A Future Muon $(g-2)$ Experiment

at a High Flux Muon Facility

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Outline of the Talk

- Introduction to muon $(g-2)$
- The theory, including beyond the SM
- E821 at Brookhaven
- Systematic Errors and what we must do better.
- A close look at $a_{\mu}(\mathrm{had})$
- Summary and Outlook

Magnetic Moments, g-Factors

$$\vec{\mu}_s = g_s\left(\frac{e}{2m}\right)\vec{s}$$ Magnetic Moment

$$\mu = (1 + a)\frac{e\hbar}{2m}$$ What you find in the PDT

$$a = \frac{g-2}{2}$$ the anomaly

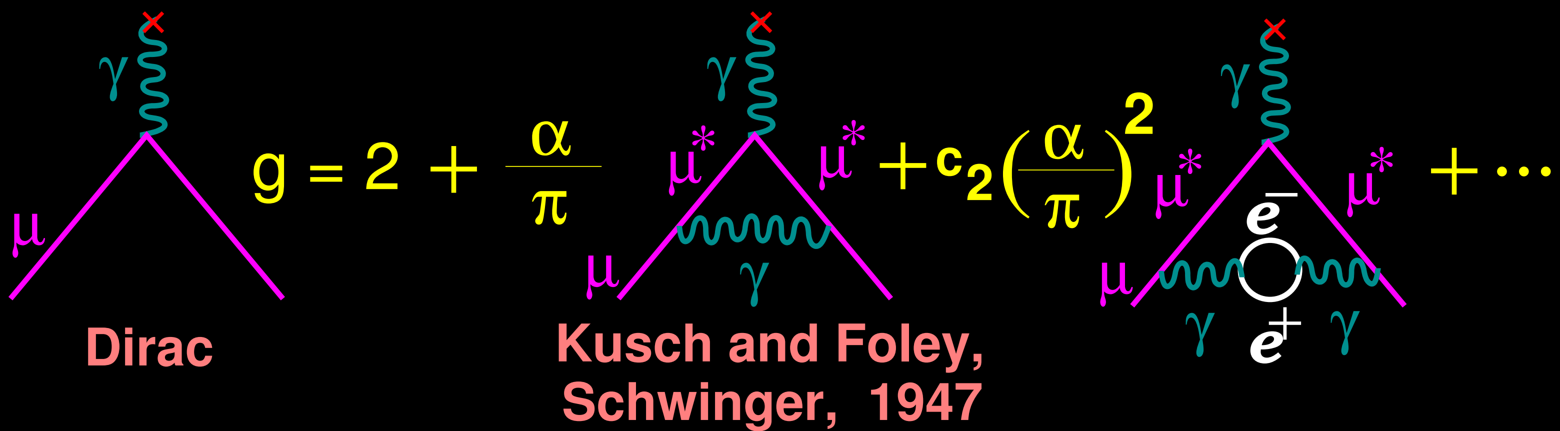
- $\vec{\mu}$ - magnetic moment;
- g - gyromagnetic ratio
- $\vec{s}$ is the spin.

Dirac Equation Predicts $g \equiv 2$

For a NR e^- in a weak $\vec{B}$-field:

$$i\hbar\frac{\partial\psi}{\partial t} = [\frac{p^2}{2m} - \frac{e}{2m}(\vec{L} + 2\vec{S})\cdot\vec{B}]\psi$$

Dirac $\Rightarrow g \equiv 2$, but in nature radiative corrections make $g \neq 2$.



In the paper where they showed $g_s \neq 2$, Kusch and Foley showed $g_\ell = 1$. PR 77, 250 (1948)

Magnetic Moments: Values

$$\mu_e = 1.001\,159\,652\,193\,\frac{e\hbar}{2m_e}$$

For comparison :

$$\mu_\mu = 1.001\,165\,923\,\frac{e\hbar}{2m_\mu}$$

$$\mu_p = 2.792\,847\,39\,\frac{e\hbar}{2m_p}$$

$$g_p = 5.5857\cdots \neq 2$$

Electric Dipole Moments?

Just as the magnetic energy is $-\vec{\mu} \cdot \vec{B}$, the electric energy is $-\vec{d} \cdot \vec{E}$, or

$$\mathcal{H} = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}$$

Transformation properties of $\vec{E}$, $\vec{B}$, $\vec{\mu}$ and $\vec{d}$:

	$\vec{E}$	$\vec{B}$	$\vec{\mu}$ or $\vec{d}$
P	-	+	+
C	-	-	-
T	+	-	-

$\vec{\mu} \cdot \vec{B}$ is even, and $\vec{d} \cdot \vec{E}$ is odd under both *P* and *T*

An EDM implies that both *P* and *T* are violated.

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Electric and Magnetic Dipole Moments for the Muon

$$\mathcal{L}_{mdm} = a_\mu \frac{e}{4m_\mu} \bar{\mu} \sigma^{\alpha\beta} F_{\alpha\beta}; \quad \mathcal{L}_{edm} = -\frac{i}{2} d_\mu \bar{\mu} \sigma^{\alpha\beta} \gamma_5 \mu F_{\alpha\beta}$$

where α, β run from 0 to 3, and d_μ is the electric dipole moment. Or we could write:

$$\mathcal{L}_{dm} = \frac{1}{2} \left[D \bar{\mu} \sigma^{\alpha\beta} \frac{1+\gamma_5}{2} + D^* \bar{\mu} \sigma^{\alpha\beta} \frac{1-\gamma_5}{2} \right] \mu F_{\alpha\beta}$$

with

$$\mathrm{Re}\, D = a_\mu \frac{e}{2m_\mu} \quad \text{and} \quad \mathrm{Im}\, D = d_\mu$$

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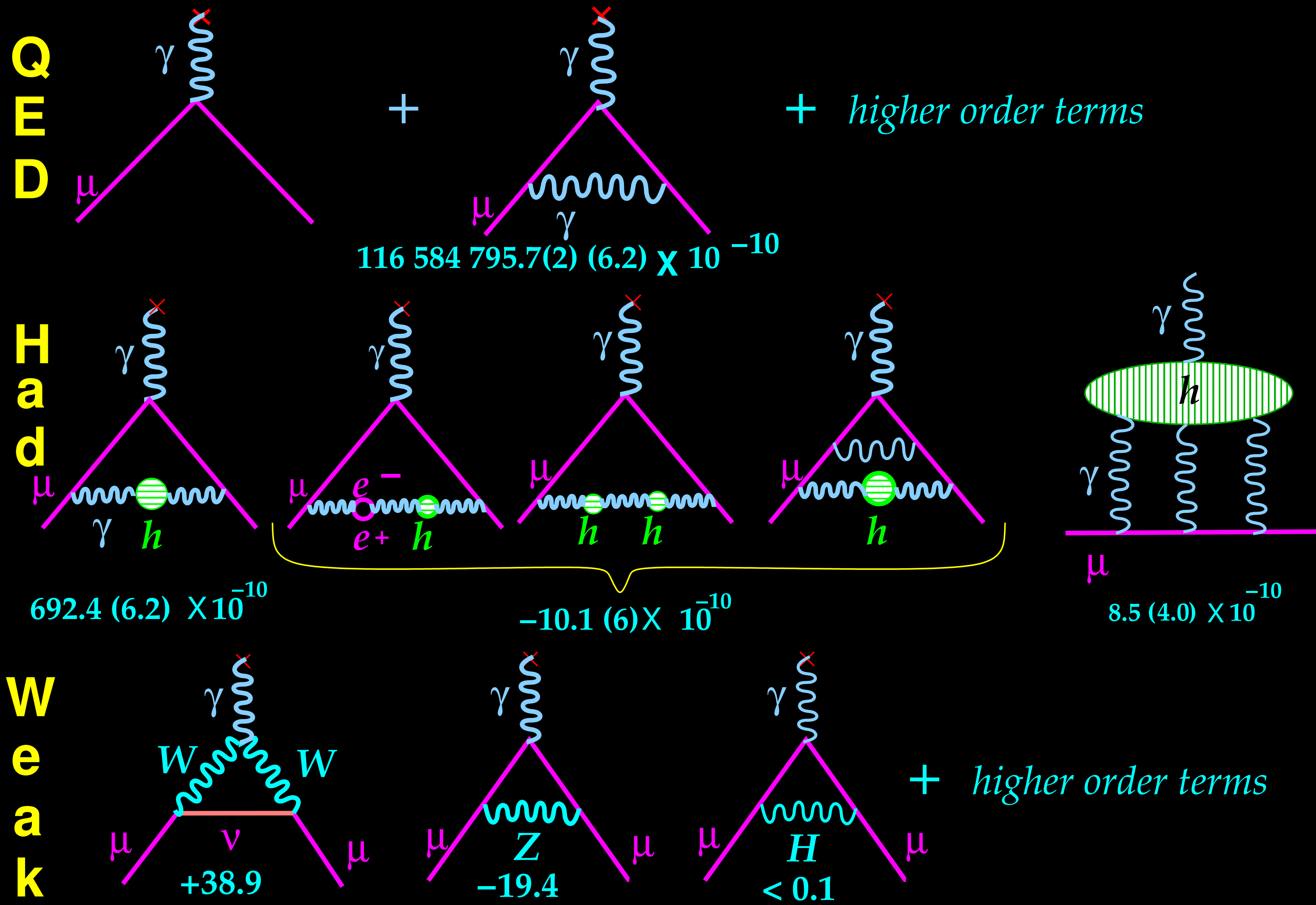
SM Theory for Muon $(g-2)$



1st + 2nd Order Weak = $-15.1\ (4) \times 10^{-10}$

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New Physics Beyond the SM?

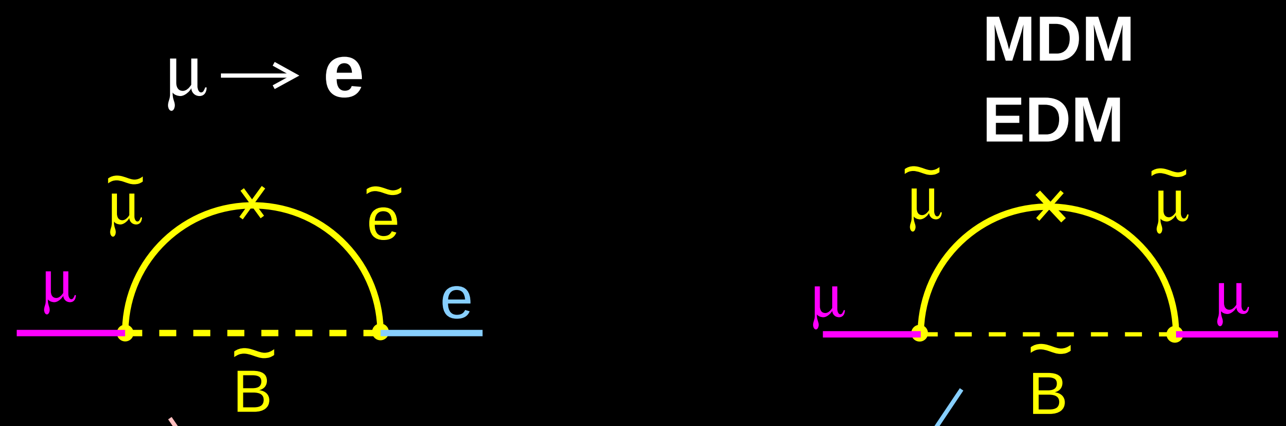
If the experimental value of a_μ/d_μ does not equal the SM value,

$$a_\mu[d_\mu](\mathrm{NP}) = a_\mu[d_\mu](\mathrm{Measured}) - a_\mu[d_\mu](\mathrm{SM})$$

As an example of non-standard model physics we use SUSY to show the connection between the MDM, EDM and Muon Conversion.

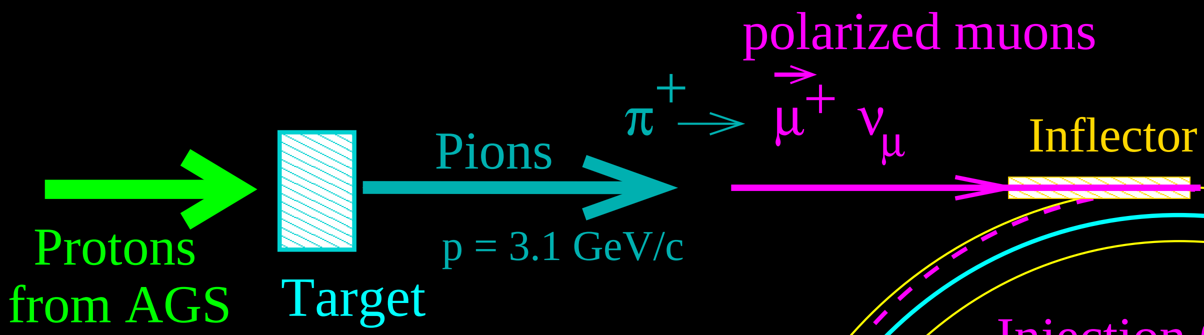
Connection Between MDM, EDM and $\mu \to e$ in SUSY

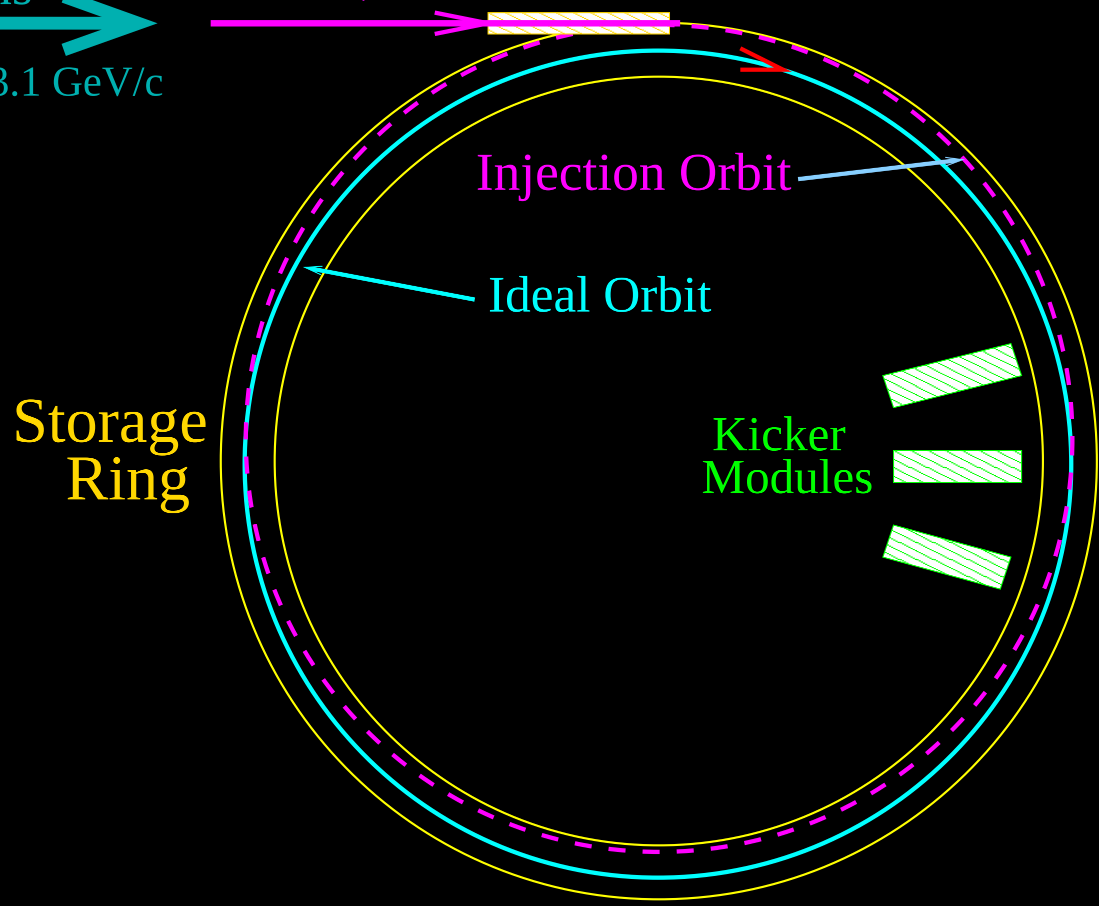
In SUSY the MDM, EDM and muon conversion are all inter-related:



$$\begin{pmatrix} m^2_{\tilde{e}\tilde{e}} & \Delta m^2_{\tilde{e}\tilde{\mu}} & \Delta m^2_{\tilde{e}\tilde{\tau}} \\ \Delta m^2_{\tilde{\mu}\tilde{e}} & m^2_{\tilde{\mu}\tilde{\mu}} & \Delta m^2_{\tilde{\mu}\tilde{\tau}} \\ \Delta m^2_{\tilde{\tau}\tilde{e}} & \Delta m^2_{\tilde{\tau}\tilde{\mu}} & m^2_{\tilde{\tau}\tilde{\tau}} \end{pmatrix}$$

The Technique for Muon $(g-2)$

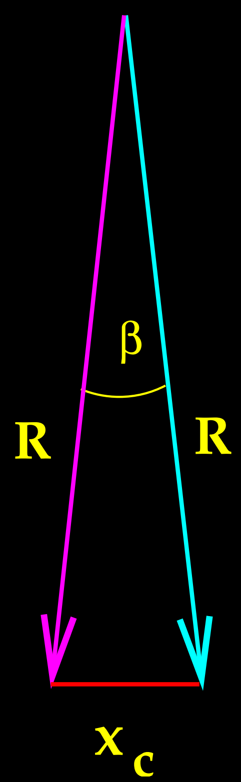




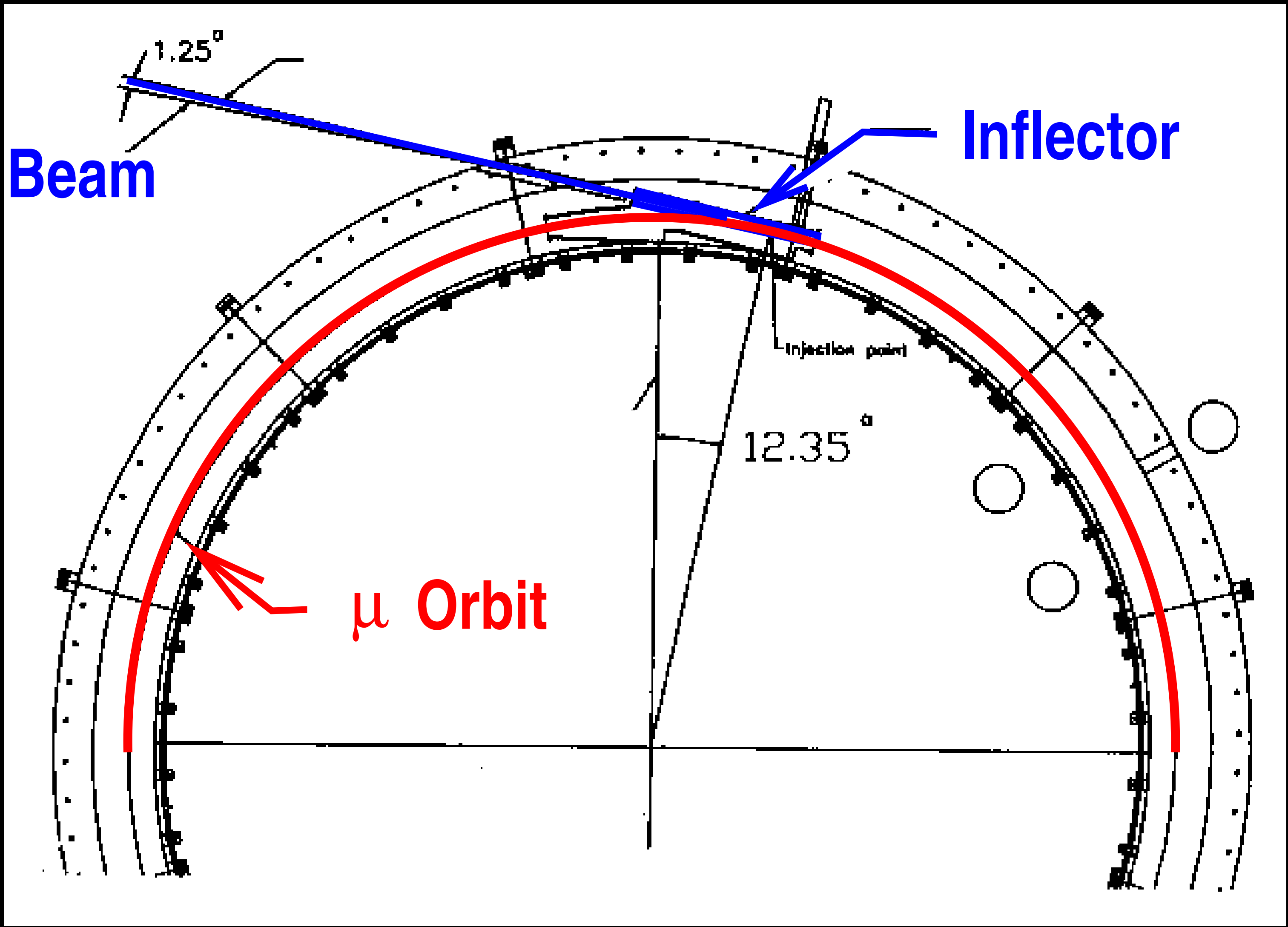
$x_c \cong 77$ mm

$\beta \cong 10$ mrad

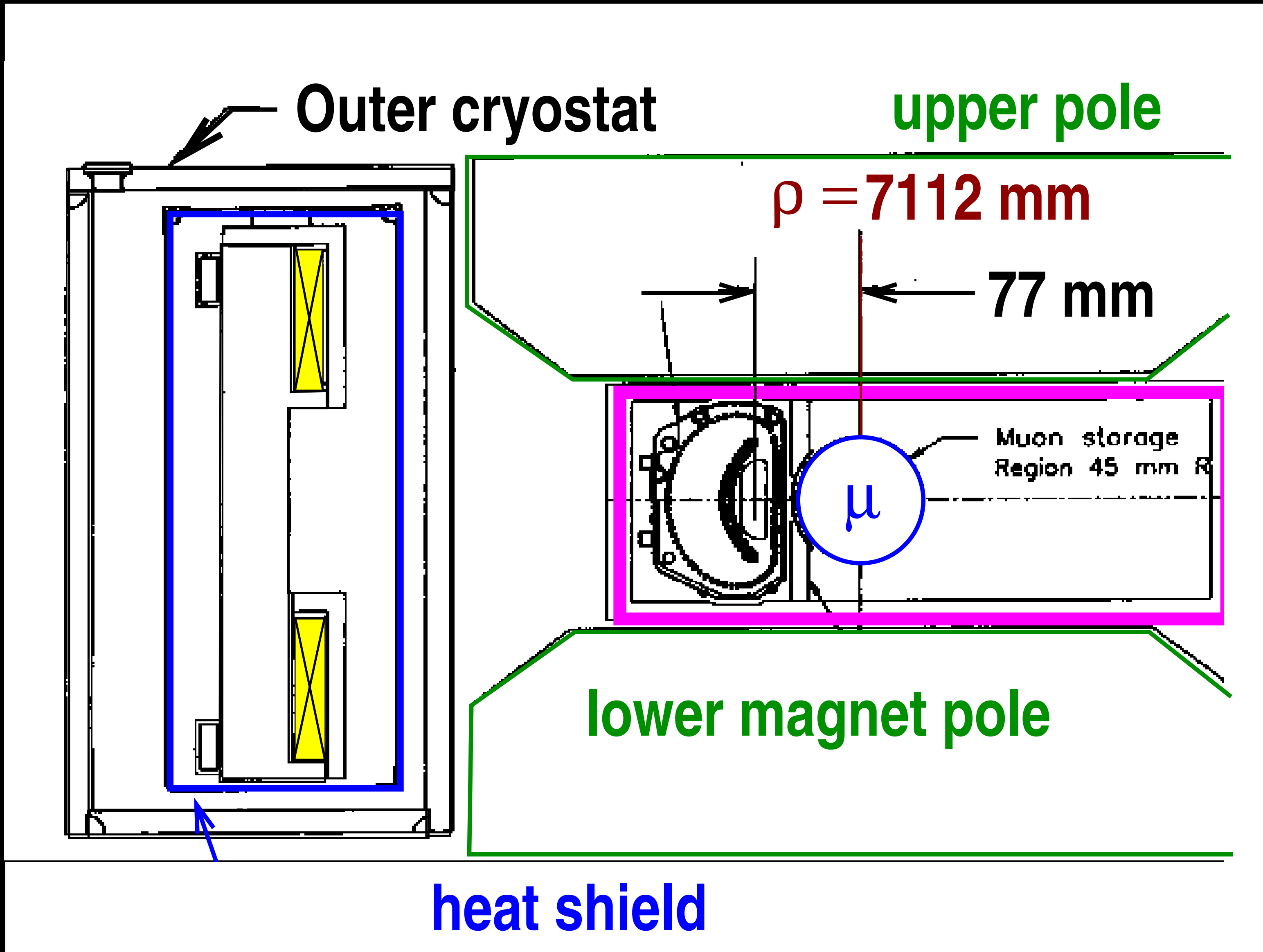
$B \cdot dl \cong 0.1$ Tm



Inflector Geometry



Inflector Exit Geometry



Spin and Momentum Precession

$$\omega_C = \frac{eB}{m\gamma} \qquad \omega_S = \frac{geB}{2m} + (1-\gamma)\frac{eB}{\gamma m}$$

$$\omega\ (\vec{S}\ \text{ relative to }\ \vec{p}) \qquad \omega_a = \omega_S - \omega_C = (\frac{g-2}{2})\frac{eB}{m}$$

n.b. If $g = 2$, $\omega_S = \omega_C$

Spin Motion in $\vec{E}$ and $\vec{B}$ Fields.

$$\vec{\omega}_a = \frac{d\Theta_R}{dt} = \frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta} \times \vec{E}\right]$$

for γ = **29.3** $\qquad \left(a_\mu - \frac{1}{\gamma^2 - 1}\right) = 0$

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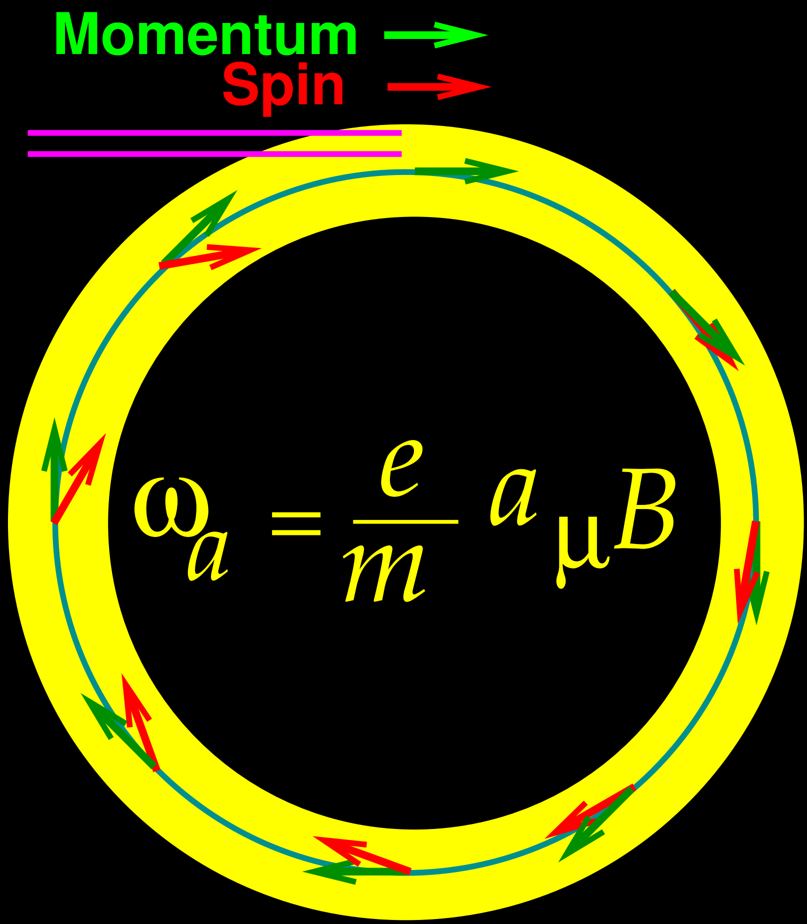
Spin and Momentum Precession

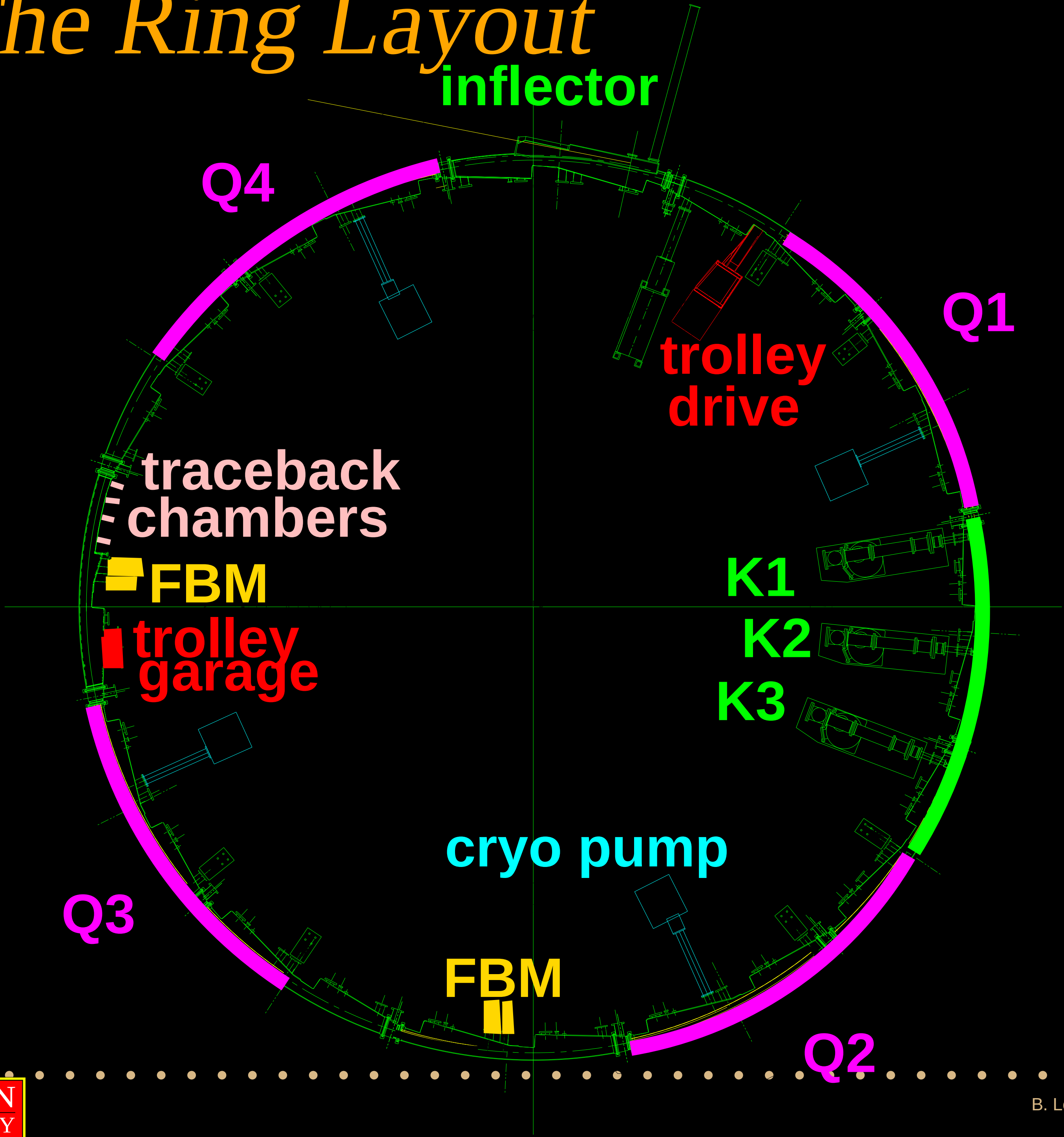


$$\omega_a = \frac{e}{m} a_\mu B$$

The highest energy decay $e^\pm$ are along the muon spin direction

In a uniform $\vec{B}$ field all muons precess at the same rate.

The Ring Layout



quads cover 43% of the ring

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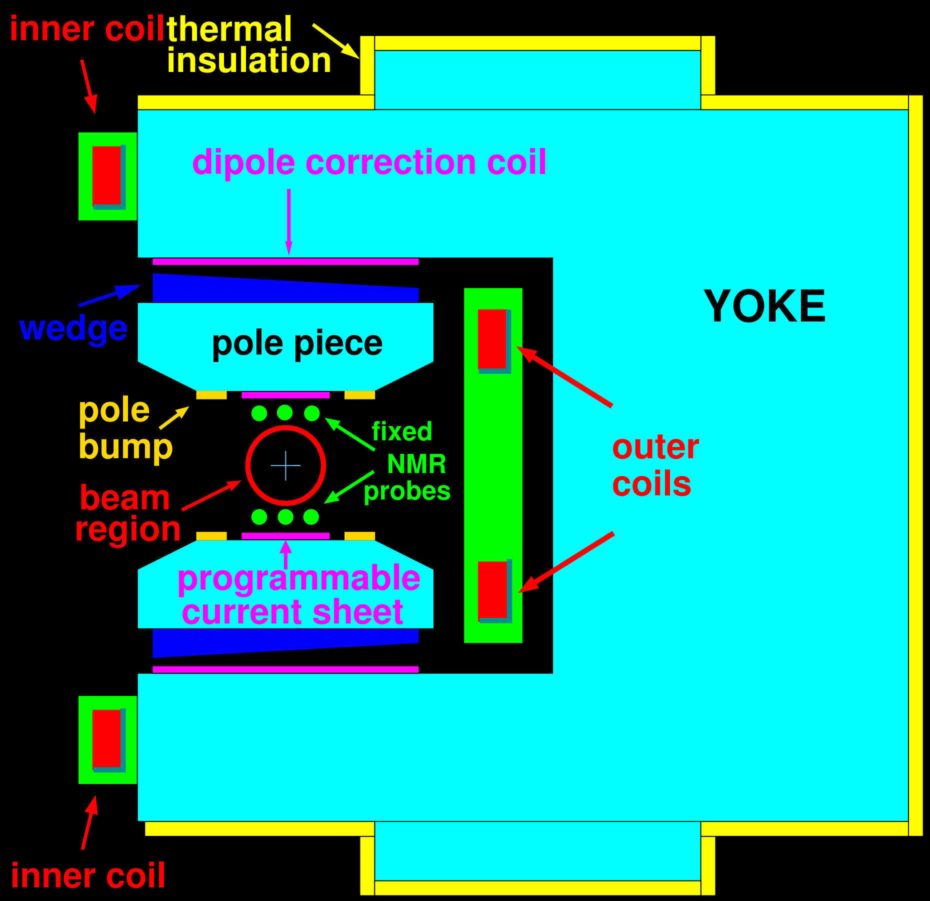
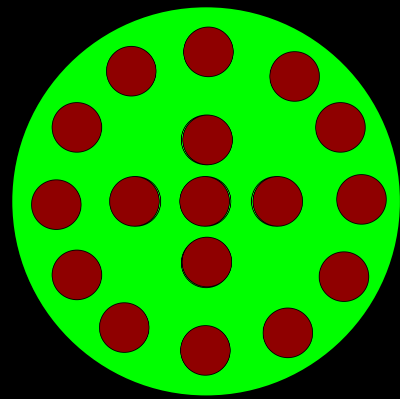
Schematic of the Magnet



g–2 Magnet in Cross Section

ρ = 7112 mm



An array of 17 NMR probes on the trolley maps the B Field in the storage region

Installation of a Pole Piece

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Storage Ring Parameters

Parameter	Value	Comments
(g-2) Frequency	$f_a \sim 0.23 \times 10^6$ Hz	$\tau_a = 4.37 \mu$s
Muon kinematics	$p_\mu = 3.094$ GeV/c $\gamma\tau = 64.4\ \mu$s	$\gamma_\mu = 29.3$
Cyclotron Period	$\tau_{cyc} = 149$ ns	
Central Radius	$\rho = 7112$ mm	$(280'')$
$B_0 = 1.451$ T	Storage Aperture	9.0 cm circle
In one lifetime:	432 revolutions around ring 14.7 *(g-2)* periods	

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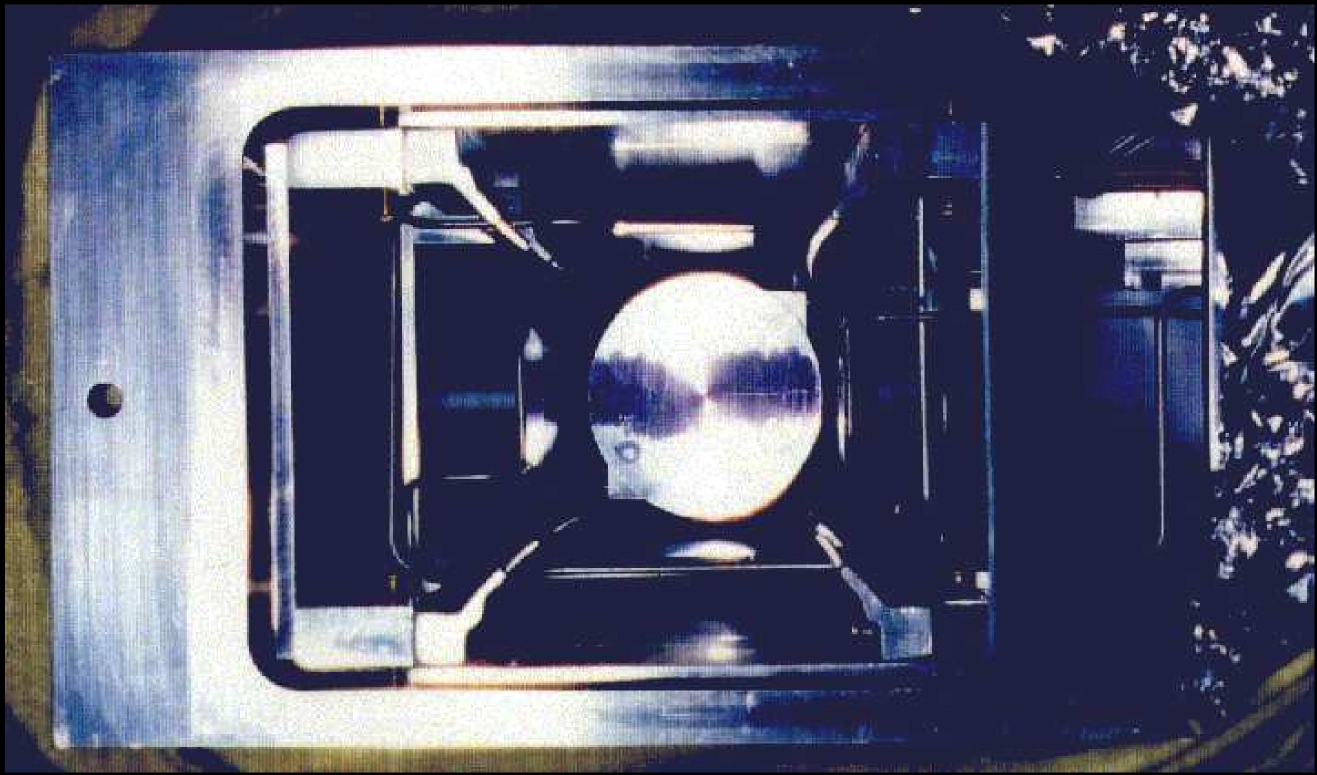
NMR Trolley with 17 Probes

$$< B >=< \int M(r,\theta)B(r,\theta)rdrd\theta >_{\phi}$$



366 fixed NMR probes monitor field stability.

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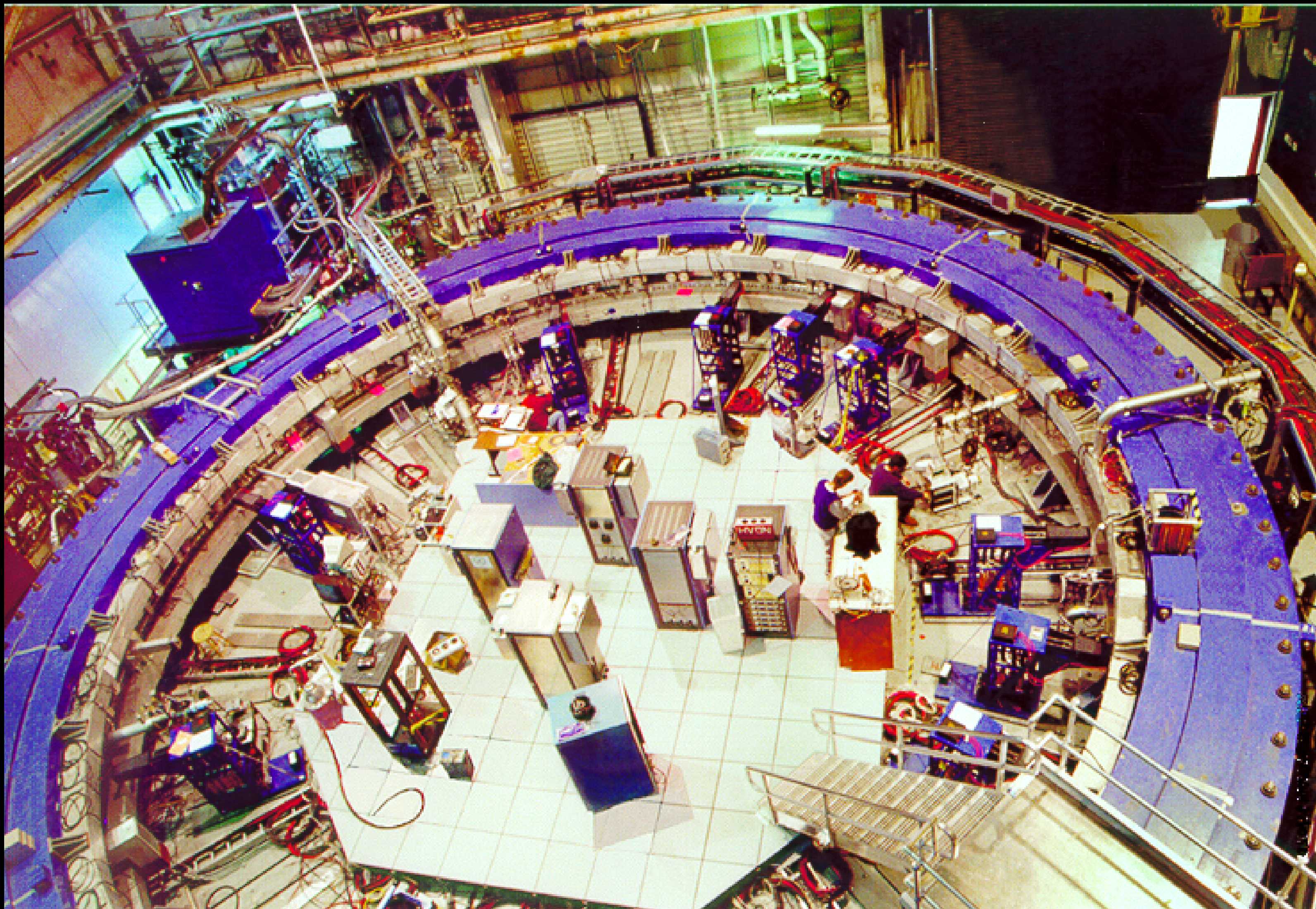
The Nude Storage Ring



Weak Focussing $\quad n = \frac{\kappa R_0}{\beta B_0}$

$\kappa =$ electric quadrupole gradient; $n \simeq 0.137$

$$\gamma m \ddot{x} + \frac{\gamma m v^2}{R_0^2}(1-n)\, x = 0; \qquad \gamma m \ddot{y} + \frac{\gamma m v^2}{R_0^2}\, n\, y = 0$$

$$f_x = f_C\sqrt{1-n} \simeq 0.929 f_C; \qquad f_y = f_C\sqrt{n} \simeq 0.37 f_C$$

Detector acceptance depends on r. The beam coherently moves radially relative to a detector with coherent betatron motion.

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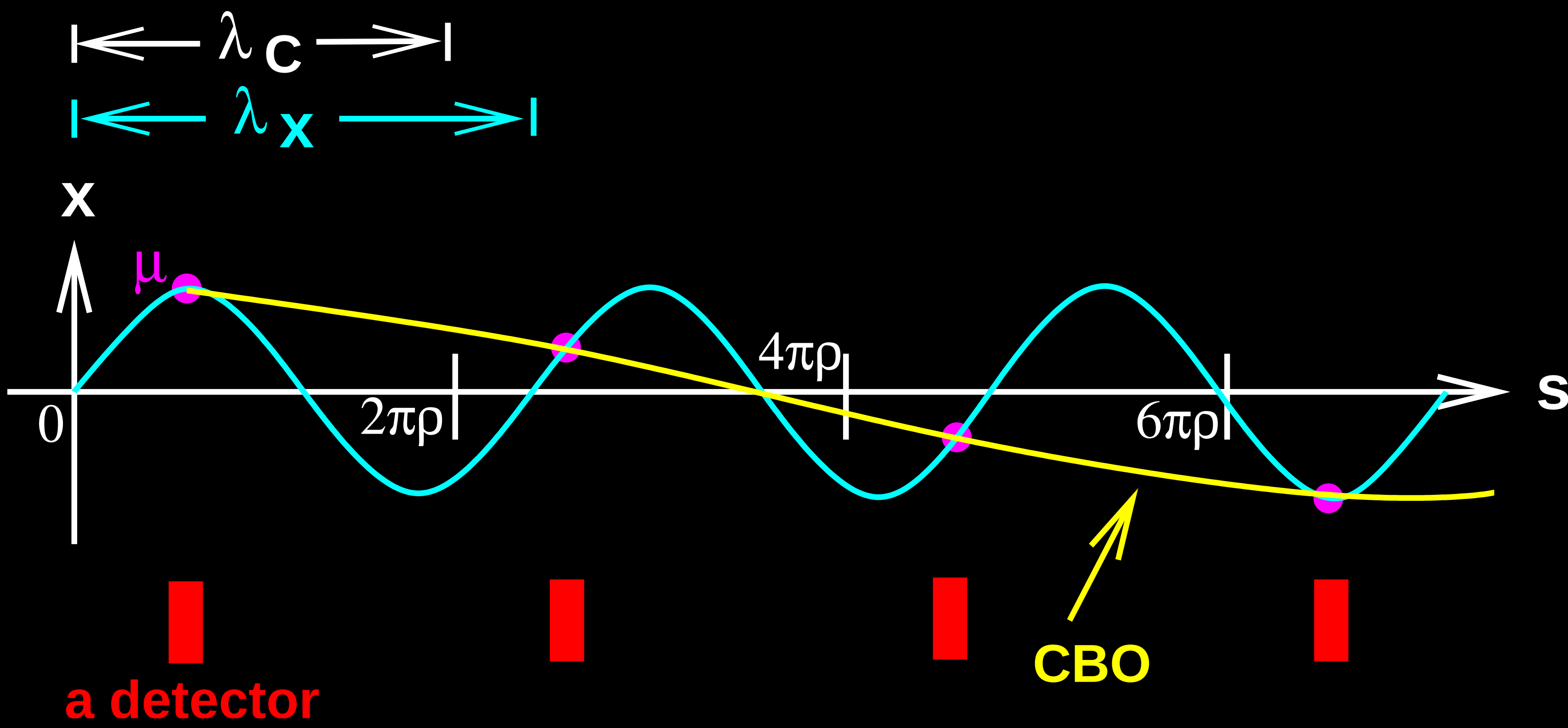
Coherent Betatron Frequency

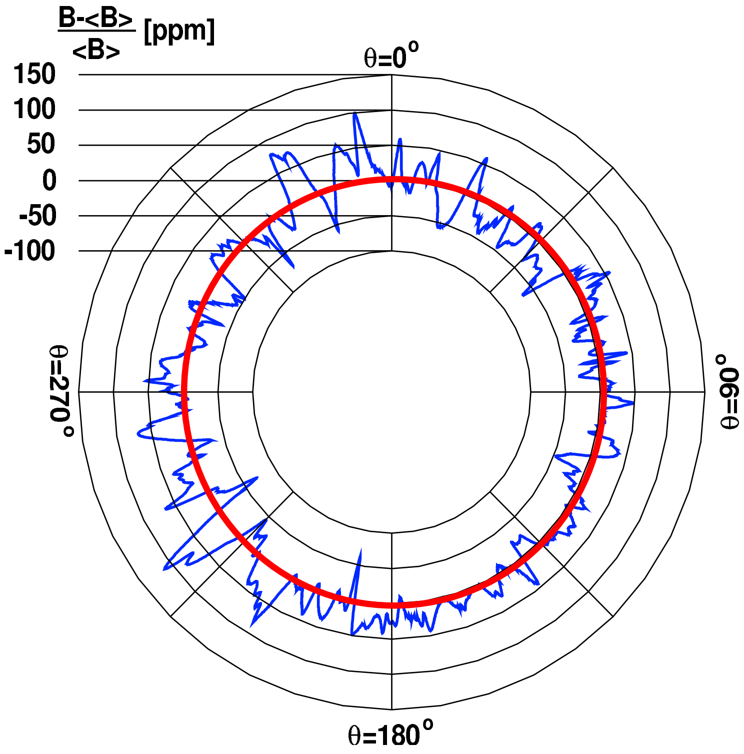


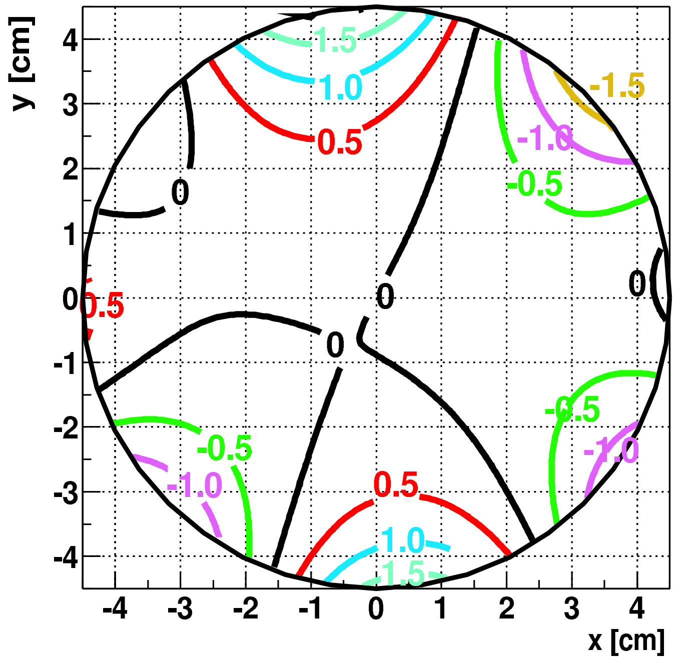
$$f_{\mathrm{CBO}} = f_C - f_x = (1 - \sqrt{1-n})f_C$$

($\lambda_{\mathrm{CBO}} \sim 14$ turns)

f_{CBO} amplitude modulates the $e^{\pm}$ signal.

B and $< B >_\phi$ are:





2001 B-field

Multipoles (ppm)

	normal	skew
Quad	-0.29	0.15
Sext	-0.71	-0.48
Octu	0.06	0.01
Decu	1.08	0.39

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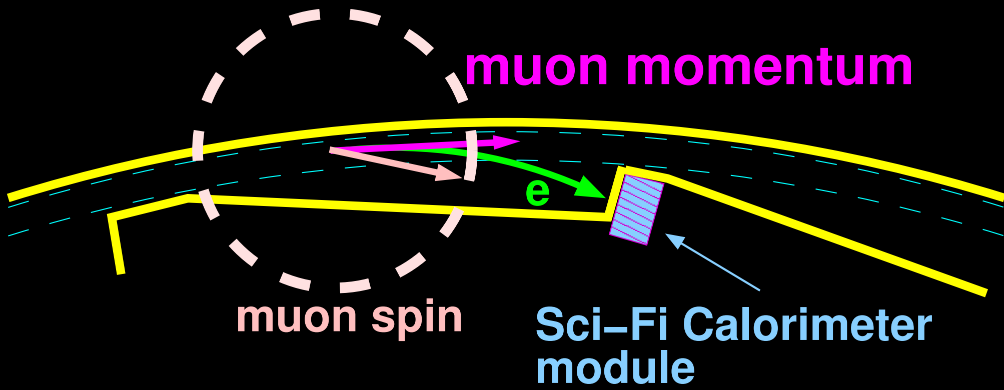
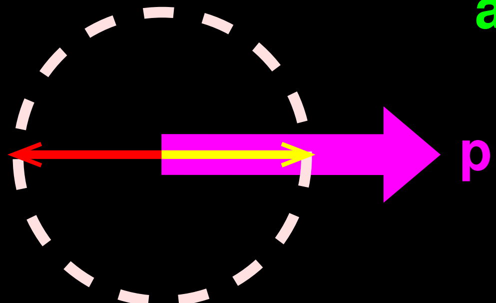
The Detector Geometry

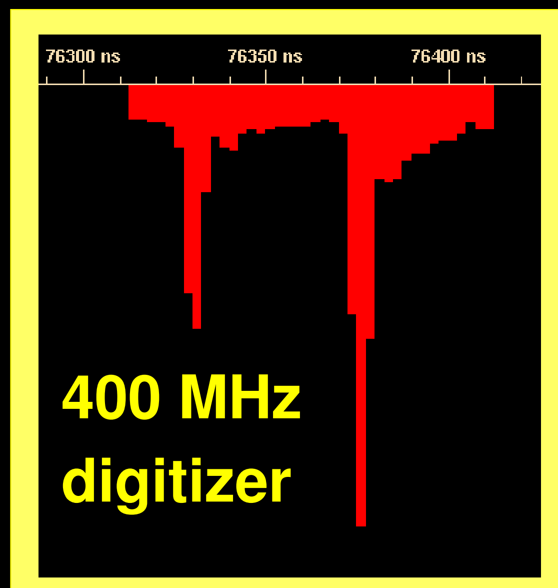


Measures Energy
and time



spin forward, more
high energy e

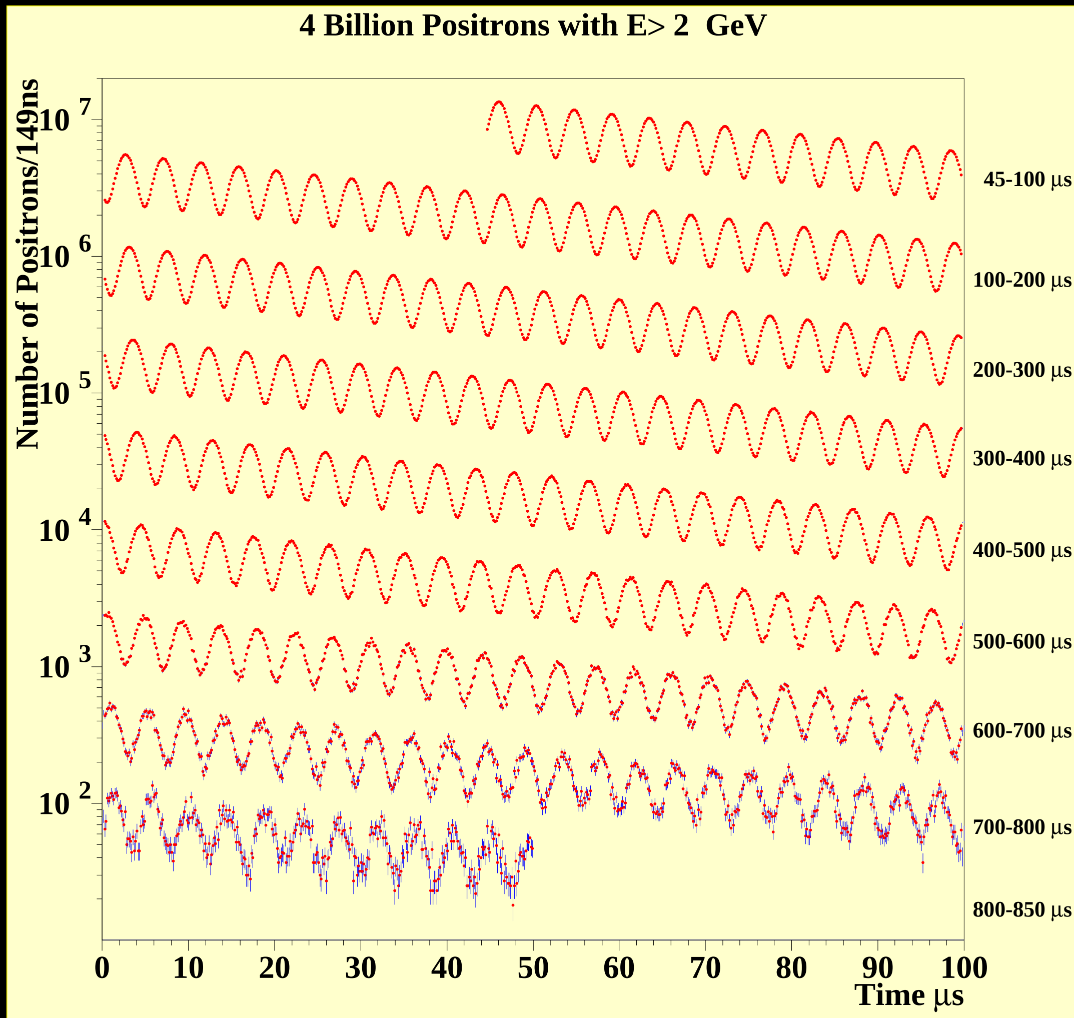
spin backward, less
high energy e



Time Spectrum, $E > 2.0 GeV$

$\sigma_{\text{stat}} \approx 0.7$ *ppm*

$$f(t) = N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$



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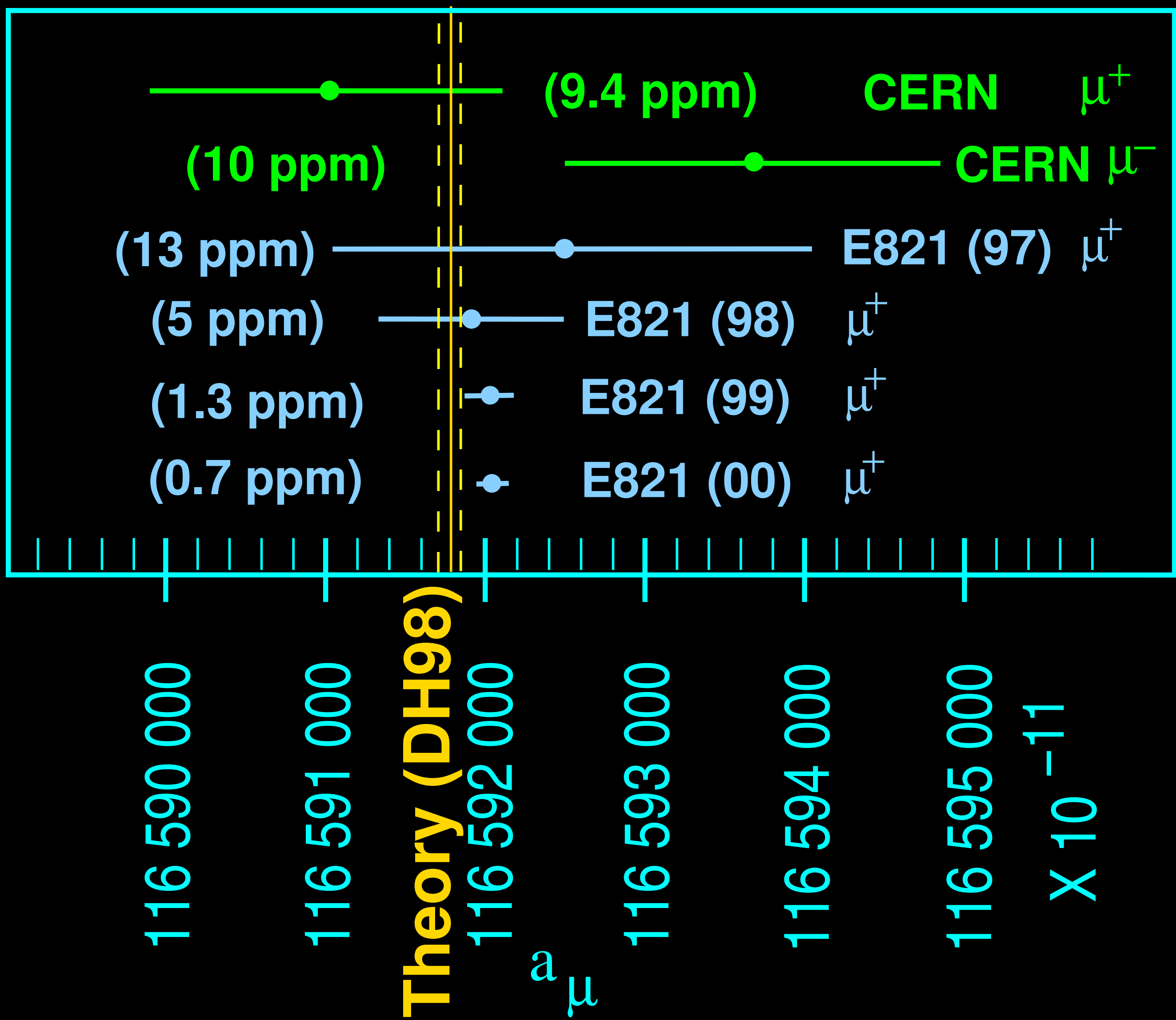
Measurements of a_μ



Present Statistical Errors

Data Set	Statistical Error (ppm)	Systematic Error (ppm)	Status (# Events)
1999 μ^+	1.25	0.5	$\dagger$, 10^9
2000 μ^+	0.6	0.43	$\dagger$, 4×10^9
2001 μ^-	~ 0.7	$\sim 0.3 - 0.4$	$*$, $\sim 3 - 4 \times 10^9$

$\dagger$Published $*$Projected

Data Set	# of Events	Statistical Error (ppm)
Total μ^+	5×10^9	0.56
Total present μ^-	$\sim 3 \times 10^9$	~ 0.7
Total present μ^+ & μ^-	$\sim 8 \times 10^9$	~ 0.44

Systematic Errors on $<\omega_p>$

From two independent analyses of $<\omega_p>$.

Source	1999 (ppm)	2000 (ppm)
Inflector Fringe Field	0.20	-
Calibration of trolley probes	0.20	0.15
Interpolation with fixed probes	0.15	0.10
Trolley measurements of B_0	0.10	0.10
Uncertainty from μ-distribution	0.12	0.03
Absolute calibration	0.05	0.05
Others†	0.15	0.10
Total systematic error on ω_p	0.4	0.24

†Higher multipoles, trolley temperature stability, kicker eddy currents.

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Systematic Errors (ppm) on ω_a

Source	1999	2000
Pile-Up	0.13	0.13
AGS Background	0.10	0.01
Lost Muons	0.10	0.10
Timing Shifts	0.10	0.02
E-field and vertical β-motion	0.08	0.03
Fitting Method / Binning	0.07	0.06
Coherent Betatron Oscillation	0.05	0.21
Beam debunching	0.04	0.04
Detector Gain Changes	0.02	0.13
Total Systematic on ω_a	0.3	0.31

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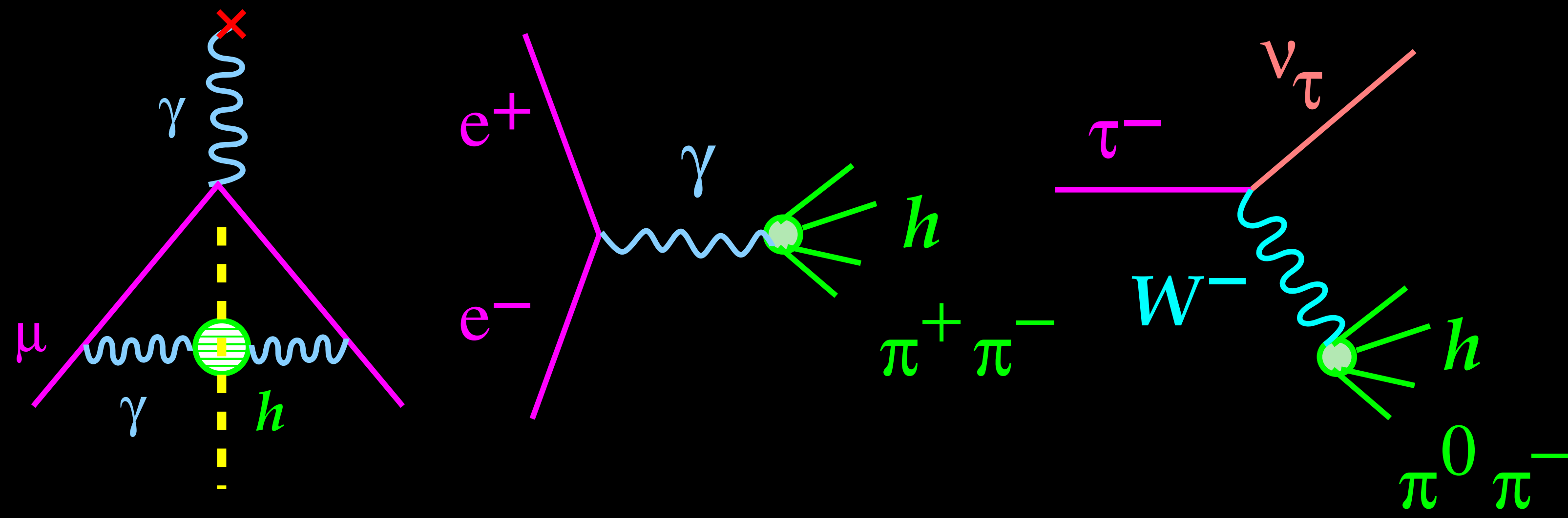
We must take a closer look at the hadronic contribution to a_μ

For an improved experiment to be meaningful, our knowledge of the hadronic contribution must be improved.

$a_\mu(\mathrm{Had})$ *from Dispersion Theory*

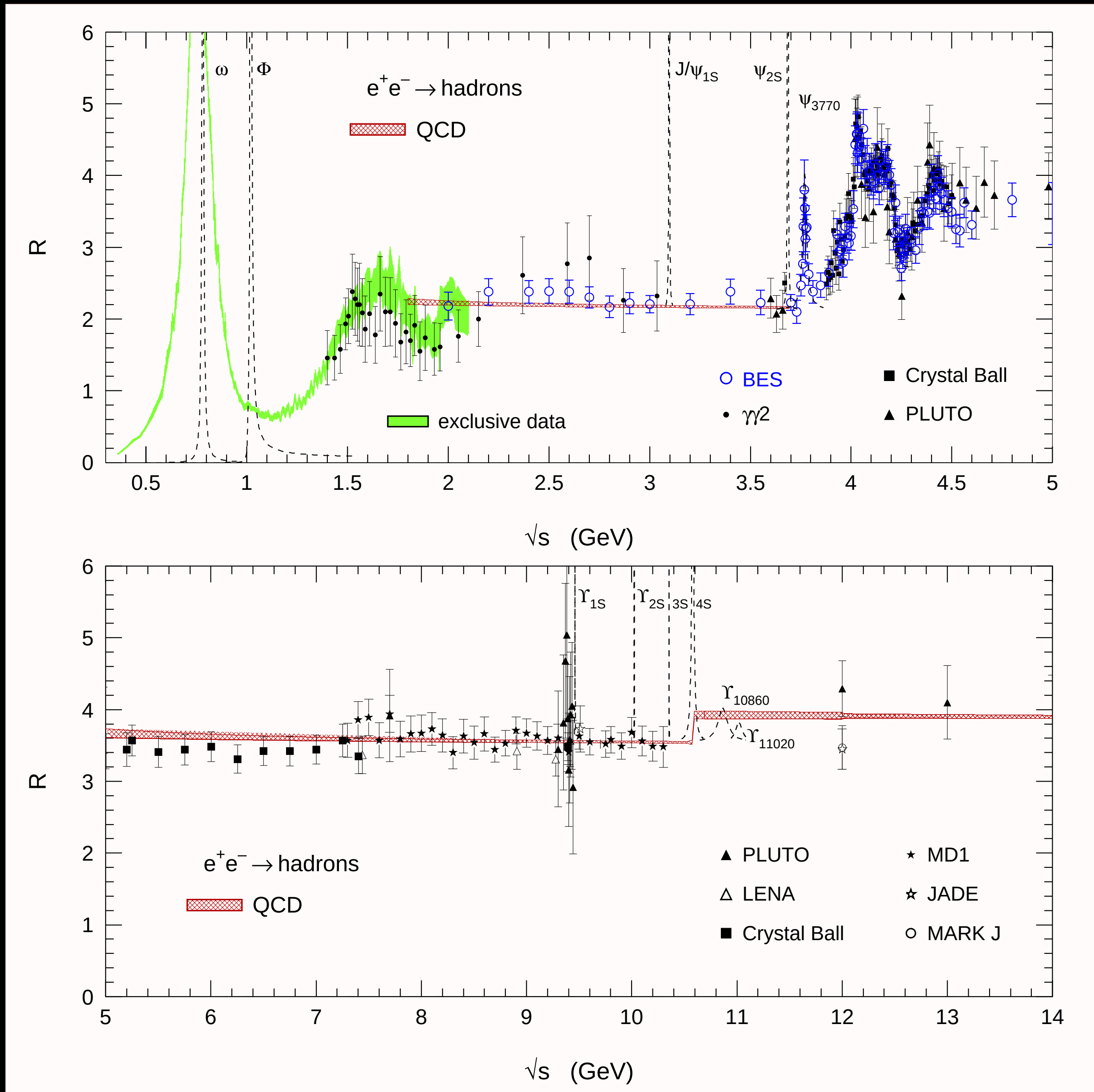


Use of τ-decays $\Rightarrow$ Isospin, CVC, no 2^{nd}-class currents, only isovector current.

$$a_\mu(\mathrm{had};1) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) R(s)$$

where $$R(s) = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$R(s)$ From e^+e^- Data



Status of Hadronic Contribution

- In Feb. 2001 E821 announces a 1.3 ppm measurement which disagrees with the theory by 2.6 σ.
- The community re-examines $a_\mu(\mathrm{Had})$.
- In October 2001, Eduardo de Rafael announces that Marc Knecht and Andreas Nyffeler at Marseille found that the sign of the hadronic light-by-light contribution is positive not negative, and the difference with theory becomes 1.6 σ.

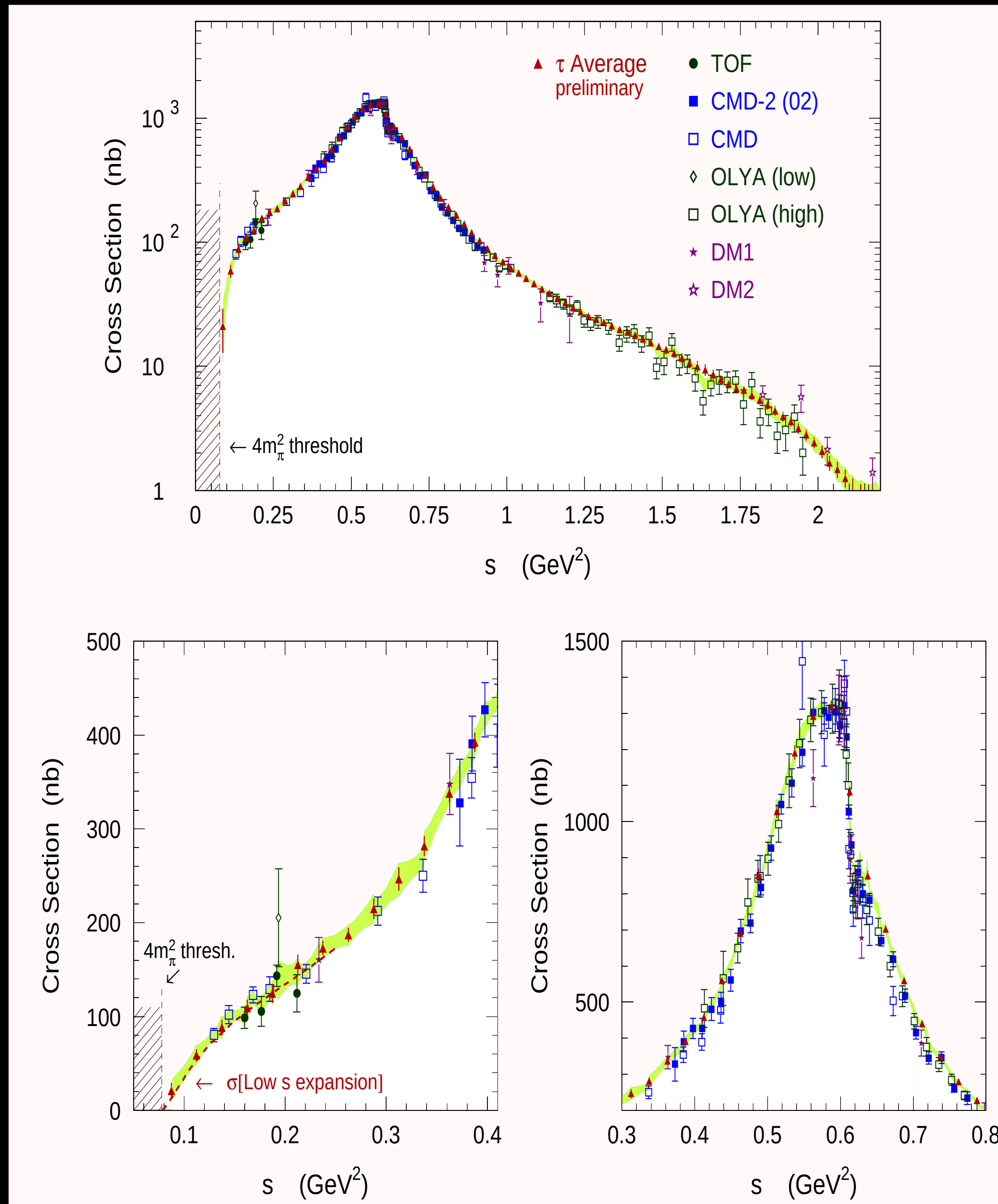
Hadronic Contribution, ctd.

- In February 2002 Novosibirsk publishes new more precise e^+e^- cross sections. Theorists re-evaluate and find that e^+e^- and τ no longer agree. The e^+e^- analysis gives a $\sim 3\sigma$ discrepancy and the τ gives $\sim 1.6\ \sigma$.
- February 2003 Novosibirsk finds a normalization error (missing radiative correction to the Bhabha cross section) and begins a re-evaluation of all of their radiative corrections and comparison with other codes.
- Let's look at the e^+e^- and the τ analyses further.

Comparison of e^+e^-; $\tau \to \pi\pi$

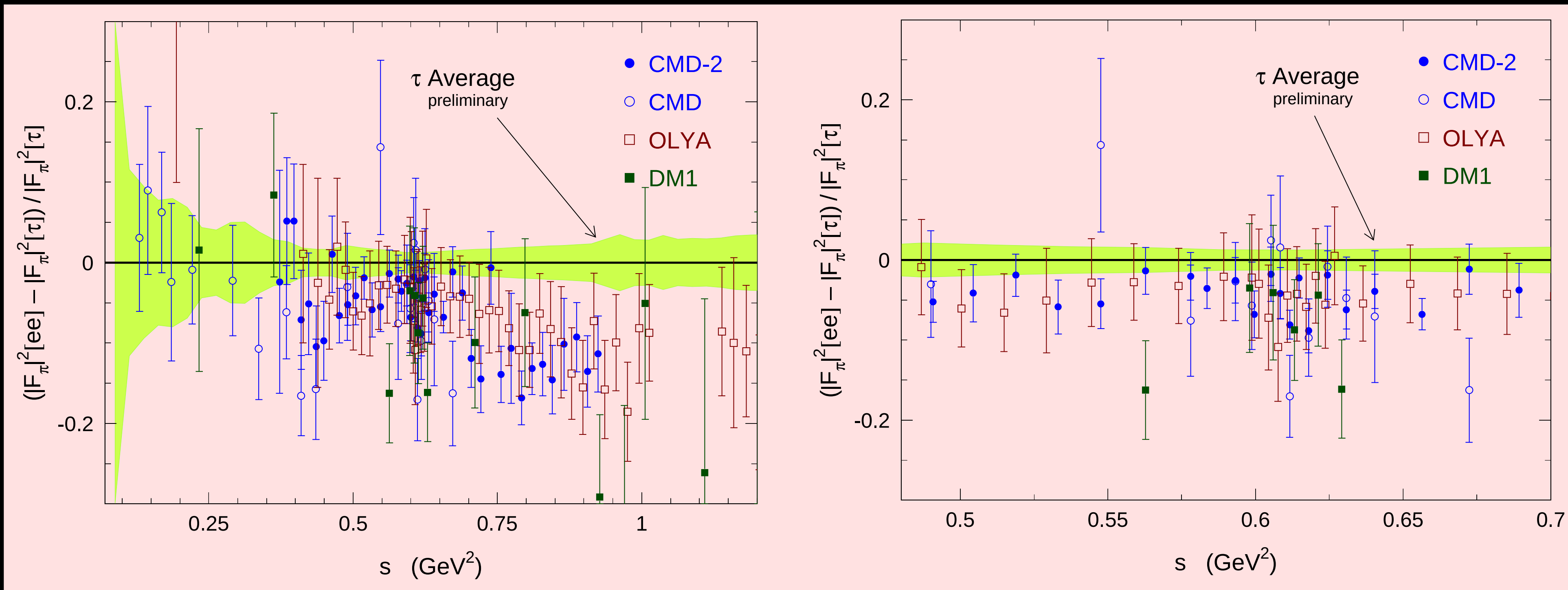
From e^+e^- and "isospin corrected" τ data, expressed as an e^+e^- cross-section.

From DEHZ hep-ph/0208177



$|F_\pi|^2$ *from* τ*-decay and* $e+e^-$

$\tau^- \rightarrow \nu_\tau \pi^- \pi^0$ vrs. $e^+e^- \rightarrow \pi^+\pi^-$



From Davier, Eidelman, Höcker, Zhang: hep-ph/0208177v3, 12 January 2003

We await the re-analysis from Novosibirsk, and additional data from DAΦNE.

Outlook

- We are left with questions, both experimental and "theoretical"

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- I assume that the “theoretical” issues will be cleared up over the next few years. We can hope that the theory will improve to the 0.1 to 0.08 ppm level, with the continued work at e^+e^- machines. DAΦNE, Novosibirsk, and τ facilities.

Outlook

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- I assume that the “theoretical” issues will be cleared up over the next few years. We can hope that the theory will improve to the 0.1 to 0.08 ppm level, with the continued work at e^+e^- machines. DAΦNE, Novosibirsk, and τ facilities.
- As we finish the analysis of our last data set, we are beginning the process of figuring out how to improve the apparatus for a next generation experiment.

Conclusions

- Whatever the final answer for a_μ it will provide an important constraint on new theories. The opportunity to improve on a_μ will constrain them further, or point to a window for new physics.

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Conclusions

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- We are now actively exploring possibilities at higher intensity facilities where we could push $(g-2)$ to further precision.

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