

Basic idea: cool muons by working
in regime where

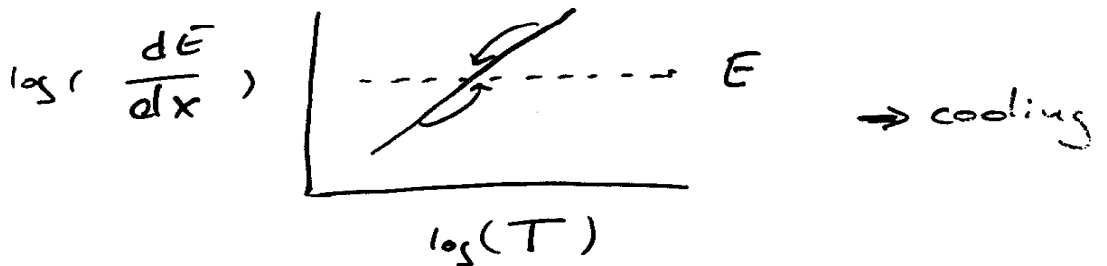
$$\frac{d\bar{E}}{dx} \propto \beta$$

for $\beta_{\mu} \leq \beta_{e^{-}}$, ionization no longer possible. Energy loss dominated by

- nuclear recoil
- excitation
- charge exchange (for μ^{+}) $\mu^{+} \rightarrow M_{\mu} \rightarrow \mu^{+}$
...

In regime where $\frac{d\bar{E}}{dx} \propto \beta$, can imagine applying an E-field which compensates $\frac{d\bar{E}}{dx}$

\Rightarrow equilibrium kinetic energy



2 23. Passage of particles through matter

23.2. Ionization energy loss by heavy particles [4-1]

Moderately relativistic charged particles other than electrons lose energy in matter primarily by ionization. The mean rate of energy loss (or stopping power) is given by the Bethe-Bloch equation,

$$-\frac{dE}{dx} = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta}{2} \right]. \quad (23.1)$$

Here T_{\max} is the maximum kinetic energy which can be imparted to a free electron in a single collision, and the other variables are defined in Table 23.1. The units are chosen so that dx is measured in mass per unit area, e.g., in $g\text{ cm}^{-2}$.

In this form, the Bethe-Bloch equation describes the energy loss of pions in a material such as copper to about 1% accuracy for energies between about 6 MeV and 6 GeV (momenta between about 40 MeV/c and 6 GeV/c). At lower energies "C/Z" corrections

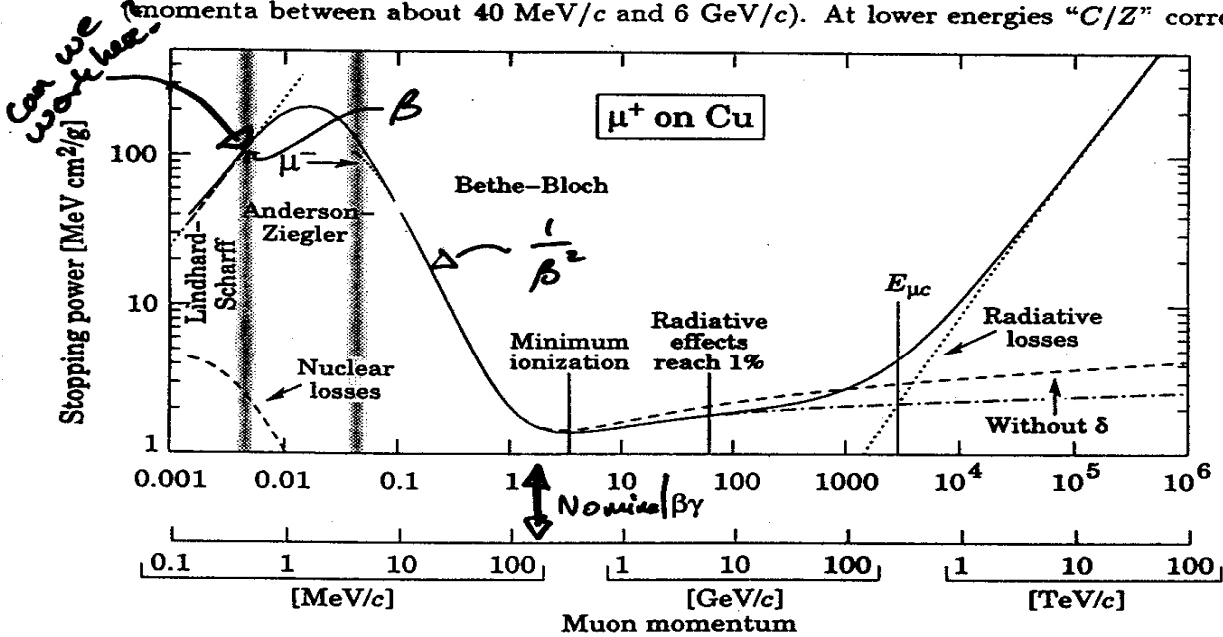


Fig. 23.1: Stopping power ($= \langle -dE/dx \rangle$) for positive muons in copper as a function of $\beta\gamma = p/Mc$ over nine orders of magnitude in momentum (12 orders of magnitude in kinetic energy) [1]. Solid curves indicate the total stopping power. Data below the break at $\beta\gamma \approx 0.1$ are taken from ICRU 49 [2], and data at higher energies are from Ref. 1. Vertical bands indicate boundaries between different approximations discussed in the text. The short dotted lines labeled " μ^- " illustrate the "Barkas effect," the dependence of stopping power on projectile charge at very low energies [3].

Issues/comments:

- $\frac{dE}{dx}$ very large in this region
 \Rightarrow need to work with gas
- Lifetime of μ^\pm
$$V_{\mu^\pm} = 0.1 \sqrt{T(\text{eV})} \text{ m} \quad \left. \vphantom{V_{\mu^\pm}} \right\} \text{TCCMC}$$

 \Rightarrow cannot stay at low T for very big distances
- Cannot have $\vec{E} \parallel \vec{p}$ or we never get below dE/dx peak
 \Rightarrow will apply $\vec{E} \perp \vec{B}$
- $\vec{E} \parallel \vec{B}$ field only works if have μ^+ , not μ^-
 \Rightarrow Helium only solⁿ
- For μ^- , need to keep T as large as possible to minimize capture

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INTRODUCTION

Charge transfer in collisions of atomic particles with gaseous atoms and molecules is one of the important processes in plasma physics, radiation physics, astrophysics, and other areas. Especially, knowledge of the total cross sections for charge transfer of hydrogen ions and atoms is necessary for thermonuclear fusion research, because this process is crucial to the containment time for plasmas, particle recycling, plasma diagnostics, and surface effects.¹ Several theoretical approaches have been developed to calculate the cross sections (see, for example, the recent review article by Morgan et al.²). These approaches, however, require time-consuming computations. For rapid evaluation of the cross sections, therefore, it is useful to have analytic formulas fitted to experimental data.

In the present work, analytic formulas† have been constructed for the total cross sections of the following

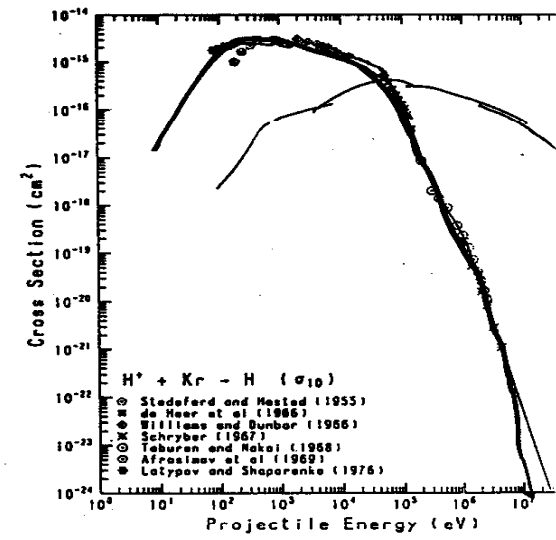
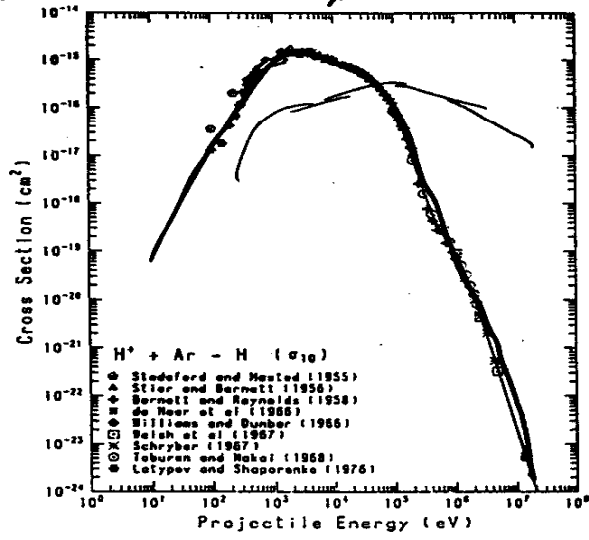
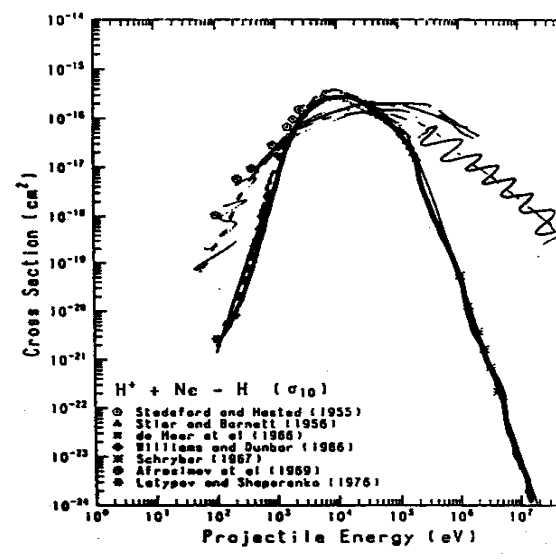
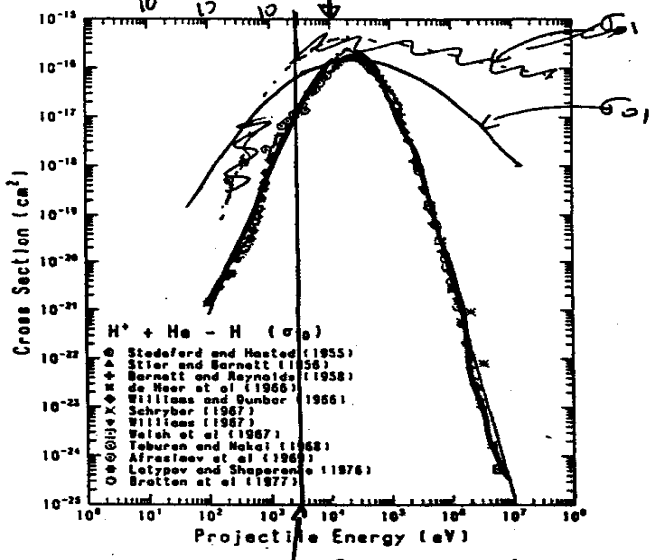
charge transfer processes:

- (1) σ_{10} for single-electron capture of H^+ .
- (2) σ_{0-1} for single-electron capture of H.
- (3) σ_{1-1} for double-electron capture of H^+ .
- (4) σ_{01} for single-electron loss of H.
- (5) σ_{-10} for single-electron loss of H^- .
- (6) σ_{-11} for double-electron loss of H^- .

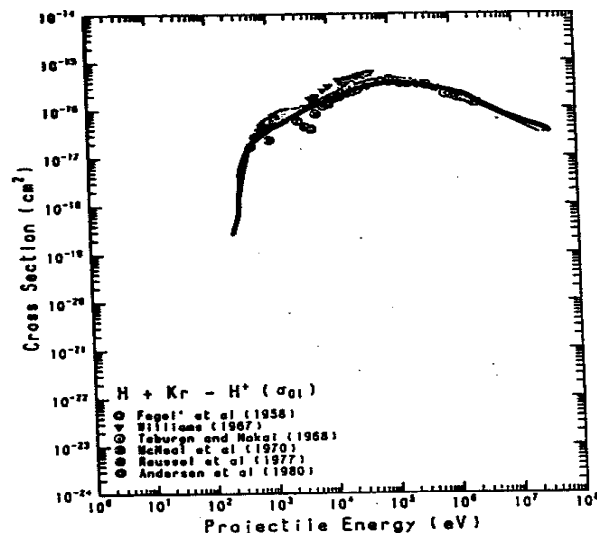
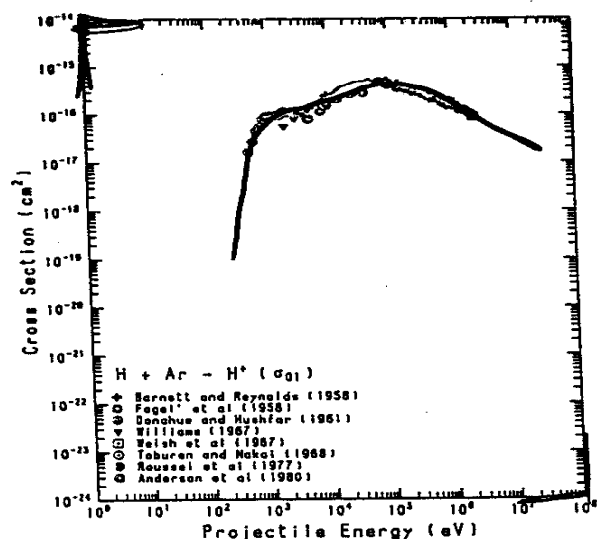
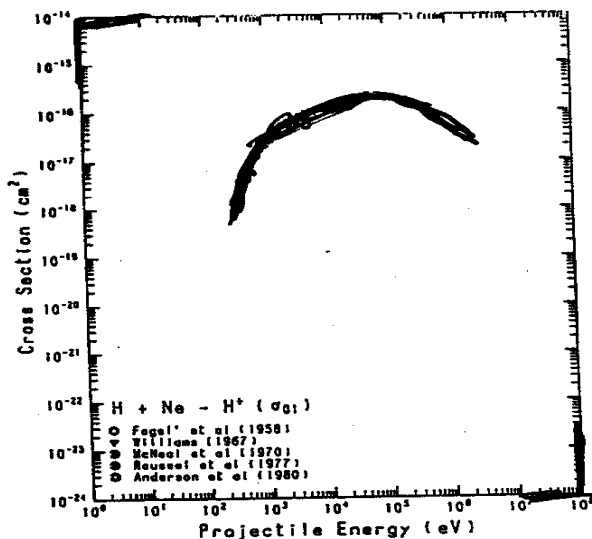
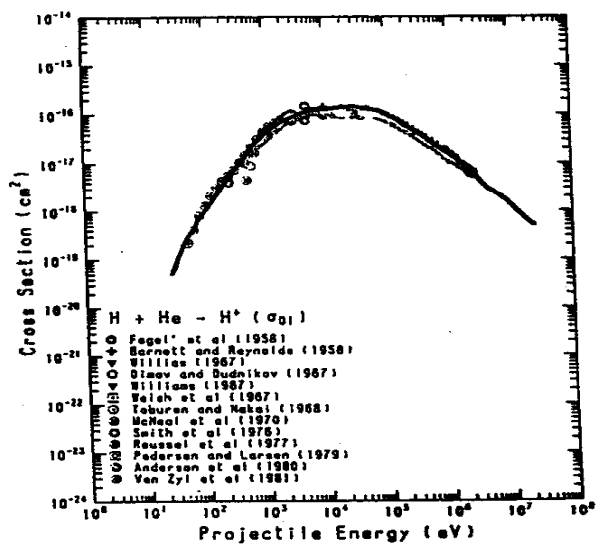
Here the subscripts represent the initial and the final charge state of the hydrogen. For each of the six processes,

† Brief accounts of the formulas were given in T. Tabata, R. Ito, Y. Nakai, A. Kikuchi, and T. Shirai, Preprints of Autumn Meeting of the Physical Society of Japan, Okayama, October 1983, p. 194; Annu. Rep. Radiat. Center Osaka Prefect. 24, 55 (1983).

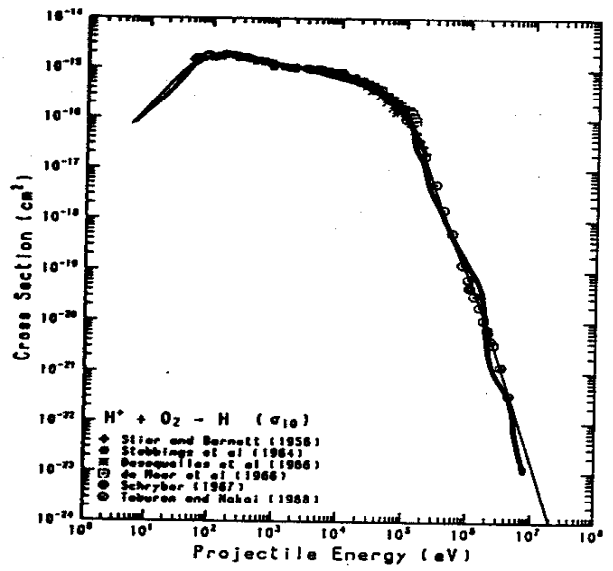
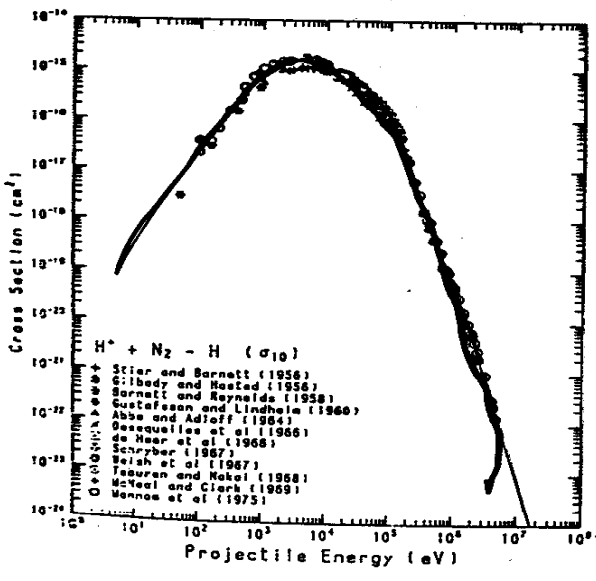
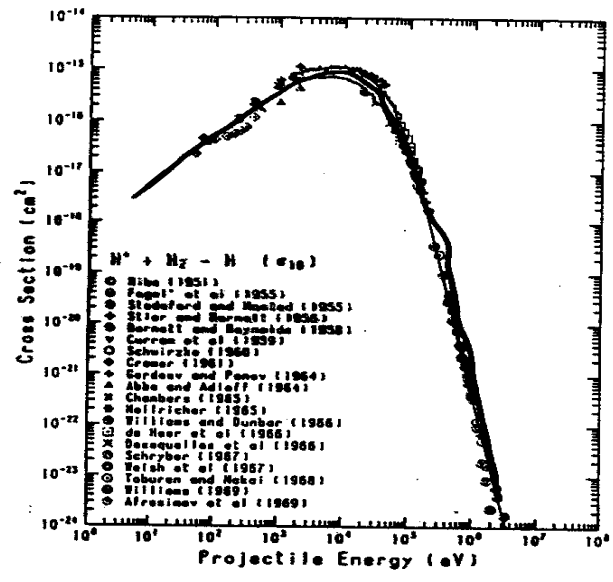
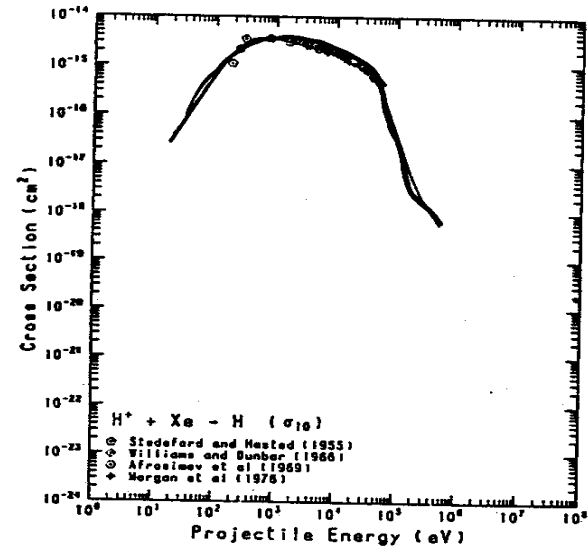
GRAPH I. Cross Section vs Energy. σ_{10} of H^+ in He, Ne, Ar, and Kr
See page 74 for Explanation of Graphs



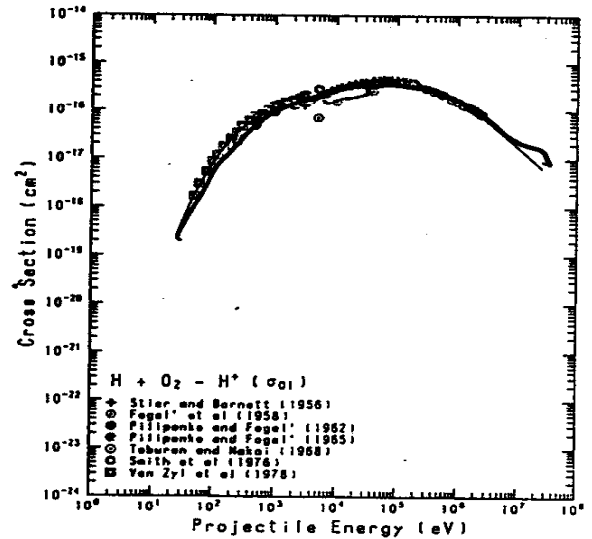
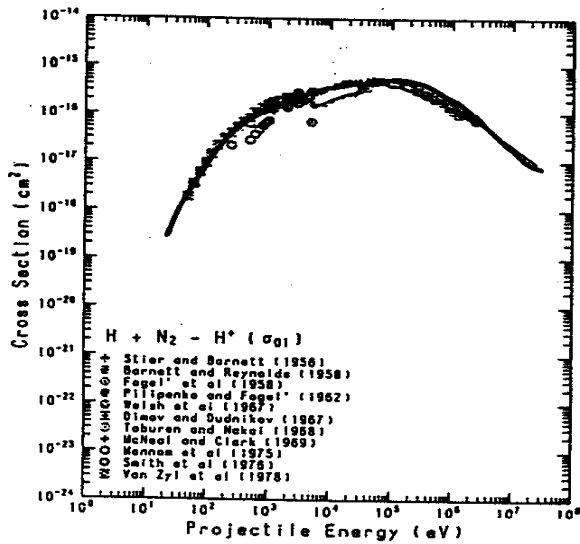
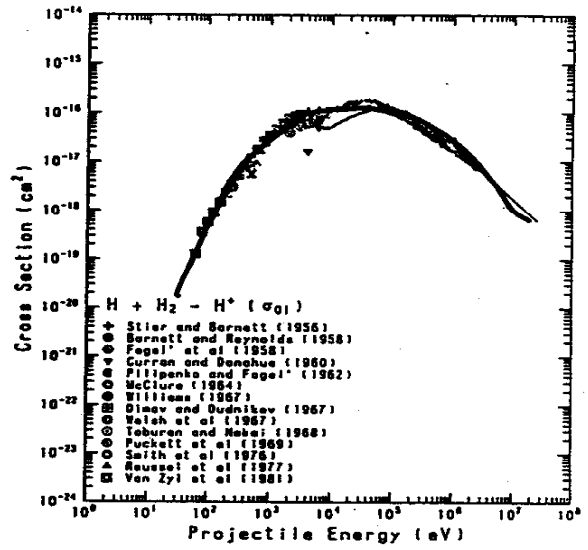
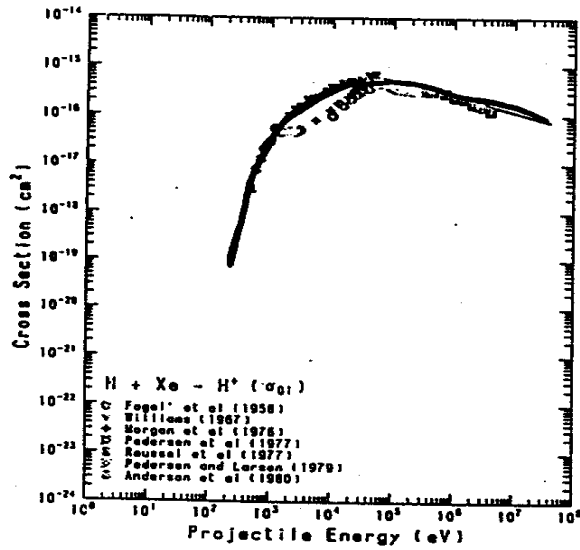
GRAPH IV. Cross Section vs Energy. σ_{01} of H in He, Ne, Ar, and Kr
See page 74 for Explanation of Graphs



GRAPH I. Cross Section vs Energy, σ_{10} of H^+ in Xe, H_2 , N_2 , and O_2
See page 74 for Explanation of Graphs



GRAPH IV. Cross Section vs Energy. σ_{01} of H in Xe, H₂, N₂, and O₂
See page 74 for Explanation of Graphs



$$\sigma_{\text{capture}} \leq \frac{(2\ell+1) \lambda^2}{4\pi}$$

Theoretical limit for capture into state of given ℓ

A. H. de Borde, Proc. Phys. Soc., London 67 (1954) 57.

Capture dominantly in μ^- such that $v_{\mu^-} = v_{e^-}$

$$n_{\mu^-} = \sqrt{\frac{m_{\mu}}{m_e}} \approx 15$$

$$\lambda^2 \sim \frac{1}{p^2} \sim \frac{1}{T}$$

$$\sigma_{\text{cap}} \leq (2\ell+1) 2\pi \frac{10^{-18} \text{ cm}^2}{T (\text{eV})} \approx \frac{2 \cdot 10^{-16} \text{ cm}^2}{T (\text{eV})}$$

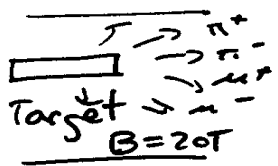
for $\rho = 10^{-4} \text{ g/cm}^3$ He, $T = 300 \text{ eV}$
 mean free path $\sim 0.1 \text{ cm}$

This is too short, but it is an lower limit. Can also lower ρ , but this will lower efficiency of cooling.

For now, proceed, but will eventually need to measure this cross section.

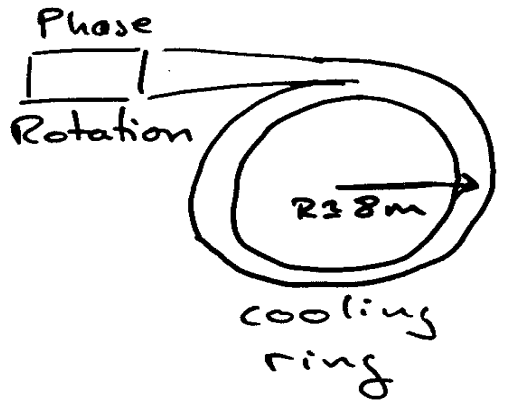
Scheme:

P →

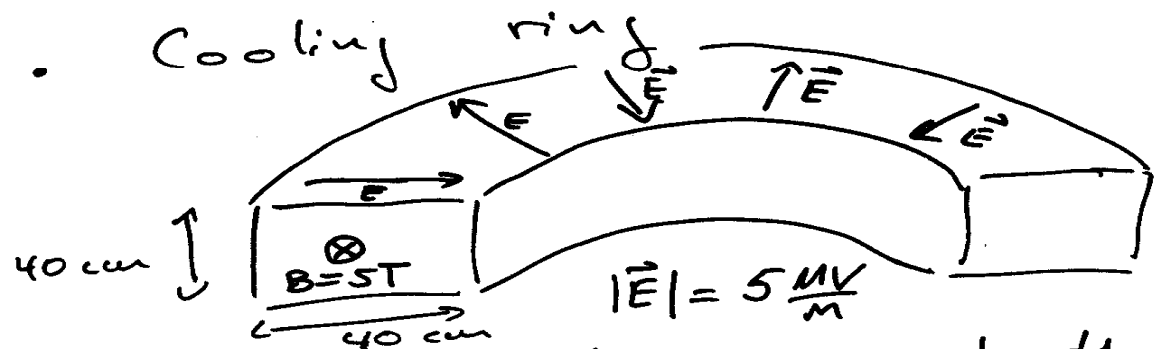


50m

 drift
 B → 5T



• Phase rotation is used to slow down μ^+

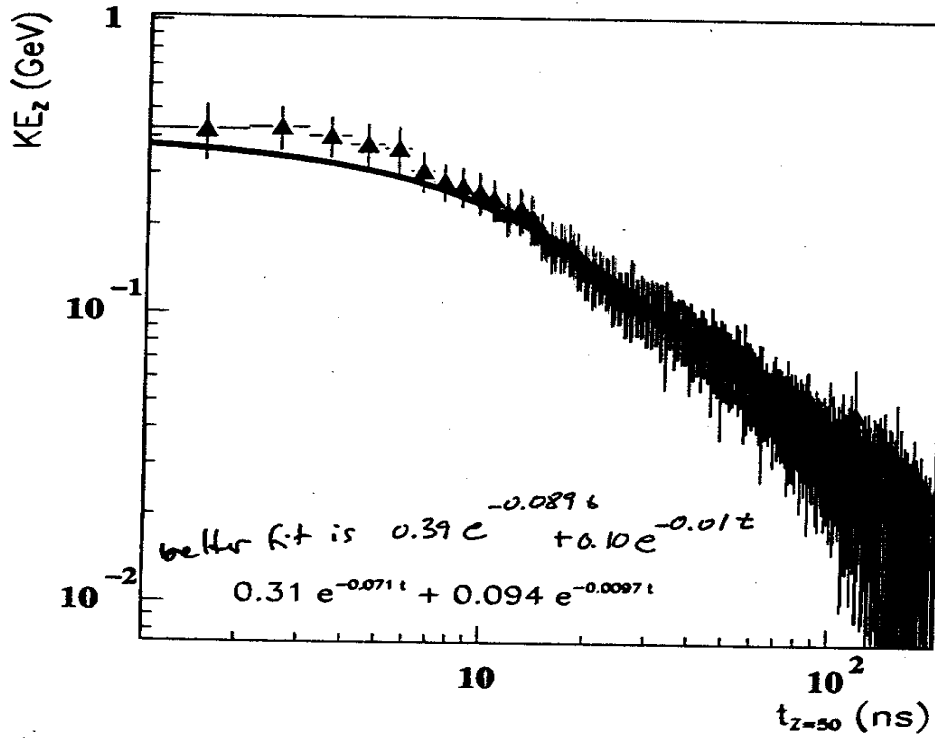


\vec{E} alternates (consistently with Maxwell eq.) so no net drift. Sweep dist $\sim 1m$

Note: cannot be DC field \rightarrow He breakdown

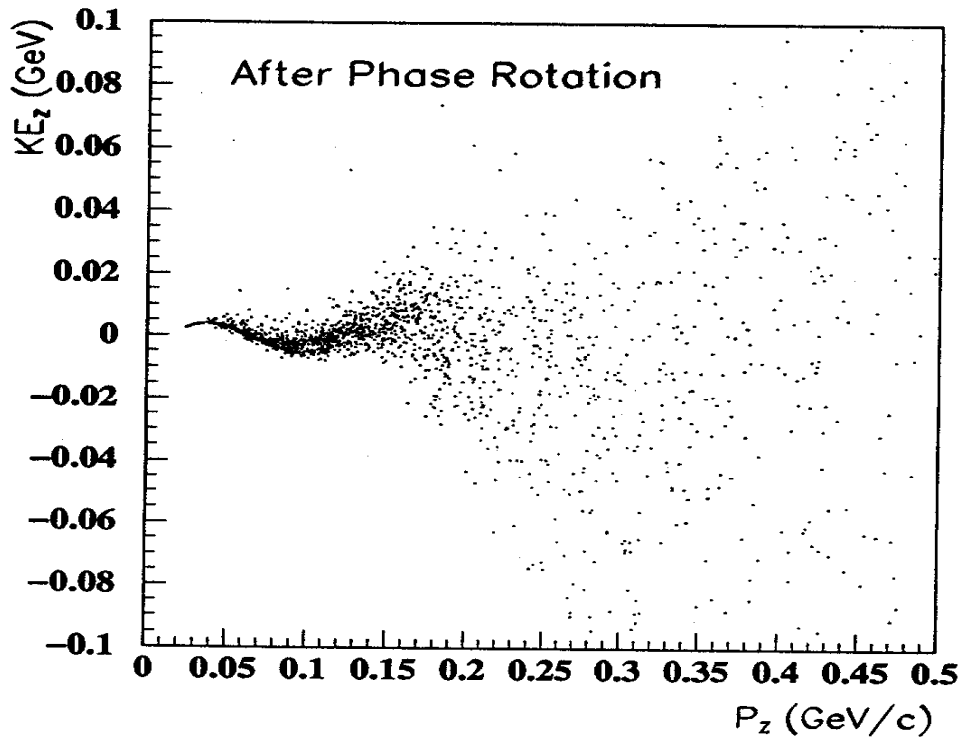
$$A_{He} = 10^{-4} \text{ g/cm}^3$$

mars3.2gev-c-50.dat

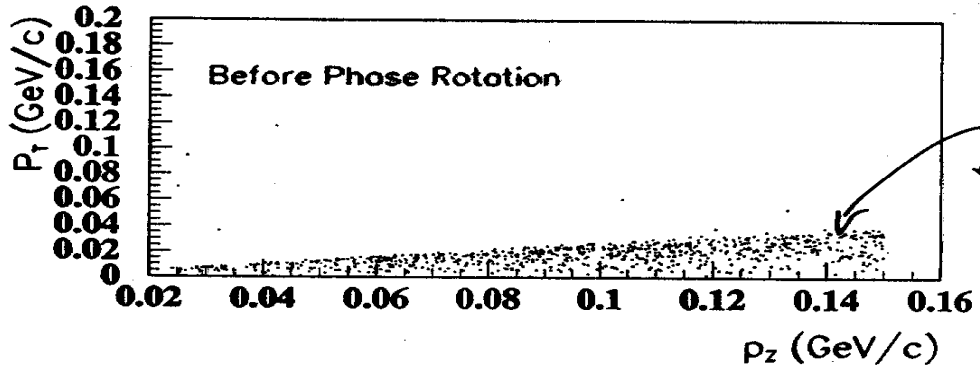


Phase rotation is just a
parametrization of KE vs t

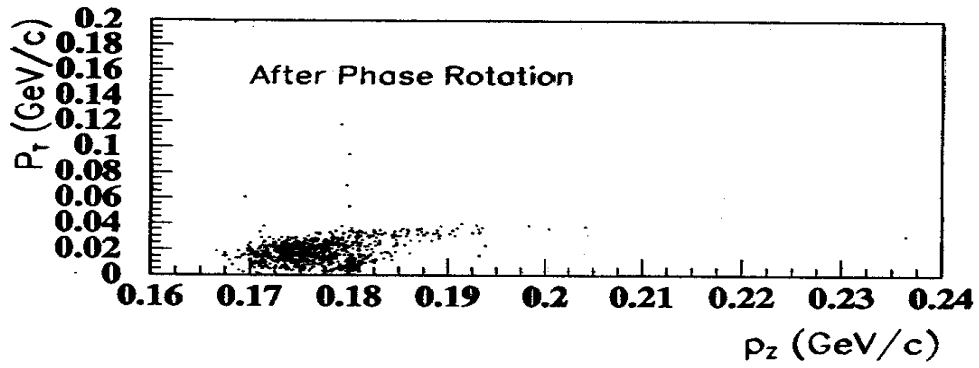
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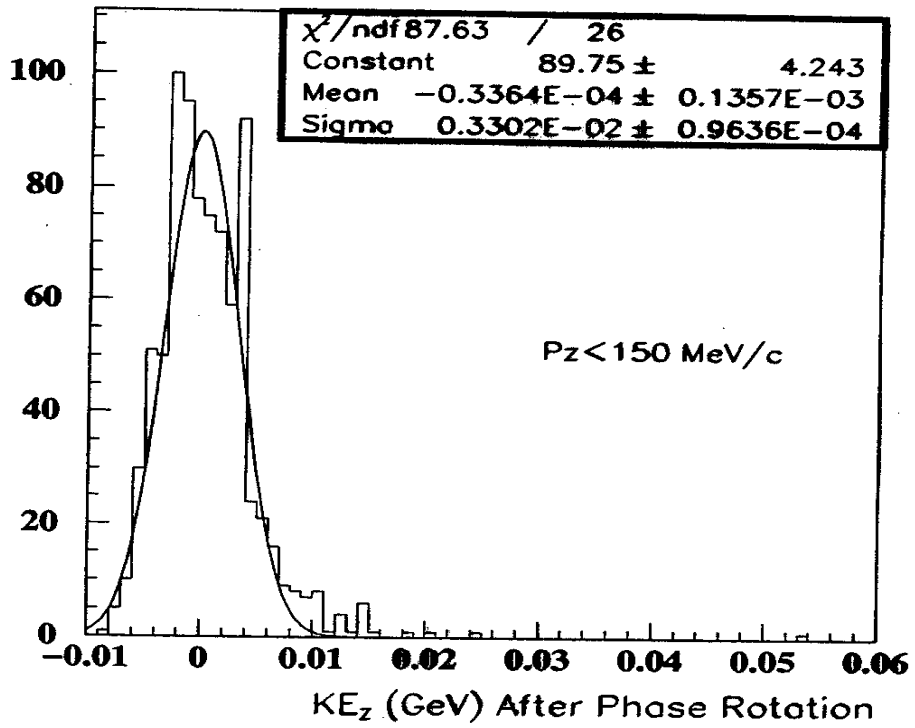
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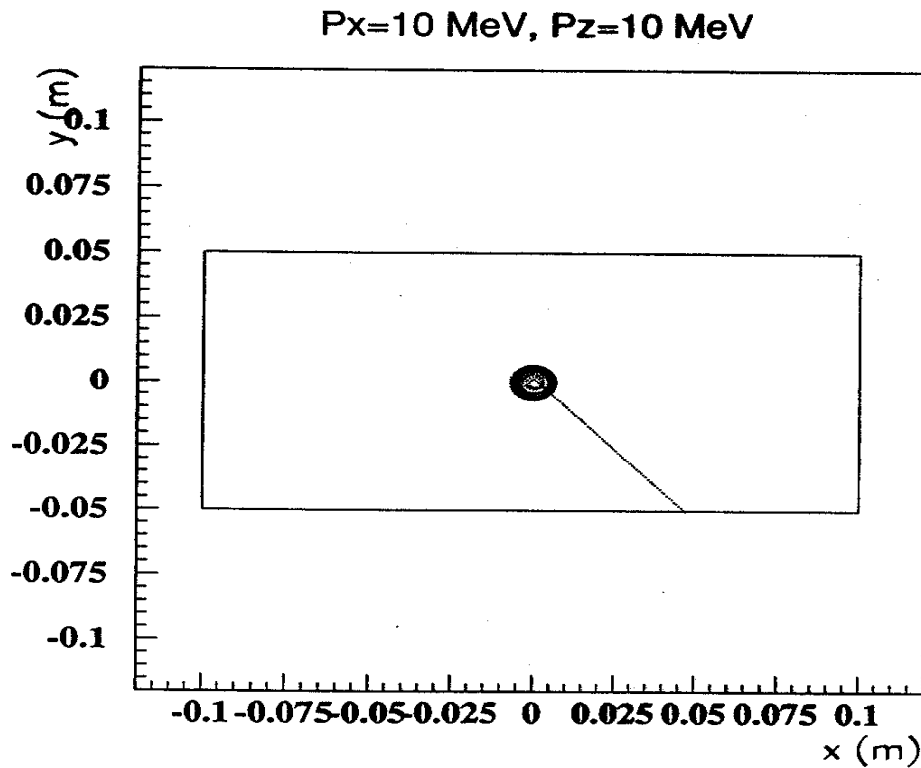
cut \rightarrow 1/ in
mars file



mars3.2gev-c-50.dat



Some pictures from simulation
(preliminary designs)

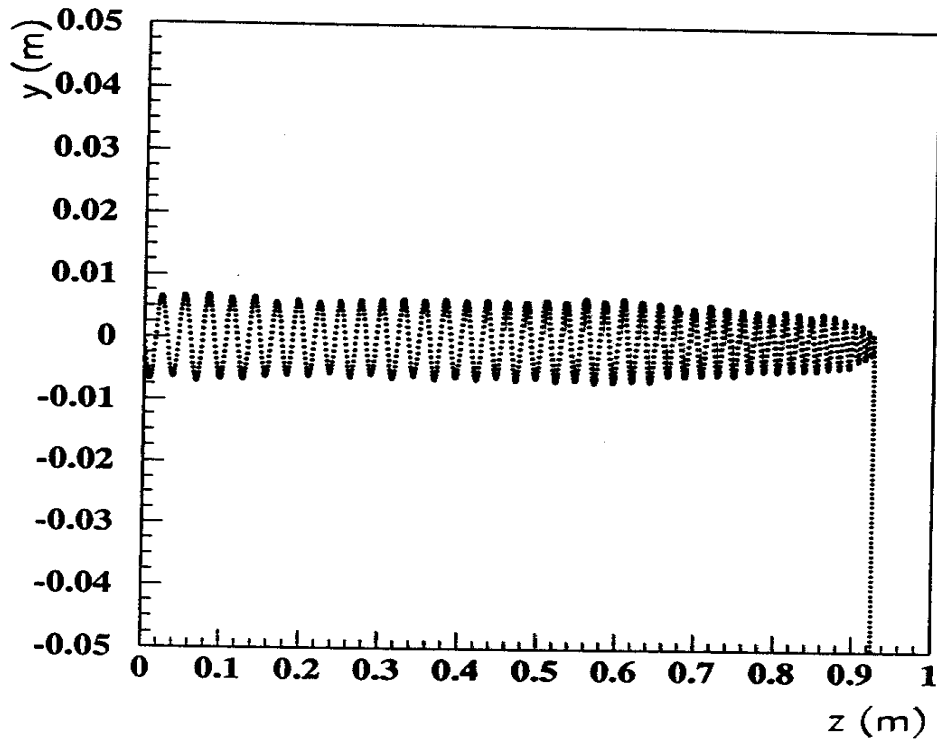


$$F_x = qE_x + qv_y B - \frac{dE}{dx} \left(\frac{\beta_x}{\beta} \right)$$

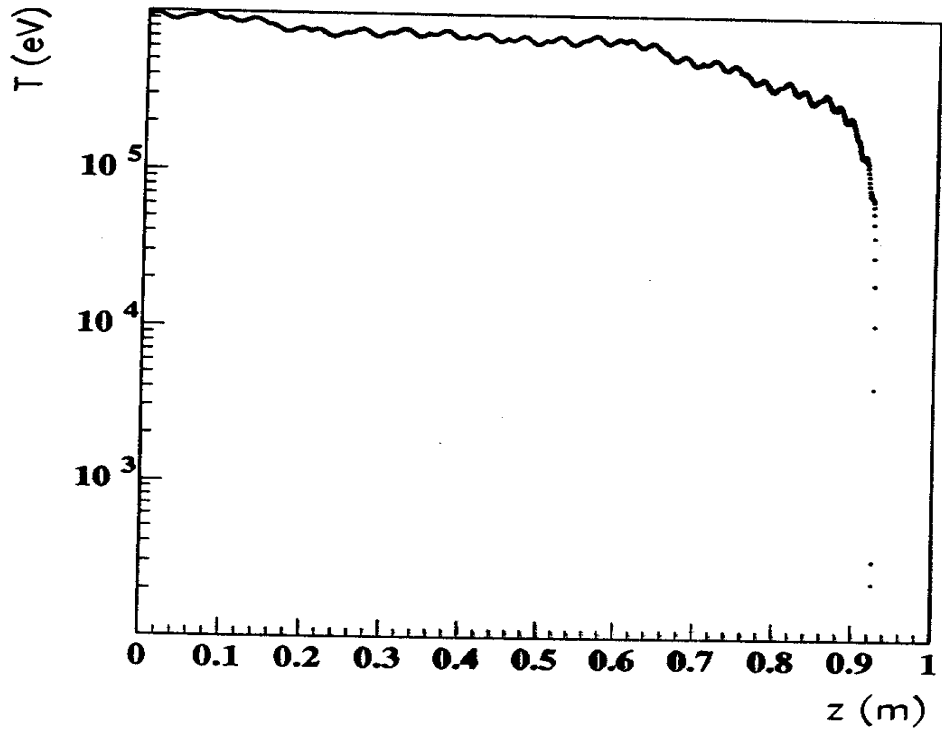
$$F_y = -qv_x B - \frac{dE}{dx} \left(\frac{\beta_y}{\beta} \right)$$

$$F_z = -\frac{dE}{dx} \left(\frac{\beta_z}{\beta} \right)$$

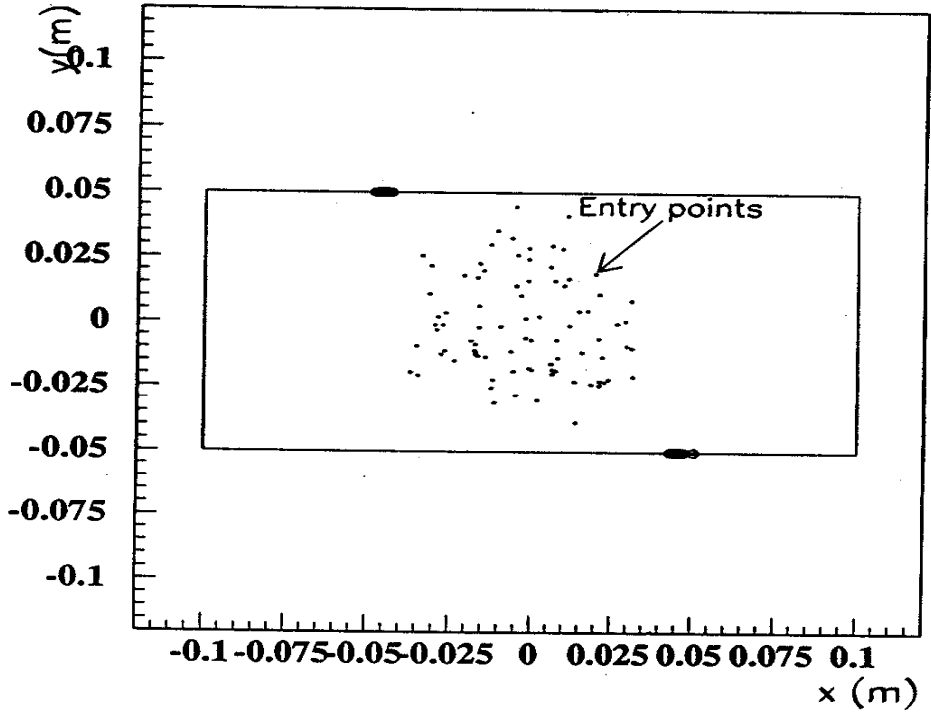
$P_x=10 \text{ MeV}, P_z=10 \text{ MeV}$



$P_x=10 \text{ MeV}, P_z=10 \text{ MeV}$

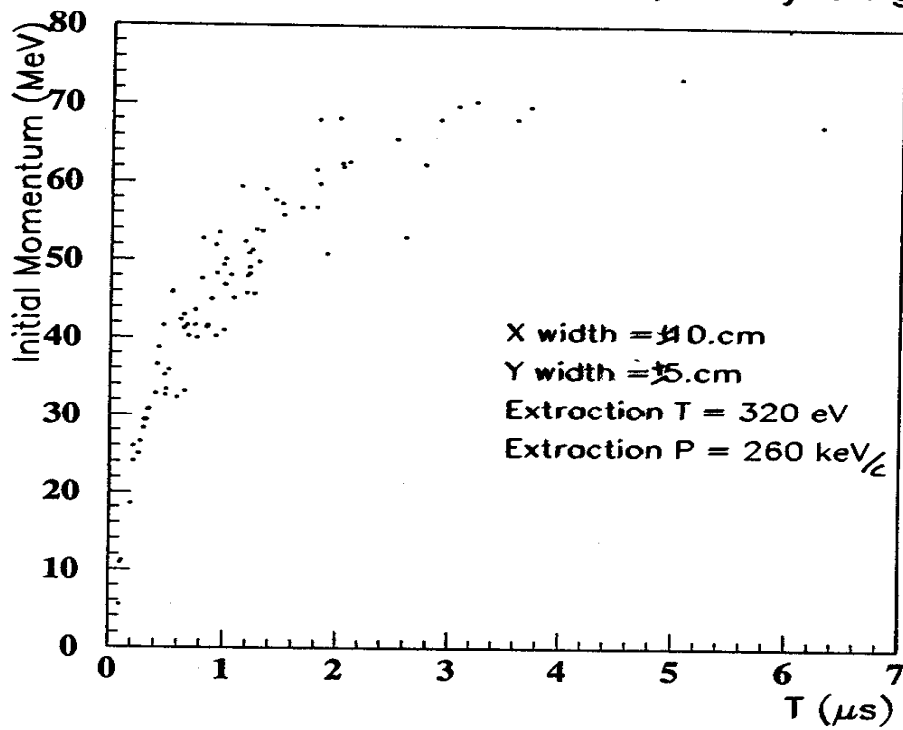


Extraction Position

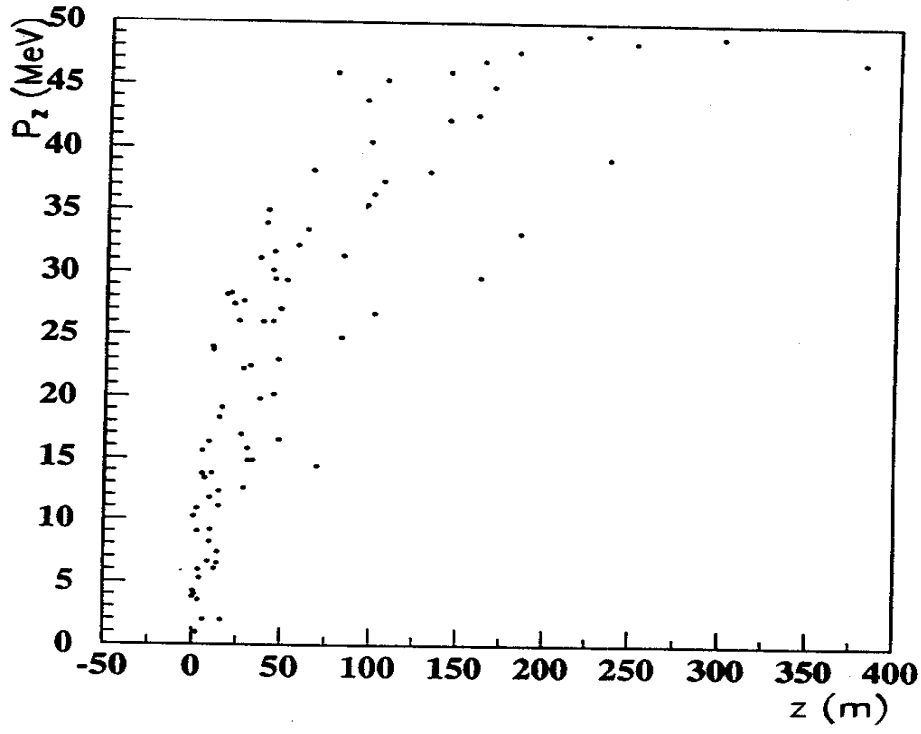


*assumes
orbits centered
on $x=y=0$*

Extraction Time B=5T, E=5MV/m, density=0.1 g/lt



Extraction Position



Some results using π^\pm, μ^\pm files from
N. Mokhov

[http://www-ep.fnal.gov/mokhov/momu/
target99/](http://www-ep.fnal.gov/mokhov/momu/target99/)

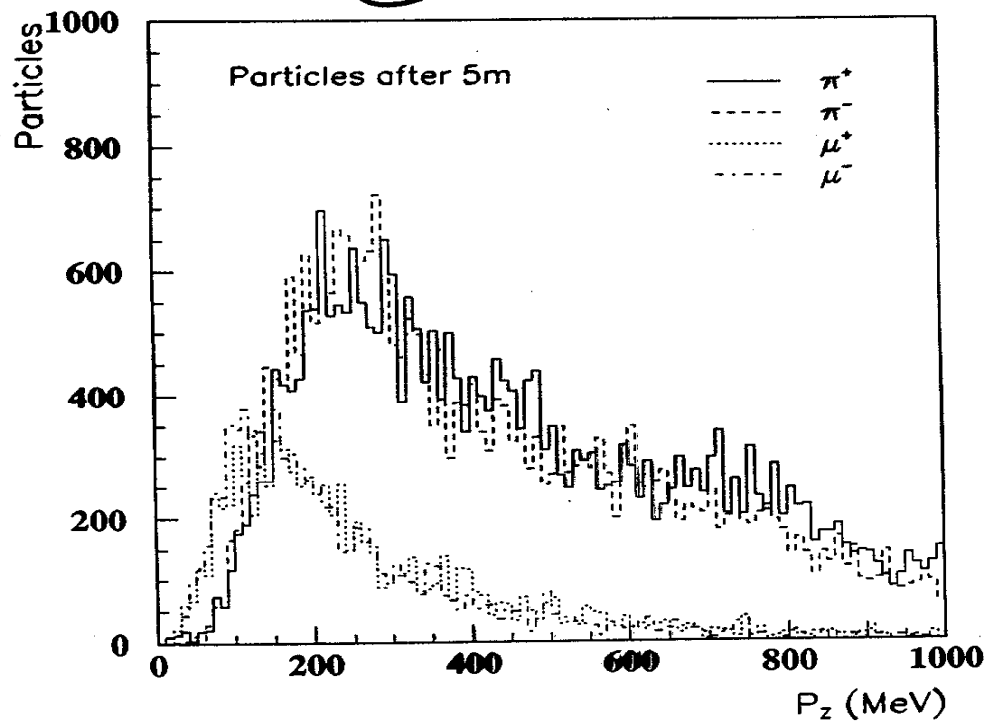
Mars3.16gev-hg-50.dat
 $\begin{matrix} \text{---} \\ E_p \end{matrix}$ $\begin{matrix} \text{---} \\ \text{target} \end{matrix}$ $\begin{matrix} \text{---} \\ 50\text{cm} \end{matrix}$ target

Not clear (to me) how many protons
generated (event number reaches 60k)

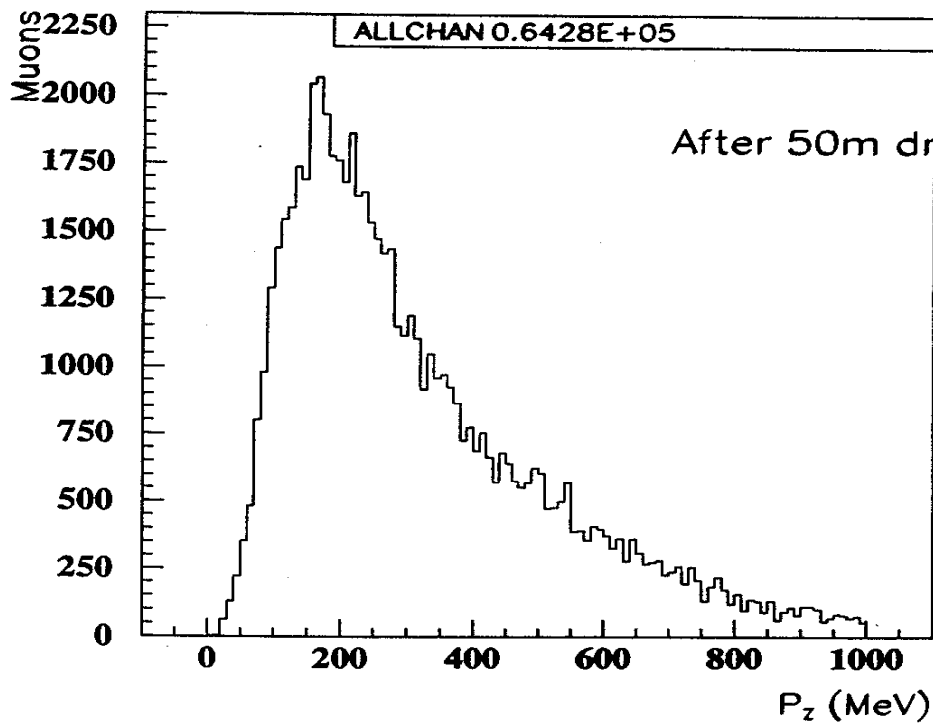
After 5m	π^+ 35000	π^- 32110	μ^+ 8348	μ^- 8465
50 m drift			34840 33820	33820 32780
phase rot ($P_z > 0$)			19080	18030
survive He drift (exit x,y or z = 200km)			7547	7393
Exit cooling channel at $T \approx 300 \text{ eV}$			4365	4866

\Rightarrow about 10% of initial π^\pm, μ^\pm result
in μ^\pm exiting He at nominal T

??
60 k 16 GeV Protons



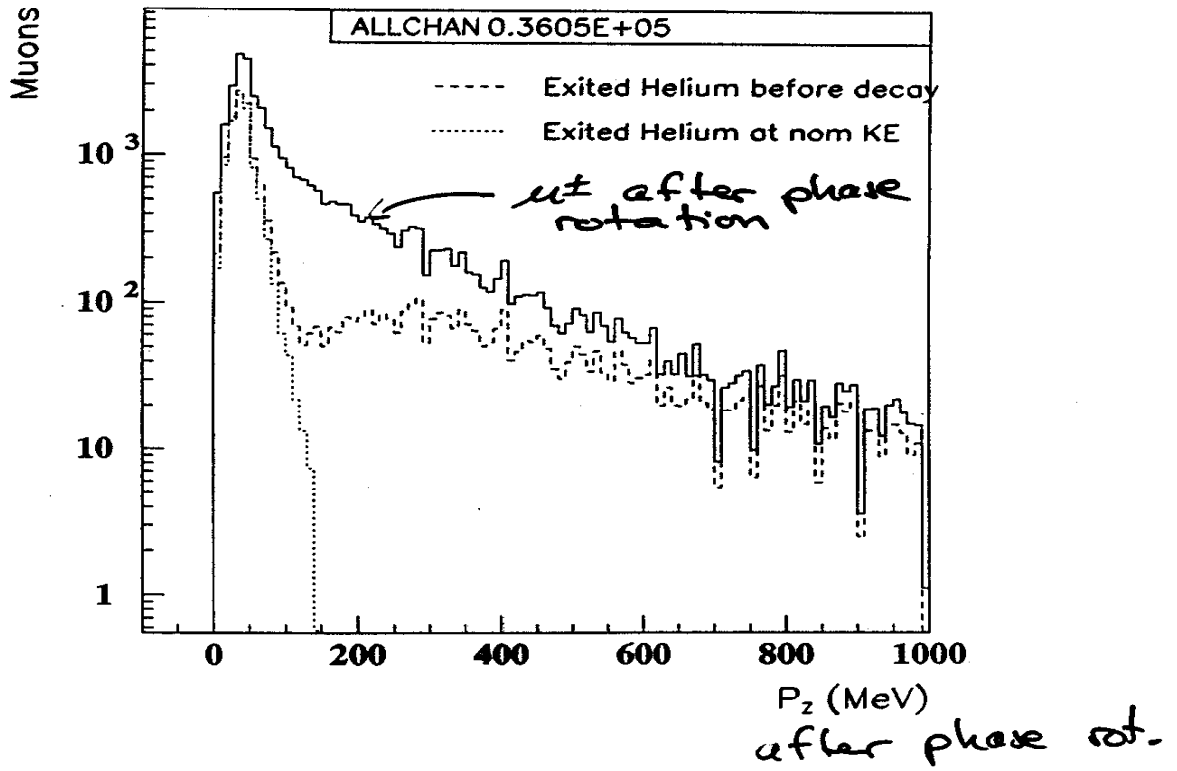
60 k 16 GeV Protons



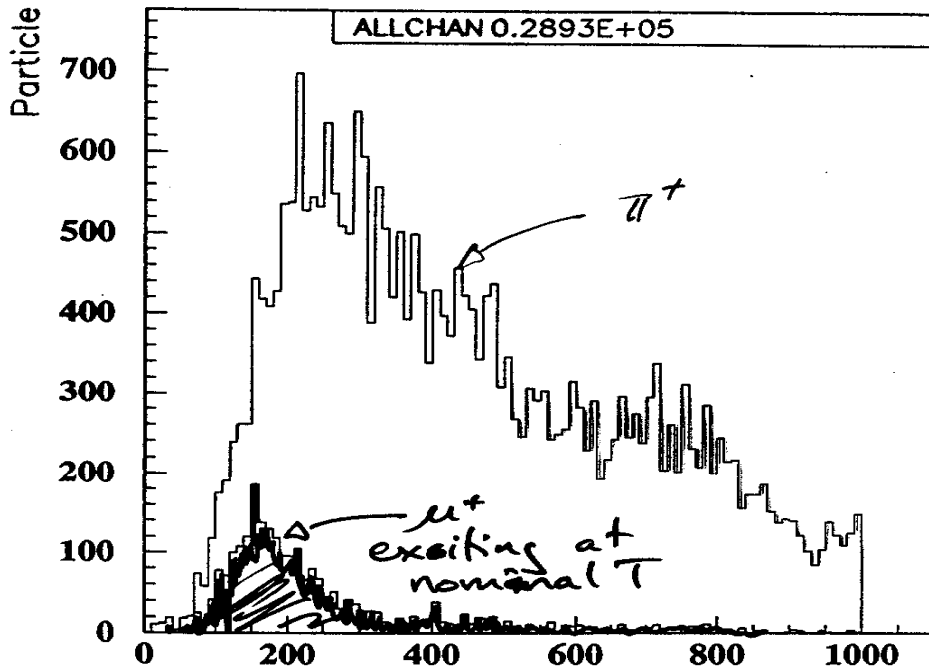
64 R

$\mu^+ + \mu^-$

60 k 16 GeV Protons



60 k 16 GeV Protons



P_z (MeV) initial momentum dist

Phase space reduction:

consider phase space of particles which eventually come out at $T = 300 \text{ eV}$.
Just for rough orders of magnitude.

initial $\sigma_x = \sigma_y \approx 2.5 \text{ cm}$ $\sigma_z = 138 \text{ cm}$ at $z = 5 \text{ m}$
 $\sigma_{p_x} = \sigma_{p_y} = 25 \text{ MeV}$ $\sigma_{p_z} = 125 \text{ MeV}$

final $\sigma_x \cdot \sigma_y = (5 \cdot 10^3 \text{ cm})(2.5 \text{ cm})$



$$\sigma_z = \sigma_t \cdot \beta \cdot c$$

$$\beta = \frac{p_z c}{m c^2} \approx 2 \cdot 10^{-3}$$

$$\sigma_t = 1 \cdot 10^{-6} \text{ s}$$

so $\sigma_z = 60 \text{ cm}$

$\sigma_{p_x} = \sigma_{p_y} = \sigma_{p_z} = ?$ $T \approx 50 \text{ eV}$
 $\rightarrow p \approx 20 \text{ keV}$

take for now $\sigma_p = 20 \text{ keV}$

exiting He channel not solved!

$$\frac{(\sigma_x \sigma_y \sigma_z \sigma_{p_x} \sigma_{p_y} \sigma_{p_z})_{\text{final}}}{(\sigma_x \sigma_y \sigma_z \sigma_{p_x} \sigma_{p_y} \sigma_{p_z})_{\text{initial}}} = 10^{-7} \quad \text{norm exitance}$$

Comments

- What are reasonable parameters for B, E ?
- How do we get μ^{\pm} out of He channel ?
- What happens after He channel ?
- μ^{\pm} capture cross sections

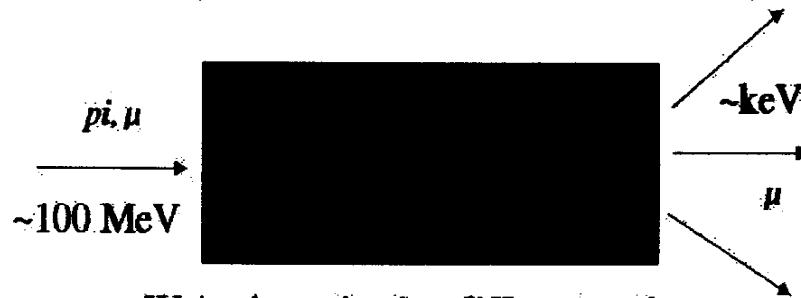
so far, looks intriguing. Please criticize and/or help!

Columbia Group Studies

L. Alhilali*, A. Boozer, A. Bozovic*, A. Caldwell, I. Shpiro*, F. Sciulli, W. Willis

*=student

Reducing the phase space occupied by the muons while keeping high efficiency is crucial for the success of a muon collider or a high intensity neutrino factory. The nominal scheme performs the cooling at $T=200$ MeV. We are investigating the possibility of reducing the phase space by bringing the muons to $T \sim 1$ keV and then reaccelerating.



We're investigating different options

Refs: K. Nagamine, At. Phys. 10, 225 (1987); K. Nagamine et al., Phys. Rev. Lett. 74, 4811 (1995).

E. Morenzoni, 'The physics and applications of low energy muons', lecture notes of SUSSP51, PSI- PR- 98- 23.

Notes:

Physics and applications of low energy muons*

Elvezio Morenzoni

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1 Introduction

Among the various nuclear solid state techniques, muon spin research (μ SR) is a relatively young technique. However, as with all fields of science, also in applied muon science novel methods and techniques have to be developed and the capabilities of the existing ones continuously extended in order to face the challenge offered by new objects of investigations. These developments in methodology are often driven by the needs of research communities. Conversely, the availability of new scientific instruments and methodologies leads to progress in scientific knowledge. An example is the discovery of the surface muon production mechanism (Pifer et al. 76). Surface muons, which originate from pions decaying at rest at the surface of a pion production target, offered decisive advantages with respect to the previously used muons obtained from the decay in flight of pions such as high degree of polarization, high stopping rate, and limited penetration depth (fraction of mm to mm) in relatively small specimens and gave a large impetus to μ SR.

In the last decade, thin films, nanomaterials, multilayered compounds, and generally objects of restricted dimensionality have emerged as critical elements in science and technology. Different methods and particle probes such as polarized electrons, neutrons, photons, and

* Lecture notes of SUSSP51, Scottish Universities Summer School in Physics, a NATO Advanced Study Institute on Muon Science, University of St. Andrews, Scotland UK, 17-28 August 1998