Straight Quadruopole Cooling Channel Simulations

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The Six Dimensional Emittance

Six-dimensional phase space vector $\vec{r} = (q_1, p_1, q_2, p_2, q_3, p_3)$. Define covariance matrix M with matrix elements

$$M_{i,j} = \sum_{\nu=1}^{N} \frac{r_i \cdot r_j}{N} - \left(\sum_{\nu=1}^{N} \frac{r_i}{N} \right) \left(\sum_{\nu=1}^{N} \frac{r_j}{N} \right)$$

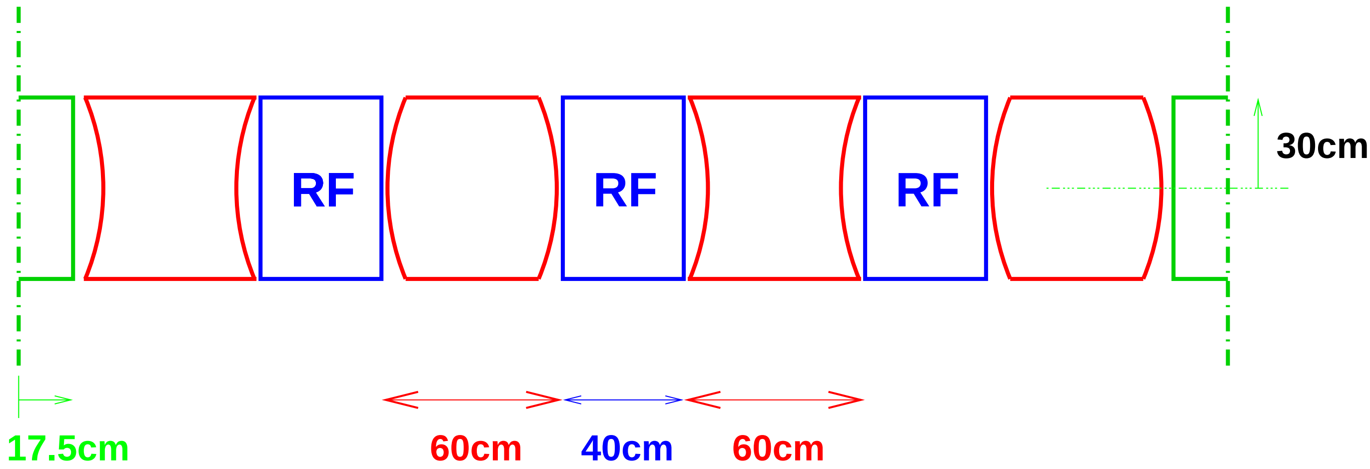
Then the 6D emittance is given as

$$\varepsilon = \sqrt{\det(M)}$$

Several comments are necessary.

- Particles have to be **outside magnetic field** (or otherwise first calculate their canonical momentum)
- Often it is appropriate to first execute **cuts**, i.e. discard particles above certain thresholds
- Reassigning of longitudinal coordinates to **proper bucket** based on particular frequency
- Careful balancing of cuts helps maximize **figure of merit**, which is based on combination of emittance and transmission

Quad Cooling Cell (4m Cell)



- Incoming Muons: 180 MeV/c to 245 MeV/c
- Magnetic Quadrupoles: k=2.88
- 35cm Liquid H Absorber: Energy loss $\approx$ 12 MeV. The same design as Study II 2.75m sFOFO cell.
- RF Cavity: Energy gain to compensate the loss. About 200 MHz, $\phi = 30°$.

Table 2. Catalog of the important optical functions for the quadrupole cooling cell as a function of momentum.

P (MeV/c)	β at absorber (m)	ν per cell (x2π rad)	β max (m)	β min (m)
155	1.57 m	.33	3.85	0.35
200	1.63	.23	3.13	0.71
245	1.91 m	.18	3.20	1.10
300	2.28 m	.15	3.45	1.37

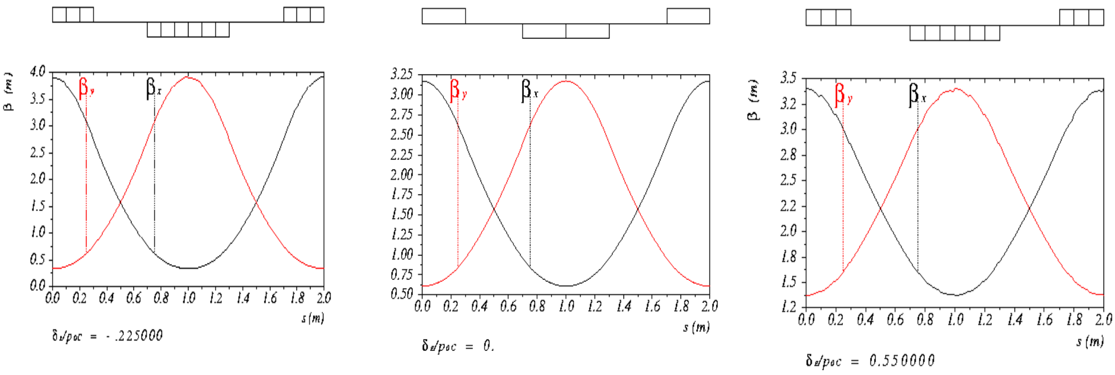


Figure 4. Plots of the beta functions for the quadrupole cooling cell at p = 155 MeV/c, 200 MeV/c and 300 MeV/c.

COSY Simulation of the Quad Channel

- **Transfer Map** Method using Differential Algebra (DA)
- **Realistic Field** Treatment using DA PDE Solver for Fringe Fields
- **Straggling:** Bethe-Bloch Formula with Vavilov Monte Carlo Kicks
- **Scattering:** Gaussian with Monte Carlo Kicks

COSY's 6D emittance tool is fully compatible with **ECALC9F** (Fernow, Penn et al.), but completely re-written in COSY language for compatibility and efficiency.

Particles from Buncher, Phase Rotator

Particle distributions provided by D. Elvira:

“The files **G4Buncher*.out** and **G4PhaseRot*.out** contain the kinematic information of a muon beam at the end of the a GEANT4 simulation of the "Neuffer" buncher and phase rotator, respectively. The input beam to the buncher was originally generated with MARS, and had previously gone through 99m of a drift section.

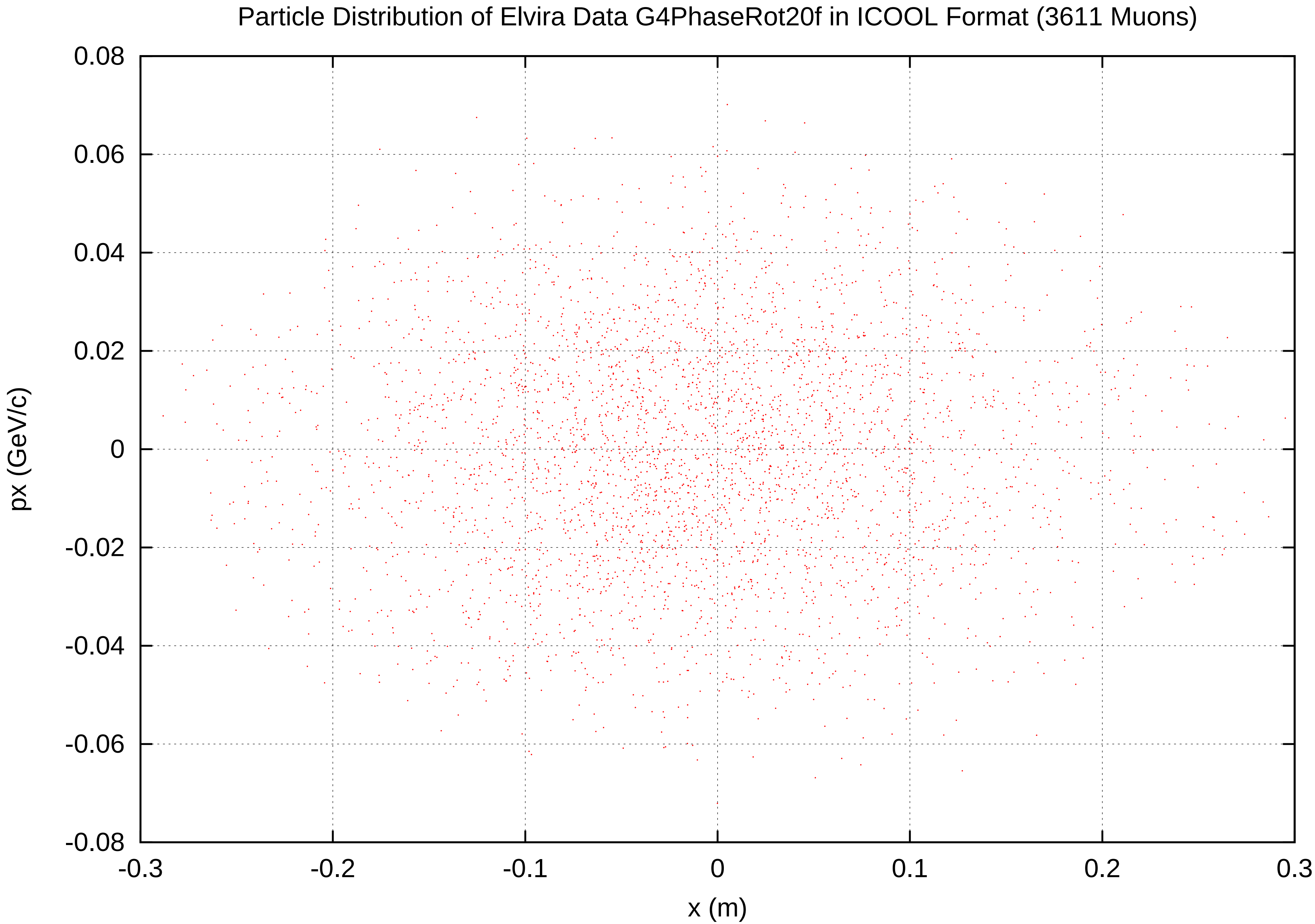
The files are NOW IN ICOOL FORMAT.

G4PhaseRot20f.out contains the beam out from a 9m phase rotator, which previously went through the 20 freq buncher. The z-location of the beam in the file is 11m, measured from the z-location of the **G4Buncher20f.out** beam. The 11m therefore include 2m of drift (from 58m to 60m) and the 9m of the phase rotator.”

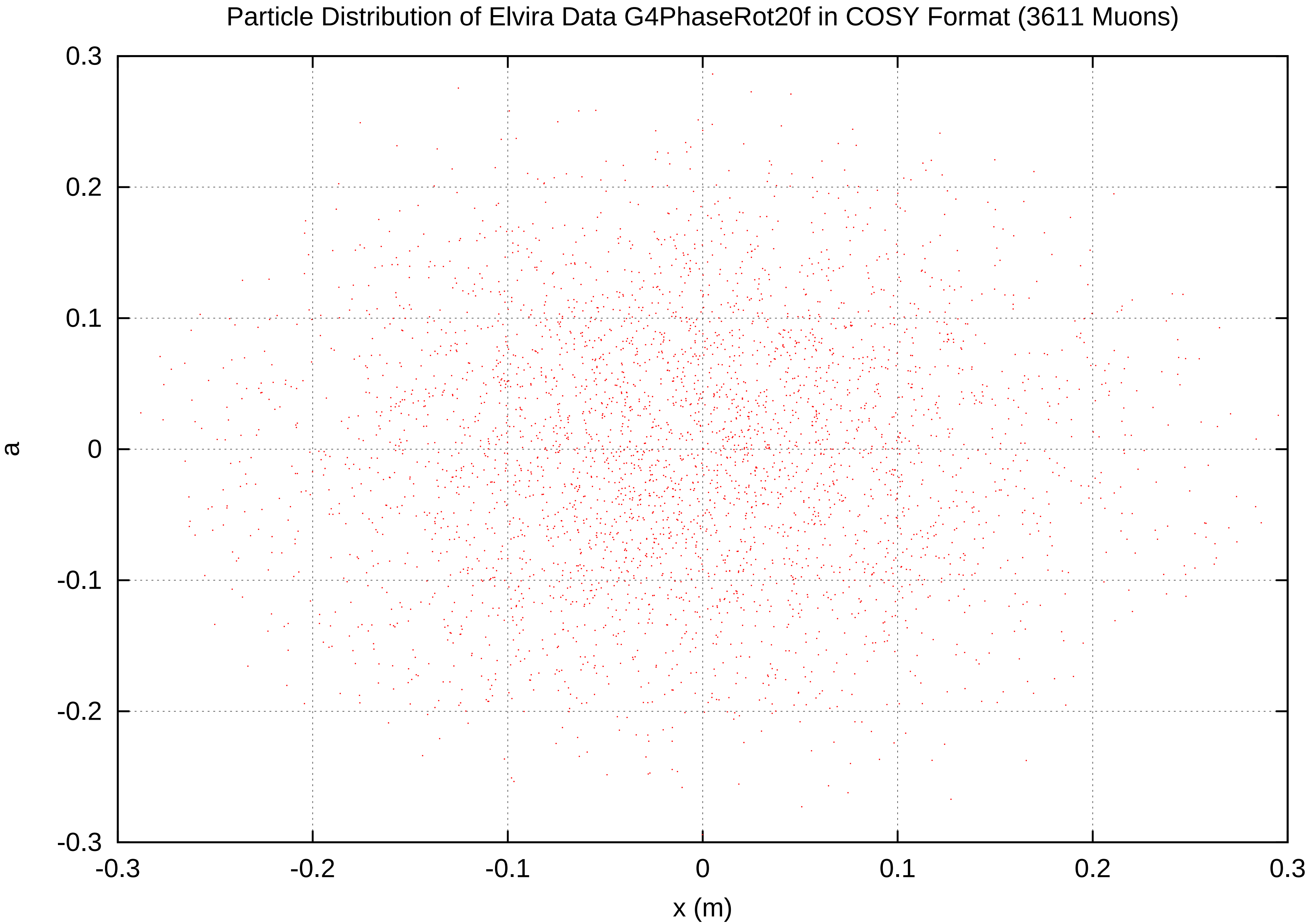
ICOOL Format to COSY Format $(x, a, y, b, l, \delta_K)$

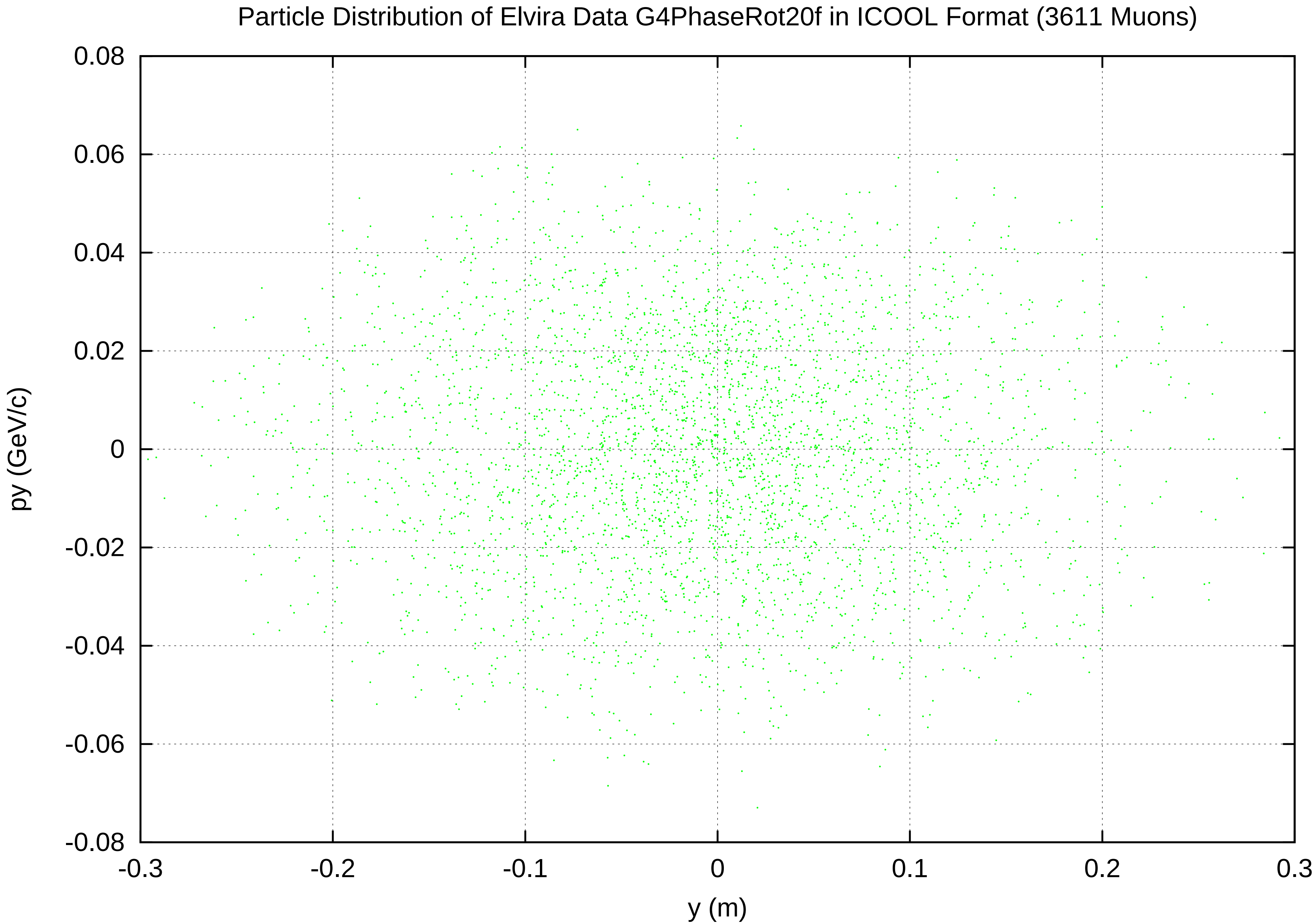
$x, \quad a = p_x/p_0, \quad y, \quad b = p_y/p_0,$

$l = -(t - t_0)v_0\gamma/(1+\gamma), \quad \delta_K = (K - K_0)/K_0.$

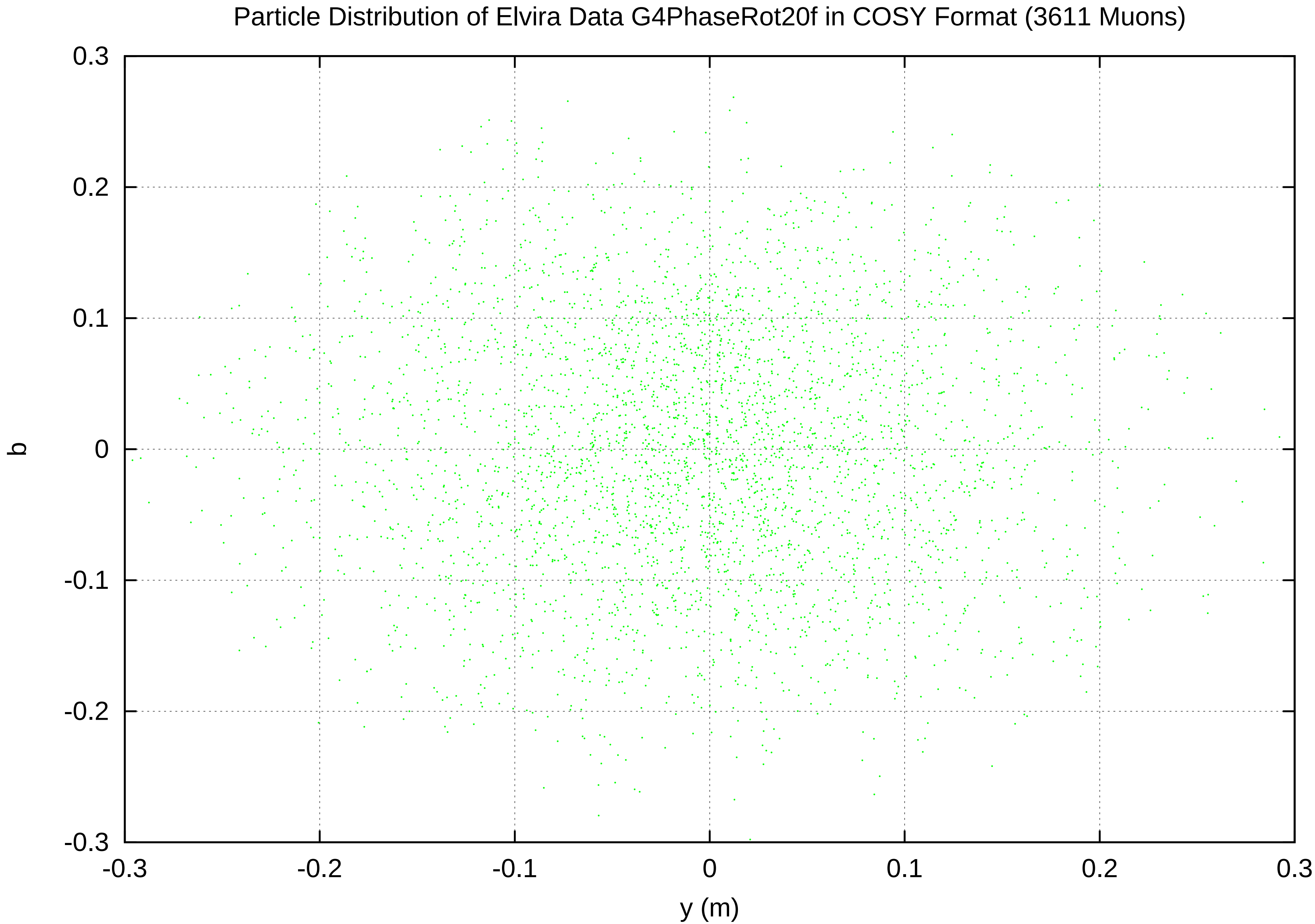


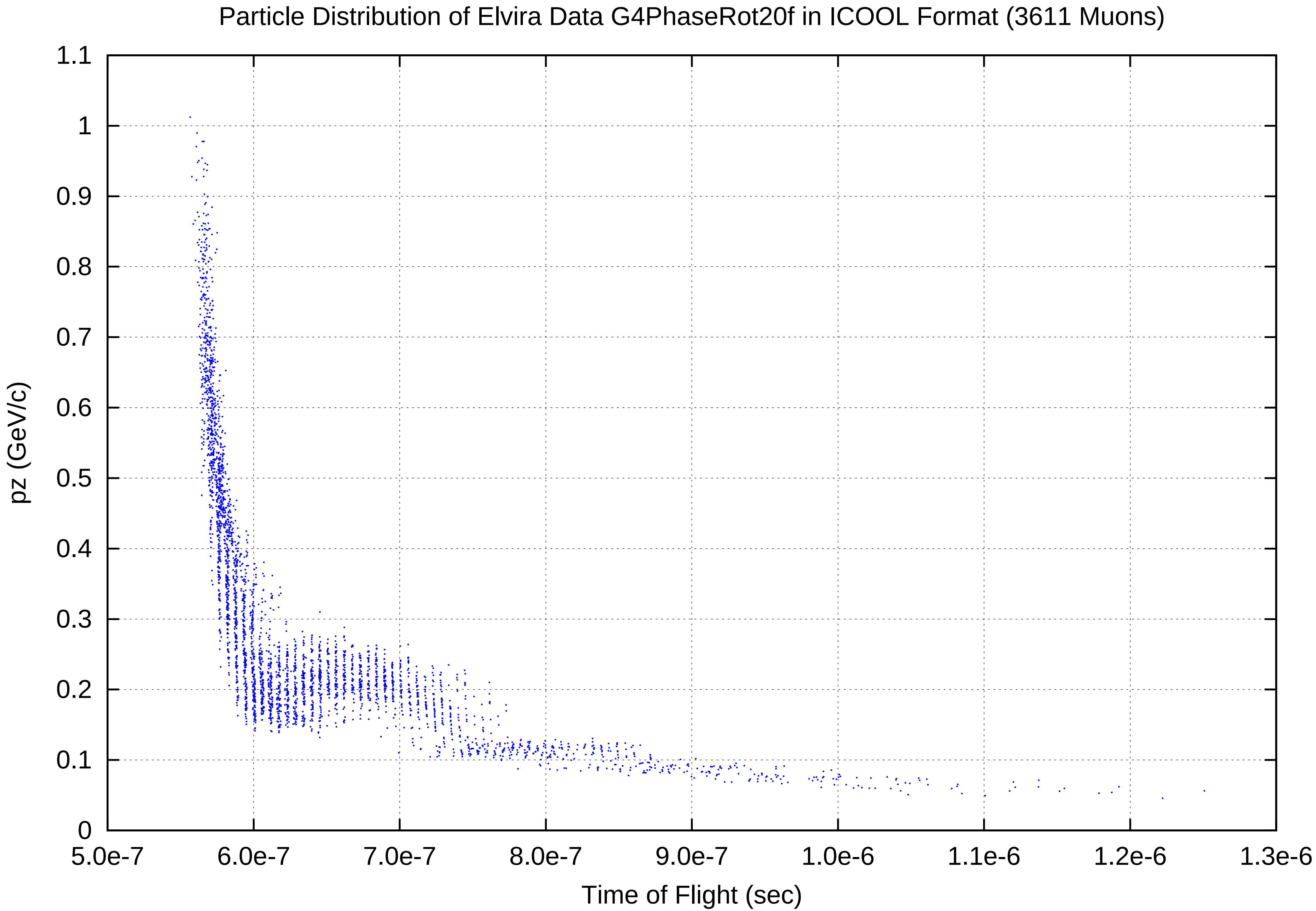
Particle Distribution of Elvira Data G4PhaseRot20f in ICOOL Format (3611 Muons)
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
px (GeV/c)
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
x (m)

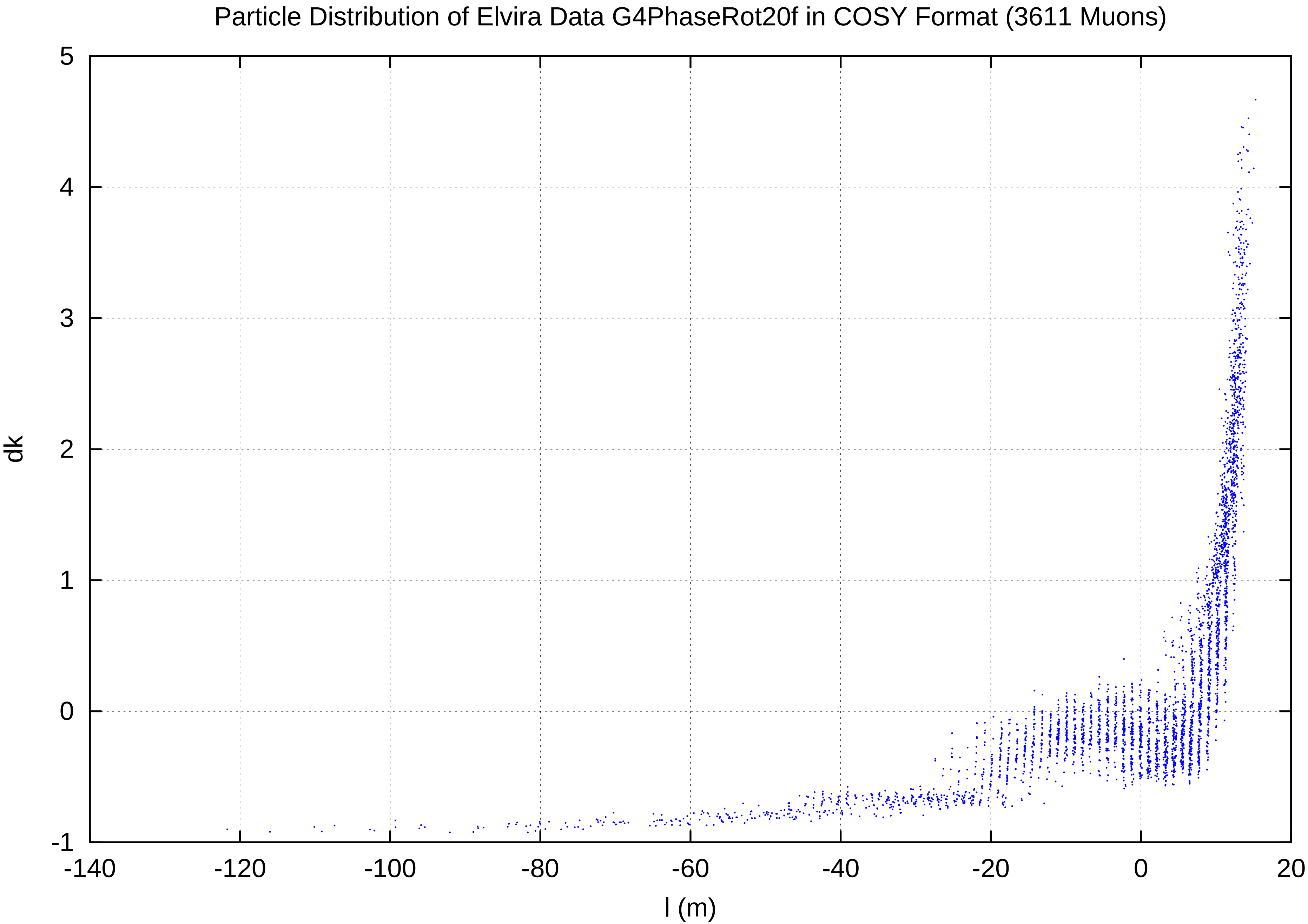

Particle Distribution of Elvira Data G4PhaseRot20f in COSY Format (3611 Muons)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
a
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
x (m)



Particle Distribution of Elvira Data G4PhaseRot20f in ICOOL Format (3611 Muons)
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
py (GeV/c)
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
y (m)


Particle Distribution of Elvira Data G4PhaseRot20f in COSY Format (3611 Muons)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
b
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
y (m)

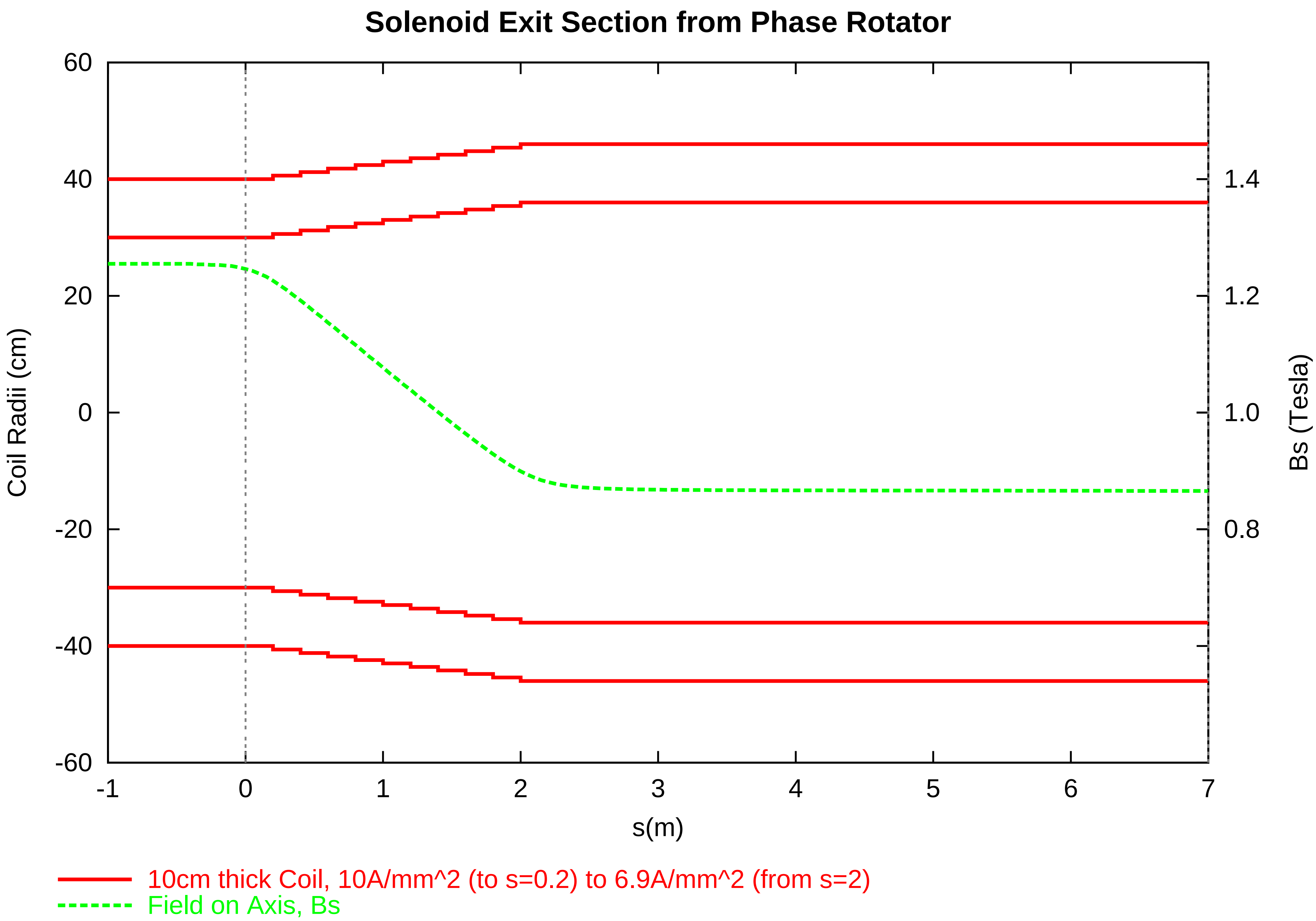

Particle Distribution of Elvira Data G4PhaseRot20f in ICOOL Format (3611 Muons)
pz (GeV/c)
Time of Flight (sec)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
5.0e-7
6.0e-7
7.0e-7
8.0e-7
9.0e-7
1.0e-6
1.1e-6
1.2e-6
1.3e-6


Particle Distribution of Elvira Data G4PhaseRot20f in COSY Format (3611 Muons)
dk
5
4
3
2
1
0
-1
-140
-120
-100
-80
-60
-40
-20
0
20
l (m)

Between the Phase Rotator and the Quad Channel

To transport the given particle distribution at the end of the phase rotator (at 1.25T solenoid) to the beginning of the quad channel cell, we have

- Solenoid (1.25T to 0.86T, 7m long. See below)
- Matching quadrupole (k=$-$8, 10cm long, and 11cm drift)
- Connecting piece of the quad channel to the beginning of the cell (a focusing quadrupole and a half size absorber with Monte Carlo kicks)



Solenoid Exit Section from Phase Rotator
Coil Radii (cm)
60
40
20
0
-20
-40
-60
1.4
1.2
1.0
0.8
Bs (Tesla)
-1
0
1
2
3
4
5
6
7
s(m)
10cm thick Coil, 10A/mm^2 (to s=0.2) to 6.9A/mm^2 (from s=2)
Field on Axis, Bs

Processing the Data for the Transfer Map Treatment

For treatment by transfer maps, the distribution in phase space should be confined so that power series converges quickly.

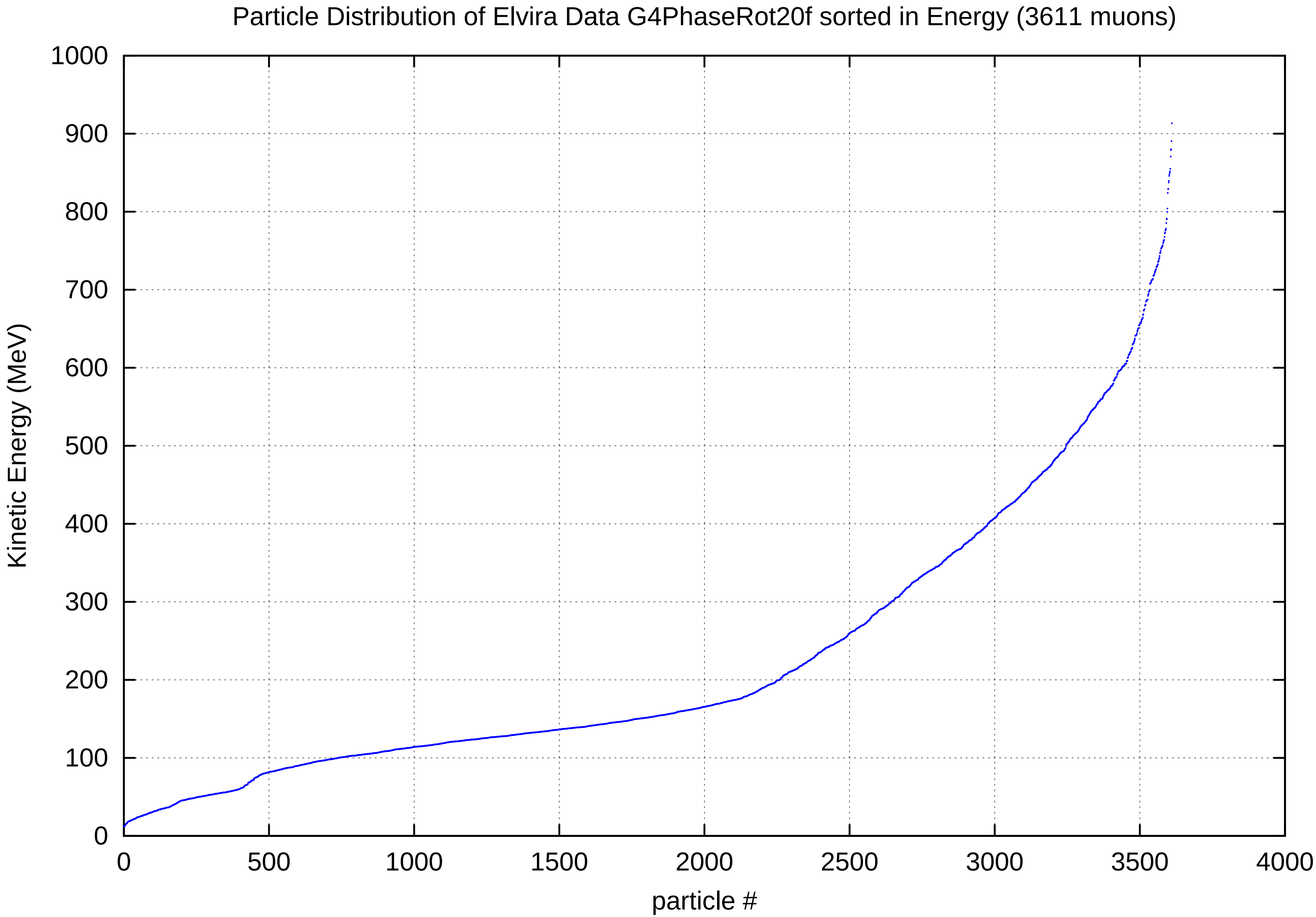
The given distribution is **extremely wide** in l - δ_K, thus this is a challenge for the transfer map method. We

- **Pre-sort** the particles by energy
- **Divide** particles to small groups
- **Keeping** energy and time-of-flight of the particles in each group very close
- Find the precise **frequency** of the system.

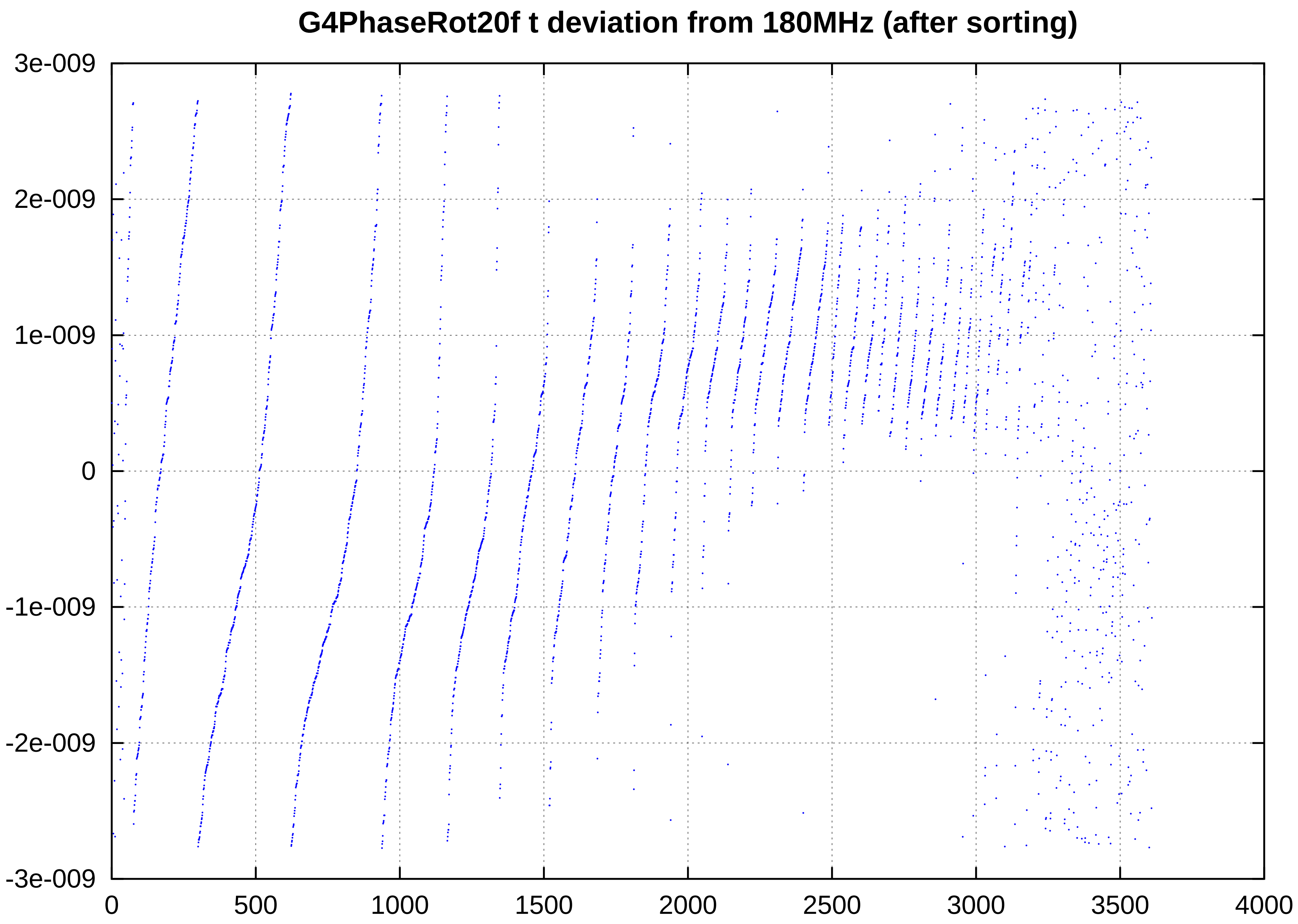
Observe: The bunch width is about 5.5×10^{-9} sec. Thus, for p_0=245 MeV/c, f=179 M Hz.

Choose: The reference energy of a group: the average energy, and reference time-of-flight of a group: 0.

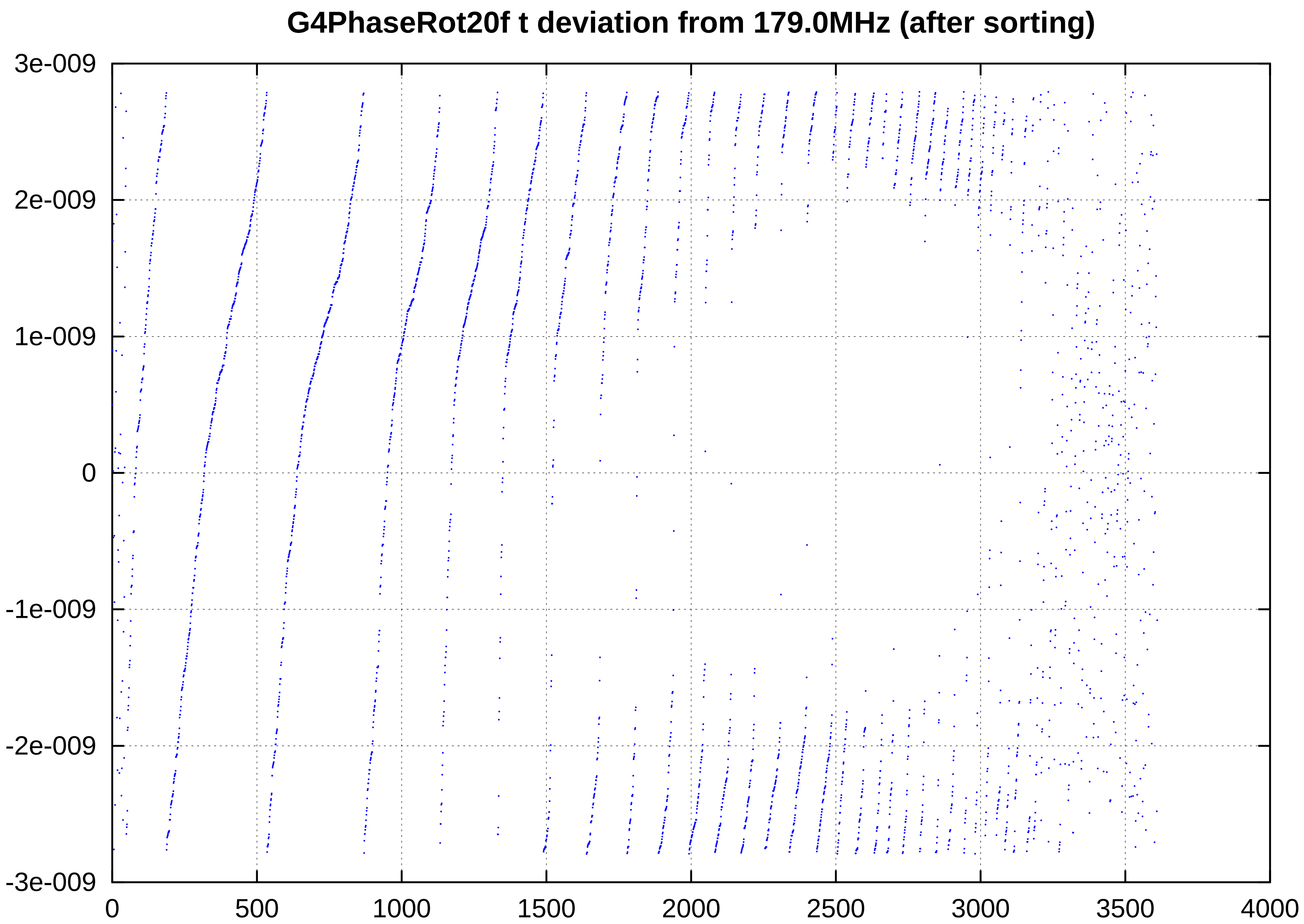
Then, we are **in business** to treat each group in one shot by transfer maps.



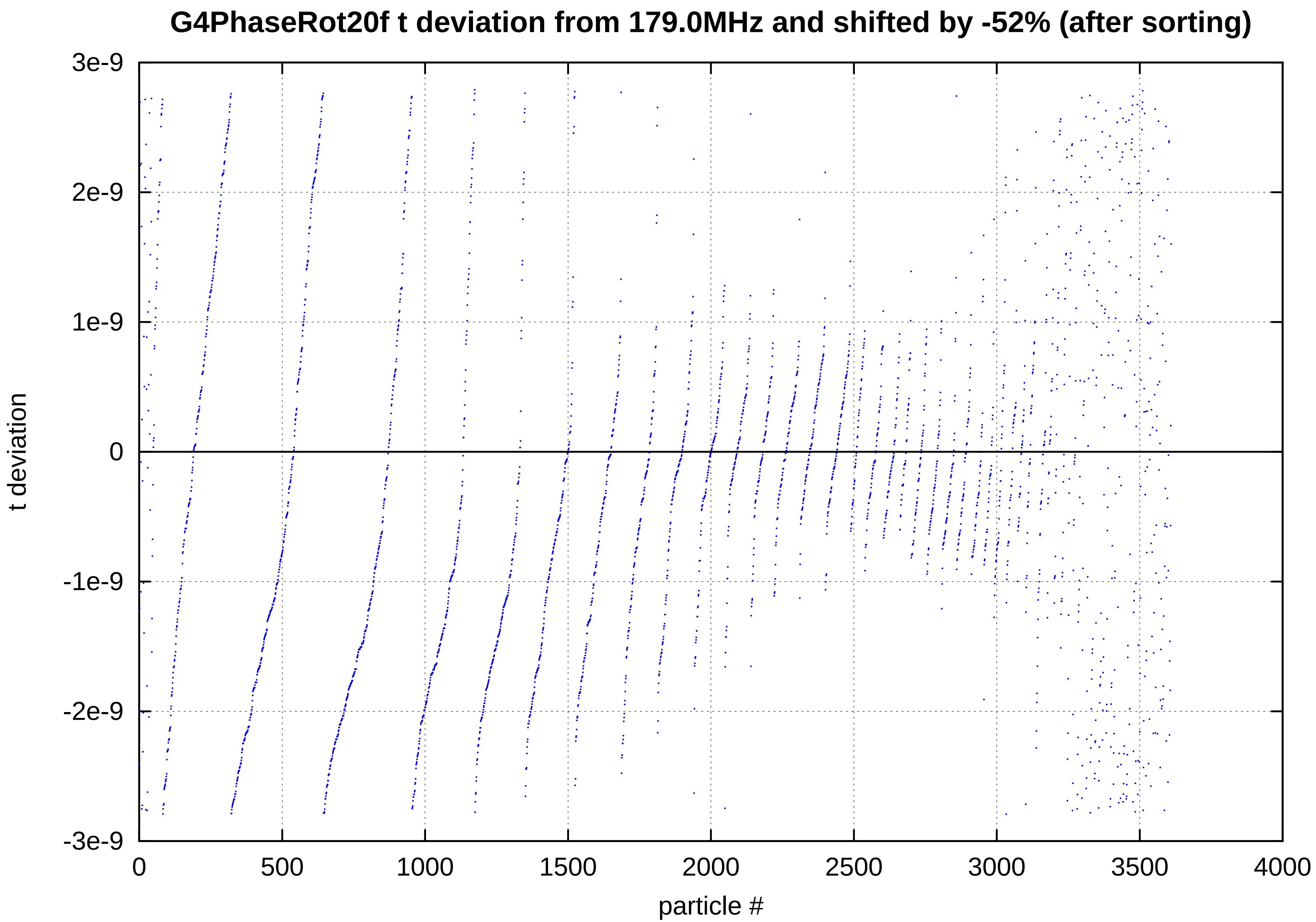
Particle Distribution of Elvira Data G4PhaseRot20f sorted in Energy (3611 muons)
Kinetic Energy (MeV)
particle #
0
100
200
300
400
500
600
700
800
900
1000
0
500
1000
1500
2000
2500
3000
3500
4000

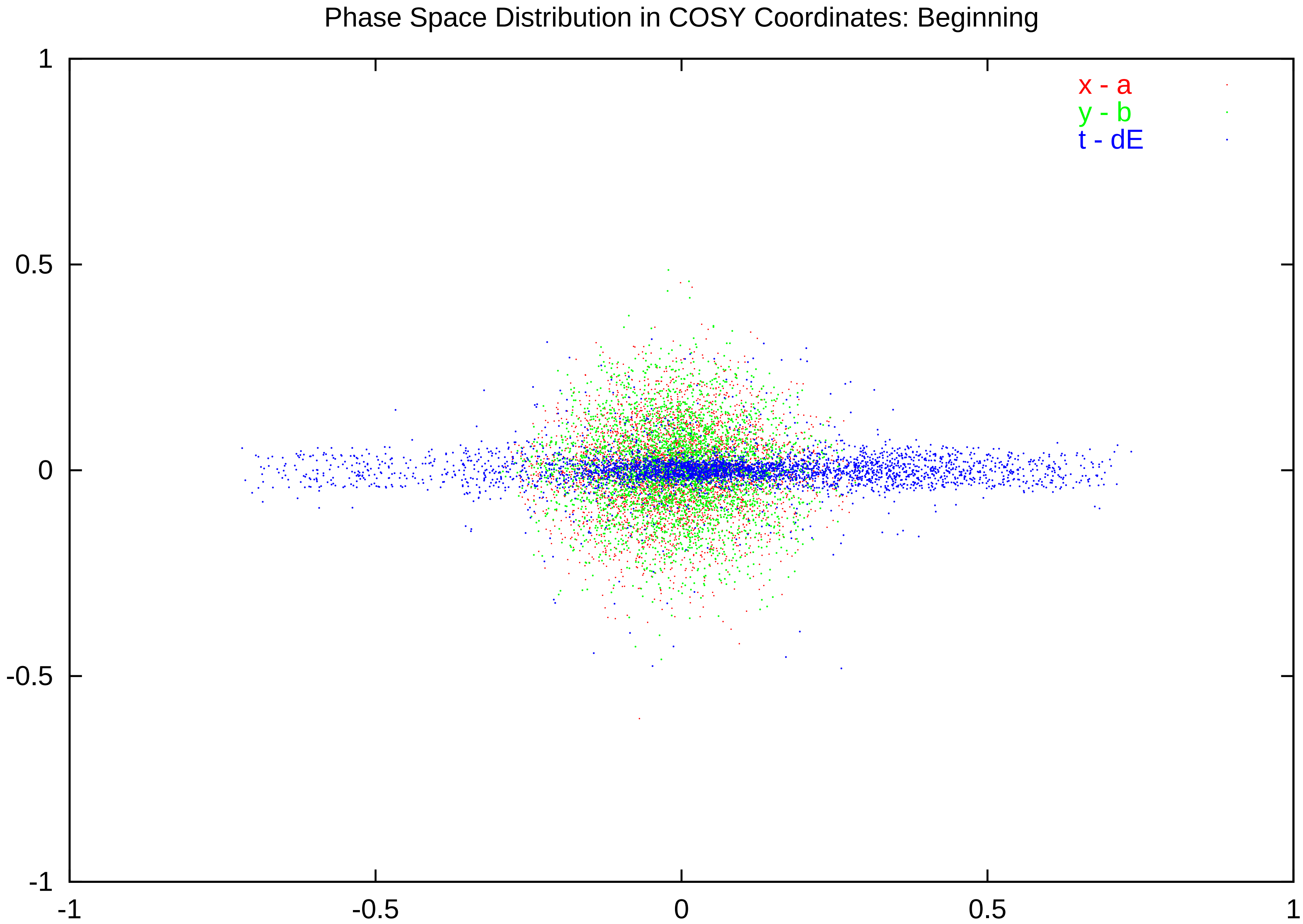


G4PhaseRot20f t deviation from 180MHz (after sorting)
3e-009
2e-009
1e-009
0
-1e-009
-2e-009
-3e-009
0
500
1000
1500
2000
2500
3000
3500
4000

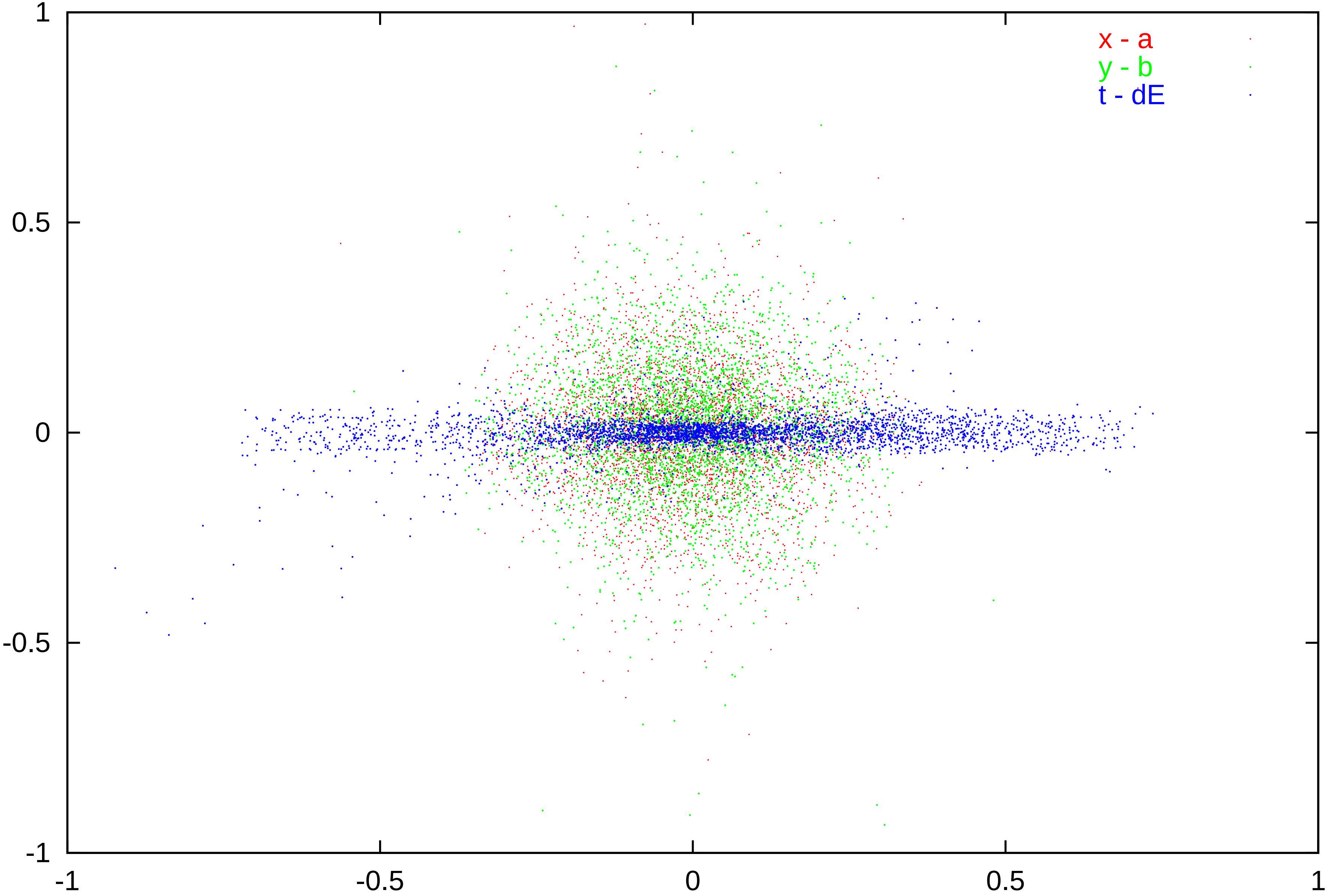


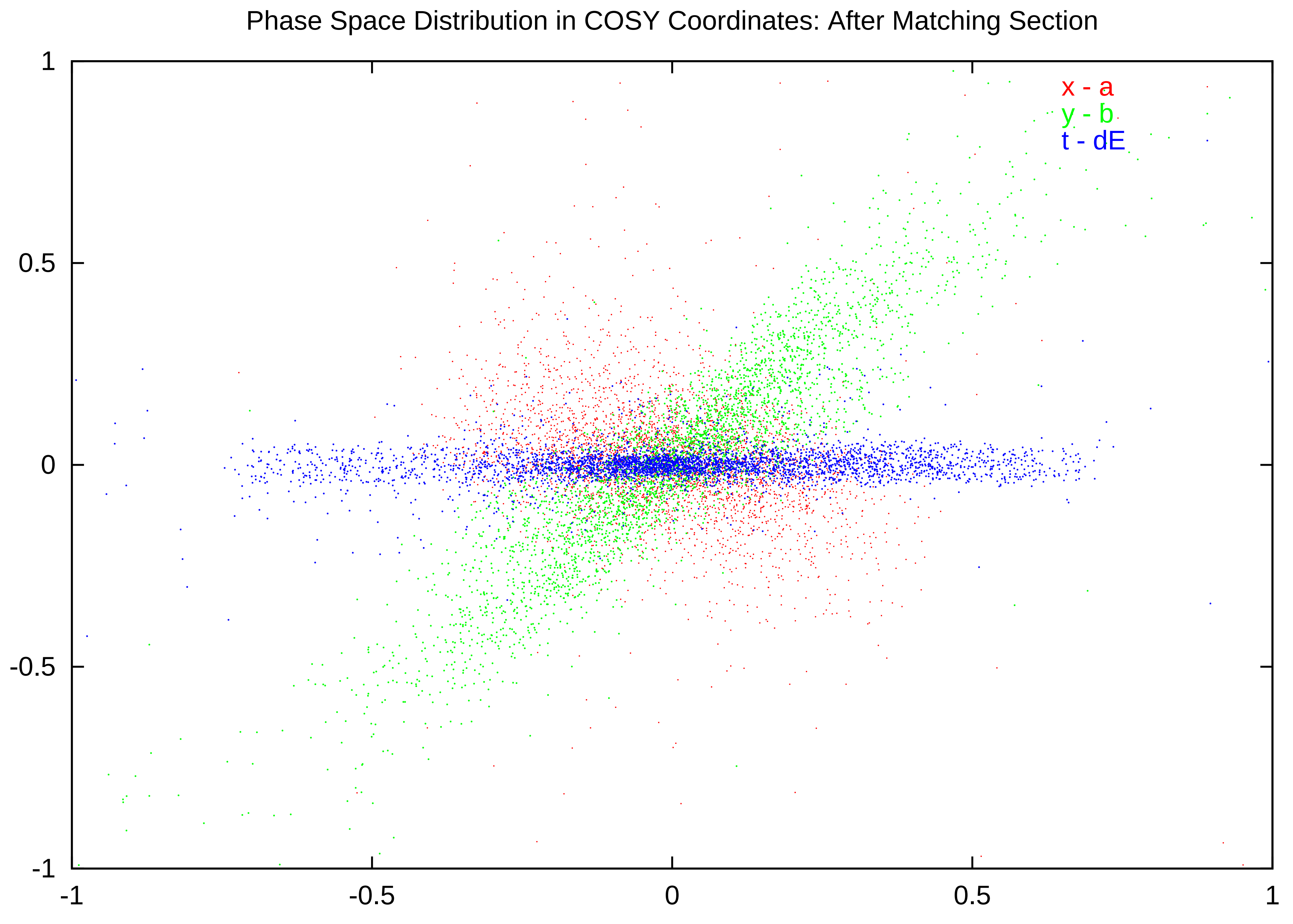
G4PhaseRot20f t deviation from 179.0MHz (after sorting)
3e-009
2e-009
1e-009
0
-1e-009
-2e-009
-3e-009
0
500
1000
1500
2000
2500
3000
3500
4000

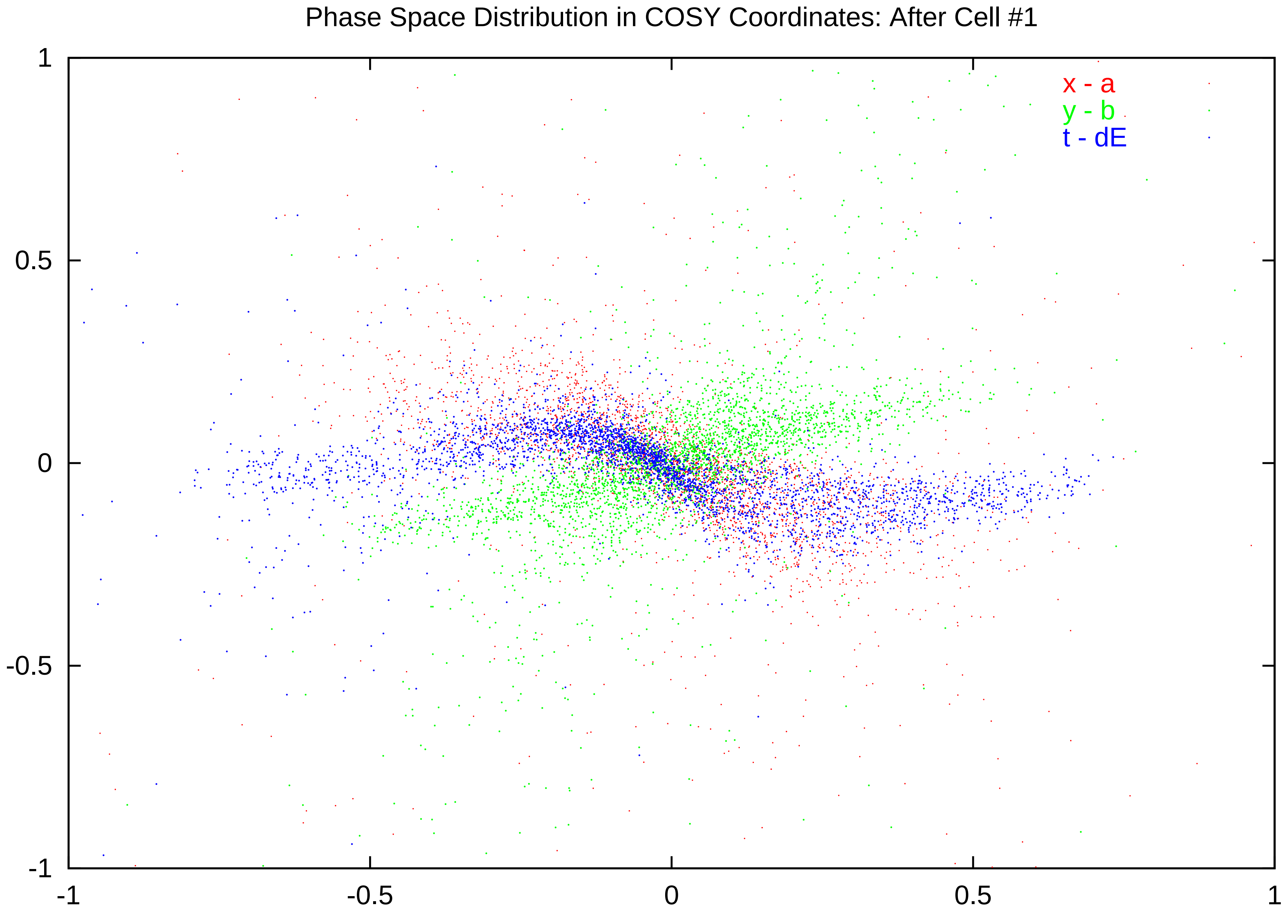

G4PhaseRot20f t deviation from 179.0MHz and shifted by -52% (after sorting)
t deviation
particle #
3e-9
2e-9
1e-9
0
-1e-9
-2e-9
-3e-9
0
500
1000
1500
2000
2500
3000
3500
4000

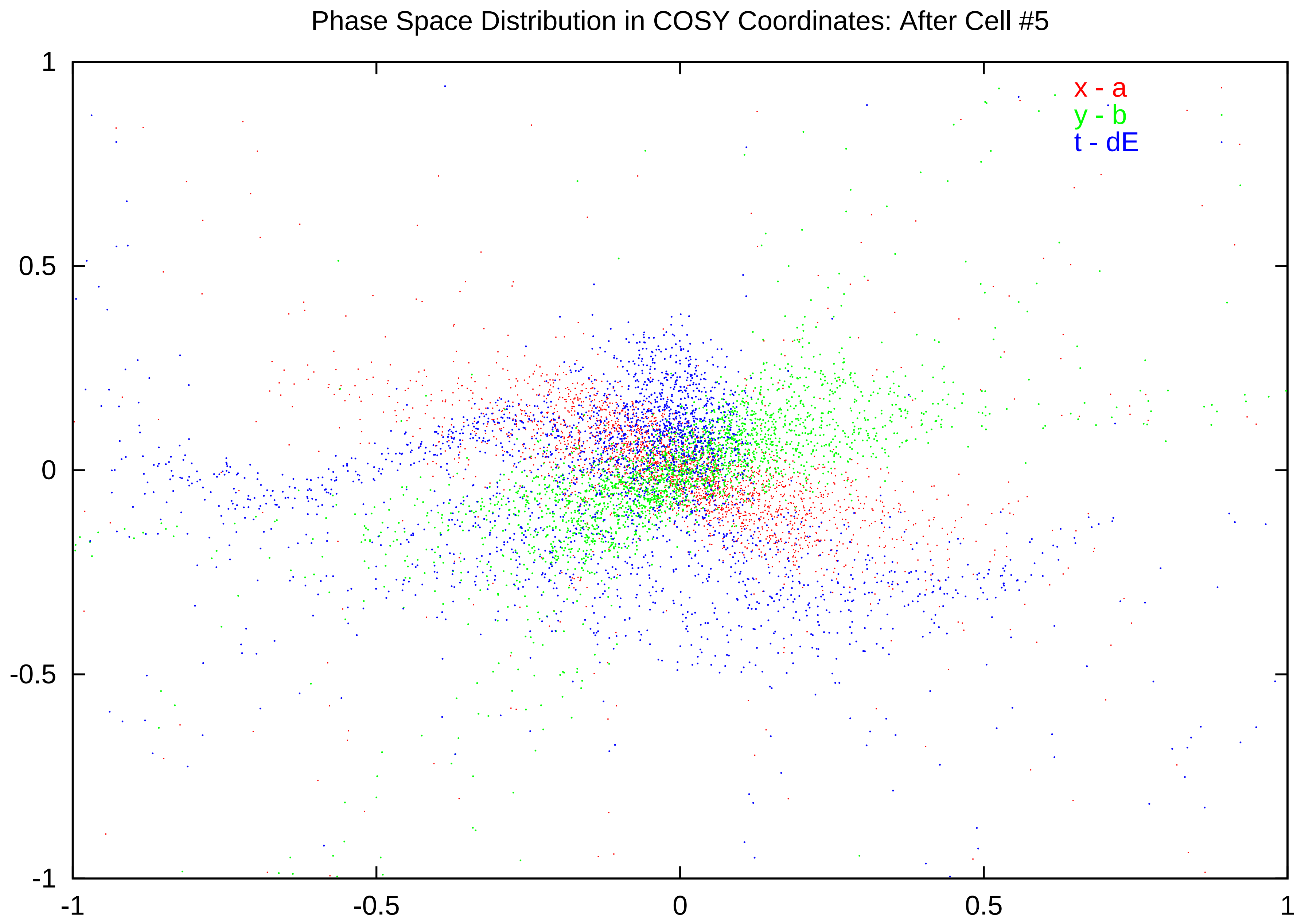

Phase Space Distribution in COSY Coordinates: Beginning
x - a
y - b
t - dE
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1

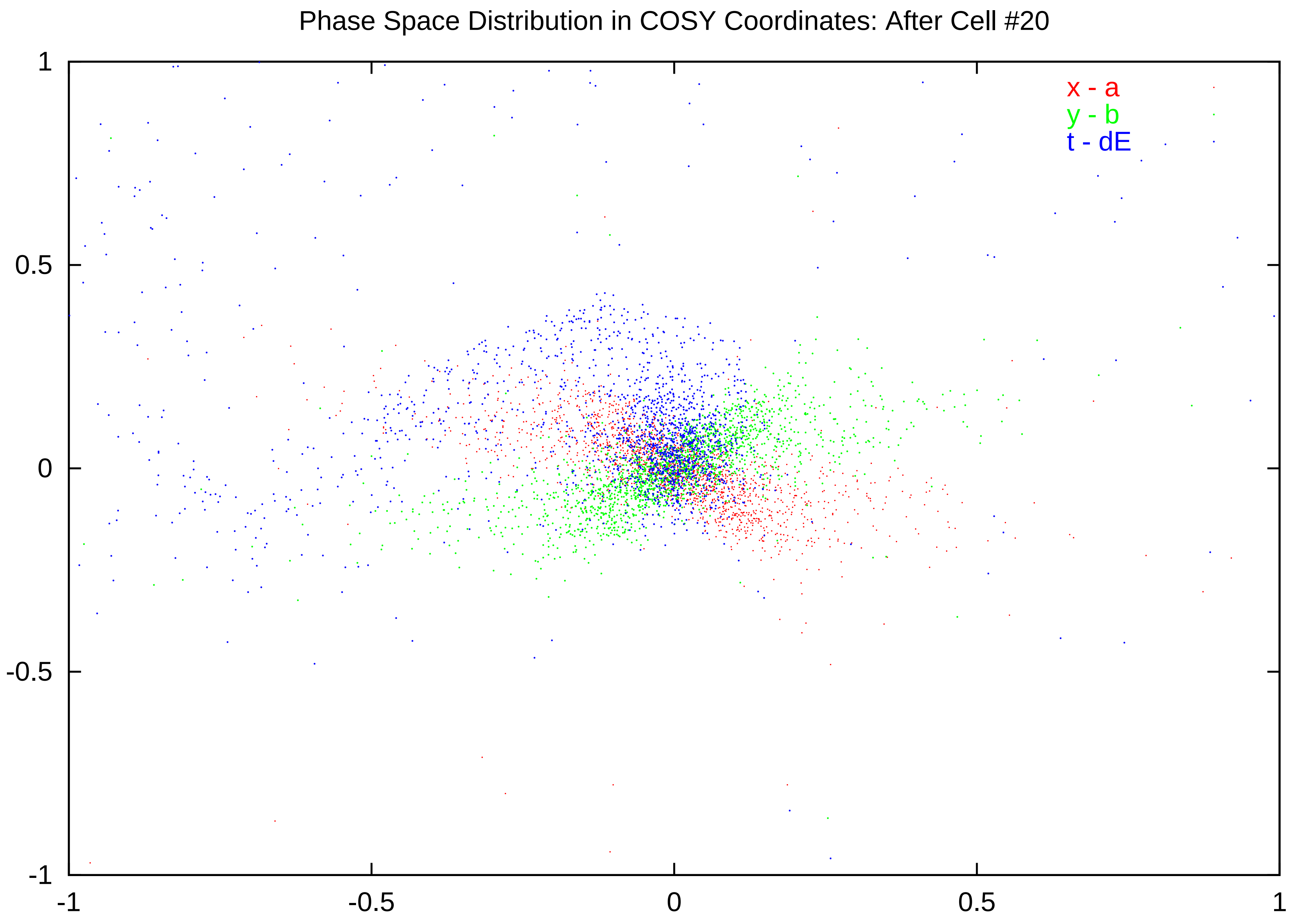
Phase Space Distribution in COSY Coordinates: After Solenoid Section

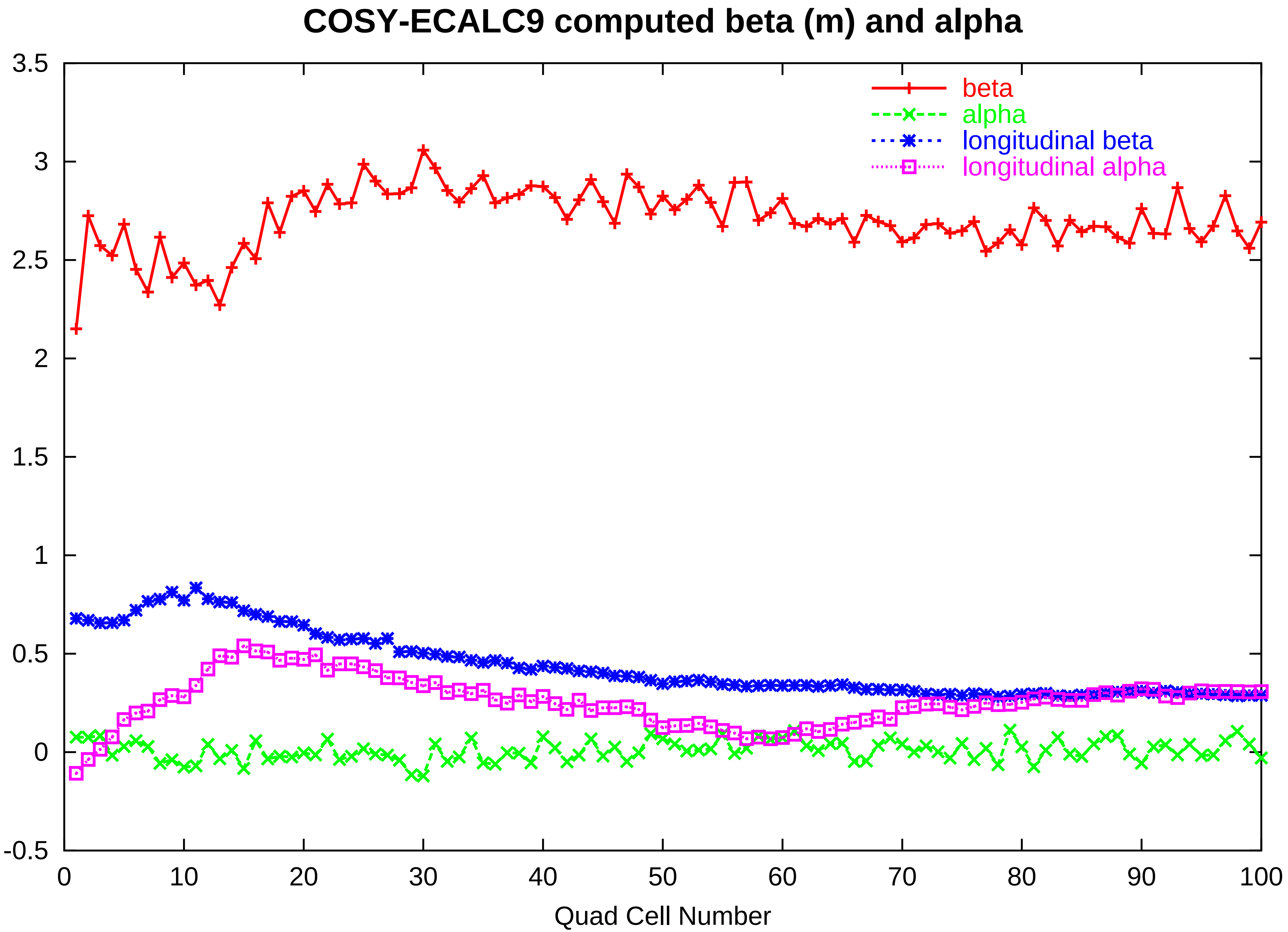


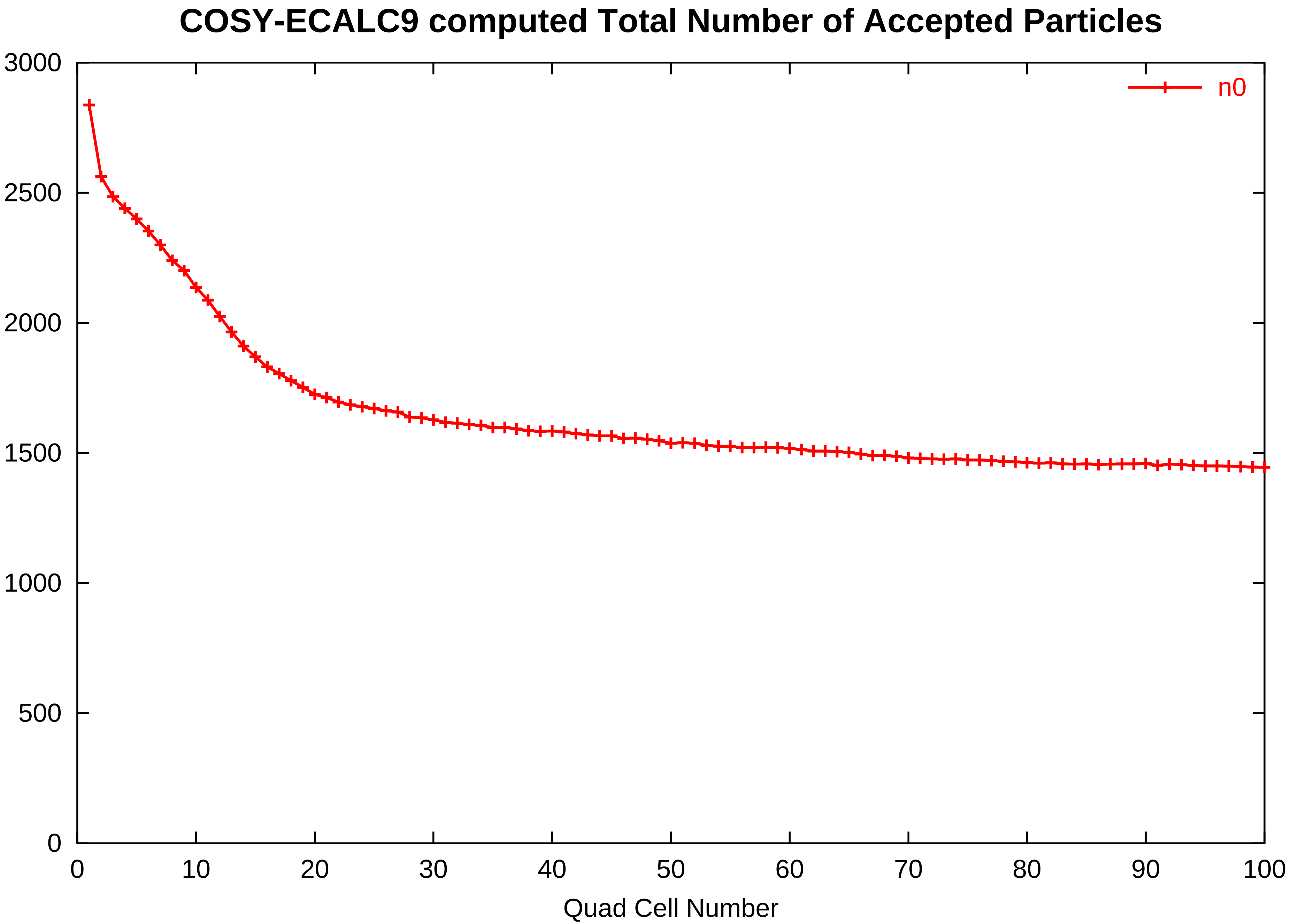

Phase Space Distribution in COSY Coordinates: After Matching Section
x - a
y - b
t - dE
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1

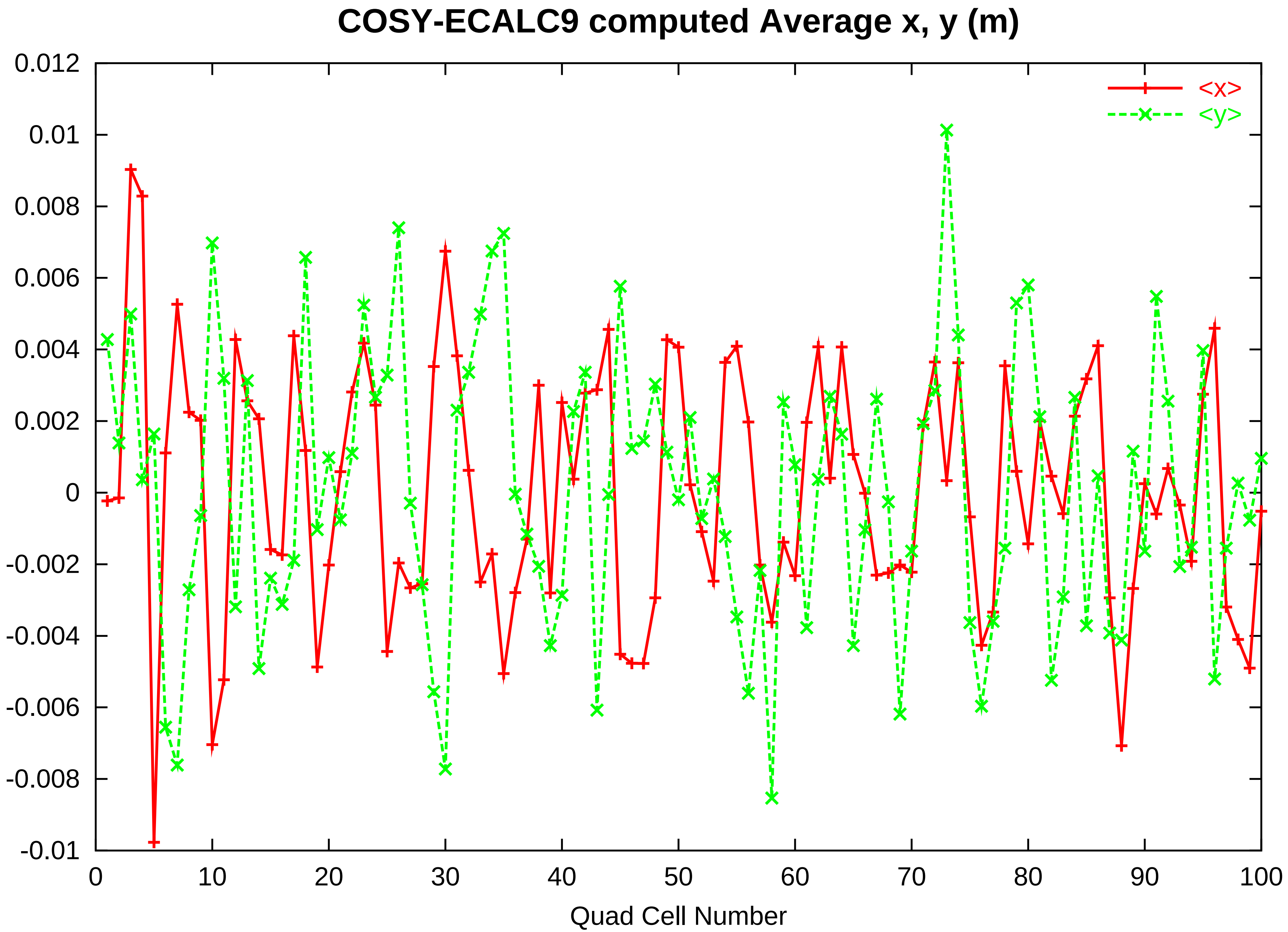

Phase Space Distribution in COSY Coordinates: After Cell #1
x - a
y - b
t - dE
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1


Phase Space Distribution in COSY Coordinates: After Cell #5
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
x - a
y - b
t - dE

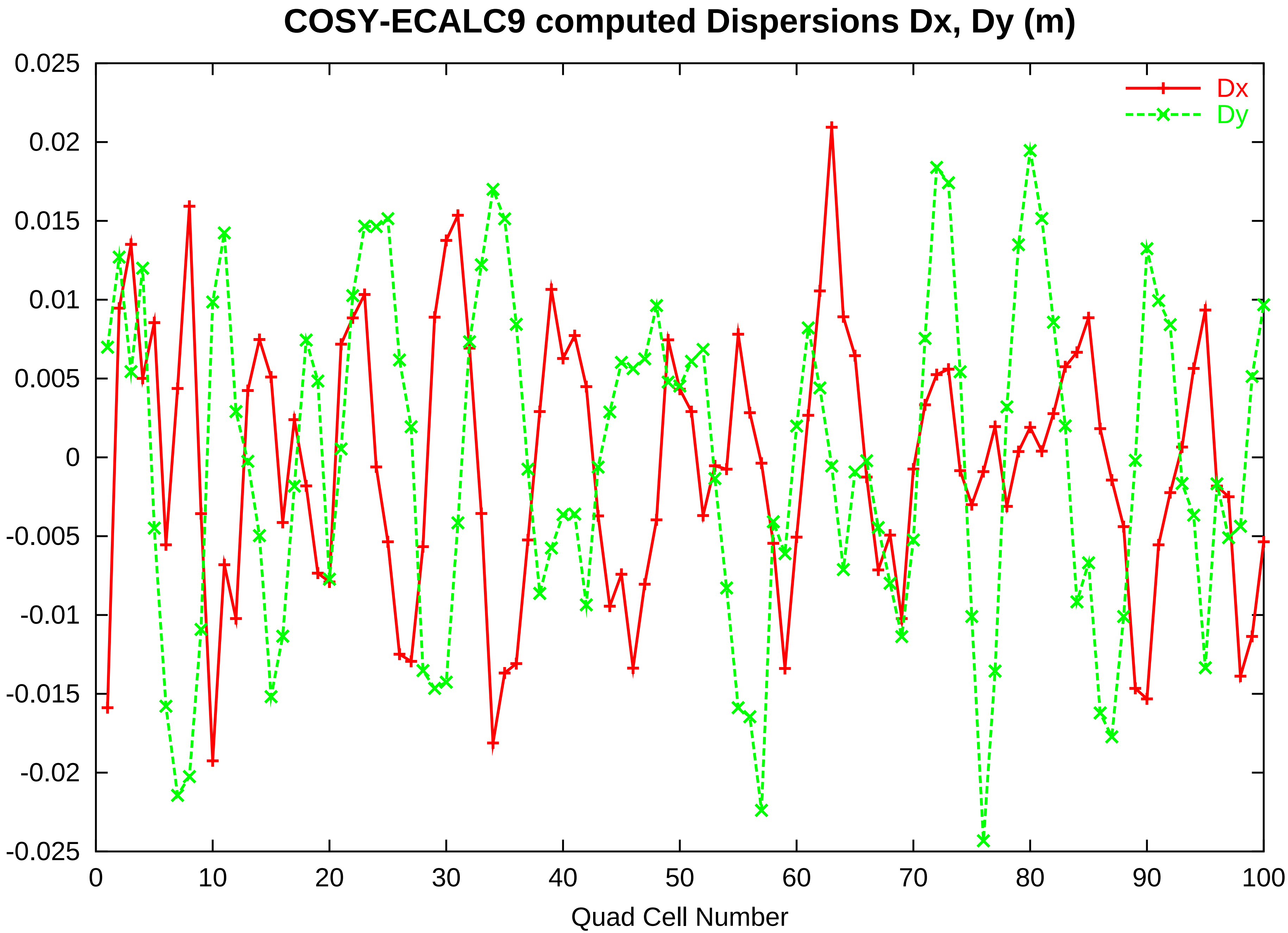

Phase Space Distribution in COSY Coordinates: After Cell #20
x - a
y - b
t - dE
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1

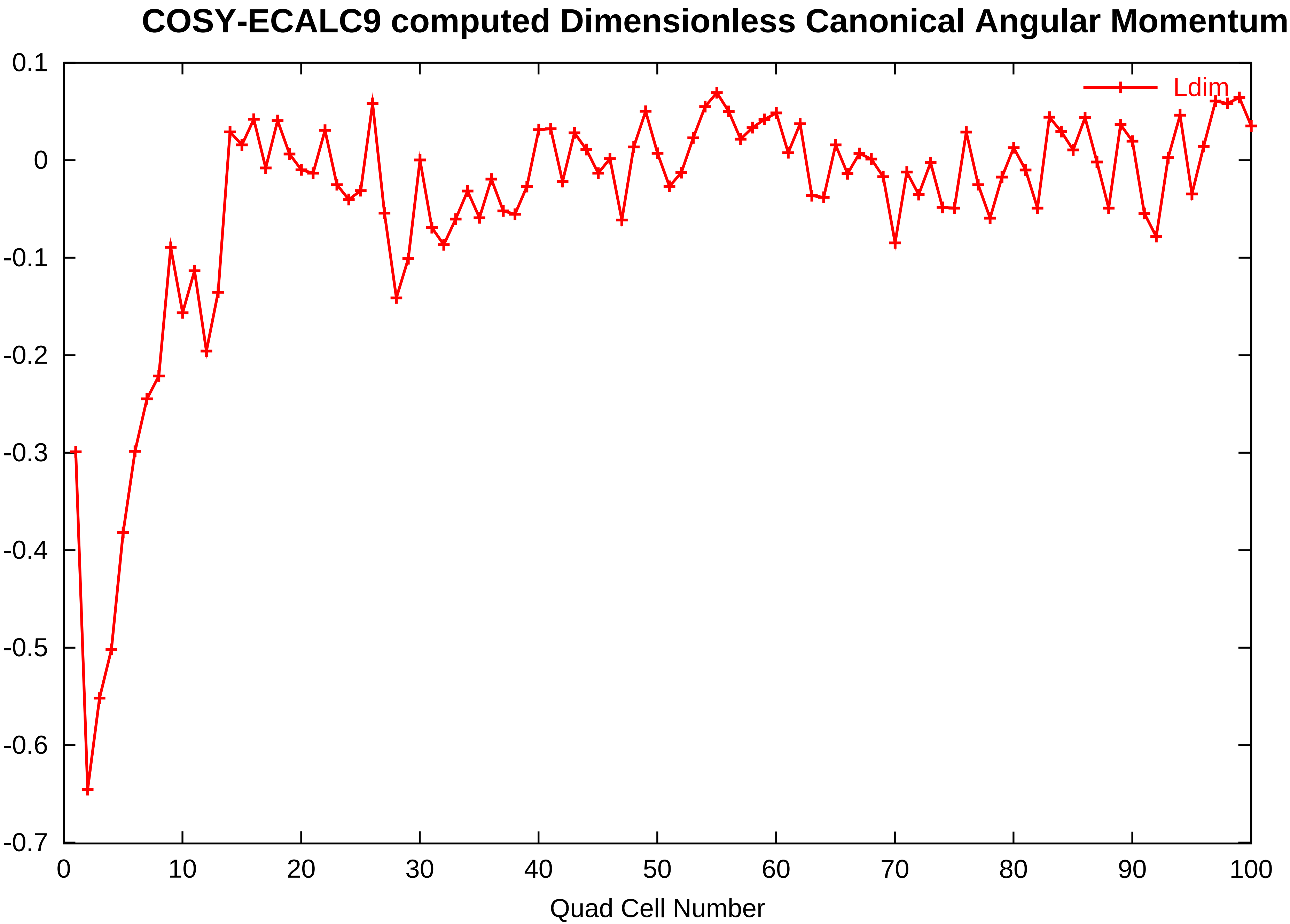

COSY-ECALC9 computed beta (m) and alpha
beta
alpha
longitudinal beta
longitudinal alpha
Quad Cell Number
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0
10
20
30
40
50
60
70
80
90
100

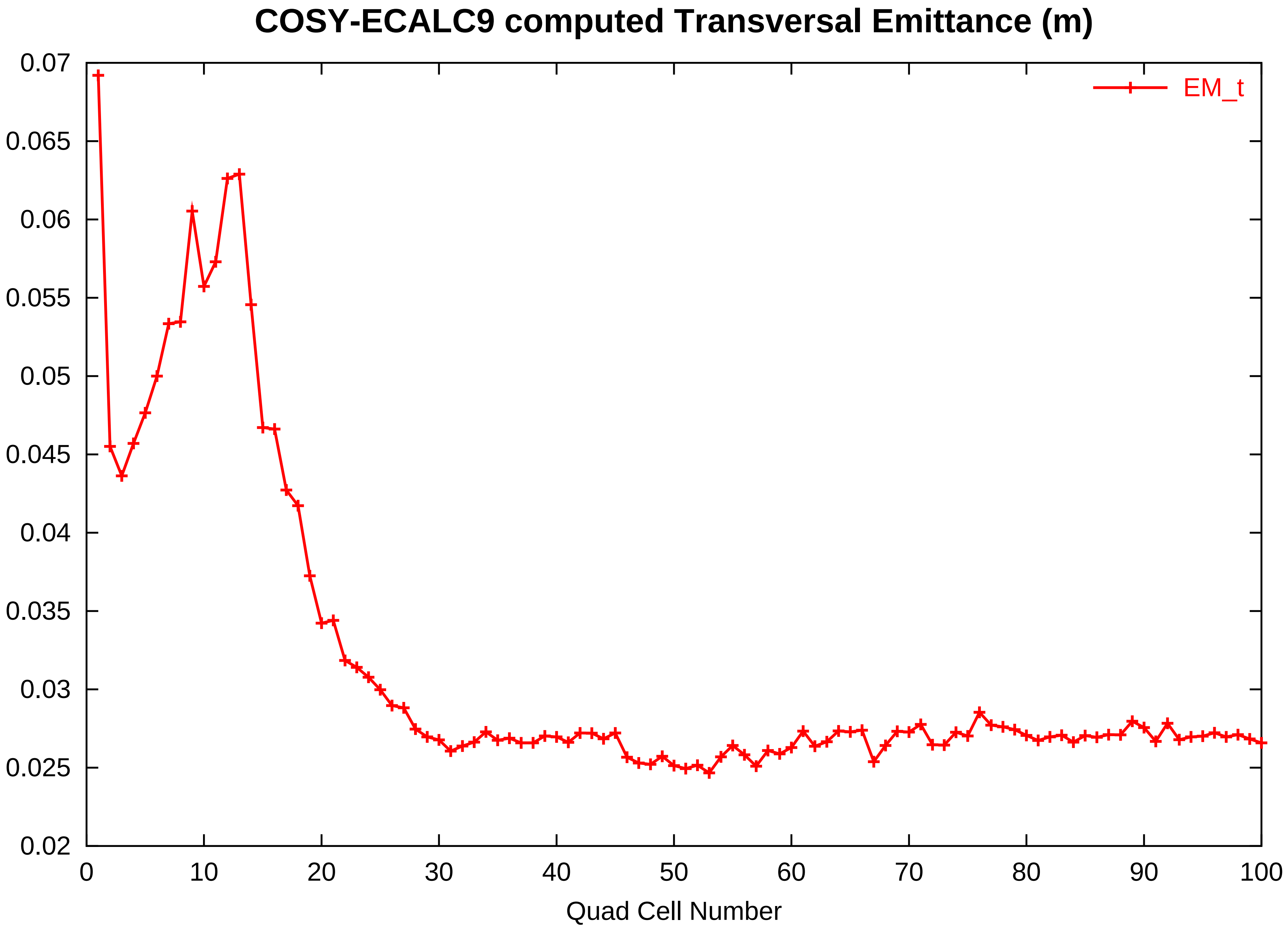

COSY-ECALC9 computed Total Number of Accepted Particles
n0
3000
2500
2000
1500
1000
500
0
0
10
20
30
40
50
60
70
80
90
100
Quad Cell Number

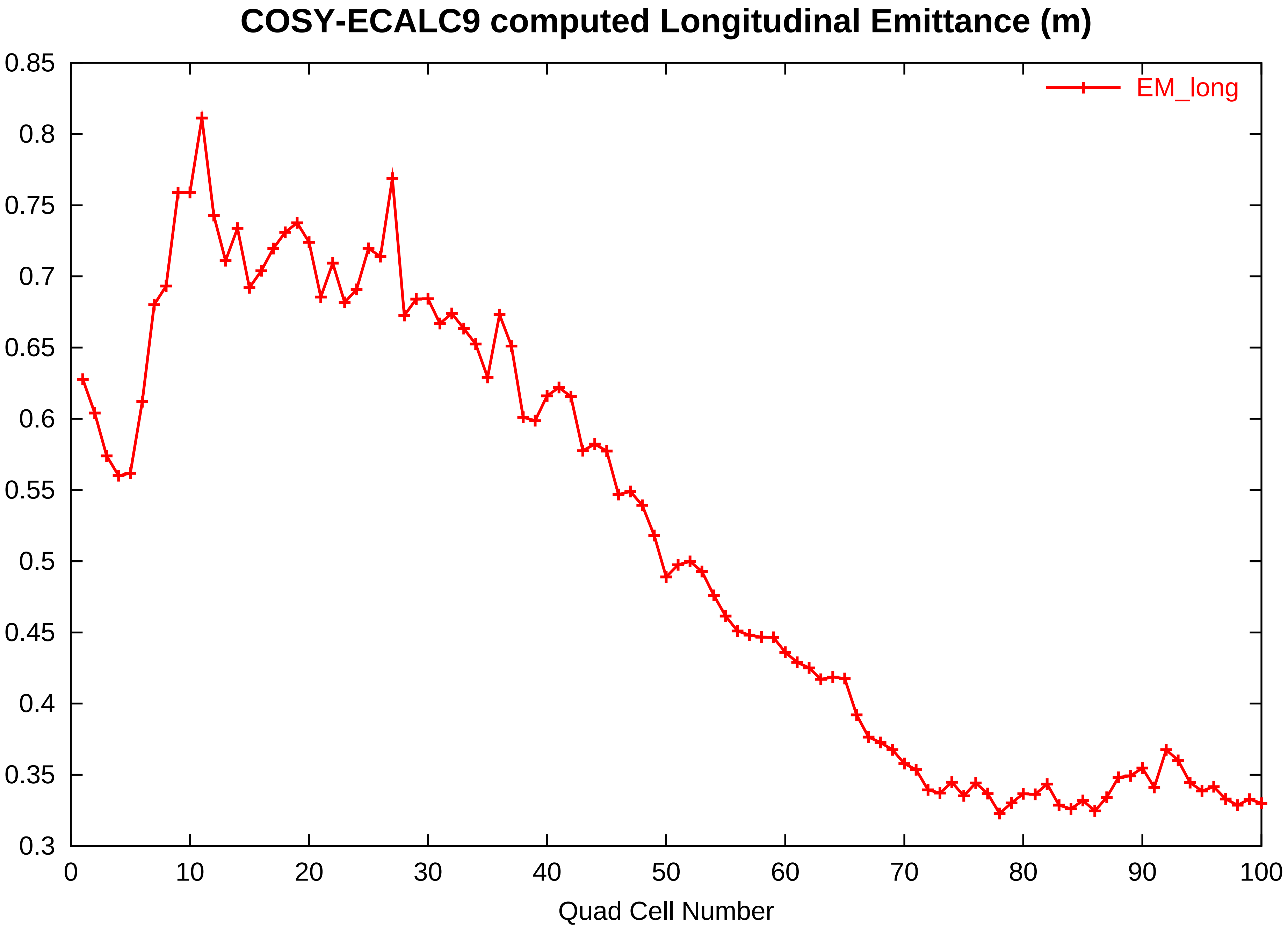


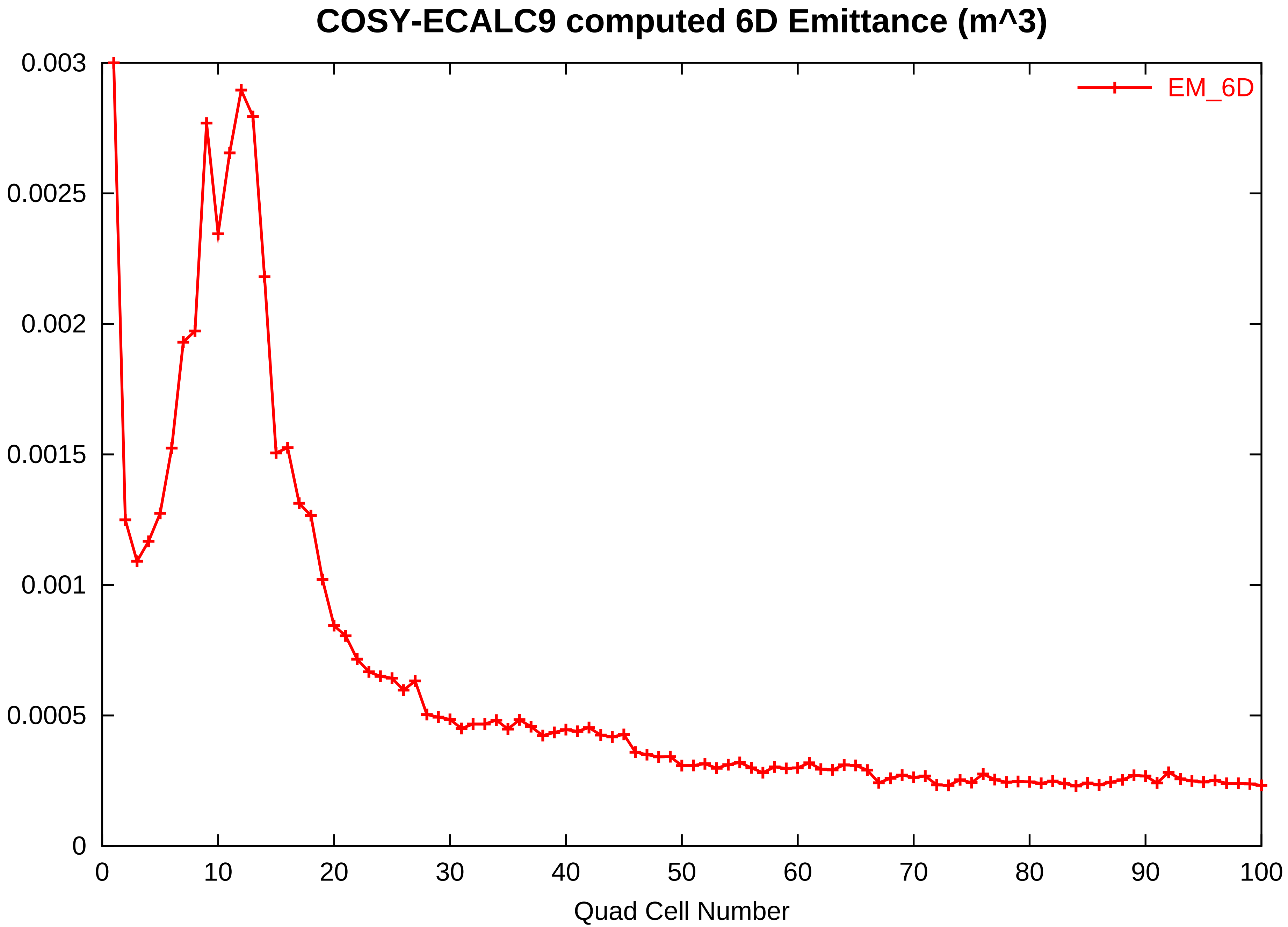
COSY-ECALC9 computed Average x, y (m)
0.012
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
0
10
20
30
40
50
60
70
80
90
100
Quad Cell Number
<x>
<y>


COSY-ECALC9 computed Dispersions Dx, Dy (m)
Dx
Dy
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
0
10
20
30
40
50
60
70
80
90
100
Quad Cell Number


COSY-ECALC9 computed Dimensionless Canonical Angular Momentum
Ldim
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0
10
20
30
40
50
60
70
80
90
100
Quad Cell Number


COSY-ECALC9 computed Transversal Emittance (m)
EM_t
0.07
0.065
0.06
0.055
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0
10
20
30
40
50
60
70
80
90
100
Quad Cell Number


COSY-ECALC9 computed Longitudinal Emittance (m)
EM_long
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0
10
20
30
40
50
60
70
80
90
100
Quad Cell Number



COSY-ECALC9 computed 6D Emittance (m^3)
EM_6D
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
0
10
20
30
40
50
60
70
80
90
100
Quad Cell Number