Theoretical Study on the Lepton Flavor Violating $\mu$-e Conversion in Nuclei

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Plan

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# 1. Introduction

**the Standard Model**
- Massive neutrinos?
- Hierarchy problem?
- Charge quantization?

**no LFV**

**Extentions of the SM**
- Right handed neutrinos
- Supersymmetry
- Grand Unification

**LFV Search**
- $\mu \rightarrow e\gamma$
- $\mu-e$ conversion
- $\mu \rightarrow 3e$
- $\tau \rightarrow \mu\gamma$ etc...

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**Experimental Limits on the Branching Ratios**

<table>
<thead>
<tr>
<th></th>
<th>Present</th>
<th>Future</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu \rightarrow e\gamma$</td>
<td>$1.2 \times 10^{-11}$ (MEGA 1999)</td>
<td>$10^{-14}$ (PSI MEG) $10^{-15}$ (J-PARC)</td>
</tr>
<tr>
<td>$\mu-e$ conversion</td>
<td>$6.1 \times 10^{-13}$ (SINDRUM II 1998)</td>
<td>$2 \times 10^{-17}$ (MECO) $10^{-18}$ (J-PARC)</td>
</tr>
<tr>
<td>$\tau \rightarrow \mu\gamma$</td>
<td>$1.0 \times 10^{-6}$ (Belle 2001)</td>
<td>$10^{-8}$ (Belle)</td>
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<tr>
<td>$\mu^+ \rightarrow e^+e^+e^-$</td>
<td>$1.0 \times 10^{-12}$ (CDF)</td>
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**μ-e Conversion in nuclei:**
(Neutrinoless muon capture)

\[ \mu^- + \frac{A}{Z} N_i \rightarrow e^- + \frac{A}{Z} N_f \]

(cf. Ordinary muon capture:
\[ \mu^- + \frac{A}{Z} N_i \rightarrow \nu_\mu + \frac{A}{Z-1} N_f \])

**Coherent process is dominant.**

\[ \frac{A}{Z} N_i = \frac{A}{Z} N_f = \text{Ground state} \]

▶ **a) Initial state — muonic atom**

\[ a_B^{(\mu)} \approx \frac{1}{200} a_B^{(e)} \]

▶ **b) Final state — monochromatic electron**

\[ E_e = m_\mu - E_{\text{binding}} - E_{\text{recoil}} \]

\[ \approx m_\mu - E_{\text{binding}} \]
Why $\mu - e$ conversion?

1. Non-Photonic + Photonic interaction

```
µ --- e
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- Contribution from
  - 4-Fermi effective interaction
  - Photon-penguin interaction

More info on THEORY

2. Monochromatic electron signal

Intense beam = Higher precision

Clean, precise EXPERIMENT

Why muon ring?

- Free from $\pi$ and $e$ in the beam
- Extinguishes prompt backgrounds:
  - Radiative $\pi^-$ capture
    \[ \pi^- + (A, Z) \rightarrow (A, Z-1) + \gamma \]
    \[ \rightarrow e^+ e^- \]
    undetected
  - $e$ in the beam scattered off the target

Lifts restriction on the selection of target nuclei! — Need not to worry about the lifetime of muonic atoms.
Electron momentum distribution@SINDRUM

http://sindrum2.web.psi.ch/home/lead92.html

Target: Pb
# Studies on LFV $\mu$-$e$ conversion

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<tbody>
<tr>
<td>$\mu$ wave function</td>
<td>Constant</td>
<td>Dirac eq.</td>
<td>Dirac eq.</td>
<td>Schrödinger eq.</td>
<td>Dirac eq.</td>
</tr>
<tr>
<td>e wave function</td>
<td>Plane wave</td>
<td>Dirac eq.</td>
<td>Dirac eq.</td>
<td>Plane wave</td>
<td>Dirac eq.</td>
</tr>
<tr>
<td>$q^2$</td>
<td>$q^2 = -m_\mu^2$</td>
<td>$q^2 = -m_\mu^2$</td>
<td>Electric fields</td>
<td>$q = m_\mu - E_{bind}$</td>
<td>Electric fields</td>
</tr>
<tr>
<td>LFV interaction</td>
<td>Photonic</td>
<td>All</td>
<td>Photonic + Vector</td>
<td>4-Fermi</td>
<td>All</td>
</tr>
<tr>
<td>$\rho^{(p,n)}(r)$</td>
<td>Approximate formula</td>
<td>Approximate formula</td>
<td>Experimental data (3 points)</td>
<td>Experimental data</td>
<td>Experimental data</td>
</tr>
</tbody>
</table>
We will be able to choose the target nucleus.

Which is the best, then,

- to discover the $\mu - e$ conversion?
- to investigate the new physics, physics beyond the Standard Model?

Thorough investigation is necessary.
2. Methods

Proton distribution in nuclei

Charge distribution = Proton distribution

Maxwell equation
Dirac equation — Relativistic

Wave functions of initial $\mu$ & final $e$

Proton & neutron distribution in nuclei

Overlap integrals appearing in the $\mu$-$e$ transition amplitude

$\mu$-$e$ transition amplitude

$\mu$-$e$ transition rate
Input data

Proton density in the nucleus: $\rho^{(p)}(r)$
--- Electron scattering experiments
Determined very precisely (within a few %). Various nuclei are measured.

Neutron density in the nucleus: $\rho^{(n)}(r)$
--- Scattering experiments (proton, $\alpha$, $\pi^+$)
--- Spectroscopy ($\pi$ atom, $\bar{p}$ atom)
(Relatively) poorly determined.
Not many data are available.

We also present the result obtained from $\rho^{(n)}(r) = \rho^{(p)}(r)$ assumption.

No justification for heavy nuclei.

Provides the tendency of the $Z$-dependence of the conversion amplitude/rate, thanks to the large data set.
LFV interaction Lagrangian

\[ \mathcal{L}_{\text{int}} = -\frac{4G_F}{\sqrt{2}} (m_\mu A_R \bar{\mu} \sigma^{\mu\nu} P_L \epsilon F_{\mu\nu} + m_\mu A_L \bar{\mu} \sigma^{\mu\nu} P_R \epsilon F_{\mu\nu} + \text{h.c.}) \]

Dipole (photonic)

\[ -\frac{G_F}{\sqrt{2}} \sum_{q=u,d,s} \left[ \left( g_{LS(q)} \bar{e} P_R \mu + g_{RS(q)} \bar{e} P_L \mu \right) \bar{q} q \right] \]

Scalar

Pseudoscalar

Vector

Axial vector

Tensor

Electric field

\[ E(r) = \frac{Ze}{r^2} \int_0^r dr' r'^2 \rho^{(p)}(r') \]

Electric potential

\[ V(r) = -e \int_r^\infty dr' E(r') \]

Transition amplitude

\[ M = \langle f | V | i \rangle \]

Electric field

\[ M = \frac{4G_F}{\sqrt{2}} \int d^3x \left( m_\mu A_R \bar{\epsilon}^{\mu(\tau)} \sigma^{\alpha\beta} P_R \psi^{(1)a}_{L} \psi^{(2)a}_{L} + m_\mu A_L \bar{\epsilon}^{\mu(\tau)} \sigma^{\alpha\beta} P_L \psi^{(1)a}_{L} \psi^{(2)a}_{L} \right) \langle N | i_{F_{\alpha\beta}} | N \rangle 
+ \frac{G_F}{\sqrt{2}} \sum_{q=u,d,s} \int d^3x \left[ \left( g_{LS(q)} \bar{\epsilon}^{\mu(\tau)} \sigma^{\alpha\beta} P_R \psi^{(1)a}_{L} \psi^{(2)a}_{L} + g_{RS(q)} \bar{\epsilon}^{\mu(\tau)} \sigma^{\alpha\beta} P_L \psi^{(1)a}_{L} \psi^{(2)a}_{L} \right) \langle N | q q | N \rangle 
+ \left( g_{LV(q)} \bar{\epsilon}^{\mu(\tau)} \gamma^\alpha P_L \psi^{(1)a}_{L} \psi^{(2)a}_{L} + g_{RV(q)} \bar{\epsilon}^{\mu(\tau)} \gamma^\alpha P_R \psi^{(1)a}_{L} \psi^{(2)a}_{L} \right) \langle N | q \gamma^\alpha q | N \rangle 
+ \left( g_{LA(q)} \bar{\epsilon}^{\mu(\tau)} \gamma^\alpha P_L \psi^{(1)a}_{L} \psi^{(2)a}_{L} + g_{RA(q)} \bar{\epsilon}^{\mu(\tau)} \gamma^\alpha P_R \psi^{(1)a}_{L} \psi^{(2)a}_{L} \right) \langle N | q \gamma^\alpha q | N \rangle 
+ \frac{1}{2} \left( g_{LT(q)} \bar{\epsilon}^{\mu(\tau)} \sigma^{\alpha\beta} P_R \psi^{(1)a}_{L} \psi^{(2)a}_{L} + g_{RT(q)} \bar{\epsilon}^{\mu(\tau)} \sigma^{\alpha\beta} P_L \psi^{(1)a}_{L} \psi^{(2)a}_{L} \right) \langle N | q \sigma_{\alpha\beta} q | N \rangle \right] \]

Transition rate

\[ \omega_{\text{conv}} = |M|^2 \]
3. Numerical Results

Dirac equation for the radial part of the wave function

Wave function: \( \psi_{nm}(\vec{r}) = \frac{1}{r} \left( \frac{u_1(r)}{iu_2(r)} \chi_{\kappa}^m \right) Y_1^m(\theta, \phi) \)

\[
\frac{d}{dr} \left( \begin{array}{c} u_1(r) \\ u_2(r) \end{array} \right) = \left( \begin{array}{cc} -\kappa/r & W - V + m_\mu \\ -(W - V - m_\mu) & \kappa/r \end{array} \right) \left( \begin{array}{c} u_1(r) \\ u_2(r) \end{array} \right)
\]

\( W \): Energy eigenvalue
\( \kappa \equiv -(\vec{L}\vec{\sigma} + 1) = -1 \) for 1S state

Initial state:
The wavefunction of \( \mu \)

Final state:
The wavefunction of \( e \)

Conversion rate --- written in terms of five Overlap integrals

\[
\omega_{\text{conv}} = 2G_F^2 \left| A_R^* D + \tilde{\gamma}_{LS}^{(p)} S^{(p)} + \tilde{\gamma}_{LS}^{(n)} S^{(n)} + \tilde{\gamma}_{LV}^{(p)} V^{(p)} + \tilde{\gamma}_{LV}^{(n)} V^{(n)} \right|^2
+ 2G_F^2 \left| A_L^* D + \tilde{\gamma}_{RS}^{(p)} S^{(p)} + \tilde{\gamma}_{RS}^{(n)} S^{(n)} + \tilde{\gamma}_{RV}^{(p)} V^{(p)} + \tilde{\gamma}_{RV}^{(n)} V^{(n)} \right|^2
\]
Transition amplitudes — Overlap integrals

"$\rho^{(n)}(r) = \rho^{(p)}(r)$" assumption

\[ D = \frac{4}{\sqrt{2}} m_\mu \int_0^\infty dr r^2 [E(-r)][g_\mu^- f_\mu^- + f_\mu^- g_\mu^-] \]

\[ S^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr r^2 Z \rho^{(p)} [g_\mu^- g_\mu^- - f_\mu^- f_\mu^-] \]

\[ S^{(n)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr r^2 (A - Z) \rho^{(n)} [g_\mu^- g_\mu^- - f_\mu^- f_\mu^-] \]

\[ V^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr r^2 Z \rho^{(p)} [g_\mu^- g_\mu^- + f_\mu^- f_\mu^-] \]

\[ V^{(n)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr r^2 (A - Z) \rho^{(n)} [g_\mu^- g_\mu^- + f_\mu^- f_\mu^-] \]

Larger difference for heavy nuclei.

Useful to distinguish the models beyond the SM.
Conversion branching ratios

$$\omega_{\text{conv}} = 2G_F^2 |A_R^* D + \tilde{g}_L^{(p)} S^{(p)} + \tilde{g}_L^{(n)} S^{(n)} + \tilde{g}_L^{(p)} V^{(p)} + \tilde{g}_L^{(n)} V^{(n)}|^2$$
$$+ 2G_F^2 |A_L^* D + \tilde{g}_R^{(p)} S^{(p)} + \tilde{g}_R^{(n)} S^{(n)} + \tilde{g}_R^{(p)} V^{(p)} + \tilde{g}_R^{(n)} V^{(n)}|^2$$

$$\text{Br}(\mu^{-} + \frac{Z}{A} N \rightarrow e^{-} + \frac{Z}{A} N) \equiv \frac{\Gamma(\mu^{-} + \frac{Z}{A} N_i \rightarrow e^{-} + \frac{Z}{A} N_f)}{\Gamma(\mu^{-} + \frac{Z}{A} N_i \rightarrow \nu_{\mu} + \frac{Z-1}{A} N'_f)}$$

"\rho^{(n)}(r) = \rho^{(p)}(r)" \text{ assumption}

Z $\sim$ (30 — 60) gives the largest rate.

The model of new physics can be distinguished through the measurement of $\mu - e$ rate for several kinds of nuclei.
\( \rho^{(n)}(r) \) from the proton scattering experiment

Transition amplitudes — Overlap integrals

Conversion branching ratios
$\rho^{(n)}(r)$ from the pionic atom spectroscopy

Transition amplitudes — Overlap integrals

Conversion branching ratios
4. Discussion

The ambiguities are chiefly due to the poor knowledge on the **neutron distribution in nuclei**.

Scattering experiments, Spectroscopy, ...

\[ S^{(n)} \text{ and } V^{(n)} \text{suffers from the ambiguity.} \]

- **Light nuclei** \[ Z < (\sim 50) \]
  - Agrees anyway within (a few) \%.
    - (including "\( \rho^{(n)}(r) = \rho^{(p)}(r) \)" assumption)

- **Heavy nuclei** \[ Z > (\sim 50) \]
  - Results of recent scattering experiments on Pb
    - Experimental error leads to (a few) \% of ambiguity.
  - Spectroscopy gives (10 – 20)\% smaller values.

Spectroscopic methods gives info on the neutron distribution at the **peripheral** region, which is **extrapolated** into inside the nucleus under a certain assumption.

Neutron density distribution is known for limited species of nucleus at present.
5. Summary

- LFV $\mu - e$ conversion search is clean and informative.

- Muon ring enables the $\mu - e$ conversion search with various target nuclei.

- Nuclei with $Z \sim (30 - 60)$ gives the largest $\mu - e$ conversion rate.

- The measurements with different nuclei are useful to distinguish the theoretical models with LFV.

- Ambiguity is small for light nuclei (within a few %).

- Recent scattering experiments on Pb gives us a hope that ambiguity can be made small for heavy nuclei also.