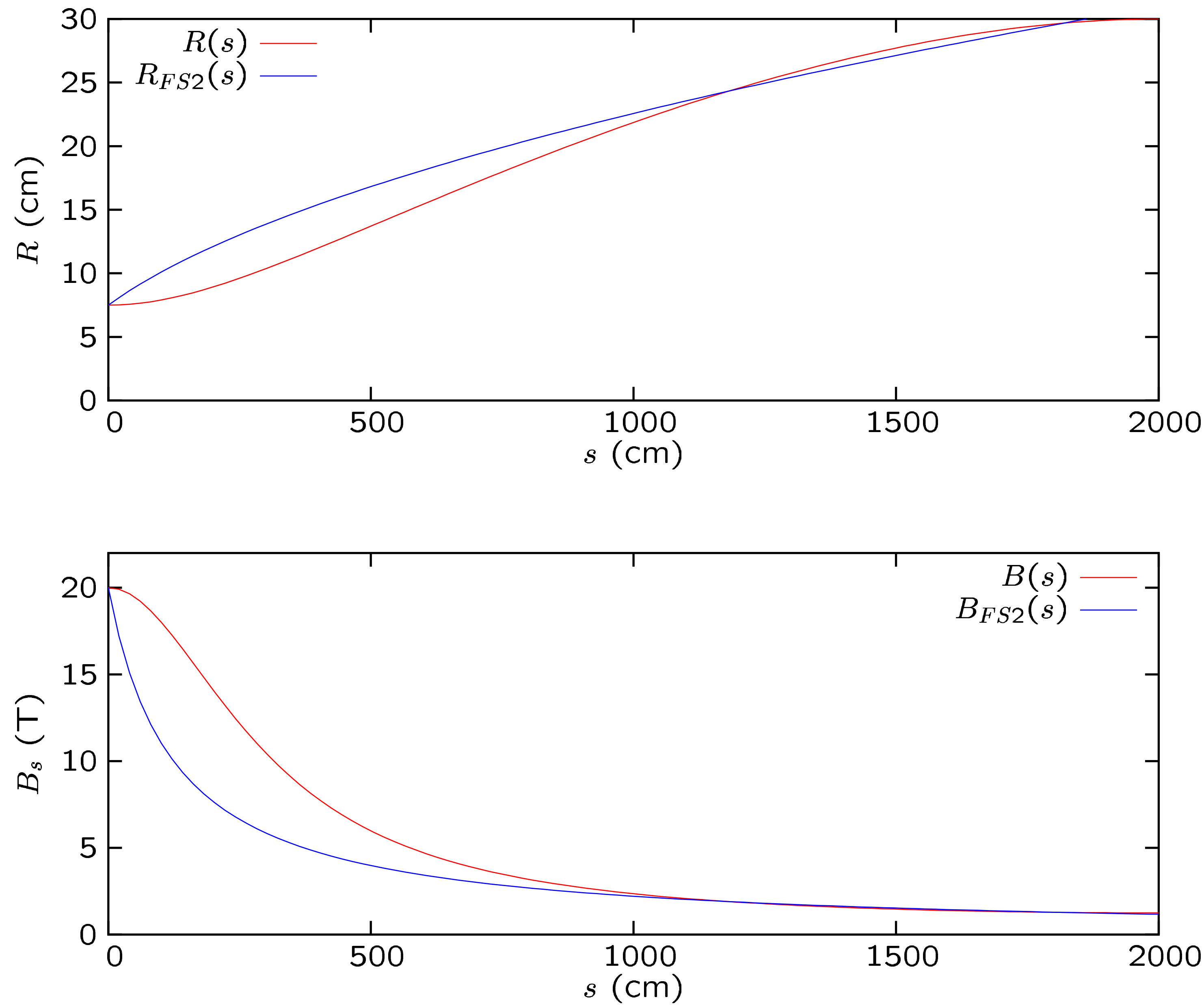
  ...Provides only one parameter to optimize $(s_2 - s_1)$

- New design...

  ...Generalized choice for radius function:

$$R(s) = \left(\alpha_0 + \alpha_1 s + \alpha_2 s^2 + \alpha_3 s^3\right)^{\frac{1}{k}}$$

  ...Provides three parameters to optimize $(s_2 - s_1,\ k,\ \lambda)$



30
25
20
15
10
5
0
$R(s)$
$R_{FS2}(s)$
R (cm)
0
500
1000
1500
2000
s (cm)
20
15
10
5
0
$B(s)$
$B_{FS2}(s)$
B_s (T)
0
500
1000
1500
2000
s (cm)

Muon Capture & Containment:

- Decay solenoid...

  ...About 40 m long with $B = 1.25$ T uniform field

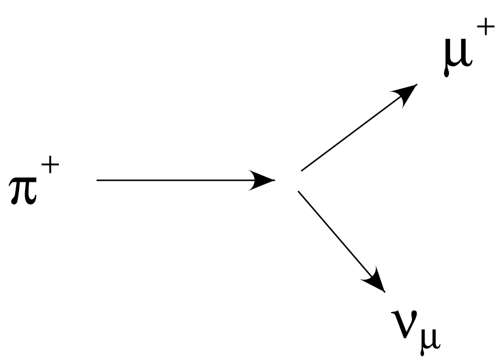
  ...Contains pions in a $R = 30$ cm radius beampipe with

$$p_T < e\,B\left(\frac{R}{2}\right) = 56 \text{ MeV/c}$$

- Pion decay...

  ...Acts like an extended target to the beam!

  ...Gives a transverse momentum kick to the muon



$$\begin{aligned}\langle p_T \rangle &\sim \frac{1}{2}\,m_\pi c\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\langle \sin^2\theta \rangle \\ &= \frac{1}{4}\,m_\pi c\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \\ &\approx 15 \text{ MeV/c}\end{aligned}$$

Muon Capture & Containment (continued):

- Muon capture...

...Average p_T kick is independent of B

...Increasing B should improve muon capture

...With BR^2 constant:

$$B = 1.25\text{ T}, 2.00\text{ T}, 3.00\text{ T}$$

and

$$R = 30\text{ cm}, 23.7\text{ cm}, 19.4\text{ cm}$$

...Requires a final adiabatic section to take muon beam down to 1.25 T

Optimization:

- Tuneable parameters...

    ...Length of adiabatic section: $s_2 - s_1$

    ...Slope of field lines at upstream end: λ

    ...Radius function exponent parameter: k

    ...Field strength of decay channel: B

- Expectations...

    ...Optimal length is as long as possible!

    ...Optimal field strength is as strong as possible!

    ...Should λ be small? How small?

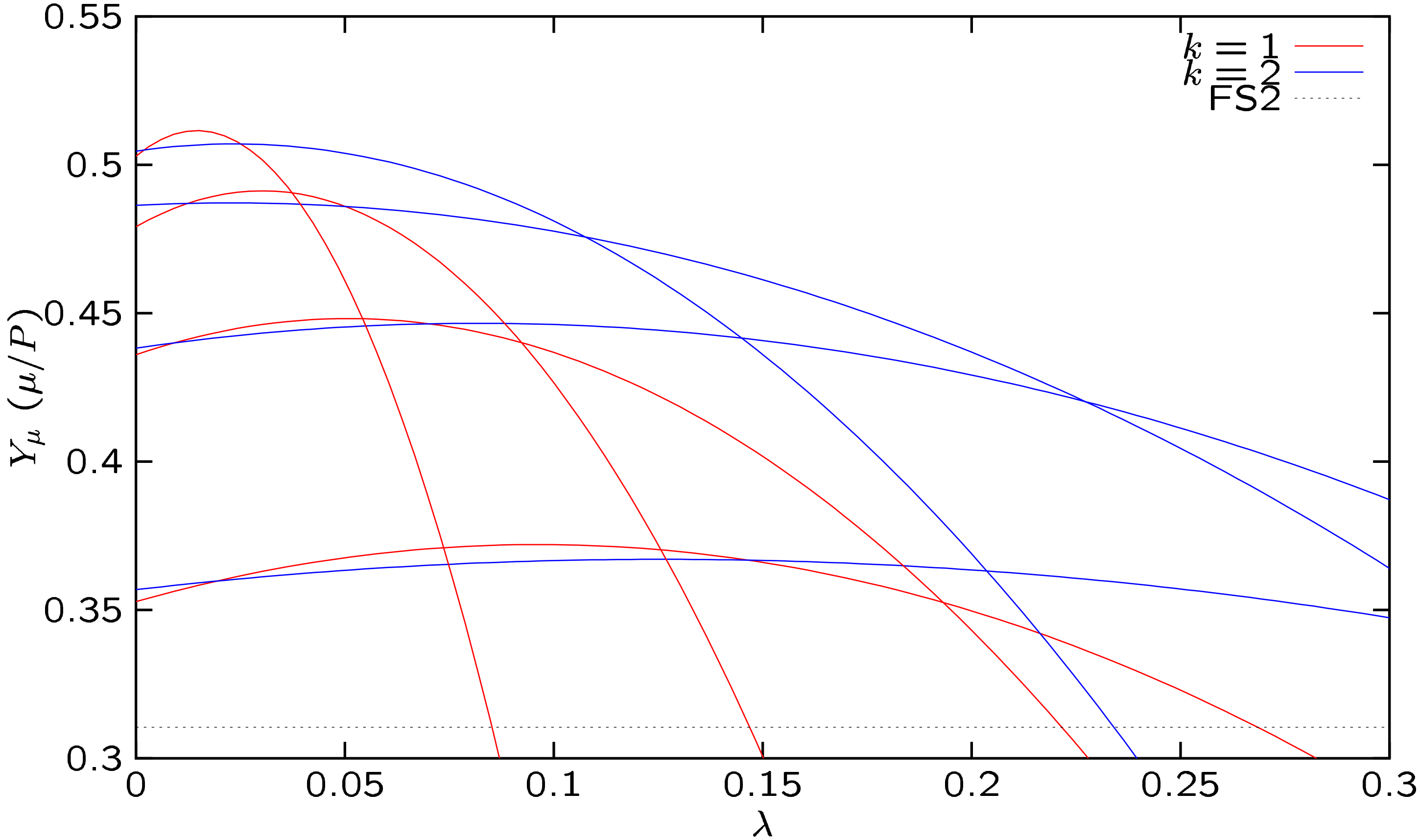
    ...How does the yield depend on k?

Analysis:

- For $B = 1.25$ T, consider:

  $s_2 - s_1 = 250, 500, 1000, 2000$ cm

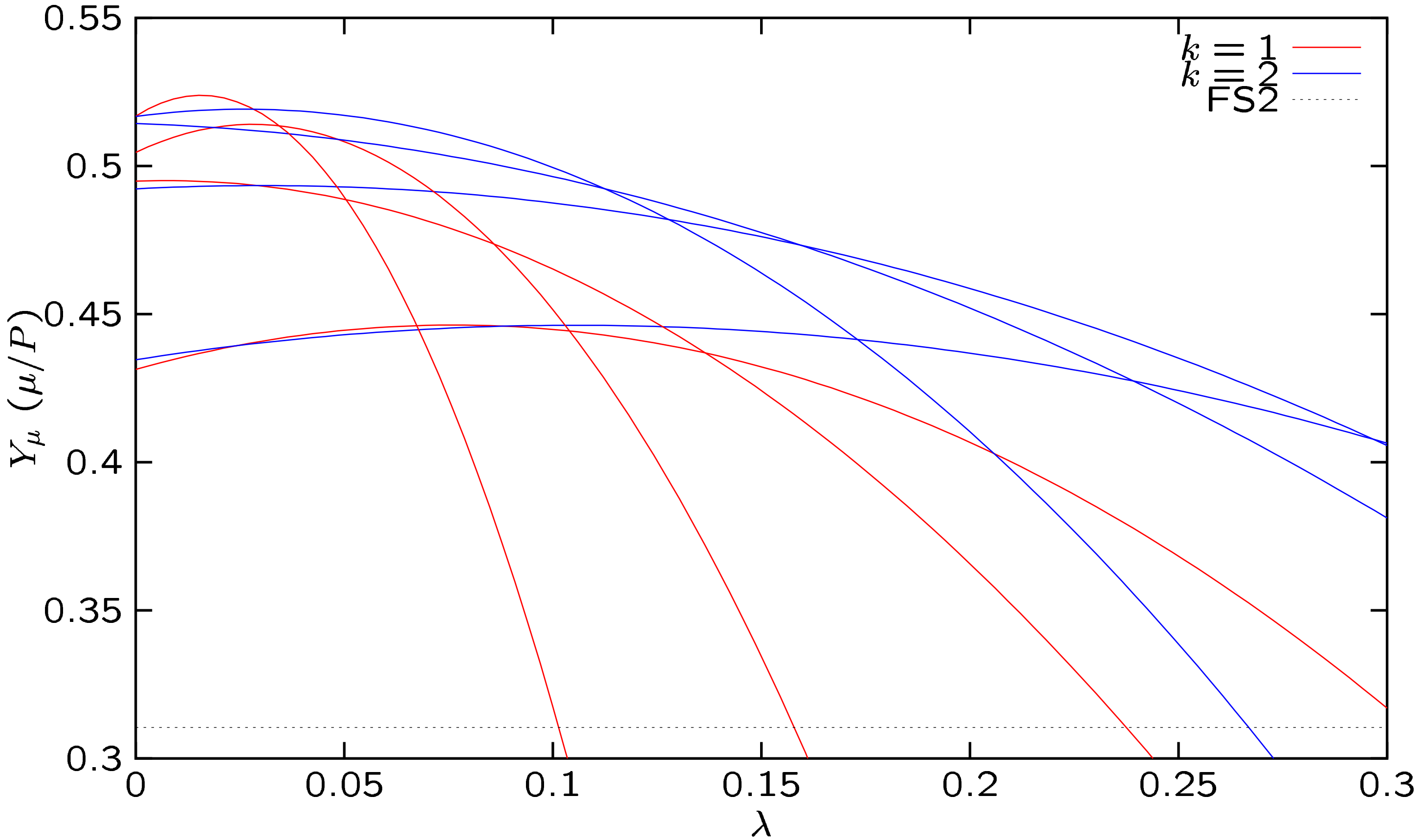
  $\lambda = 0 \rightarrow 0.30$ and $k = 1, 2$



- For $B = 2.00$ T, consider:

  $s_2 - s_1 = 250, 500, 1000, 2000$ cm

  $\lambda = 0 \rightarrow 0.30$ and $k = 1, 2$



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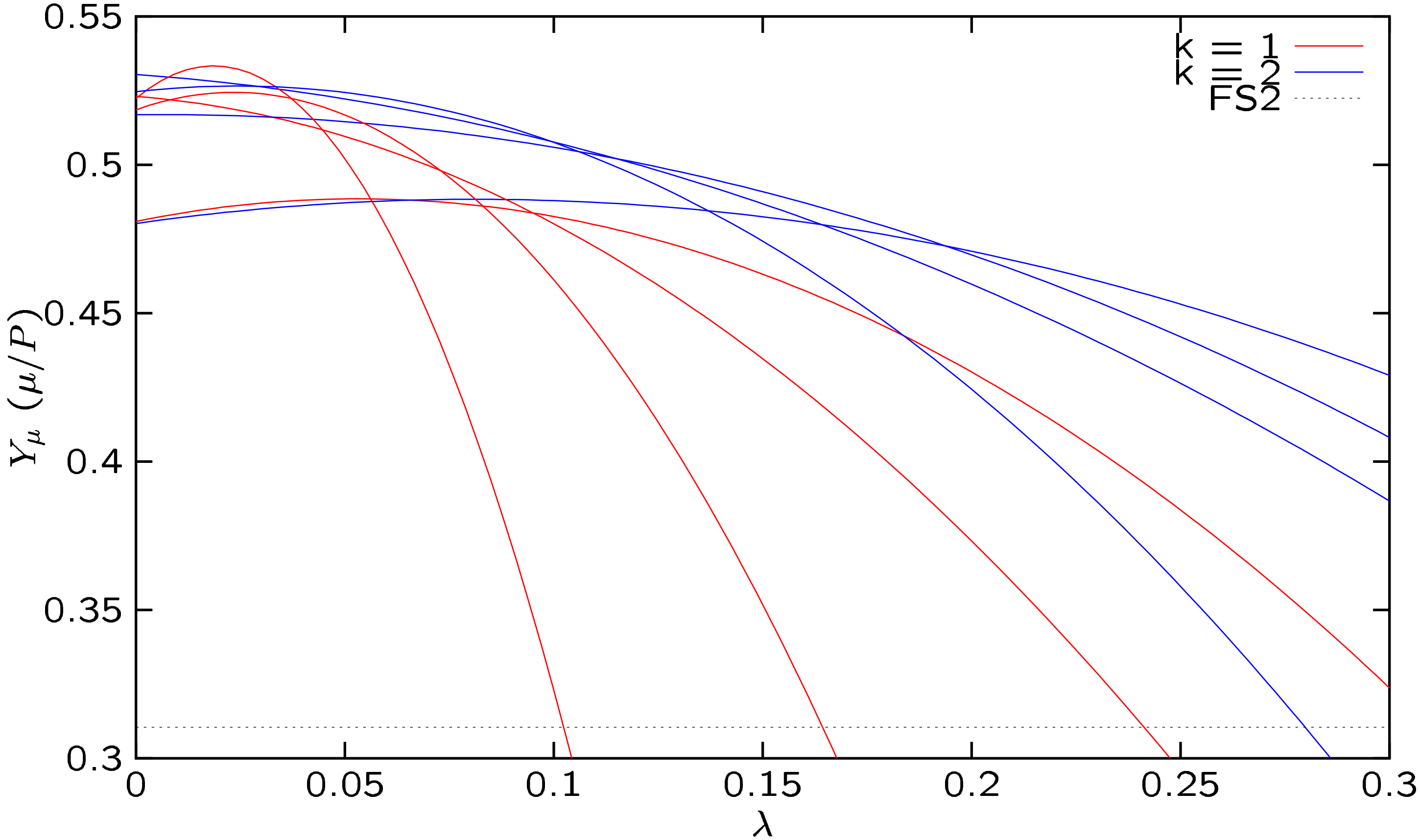

Performance-to-cost Ratio:

- Define a merit factor:

$$f = \frac{Y_\mu}{W}$$

where W is the energy stored in the magnetic field

- For FS2:

$$f_{FS2} = \frac{1}{1.9}\left(\frac{0.59\ \mu/P}{15\ \mathrm{MJ}}\right) = 0.021\ \mu/P\,\mathrm{MJ}^{-1}$$

- Normalized merit factor:

$$f/f_{FS2}$$

should roughly measure comparable performance-to-cost ratio

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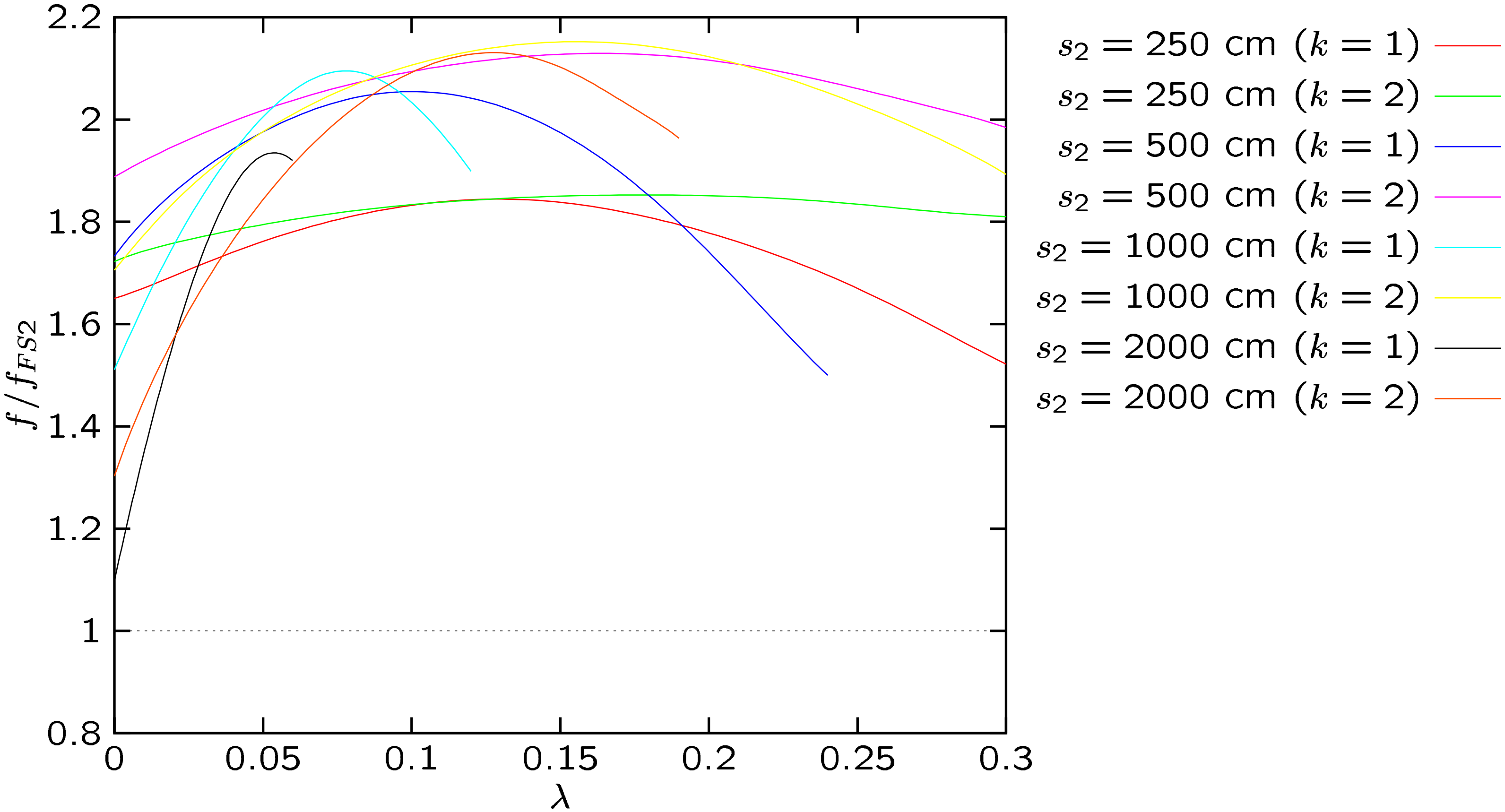

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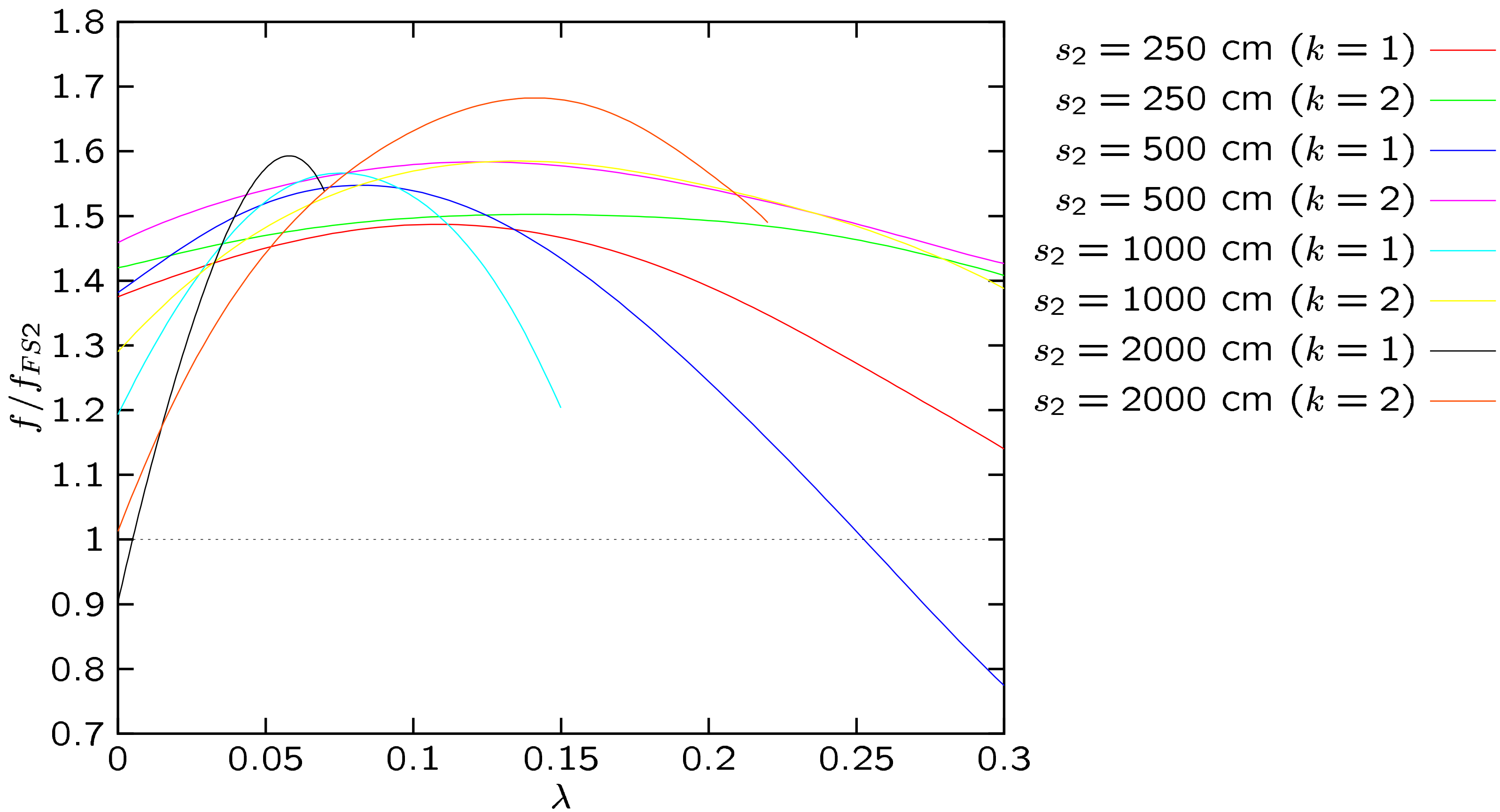

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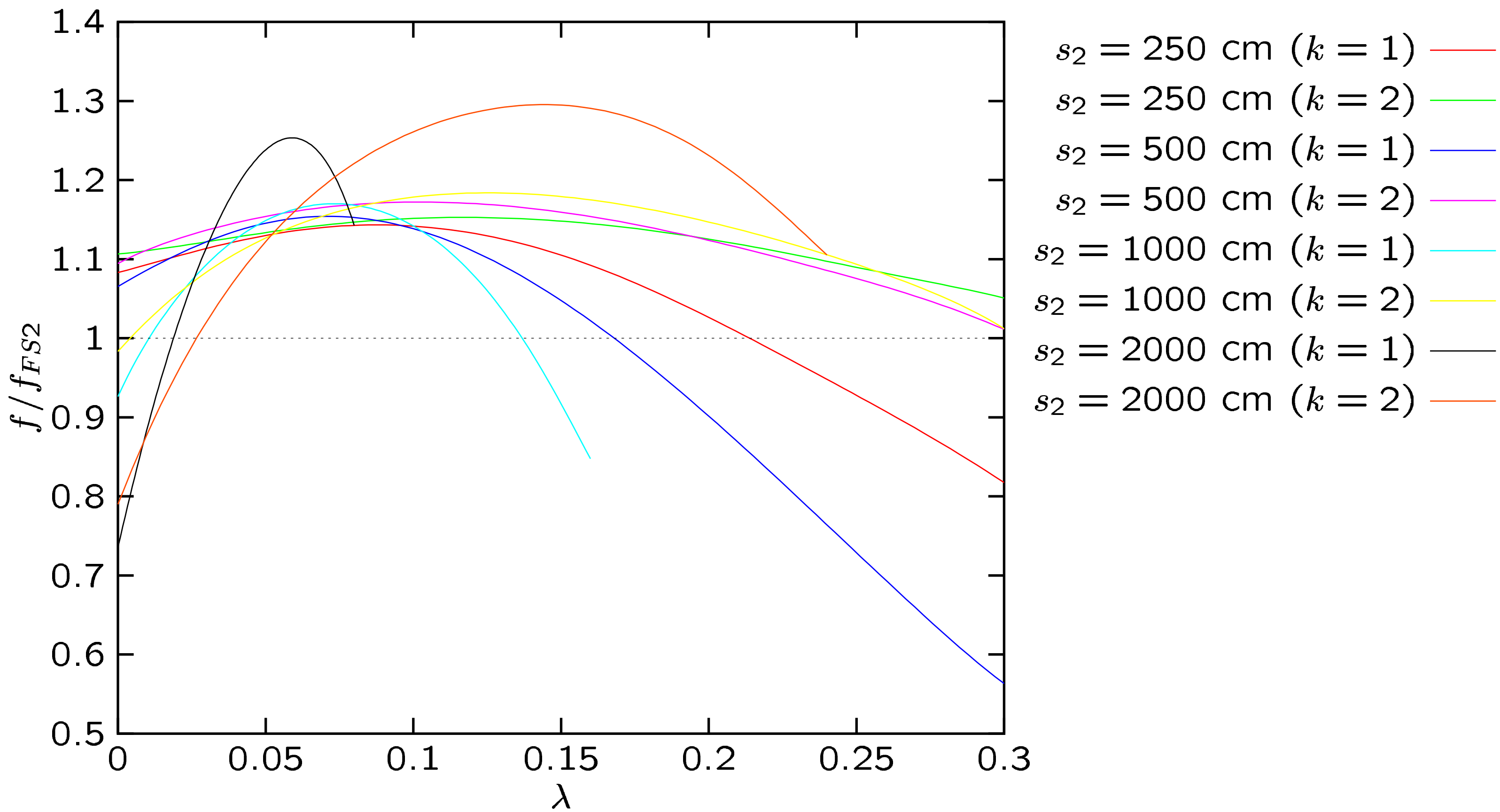

Optimal designs:

- Performance-based only:

  ...$k = 2$ and $\lambda = 0$

  ...$B = 2.00$ T

  ...$s_2 - s_1 = 2000$ cm

  ...$Y_\mu \approx 0.53$ (*Verified!*)

  ...$\approx 67\%$ more expensive

- Performance-to-cost ratio:

  ...$k = 2$ and $\lambda \approx 0.10$

  ...$B = 1.25$ T

  ...$s_2 - s_1 = 1000$ cm

  ...$Y_\mu \approx 0.47$ (*Unverified!*)

  ...$\approx 30\%$ less expensive