Strange form factors in $\nu$ scattering

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- Looking for strangeness with neutrino scattering
- Interesting observables
- Elastic NC $\nu$–nucleon scattering
- The neutrino asymmetry
- The BNL - 734 experiment
- Future perspectives: Fermilab
- Elastic scattering on $S = T = 0$ nuclei
- Comments and conclusions
The measurement of NC neutrino cross sections

\[ \nu_\mu(\bar{\nu}_\mu) + N \rightarrow \nu_\mu(\bar{\nu}_\mu) + N \]  \hspace{1cm} (1)

is very important tool for the determination of
the matrix elements of the strange current:

\[ \langle p, s | \bar{S}\gamma^\alpha\gamma^5 S|p, s \rangle = 2M s^\alpha g^s_A \]

\(S, \bar{S}\) strange quark fields
\(|p, s\rangle\) proton (momentum, spin) state vector.

**CC processes** also considered:

\[ \nu_\mu + n \rightarrow \mu^- + p , \]
\[ \bar{\nu}_\mu + p \rightarrow \mu^+ + n . \] \hspace{1cm} (2)

Nucleon currents involved:

\[ J^Z_\alpha = V^3_\alpha + A^3_\alpha - 2 \sin^2 \theta_W J^{em}_\alpha - \frac{1}{2} V^s_\alpha - \frac{1}{2} A^s_\alpha \]

\[ J^W_\alpha = V_{ud}\langle N | \bar{U}\gamma_\alpha(1 + \gamma_5)D | N \rangle \]
**Weak neutral current and s̅s content**

One nucleon matrix element of axial quark current:

\[
\langle p, s | \bar{q} \gamma^\alpha \gamma^5 q | p, s \rangle = 2 M s^\alpha g_A^q
\]

constants \(g_A^u, g_A^d, g_A^s\) determined from:

- **QCD sum rule** (polarized structure function)

\[
\Gamma_1^p = \int_0^1 dx g_1^p(x) = \frac{1}{2} \left( \frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right)
\]

- relation \(g_A = g_A^u - g_A^d\)

with \(g_A = 1.2573 \pm 0.0028\) from neutron decay

- relation \(3F - D = g_A^u + g_A^d - 2g_A^s\)

\(F, D\) from semileptonic decay of hyperons.

Determination of various \(g_A^q\) subject to several assumptions (small \(x\) extrapolation, QCD corrections, SU(3) invariance, etc.)
Notations

One nucleon matrix elements of the vector and axial NC:

\[ p(n) \langle p' | V^{NC}_\alpha | p \rangle_{p(n)} = \overline{u}(p') \left[ \gamma_\alpha F_1^{NC;p(n)}(Q^2) + \right. \]
\[ \left. + \frac{i}{2M} \sigma_{\alpha \beta} q^\beta F_2^{NC;p(n)}(Q^2) \right] u(p) \]
\[ p(n) \langle p' | A^{NC}_\alpha | p \rangle_{p(n)} = \overline{u}(p') \gamma_\alpha \gamma_5 G_A^{NC;p(n)} u(p) \]

where the NC form factors are given by

\[ F_{1,2}^{NC;p(n)}(Q^2) = \pm \frac{1}{2} \left\{ F_{1,2}^p(Q^2) - F_{1,2}^n(Q^2) \right\} - \]
\[ -2 \sin^2 \theta_W F_{1,2}^{p(n)}(Q^2) - \frac{1}{2} F_{1,2}^s(Q^2) \]
\[ G_A^{NC;p(n)}(Q^2) = \pm \frac{1}{2} G_A(Q^2) - \frac{1}{2} G_A^s(Q^2) \]

Equivalently, NC Sachs form factors are used:

\[ G_E^{NC;p(n)}(Q^2) = \pm \frac{1}{2} \left\{ G_E^p(Q^2) - G_E^n(Q^2) \right\} - \]
\[ -2 \sin^2 \theta_W G_E^{p(n)}(Q^2) - \frac{1}{2} G_E^s(Q^2) \]
\[ G_M^{NC;p(n)}(Q^2) = \pm \frac{1}{2} \left\{ G_M^p(Q^2) - G_M^n(Q^2) \right\} - \]
\[ -2 \sin^2 \theta_W G_M^{p(n)}(Q^2) - \frac{1}{2} G_M^s(Q^2) \]
Interesting Observables

Consider $\nu$–proton elastic cross sections or $\nu$–nucleus elastic and inelastic cross sections

NC over CC ratio:

$$R_{NC/CC}(Q^2) = \frac{(d\sigma/dQ^2)^{NC}_\nu}{(d\sigma/dQ^2)^{CC}_\nu}$$

(3)

Asymmetry:

$$A(Q^2) = \frac{\left(\frac{d\sigma}{dQ^2}\right)^{NC}_\nu - \left(\frac{d\sigma}{dQ^2}\right)^{NC}_{\bar{\nu}}}{\left(\frac{d\sigma}{dQ^2}\right)^{CC}_\nu - \left(\frac{d\sigma}{dQ^2}\right)^{CC}_{\bar{\nu}}}$$

(4)

Ratio p/n

$$R^\nu_{p/n}(Q^2) = \frac{\left(\frac{d\sigma}{dQ^2}\right)^{NC}_{(\nu,p)}}{\left(\frac{d\sigma}{dQ^2}\right)^{NC}_{(\nu,n)}}$$

(5)
Elastic NC $\nu$–nucleon scattering

Differential cross sections:

$$\left( \frac{d\sigma}{dQ^2} \right)_{\nu(\bar{\nu})}^{NC} = \frac{G_F^2}{2\pi} \left[ \frac{1}{2} y^2 (G_M^{NC})^2 + \right.$$

$$+ \left( 1 - y - \frac{M}{2E} y \right) \frac{(G_E^{NC})^2 + \frac{E}{2M} y (G_M^{NC})^2}{1 + \frac{E}{2M} y}$$

$$+ \left( \frac{1}{2} y^2 + 1 - y + \frac{M}{2E} y \right) (G_A^{NC})^2$$

$$\pm 2y \left( 1 - \frac{1}{2} y \right) G_M^{NC} G_A^{NC} \right].$$

with

$$y = \frac{\mathbf{p} \cdot \mathbf{q}}{\mathbf{p} \cdot \mathbf{k}} = \frac{Q^2}{2p \cdot k}$$

$E$ is the energy of neutrino (antineutrino) in the laboratory system.

Note
To obtain information on the strange form factors it is compulsory to know the axial CC form factors $G_A$ (via charge-exchange quasi–elastic processes).
The $\nu - \bar{\nu}$ asymmetry

The neutrino-antineutrino asymmetry in $\nu(\bar{\nu})$-nucleon elastic scattering reads:

$$A_{p(n)} = \frac{1}{4} \left( \pm 1 - \frac{G_A^s}{G_A} \right) \times$$

$$\times \left( \pm 1 - 2 \sin^2 \theta_W \frac{G_M^{p(n)}}{G_M^3} - \frac{1}{2} \frac{G_M^s}{G_M^3} \right).$$

Thus, in the asymmetry $A$ the strange axial and vector form factors enter in the form of ratios, $G_A^s / G_A$ and $G_M^s / G_M^3$.

Taking into account only terms which linearly depend on the strange form factors:

$$A_{p(n)} = A_{p(n)}^0 \mp \frac{1}{8} \frac{G_M^s}{G_M^3} \mp \frac{G_A^s}{G_A} A_{p(n)}^0$$

with

$$A_{p(n)}^0 = \frac{1}{4} \left( 1 \mp 2 \sin^2 \theta_W \frac{G_M^{p(n)}}{G_M^3} \right)$$
Fig. 1 – Plot of $4|V_{ud}|^2 A_p$ as a function of $Q^2$. The shadowed area corresponds to the present uncertainty in the magnetic form factors. The other curves were obtained using a dipole form for $F_A^s(Q^2)$ with $g_A^s = -0.15$. The dashed (dotted) curve was obtained with $G_M^s(Q^2) = 0$ utilizing our fit (respectively, the WT2 fit) for the magnetic FF of the nucleon. The solid (dot–dashed) line was analogously obtained, setting $\mu_s = -0.3$. 
Fig. 2 — Flux averaged differential cross sections measured by Ahrens et al., PRD35 (1987). Solid curves are the best fit to the combined data using $M_A = 1.06$ GeV and $\sin^2 W = 0.220$. The same data are compatible (at 90% CL) with $-0.25 \leq G^s_A(0) \leq 0$ if the axial cutoff is constrained to $M_A = 1.032 \pm 0.036$ GeV.
Information about the above mentioned ratios has been obtained by the BNL - 734 experiments. They measured:

\[
R_\nu = \frac{\langle \sigma \rangle_{(\nu p \rightarrow \nu p)}}{\langle \sigma \rangle_{(\nu n \rightarrow \mu^- p)}} = 0.153 \pm 0.007 \pm 0.017
\]

\[
R_{\nu} = \frac{\langle \sigma \rangle_{(\bar{\nu} p \rightarrow \bar{\nu} p)}}{\langle \sigma \rangle_{(\bar{\nu} p \rightarrow \mu^+ n)}} = 0.218 \pm 0.012 \pm 0.023
\]

\[
R = \frac{\langle \sigma \rangle_{(\bar{\nu} p \rightarrow \bar{\nu} p)}}{\langle \sigma \rangle_{(\nu p \rightarrow \nu p)}} = 0.302 \pm 0.019 \pm 0.037
\]

where \( \langle \sigma \rangle_{\nu(\bar{\nu})} \) is a total cross section integrated over the incident neutrino (antineutrino) energy and weighted by the \( \nu(\bar{\nu}) \) flux. The first error is statistical and the second is the systematic one. In terms of these ratios, the “integrated” asymmetry reads:

\[
\langle A_p \rangle = \frac{R_\nu(1 - R)}{1 - RR_\nu/R_{\nu}}
\]

and from the experimental data we found

\[
\langle A_p \rangle = 0.136 \pm 0.008(\text{stat}) \pm 0.019(\text{syst})
\]
Fig. 3 – The ratios $R_\nu$ and $R_{\bar{\nu}}$, $R$ and $\langle A_p \rangle$ versus $\mu_s$. Both for $g_A^s = 0$ and $g_A^s = -0.15$ three choices of $\rho_s$ are shown: $\rho_s = 0$ (solid), $\rho_s = -2$ (dot–dashed) and $\rho_s = +2$ (dashed). The shadowed regions correspond to the experimental band.
Fig. 4 – The ratios $R_\nu$, $R_{\bar{\nu}}$, $R$ and $\langle A_p \rangle$ versus $g_A^s$: the sensitivity to $M_A$ is shown: $M_A = 1.032$ GeV (solid), $M_A = 1.068$ GeV (dashed) and $M_A = 0.996$ GeV (dot–dashed). Here $\mu_s = \rho_s = 0$. 
Fig. 5 – Ratio of the integrated inelastic $\nu$ ($\bar{\nu}$)–nucleus scattering cross sections, with emission (and detection) of a proton or neutron, calculated in RFG. $E_{\nu(\bar{\nu})} = 1$ GeV.
We have considered, for the NuMi low-energy neutrino flux:

- The ratio of NC and CC elastic $\nu p$ scattering

1. It is sensitive to $g_A^s$, but not much affected by the cutoff mass of the axial form factors, assumed in the dipole form:

$$G_A(Q^2) = \frac{1.26}{(1 + Q^2/M_A^2)}, \quad G_A^s(Q^2) = \frac{g_A^s}{(1 + Q^2/M_A^2)}$$

2. Different parameterizations of the e.m. form factors do not sensibly affect the ratio

3. The interference between axial and vector strange form factors (in particular: magnetic strange ff) can hinder the effect of $g_A^s$ alone.

4. The sensitivity to the flux is negligible, because of ratio

5. Nuclear effects (again negligible, because of ratio)

- The ratio of NC and CC elastic $\bar{\nu} p$ scattering (if possible, gives great complementary information)
• The ratio of NC and CC elastic \( \nu p \) integrated cross sections
  (integration over \( Q^2 \) between 0.15 and 0.64 GeV\(^2\).)

  The ratio \( \sigma_{\nu p}^{NC}/\sigma_{\nu}^{CC} \) as a function of \( g_A^s \), shows the correlation with other effects, e.g.:
  1. Different values of \( M_A \)
  2. Different values of \( \mu_s \)
  3. Different cutoff in strange ff

• The ratio of proton to neutron yield in quasi-elastic neutrino scattering on \( ^{12}C \)
  1. Sensitivity to \( g_A^s \)
  2. Interference between axial and vector strange ff
  3. Elastic versus relativistic Fermi gas
Fig. 6 – Ratio of elastic NC/CC $\nu$-proton scattering: effects of the axial strange form factor
Fig. 7 – Sensitivity to the parameterization of electromagnetic form factors.
Fig. 8 – Sensitivity to the vector strange form factors

\[ R_{NC/CC}(Q^2) \text{ neutrino--proton} \]

\[ Q^2 (\text{GeV}^2) \]

le beam

\[ M_A=1.026 \text{ GeV} \]

\[ g_A^s = 0 \]

\[ g_A^s = -0.12 \]

\[ \mu_s = 0.44, \rho_s = 0 \]

\[ \mu_s = -0.42, \rho_s = 0 \]
Fig. 9 – Example of the ratio of $\bar{\nu}$–proton cross sections.

\[ R_{\text{NC/CC}}(Q^2) \text{ antineutrino--proton} \]

- $g_A = -0.12$
- $g_A = 0$

$M_A(\text{GeV}) = 1.047$
- $= 1.026$
- $= 1.005$
Fig. 10 – Sensitivity of $\bar{\nu}$–proton to vector strange form factors
Fig. 11 – Ratio of total cross sections versus $g_A^s$: sensitivity to $\mu_s$
Fig. 12 – Ratio of total cross sections versus $g_A^s$: elastic versus inelastic on $^{12}$C.
Fig. 13 – Ratio of proton to neutron yields in quasi-elastic NC neutrino scattering on $^{12}$C
Fig. 14 – Ratio of proton to neutron yields in quasi-elastic NC neutrino scattering on $^{12}$C: sensitivity to $\mu_s$
**Elastic scattering on \( S = T = 0 \) nuclei**

Consider the processes

\[
\nu (\bar{\nu}) + A \longrightarrow \nu (\bar{\nu}) + A
\]

The axial current, \( A_{\alpha}^{NC} \), and the isovector part of the vector NC, \( V_{\alpha}^3 (1 - 2\sin^2 \theta_W) \), do not contribute. Cross sections are given by:

\[
\frac{d\sigma_{\nu}}{dQ^2} = \frac{d\sigma_{\bar{\nu}}}{dQ^2} = \frac{G_F^2}{2\pi} \left( 1 - \frac{p \cdot q}{M_A E} - \frac{Q^2}{4E^2} \right) \left[ F^{NC}(Q^2) \right]^2
\]

with

\[
F^{NC}(Q^2) = -2\sin^2 \theta_W F(Q^2) - \frac{1}{2} F^s(Q^2)
\]

The e.m. FF of the nucleus \( F(Q^2) \) can be determined from elastic scattering of unpolarized electrons:

\[
\frac{d\sigma_e}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left( 1 - \frac{p \cdot q}{M_A E} - \frac{Q^2}{4E^2} \right) \left[ F(Q^2) \right]^2.
\]

Hence the strange form factor of the nucleus can be obtained from measurable cross sections:

\[
F^s(Q^2) = \pm 2F(Q^2) \times \frac{2\sqrt{2\pi\alpha}}{G_F Q^2} \sqrt{\frac{d\sigma_{\nu}/dQ^2}{d\sigma_e/dQ^2}} \mp 2\sin^2 \theta_W
\]
or, equivalently:

\[ F^s(Q^2) = \pm 2 \frac{1}{\sqrt{1 - \frac{p\cdot q}{M_A E} - \frac{Q^2}{4E^2}}} \times \]

\[ \times \left\{ \sqrt{\frac{2\pi}{G^2_F}} \frac{d\sigma_\nu}{dQ^2} \pm 2\sin^2\theta_W \sqrt{\frac{Q^4}{4\pi\alpha^2}} \frac{d\sigma_e}{dQ^2} \right\}. \]

The observation of the process of the scattering of neutrino on nuclei requires the measurement of the small recoil energy of the final nucleus. It could be easier to detect the process of scattering of neutrinos and electrons on nuclei if the nucleus undergoes a transition to excited states, for example:

\[ \nu + {}^4\text{He} \rightarrow \nu + {}^4\text{He}^* \]

\[ e + {}^4\text{He} \rightarrow e + {}^4\text{He}^* \]

where \(^4\text{He}^*\) is the excited state of \(^4\text{He}\) with with \(S = 0\) and \(T = 0\) and excitation energy of 20.1 MeV. This state can decay into \(p\) and radioactive \(^3\text{H}\).
Conclusions

- The experiments of $\nu$-proton NC and CC scattering are highly interesting for the determination of $\Delta s \equiv g_A^s$.

- Problems of interference with strange vector form factors can be resolved by complementary experiments (PV electron scattering).

- If feasible, $\bar{\nu}$ scattering would offer relevant and complementary information and:

- would allow the determination of the neutrino asymmetry (a unique tool for unambiguous determination of $\Delta s$)

For reference, see:

Fig. 15 – Sensitivity to flux.

$R_{NC/CC}(Q^2)$ neutrino--proton flux averaged, le beam

- $E_\nu(\text{GeV}) = 1.5$
- $E_\nu(\text{GeV}) = 3$
- $E_\nu(\text{GeV}) = 7$
Fig. 16 – Elastic versus Fermi gas (for $^{12}\text{C}$, with $p_F = 225 \text{ MeV}$, $\epsilon_B = 25 \text{ MeV}$)
Fig. 17 – Ratio of total cross sections versus $g_A^s$: sensitivity to the axial cutoff
Fig. 18 – Ratio of total cross sections versus $g_A^s$: sensitivity to strange axial cutoff
Fig. 19 – Ratio of proton to neutron yields in quasi-elastic NC neutrino scattering on $^{12}\text{C}$: “elastic” versus RFG
Fig. 20 –Schematic representation for the amplitude, in Born approximation, of the neutrino–nucleus scattering.