Decay Channel Optimization using MARS

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From work in collaboration with C. Johnstone and MARS (i.e., Nikolia Mokhov)

Fermi National Accelerator Laboratory
Pion/Muon Production Overview:

- Choosing a target...
  ...Mercury jet (high density, but difficult to work with)
  ...Graphite rod (lower density, but easy to work with)
  ...other possibilities?

- The pion decay channel...
  ...Dominant decay mode ($\sim 100\%$):
    \[
    \pi^+ \rightarrow \mu^+ \nu_\mu \\
    \pi^- \rightarrow \mu^- \bar{\nu}_\mu
    \]
  ...Lifetime: $\tau = 26$ ns (or $c\tau = 7.8$ m)
  ...After a 40 m decay channel:
    $\sim 99\%$ of pions with $p = 150$ MeV/c decay
    $\sim 95\%$ of pions with $p = 250$ MeV/c decay
...Use solenoids to contain the beam

- Matching sections...
  
  ...To match from target region into decay channel, use adiabatic magnetic matching sections

...Use something else to match into the buncher/linac

(see C. Johnstone)
The Target Region:

- Using Feasibility Study I target design...

  ...Graphite rod (80 cm long, 1.5 cm diamater) at a 50 mrad angle w.r.t. central axis (parallel to incident proton beam)

  ...1 MW incident proton beam power with 16 GeV protons in pulses at 15 Hz

  ...Allows for $\sim 5 \times 10^{12}$ muons per pulse if $\sim 0.2$ muons produced per proton on target

  ...Currently, leaving target design unchanged
• “Large” capture solenoid...

...Chosen to capture pions of given $p_T$,

$$(p_T)_{MAX} = e B_0 \left( \frac{R_0}{2} \right)$$

where $B_0$ is the strength of the capture solenoid, and $R_0$ is the radius of the beampipe (one-half the full aperture) at the target ($z = 0$ cm)

...Aperture limited to 60 cm at the end of the decay channel (beginning of buncher/linac)

...Field strength in buncher set at $B = 1.25$ T

...Hence, magnetic flux fixed at the end of the decay channel:

$$\Phi \approx B (\pi R^2) = 0.353 \text{ Wb}$$
The capture solenoid...

...By matching the sections adiabatically, magnetic flux is conserved throughout the decay channel:

$$\Phi_0 \approx B_0 (\pi R_0^2) = \Phi$$

...Hence, increasing the captured pion $\langle p_T \rangle_{MAX}$ means decreasing the initial aperture:

$$\langle p_T \rangle_{MAX} = \frac{e \Phi}{2\pi R_0}$$

...How high should $\langle p_T \rangle_{MAX}$ be?
...How about \( \sim 225 \) MeV/c?
• The capture solenoid (continued)...

...With the choice \((p_T)_{MAX} = 225\) MeV/c, we find

\[
R_0 = \frac{e \Phi}{2\pi (p_T)_{MAX}} = 7.5 \text{ cm}
\]

\[
B_0 = \frac{\Phi}{\pi R_0^2} = 20 \text{ T}
\]

...Strong! But is it too strong? Let’s hope not...

...If too strong, then only option is magnetic horns?
Matching Sections:

- How to match solenoids into solenoids...

  ...Use solenoids!

  ...Simultaneously change field strength and aperture with $z$, keeping the total flux through the beampipe fixed:

  \[ B_0 R_0^2 = B(z) R(z)^2 \]

  ...Change must be “slow” in order to be adiabatic:

  \[ a \ll R_B, \ B \left( \frac{\partial B}{\partial z} \right)^{-1} \]

  where $a$ is the particle’s Larmor radius of orbit, and $R_B$ is the radius of curvature of the field lines
...Design half-aperture, $R(z)$, with these constraints and fit $B(z)$ accordingly (using short solenoids) such that:

$$B(z) = B_0 \left( \frac{R_0}{R(z)} \right)^2$$

is the field strength on axis.
• Adiabatic matching sections...

...Should hold pions with $P_T < 225$ MeV/c

...Easily understood in terms of the adiabatic invariants:

$$B a^2 \quad \frac{p_T^2}{B}$$

(See Jackson, Sect. 12.6)

...As the particle moves along $z$, $B$ decreases, $p_T$ decreases and $a$ increases

...Since kinetic energy is conserved, the longitudinal momentum $p_z$ increases

**BOTTOM LINE:**

It decreases the divergence of the beam at the cost of increased spot size!
• Designing adiabatic matching sections...

...Determine constraints on aperture of adiabatic region:

\[ R(z_1) \equiv R_1 \quad \left( \frac{\partial R}{\partial z} \right) \Big|_{z_1} \equiv \lambda_1 \]

\[ R(z_2) \equiv R_2 \quad \left( \frac{\partial R}{\partial z} \right) \Big|_{z_2} \equiv \lambda_2 \]

...Simple choice for \( R(z) \):

\[ R(z) \equiv \left( \alpha_0 + \alpha_1 z + \alpha_2 z^2 + \alpha_3 z^3 \right)^{\frac{1}{k}} \]

Solve for \( \alpha_i(k) \)'s

...Easy! But how do we choose \( k \)?
Minimize curvature & maximize length!
(i.e., choose \( k \approx 1 \))
...Consider a “short” section (240 cm):

...And a “long” section (720 cm):
...Distributions at the end of the sections include losses due to pion decay

...Scale them according to the pion’s momentum!
Scaled momentum distributions in short section:

![Graph showing pion momentum distributions over MeV/c](image-url)
...Scaled momentum distributions in long section:
...Scaled radial distributions:
Scaled angular distributions:
\textbf{Final Muon Yields for } p = 100 – 350 \text{ MeV/c:}

<table>
<thead>
<tr>
<th></th>
<th>SHORT (240 cm)</th>
<th>LONG (720 cm)</th>
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</thead>
<tbody>
<tr>
<td>$\mu^+/P$</td>
<td>0.163</td>
<td>0.181</td>
</tr>
<tr>
<td>$\mu^−/P$</td>
<td>0.154</td>
<td>0.170</td>
</tr>
<tr>
<td>$\mu^{±}/P$</td>
<td>0.317</td>
<td>0.352</td>
</tr>
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</table>

(SHORT section equivalent to FS1 geometry)

\textbf{RMS emittance for various } p \pm 20 \text{ MeV/c:}

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<td>1.78\pi \text{ cm rad}</td>
<td>1.76\pi \text{ cm rad}</td>
</tr>
<tr>
<td>200 MeV/c</td>
<td>1.36\pi \text{ cm rad}</td>
<td>1.38\pi \text{ cm rad}</td>
</tr>
<tr>
<td>300 MeV/c</td>
<td>.941\pi \text{ cm rad}</td>
<td>.911\pi \text{ cm rad}</td>
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</table>
• Further improvements...

...In the decay channel:

\[(p_T)_{MAX} \sim 28 \text{ MeV/c}\]

...During pion decay:

\[
\langle p_T \rangle \sim \frac{1}{2} m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \langle \sin^2 \theta \rangle
\]

\[
= \frac{1}{4} m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)
\]

\[\approx 15 \text{ MeV/c}\]

...Increase \((p_T)_{MAX} \rightarrow\) Increase \(B\)

...In a 5 T decay channel:

\[(p_T)_{MAX} \sim 56 \text{ MeV/c}\]

with a 30 cm (\(R = 15 \text{ cm}\)) aperture
...A “short” initial section (240 cm):

...Or a “long” initial section (720 cm):
...Follow with a 47 m decay channel, with a field strength of 5 T

...End with another adiabatic matching section, matching from 5 T to 1.25 T:

...Called a DUET configuration
...Final Muon Yields for $p = 100 – 350$ MeV/c:

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<td>$\mu^+/P$</td>
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<td>0.181</td>
<td>0.192</td>
<td>0.193</td>
</tr>
<tr>
<td>$\mu^-/P$</td>
<td>0.154</td>
<td>0.170</td>
<td>0.183</td>
<td>0.185</td>
</tr>
<tr>
<td>$\mu^\pm/P$</td>
<td>0.317</td>
<td>0.352</td>
<td>0.375</td>
<td>0.378</td>
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...RMS emittance for various $p \pm 20$ MeV/c:

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<td>1.78\pi cm rad</td>
<td>1.76\pi cm rad</td>
<td>1.42\pi cm rad</td>
<td>1.30\pi cm rad</td>
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<td>1.36\pi cm rad</td>
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<td>.908\pi cm rad</td>
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• Conclusions:

...Losses mostly due to pion decay

...Increasing decay channel field strength significantly increases yields

...Adiabaticity effects are large for low field strength channels

...Phase space density depends:

○ Weakly on adiabatic effects

○ Strongly on channel field strength (for low $p$, but not for high $p$)

...Improved carbon target yields by 18% over FS1 results (for both $\mu^+$ and $\mu^-$)
• Future work:

... “Orange”-type capture magnet
    (Only captures one sign)

... Compare to horn & funnel designs
    (Only captures one sign, though)

... “Real” solenoid designs & fields
Initial momentum distribution:
Initial radial distributions:

Number of Pions per Unit Annulus (per Proton on Target)

Distance from Central Axis (cm)

- 200 MeV/c
- 300 MeV/c
- 400 MeV/c
Initial angular distributions:
Final momentum distributions:
Final radial distributions:
Final angular distributions:
Final DUET momentum distributions:
Final DUET radial distributions:

![Graph showing number of muons per unit annulus (per proton on target) versus distance from central axis (cm). The graph includes data for different momenta: 200 MeV/c (240 cm), 200 MeV/c (720 cm), 400 MeV/c (240 cm), 400 MeV/c (720 cm).]
Final DUET angular distributions:

Number of Muons per Unit Solid Angle (per Proton on Target) vs. Angle from Central Axis (radian) for 200 MeV/c (240 cm), 200 MeV/c (720 cm), 400 MeV/c (240 cm), and 400 MeV/c (720 cm).
### Final Muon Yields ($p = 100 – 350$ MeV/c):

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<td>$\mu^+/P$</td>
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<td>0.1814</td>
<td>0.1922</td>
<td>0.1931</td>
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<tr>
<td>$\mu^-/P$</td>
<td>0.1535</td>
<td>0.1701</td>
<td>0.1827</td>
<td>0.1851</td>
</tr>
<tr>
<td>$\mu^{\pm}/P$</td>
<td>0.3165</td>
<td>0.3515</td>
<td>0.3749</td>
<td>0.3781</td>
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**Mean & RMS Emittance \((p \pm 20 \text{ MeV/c})\):**

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<td>(1.757\pi \text{ cm rad})</td>
<td>(1.418\pi \text{ cm rad})</td>
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<tr>
<td>100 MeV/c</td>
<td>(1.077\pi \text{ cm rad})</td>
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<td>(1.666\pi \text{ cm rad})</td>
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<td>(.8719\pi \text{ cm rad})</td>
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<td>(.8438\pi \text{ cm rad})</td>
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