Proton FFAG Accelerator Work at Brookhaven National Laboratory

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Summary

- Design of a Proton FFAG Accelerator
- FFAG Accelerator for AGS Upgrade
- 1-GeV 10-MW FFAG Proton Driver
- FFAG Proton Driver for Neutrino Factory
- FFAG Medical Accelerator
- e-RHIC
- FFAG for Synchrotron Light Source
- FFAG Electron Model (for Protons)
- Acceleration by Harmonic-Number Jump
- RIA
To avoid the problem of frequency modulation for acceleration of low-energy beams over a too short period of time, and to boost acceleration rate.

The method allows the use of constant frequency acceleration using superconducting cavities, despite the fact that the beam velocity may vary considerably.

The accelerating voltage and RF phase need to be programmed accordingly.

We studied first the motion of Synchronous particles, and then of those with deviating initial conditions.

We estimated the area and height of the RF buckets that are to contain the beam bunches with the added condition of the HNJ.

We determined methods to create the program of energy gain as required by the HNJ method, including the effect of the cavity Transit Time Factor (TTF).
Assume the beam as a sequence of point-like bunches (synchronous, reference).

The energy gain is adjusted for a change in the travel period $T_n$ in the following arc so that the reference particle is pushed forward or back exactly by $\Delta h$ harmonics.

$$T_n = h_n T_{RF} \quad T_{n-1} = h_{n-1} T_{RF} \quad h_n - h_{n-1} = -\Delta h$$

$$\Delta E_n = \beta_n^2 \gamma_n^3 E_0 \frac{\Delta h}{h_n} (1 - \alpha_{pn} \gamma_n^2)$$

The ring is made of $N$ RF cavities equally spaced.

$E_n = \text{total energy}$

$T_n = h_n T_{RF}$

$\Delta E_n = (Q \frac{eV_n}{A}) \sin (\omega_{RF} t_n)$

$\quad = (Q \frac{eV_n}{A}) \sin (\phi_n)$
Any other particle

\[ t_n = t_n + \tau_n \quad \Delta E_n = (Q eV_n / A) \sin (\omega_{RF} t_n) \]

\[ \varepsilon_n = E_n - E_n \]

\[ \Delta \varepsilon_n = (Q eV_n / A) \left[ \sin (\phi_n + \omega_{RF} \tau_n) - \sin (\phi_n) \right] \]

\[ \sim (Q eV_n / A) \cos \phi_n \omega_{RF} \tau_n \]

\[ \Delta \tau_n = \tau_n - \tau_{n-1} \]

\[ = - (1 - \alpha_p \gamma_n^2) T_n \varepsilon_n / \beta_n^2 \gamma_n^3 E_0 \]

Small-Amplitude Oscillations

\[ \Delta^2 \tau_n / \Delta n^2 + \Omega_n^2 \tau_n = 0 \quad \text{with} \quad \Omega_n^2 = 2 \pi \Delta h / \tan \phi_n \]
RF Buckets with Harmonic-Number Jump

The Hamiltonian

\[ H = \left( \frac{Q \, e \, V_n}{A \, \omega_{RF}} \right) \left[ \cos \left( \phi_n + \omega_{RF} \tau_n \right) + \omega_{RF} \tau_n \sin \left( \phi_n \right) \right] + \]

\[ - \left( 1 - \alpha_{pn} \gamma_n^2 \right) T_n \epsilon_n^2 / \left( 2 \beta_n^2 \gamma_n^3 E_0 \right) \]

\[ \cos \left( \phi_n + \phi_{1,2n} \right) + \cos \left( \phi_n \right) + \]

\[ (\phi_{1,2n} - \pi + 2 \phi_n) \sin \left( \phi_n \right) = 0 \]
RF Buckets with Harmonic-Number Jump

Bucket Area

\[ B_n = \left( \frac{8}{w_{RF}} \right) \frac{2 Q e V_n b_n^2 g_n^3 E_0}{A \pi h_n (1 - a_{pn} g_n^2)} \right)^{1/2} I(\phi_{1n}, \phi_{2n}) \]

\[ I(\phi_{1n}, \phi_{2n}) = \int \left[ \cos(\phi_n + \phi) + \phi \sin(\phi_n) + G(\phi_n) \right]^{1/2} / 4 \sqrt{2} \ d\phi \]

\[ G(\phi_n) = \cos(\phi_n) - (\pi - 2 \phi_n) \sin(\phi_n) \]

Bucket Height

\[ \Delta^2 = 2 Q e V_n \beta_n^2 \gamma_n^3 E_0 F(\phi_n) / A \pi (1 - \alpha_{pn} \gamma_n^2) h_n \]

\[ F(\phi_n) = \cos(\phi_n) - (\pi/2 - \phi_n) \sin(\phi_n) \]
To avoid beam losses, the number of bunches ought to be less than the harmonic number at all time. On the other end, because of the change of the revolution period, the number of RF buckets will vary. There is a difference between the case of acceleration below and above transition energy. Below transition energy the beam extension at injection ought to be shorter than the revolution period. That is, the number of injected bunches cannot be larger than the RF harmonic number at extraction. The situation is different when the beam is injected above the transition energy. In this case the revolution period decreases and the harmonic number increases during acceleration.
Energy Gain Programming

Energy gain at the n-th cavity

$$\Delta E_n = eV_n \sin (\phi_n) = A \beta_n^2 \gamma_n^3 E_0 \Delta h / Q h_n (1 - \alpha_{pn} \gamma_n^2)$$

$$V_n = n_c g \xi_n \text{TTF}(\beta_0/\beta, n_c) \quad \text{TTF}(x, 1) = \sin(\pi x/2) / (\pi x/2)$$

$$g = \lambda \beta_0 / 2 \quad \xi_n = \text{average axial field}$$

Two Programming Methods:

1. Constant RF Phase $\phi_n$
   - It requires the design of a RF Cavity with proper radial field profile
2. Constant average axial Field $\xi_n$
   - It requires a RF phase modulation
Radio-Isotopes Acceleration (RIA)

We have studied the use of FFAG’s for the acceleration of U-238 with charge state +28 to produce radioisotopes and exotic nuclear fragments.

A.G. Ruggiero “AGS-less RIA with FFAG Accelerators”,
BNL Internal Report, C-A/AP 238, May 2006

Because of the large variation of the beam velocity in each ring, to avoid the use of ferrite or other techniques for RF modulation, we proposed acceleration with the method of Harmonic-Number Jump (HNJ).

The ion source is an ECR capable of 30mA-electric (CW). Only one turn needs to be injected. Multi-turn injection can be avoided as well methods of beam cooling.

Space charge tune depression is less than $\Delta\nu = 0.3$ with a betatron emittance of $5.0 \pi \text{mm-mrad}$ (full value, normalized).
### Outline of the Scheme

**Type of Ions**
- Uranium

**Charge State, Q**
- +28

**Mass Number, A**
- 238

**ECR current**
- 30 mA-electric

**Injector Linac Energy**
- 6 MeV/u

**Beam Bunching Frequency**
- 201.34 MHz

**Chopping Ratio**
- 80%

**Transmission Efficiency**
- 80%

**Injected Current**
- 20 mA-electric

**Linac Pulse Length**
- 4.13 µs

**Repetition Rate**
- 1,000 pulses/s

**Linac Duty Cycle**
- 0.413 %

**No. of Injected Turns**
- 1

**No. of Ions / Cycle**
- $1.8 \times 10^{10}$

**No. of Bunches**
- 831

**No. of Ions/Bunch**
- $2.13 \times 10^{7}$

**Norm. Emittance (full)**
- $5.0 \, \pi \, \text{mm-mrad}$

**Bunch Area (full)**
- $10 \, \mu\text{eV/u-s}$

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<table>
<thead>
<tr>
<th>FFAG-1</th>
<th>FFAG-2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inject.</strong></td>
<td><strong>Transfer</strong></td>
</tr>
<tr>
<td>Circumference, m</td>
<td>807.091</td>
</tr>
<tr>
<td>Energy, MeV/u</td>
<td>6</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.1126</td>
</tr>
<tr>
<td>Rev. Freq., MHz</td>
<td>0.0418</td>
</tr>
<tr>
<td>Rev. Period, µs</td>
<td>23.919</td>
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<tr>
<td>Harmonic No.</td>
<td>4816</td>
</tr>
<tr>
<td>$\Delta E$/ Cavity, MeV/u</td>
<td>0.0201</td>
</tr>
<tr>
<td>Circ. Current, mA-e</td>
<td>3.31</td>
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<tr>
<td>RF Power, MW</td>
<td>0.0159</td>
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<tr>
<td>Beam Power, kW</td>
<td>4.04</td>
</tr>
<tr>
<td>Bunching Factor</td>
<td>4</td>
</tr>
<tr>
<td>S. C. Tune-Shift</td>
<td>0.29</td>
</tr>
</tbody>
</table>
### FFAG Rings at Injection

<table>
<thead>
<tr>
<th>FFAG-1</th>
<th>FFAG-2</th>
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<tbody>
<tr>
<td><strong>Circumference, m</strong></td>
<td>807.091</td>
</tr>
<tr>
<td><strong>Periodicity</strong></td>
<td>136</td>
</tr>
<tr>
<td><strong>Period Length, m</strong></td>
<td>5.9345</td>
</tr>
<tr>
<td><strong>Long Drift S, m</strong></td>
<td>2.5345</td>
</tr>
<tr>
<td><strong>Short Drift g, m</strong></td>
<td>0.300</td>
</tr>
<tr>
<td><strong>B\rho, kG-m</strong></td>
<td>30.13</td>
</tr>
</tbody>
</table>

#### F-Sector Magnet
- **Length, L_F, m**: 0.700, 0.701
- **Bend Field, kG**: -0.7423, -2.1644
- **Gradient, kG/m**: 25.164, 73.2661

#### D-Sector Magnet
- **Length, L_D, m**: 1.400, 1.402
- **Bend Field, kG**: 1.7367, 5.0640
- **Gradient, kG/m**: 22.0533, -64.2089

**Graph**
- **\( \beta_H \)** \( \text{in m} \), **\( \beta_V \)** \( \text{in m} \), **10x \( \eta \)** \( \text{in m vs. dist. in m} \)

**Phase Advance / Period, H / V**: 105° / 100°
**Betatron Tunes H / V**: 39.76 / 37.75
**Transition Energy, \( \gamma_T \)**: -i105.5
**Max \( \beta \) value, H / V, m**: 4.9 / 11.8
**Max dispersion, \( \eta \)**: 6.0 cm
**Chromaticity, H / V**: -0.925 / 1.814
Non-Scaling Lattice with Linear Profile

Magnetic Field $B$ in kG vs. Radial Position $x$ in cm

Momentum Closed Orbits $x$ in cm vs. Path Length $s$ in meter across one Period
# RF Cavities Parameters

<table>
<thead>
<tr>
<th></th>
<th>FFAG-1</th>
<th>FFAG-2</th>
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</thead>
<tbody>
<tr>
<td><strong>Type</strong></td>
<td>Superconducting</td>
<td></td>
</tr>
<tr>
<td><strong>Elliptical Cells</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>π-mode</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>RF 201.34 MHz</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Number of Cavities /Ring</strong></td>
<td>8 equally spaced</td>
<td>3 equally spaced</td>
</tr>
<tr>
<td><strong>Number of Cells / Cavity</strong></td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td><strong>Reference β Value, β₀</strong></td>
<td>0.196377</td>
<td>0.314049</td>
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<tr>
<td><strong>Cavity Cell Gap, cm</strong></td>
<td>14.6267</td>
<td>35.5402</td>
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<tr>
<td><strong>Cavity Diameter, cm</strong></td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td><strong>Cavity Length, m</strong></td>
<td>2.0</td>
<td>2.0</td>
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<tr>
<td><strong>Harmonic Number Jump, Δh</strong></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>RF Phase, degrees</strong></td>
<td>11.95 - 60</td>
<td>4.02 - 60</td>
</tr>
<tr>
<td><strong>Average Axial Field, MV/m</strong></td>
<td>9.792</td>
<td>25.102</td>
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<tr>
<td><strong>Acceleration Period, ms</strong></td>
<td>0.787</td>
<td>0.637</td>
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<tr>
<td><strong>Number of Cavity Crossings</strong></td>
<td>388</td>
<td>301</td>
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<tr>
<td><strong>Number of Revolutions</strong></td>
<td>50</td>
<td>100</td>
</tr>
</tbody>
</table>

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Constant RF Phase

Average Axial Field in MVolt/m vs. Radial Position x in cm

FFAG-1

FFAG-2
Acceleration by HNJ

$\Delta E_n$ in MeV/u

$\phi_n$ in degrees

$h_n$

vs. no. of cavity crossings $n$
RF Buckets Height and Area

FFAG-1

\[ \Delta \frac{p}{p} \text{ in } \% \]

Number of Cavity Crossings, \( n \)

FFAG-2

\[ \Delta \frac{p}{p} \text{ in } \% \]

Number of Cavity Crossings, \( n \)