

Measurement of the
polarization vector
of the e^+
from the decay of polarized μ^+
and its implications on

- G_F (Fermi coupling constant)
- TRI (time reversal invariance)
- PBSM (physics beyond the standard model)

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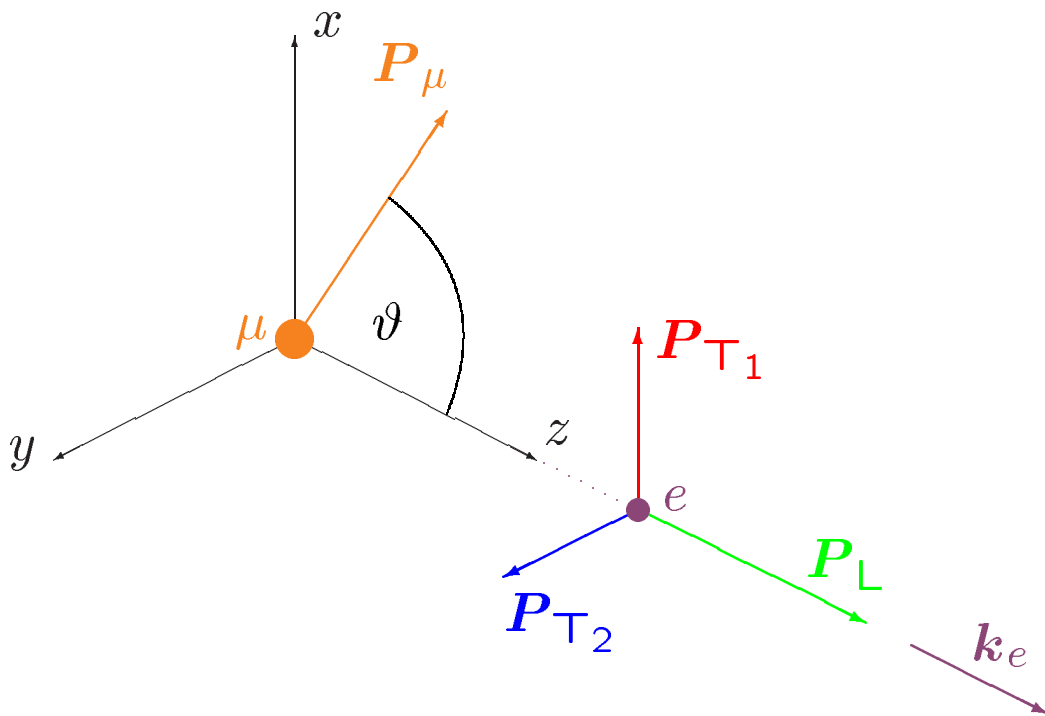
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2. Observables



$$P_{T1} = f_1(E, \vartheta, \eta, \eta'')$$

$$P_{T2} = f_2(E, \vartheta, \frac{\alpha}{A}, \frac{\beta}{A})$$

The standard model predicts:

$$\langle P_{T1}(\vartheta = 90^\circ) \rangle_E = -1.6 \times 10^{-3}$$

$$P_{T2} \equiv 0$$

A nonzero P_{T2} would signal violation of time reversal invariance. This is the only purely leptonic reaction for which TRI has been tested up to now.

3. Matrix Element

$$\mathcal{M} = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \varepsilon,\mu=R,L}} g_{\varepsilon\mu}^{\gamma} \langle \bar{e}_{\varepsilon} | \Gamma^{\gamma} | (\nu_e)_n \rangle \langle \bar{\nu}_m | \Gamma_{\gamma} | (\mu)_{\mu} \rangle$$

The index γ labels the type of interaction:

$$\Gamma^S = \text{4-scalar}$$

$$\Gamma^V = \text{4-vector}$$

$$\Gamma^T = \text{4-tensor}$$

The indices ε, μ indicate the chiralities of the spinors of the observed (charged) leptons. The chiralities n, m of the neutrinos are uniquely determined for given γ, ε and μ .

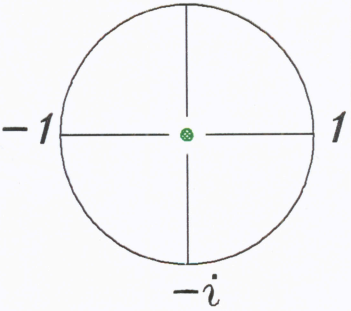
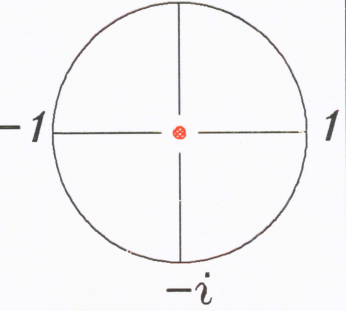
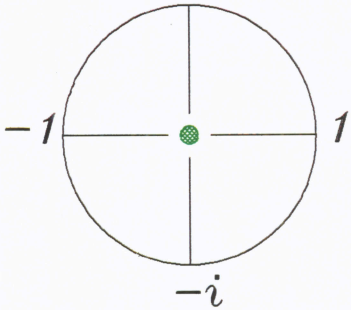
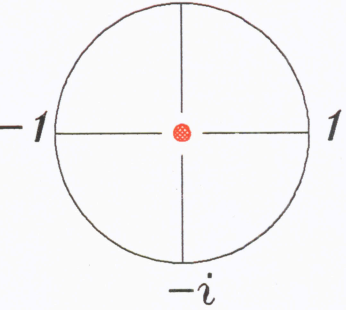
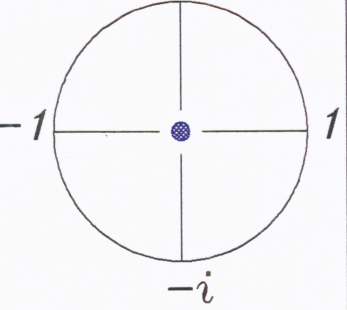
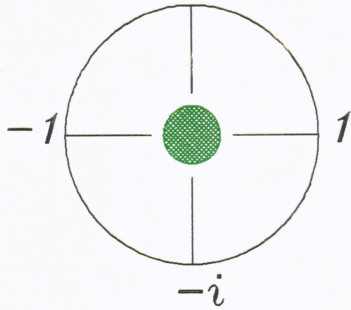
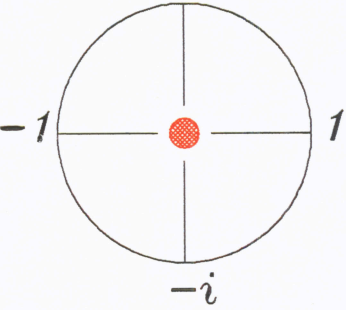
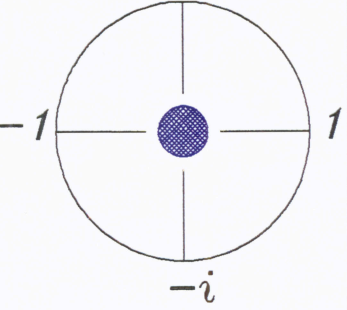
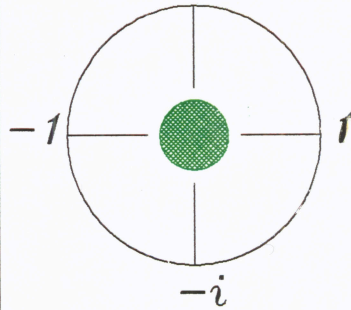
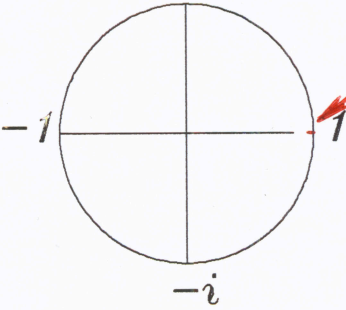
The transverse polarization component P_{T_1} yields the low energy parameter η *without* the suppression factor m_e/m_{μ} of η in the energy spectrum of the decay positron. These interference terms allow for sizeable effects.

$$\eta = \frac{1}{2} \text{Re} \{ g_{LL}^V g_{RR}^{S*} + g_{RR}^V g_{LL}^{S*} + g_{LR}^V (g_{RL}^{S*} + g_{RL}^{T*}) + g_{RL}^V (g_{LR}^{S*} + g_{LR}^{T*}) \}$$

In the standard model

$$g_{LL}^V = 1$$

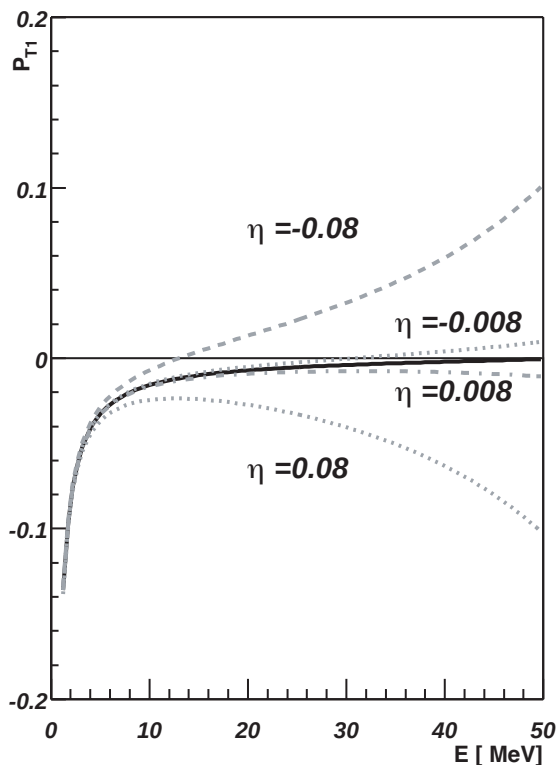
$$g_{\varepsilon\mu}^{\gamma} = 0 \quad (\text{all other interactions})$$

$g_{\varepsilon\mu}^{\gamma\iota}$	S	V	T
	i  $-i$	i $V+A$  $-i$	
$e_R \mu_R$	i  $-i$	i  $-i$	i  $-i$
	i  $-i$	i  $-i$	i  $-i$
$e_R \mu_L$	i  $-i$	i $V-A$  $-i$	
			
$e_L \mu_L$			

Assuming 1 additional interaction and knowing that

$$g_{LL}^V \approx 1,$$

one deduces:



$$P_{T_1}(E_e) \rightarrow \eta \approx \frac{1}{2} \text{Re}\{g_{RR}^S\}$$

$$P_{T_2}(E_e) \rightarrow \frac{\beta'}{A} \approx \frac{1}{4} \text{Im}\{g_{RR}^S\}$$

Main scientific interests:

P_{T_1} : Precise determination of Fermi coupling constant G_F

P_{T_2} : Test of time reversal invariance

4. Fermi coupling constant

Should be independent of masses and radiative corrections:

Universal coupling constant

$$G_F^2 = 192\pi^3 \cdot \frac{\hbar}{\tau_\mu} \cdot \frac{1}{m_\mu^5} \cdot \left\{ 1 + \frac{\alpha}{2\pi} \left(\pi^2 - \frac{25}{4} \right) \right\} \cdot \left\{ 1 - \frac{3}{5} \left(\frac{m_\mu}{m_W} \right)^2 \right\} \cdot \left\{ 1 - 4\eta \cdot \frac{m_e}{m_\mu} - 4\lambda \cdot \frac{m_{\nu\mu}}{m_\mu} + 8 \left(\frac{m_e}{m_\mu} \right)^2 + 8 \left(\frac{m_{\nu\mu}}{m_\mu} \right)^2 \right\}$$

New:

$$\lambda \approx \frac{1}{2} \text{Re} \left\{ g_{LL}^V \cdot g_{LR}^{V*} \right\}$$

In left-right symmetric models with mixing angle ζ :

$$\lambda \approx \frac{1}{2} \zeta$$

Contribution from	$\frac{\Delta G_F}{G_F} [10^{-6}]$	
	$\mu^+ \rightarrow \bar{\nu}_\mu e^+ \nu_e$	$\tau^+ \rightarrow \bar{\nu}_\tau \mu^+ \nu_\mu$
Δm_W	0.0	1
$\Delta m_{\mu,\tau}$	0.2	421
$\Delta \tau$	9.1	2 070
$\Delta(\lambda m_{\bar{\nu}})$	70.0	22 500
$\Delta \eta$	193.0	6 500
$\Delta \Gamma_{\tau \rightarrow \mu}$	-	100 000

5. Experimental method

Measure the complete polarization vector of the decay positrons:

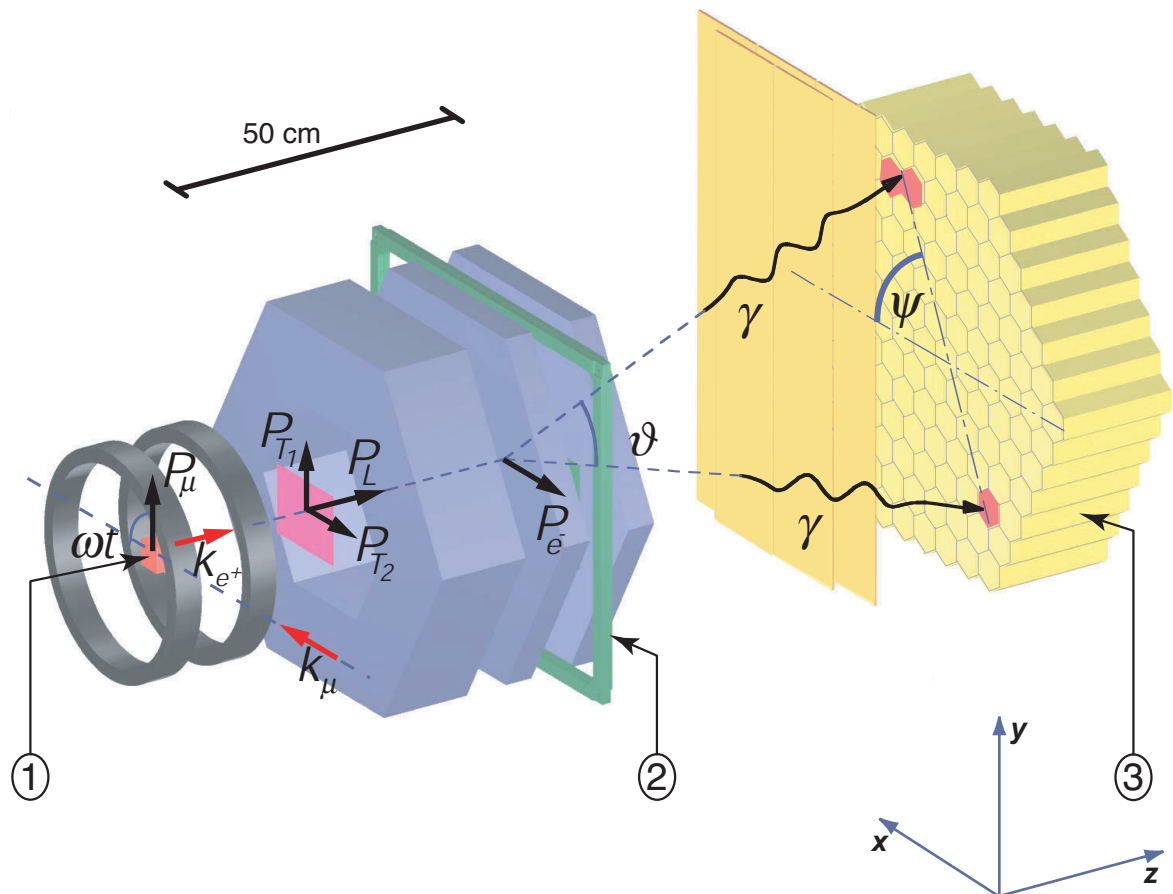
$$\boldsymbol{P}_{e^+} = \begin{pmatrix} P_{T_1} \\ P_{T_2} \\ P_L \end{pmatrix} \equiv \begin{pmatrix} P_T \cdot \cos \varphi \\ P_T \cdot \sin \varphi \\ P_L \end{pmatrix}$$

with 3 simultaneous and independent measurements:

Observable	Method
P_T	Time dependence of annihilation
φ	Remnant μ SR effect
P_L	Spatial dependence of annihilation

6. Experimental setup

- 6.1 Highly polarized μ^+ beam at μ E1 area of PSI: (91%)
- 6.2 Muon stop rate in Be target:
 $(20 - 80) \times 10^6 \text{ s}^{-1}$
- 6.3 Precession in homogeneous B field;
precession frequency = cyclotron frequency
(50.8 MHz)
- 6.4 Burst width 3.9 ns (FWHM)
 $\Rightarrow$ 80% muon polarization in Be stop target
- 6.5 Positron tracking with drift chambers
- 6.6 Annihilation with polarized e^-
- 6.7 Detection of annihilation quanta with 127 BGO crystals



- ① : Beryllium stop target within spin precession magnet
- ② : magnetized Vacoflux foil within iron return yoke
- ③ : calorimeter consisting of 127 BGO crystals

Triggering on Two Photons at a minimal Distance: The Cluster Recognition Unit

How does it work?

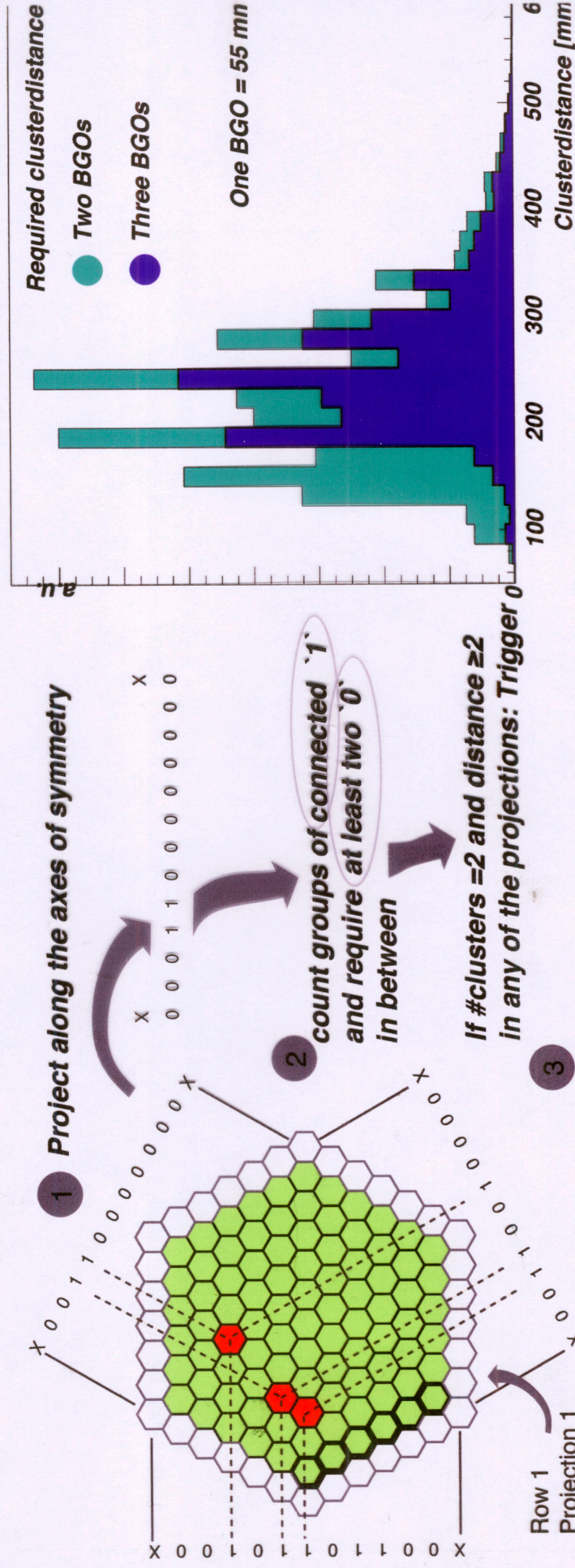
Kinematics require a
minimal cluster distance

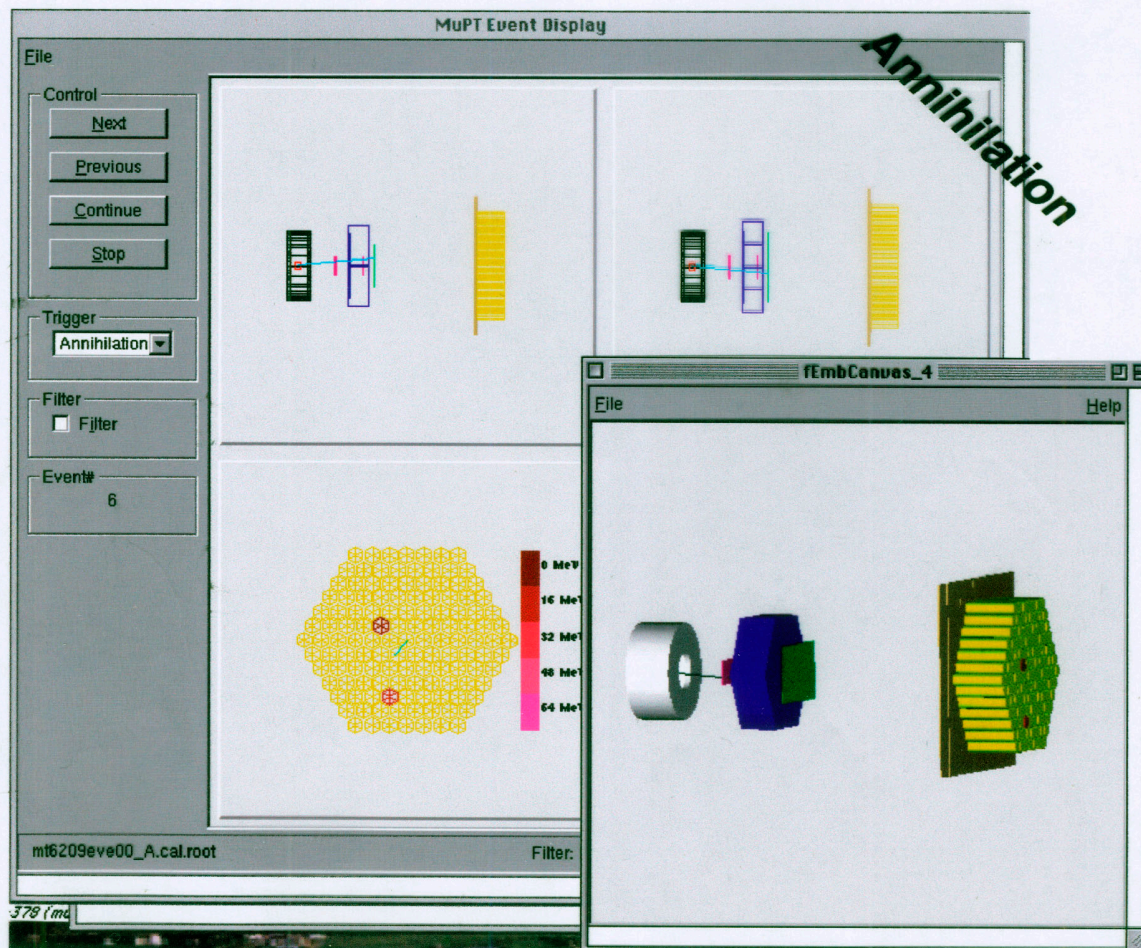
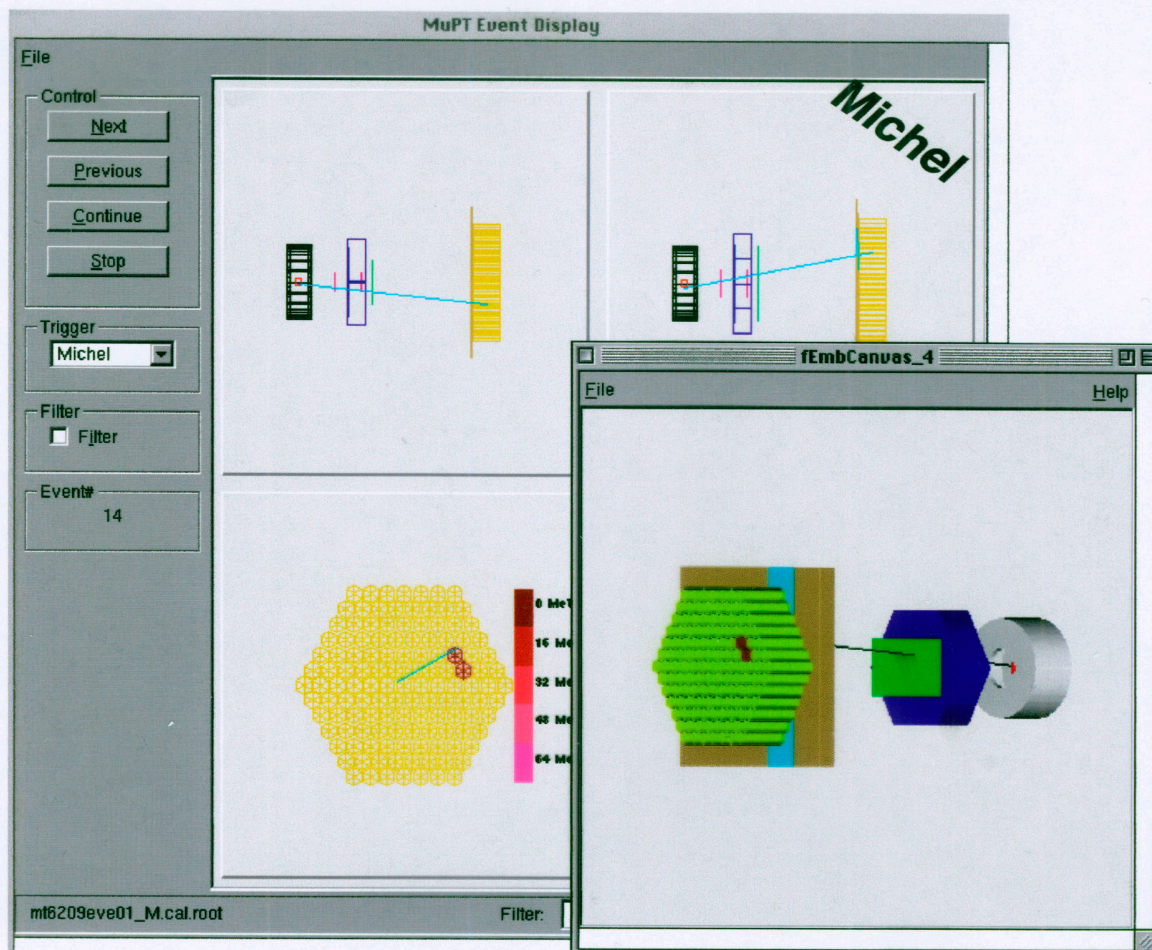
$$\cos \vartheta = 1 - m_e \frac{4}{E_{e^+}} \quad d = 2z \tan \frac{\vartheta}{2}$$

$$d_{min} = d(E_{e^+} = 50 \text{ MeV}) \approx 16 \text{ cm}$$

FPGA approach allows redefining
trigger conditions 'on the fly'

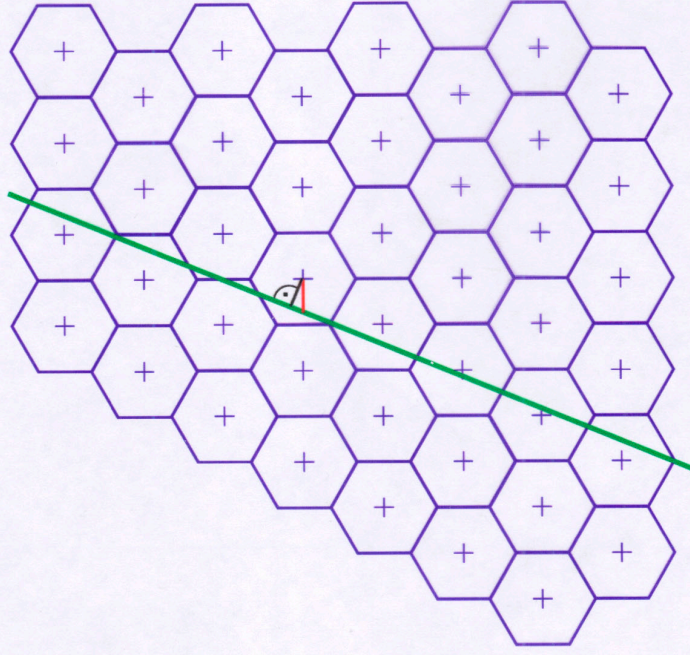
Distance between two clusters



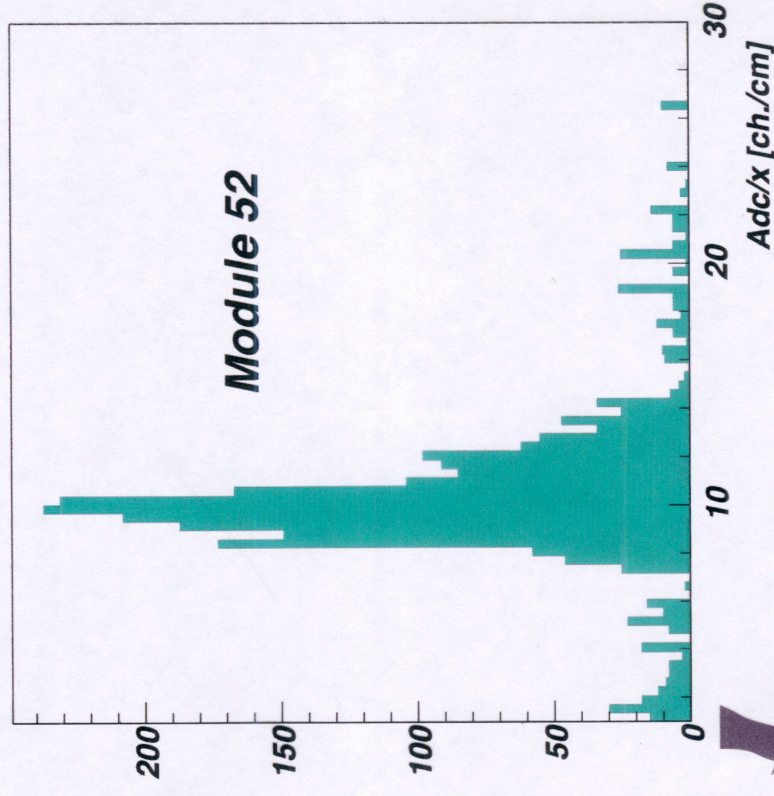


Data is Calibrated with Cosmic Muons

Reconstructing the cosmic tracks gives the tracklength in BGOs



Histogram Adc/x for cosemics



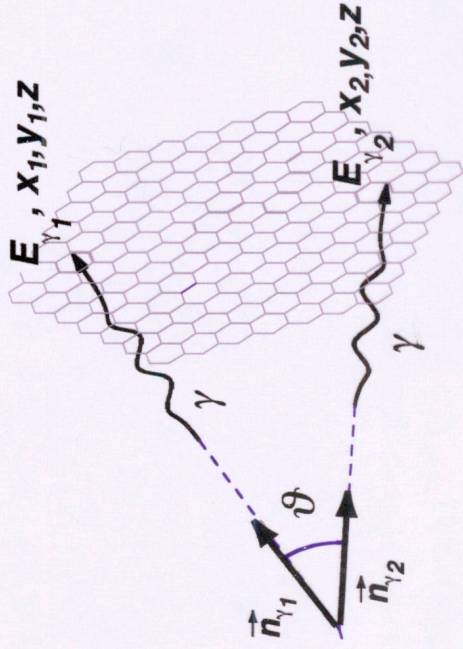
$$\frac{1}{E_i/x_i} \times \frac{ADC_i}{x_i} = \frac{ADC_i}{E_i} = c_i$$

Calibration constant
for BGO number i

Monte Carlo

Cut on the Kinematics to Extract the 'Good' Events

Calculate ϑ in two different ways:



Energy:

$$\cos \vartheta = 1 - m_e \frac{E_{\gamma_1} + E_{\gamma_2}}{E_{\gamma_1} \cdot E_{\gamma_2}}$$

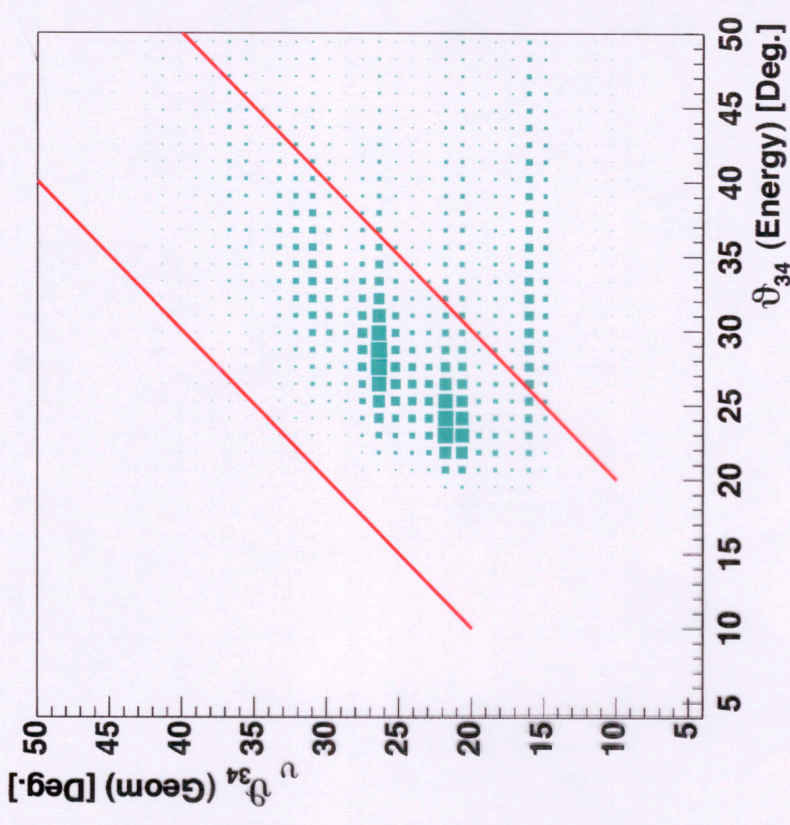
Geometry

$$\cos \vartheta = \vec{n}_{\gamma_1} \cdot \vec{n}_{\gamma_2}$$

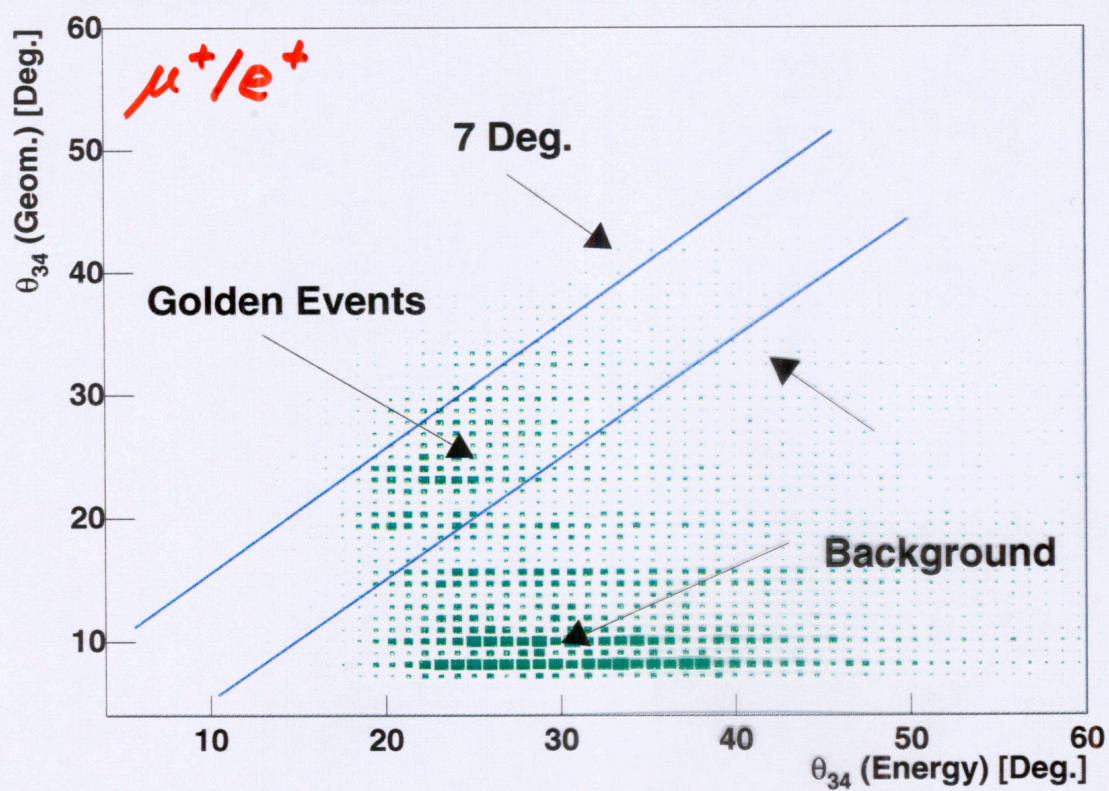
$$\vartheta^{\text{Geom}} = \vartheta^{\text{Energy}}$$

for 'good' annihilations

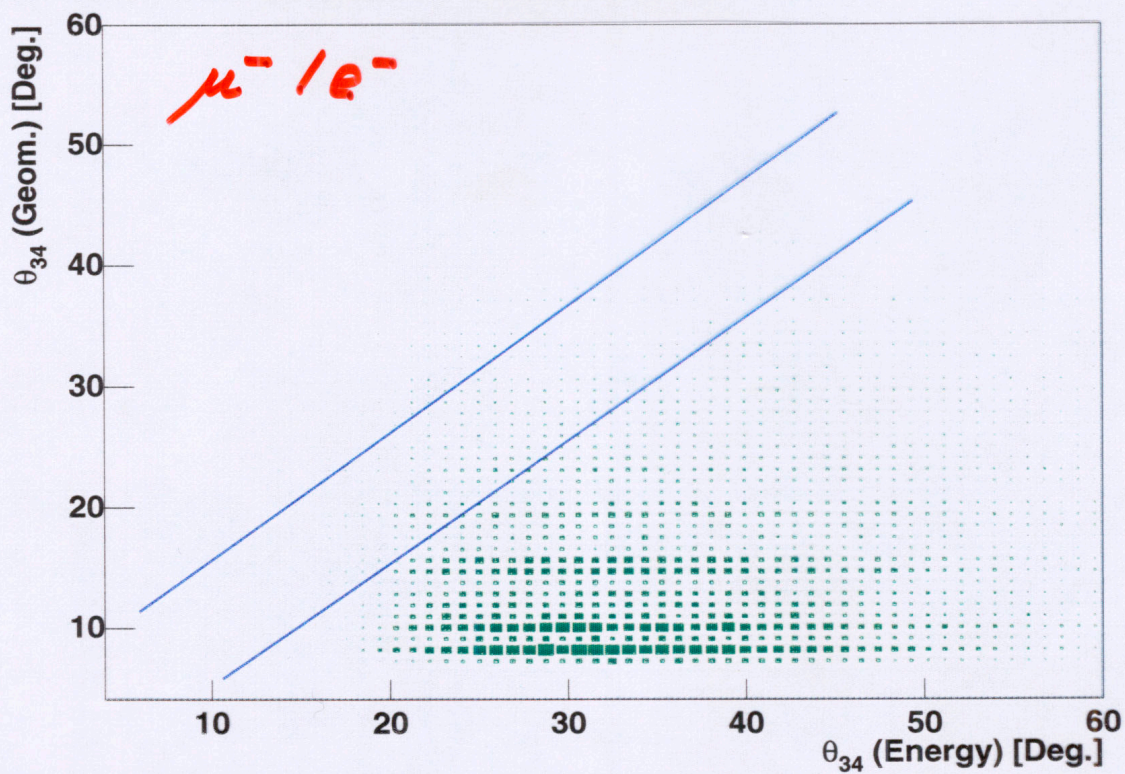
Sample from the last run



θ_{34} (Energy) vs. θ_{34} (Geom.)



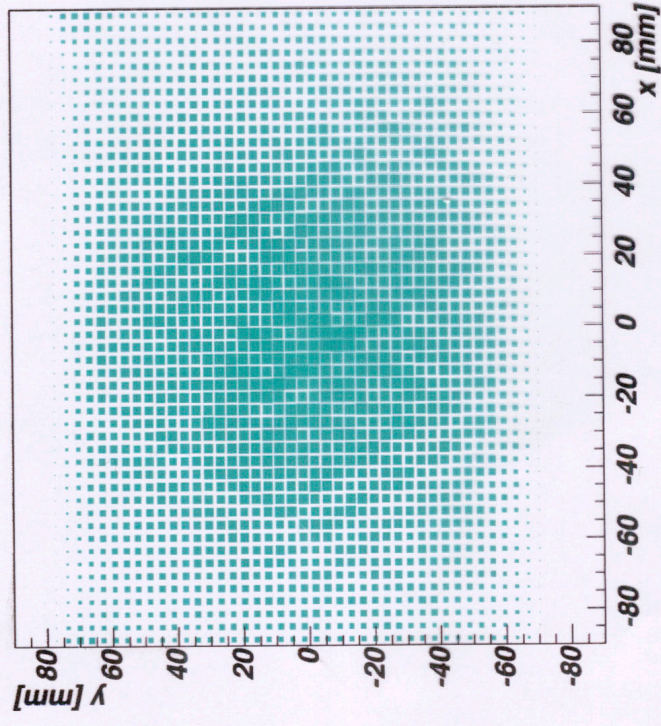
θ_{34} (Energy) vs. θ_{34} (Geom.)



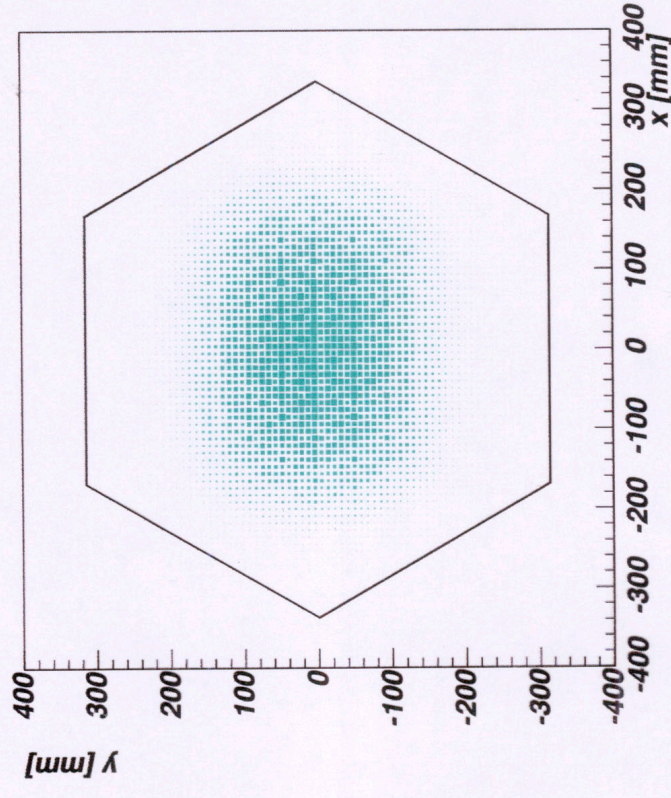
Where do `good` annihilations

come from,

and where do they go?



**reconstructed position
of annihilations on the foil**

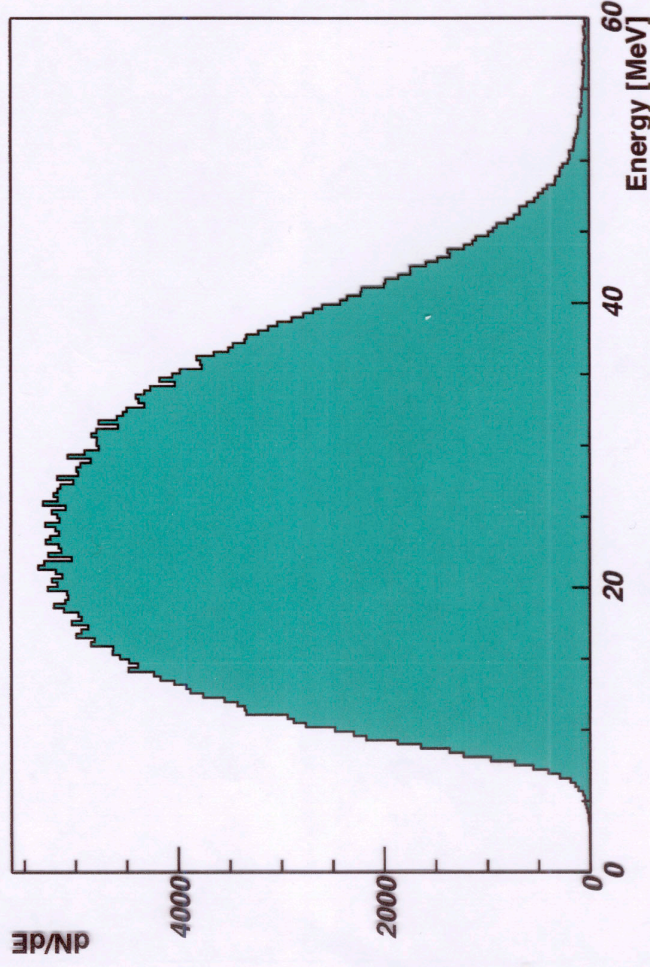


center of energy on the BGO-wall

Analysis, Step Two: From Raw Data to `Good` Events

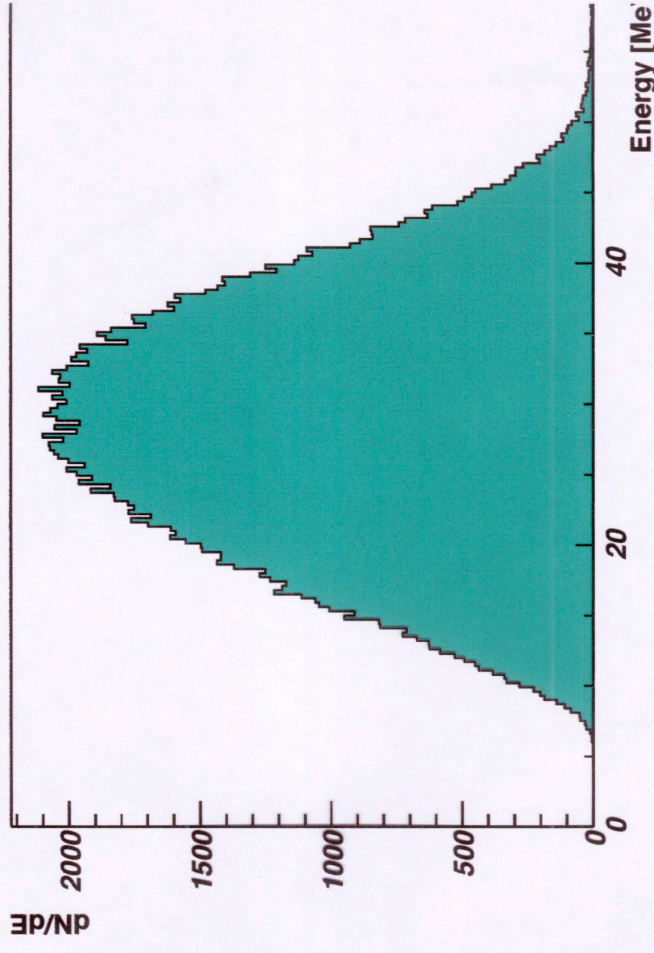
Energy ($E_{\text{tot}} = E_{\gamma_1} + E_{\gamma_2}$) spectra of annihilation events

After reconstruction ($\approx 50\%$ Eff.)



charged track reconstructed,
exactly two clusters in the BGO.

After selection ($\approx 34\%$ Eff.)



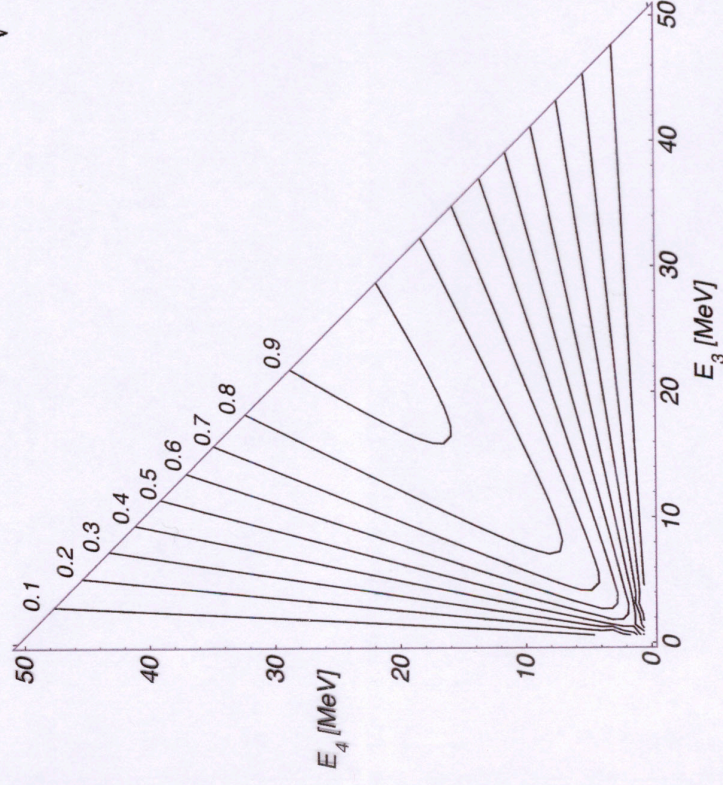
event kinematics must be
consistent with annihilation
hypothesis.

After all cuts $\approx 17\%$ `good` annihilation events remain.

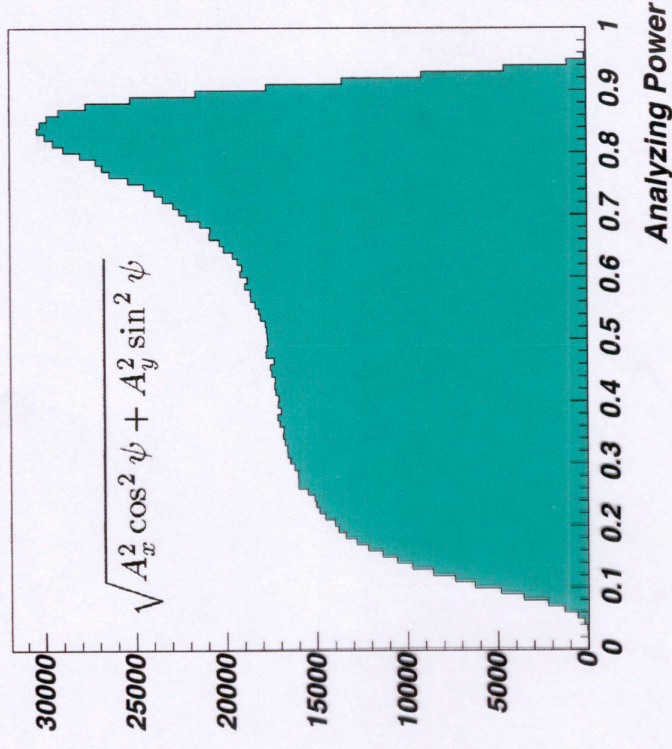
Theoretical and Experimental Analyzing Power

The analyzing power is the amplitude of the expected oscillation

$$A = S \cdot \sqrt{P_1^2 + P_2^2} \cdot \sqrt{A_x^2 \cos^2 \psi + A_y^2 \sin^2 \psi}$$



Theoretical analyzing power,
 $\psi = 90^\circ$ $P_e = 100\%$, $IP_T = 1$



Analyzing Power of 'good' annihilation events.

A_x and A_y are functions of the photon energies

8. Data analysis

$$\begin{aligned} R(t) = & F_{\text{TDC}}(t) \\ & \cdot F_{\mu}(t) \\ & \cdot F_{\mu\text{SR}}(P_{\mu}, E_1, \alpha, \chi, t) \\ & \cdot F_{\gamma\gamma}(E_3, E_4, \mathbf{P}, P_{e^-}, \alpha, \chi, \psi, t) \end{aligned}$$

Quantity	Symbol
μ^+ polarization	P_{μ}
e^+ energy	E_1
e^+ direction	(α, χ)
e^+ polarization	$\mathbf{P} = (P_x, P_y, P_z)$
e^- polarization	P_{e^-}
γ energies	E_3, E_4
analysing powers	A_x, A_y, A_z
azimuthal angle of BGO pair	ψ

$$F_{\mu\text{SR}}(P_\mu, E_1, \alpha, \chi, t) \\ = 1 + P_\mu A(E_1) \sin \chi \cos(\omega t - \alpha - \delta_0)$$

$$F_{\gamma\gamma}(E_3, E_4, P, P_{e-}, \alpha, \chi, \psi, t) \\ = 1 + P_{e-} \cdot P_\mu \cdot \cos(\omega t - \delta_0) [P_x \cdot G + P_y \cdot H] \\ + P_{e-} \cdot P_\mu \cdot \sin(\omega t - \delta_0) [P_x \cdot H - P_y \cdot G] \\ + P_{e-} \cdot P_z \cdot A_z \cos \alpha \sin \chi$$

$$G \equiv A_x \cos^2 \psi + A_y \sin^2 \psi$$

$$H \equiv (A_x - A_y) \sin \psi \cos \psi$$

How can we extract the e^+ transverse polarization (P_x, P_y) from the data?

Harmonic oscillation:

$$F(t) = 1 + a \cos \omega t + b \sin \omega t \\ = 1 + A_F \cos(\omega t - \varphi)$$

Solution: Fourier analysis; neglect multiples of ω .

#	Effect	$A_F \times 10^3$
1	Residual (Beam micro-structure & muon decay & TDC diff. nonlinearity)	8.3
2	μ SR (decay asymmetry)	210.0
3	ΔP_T	0.3

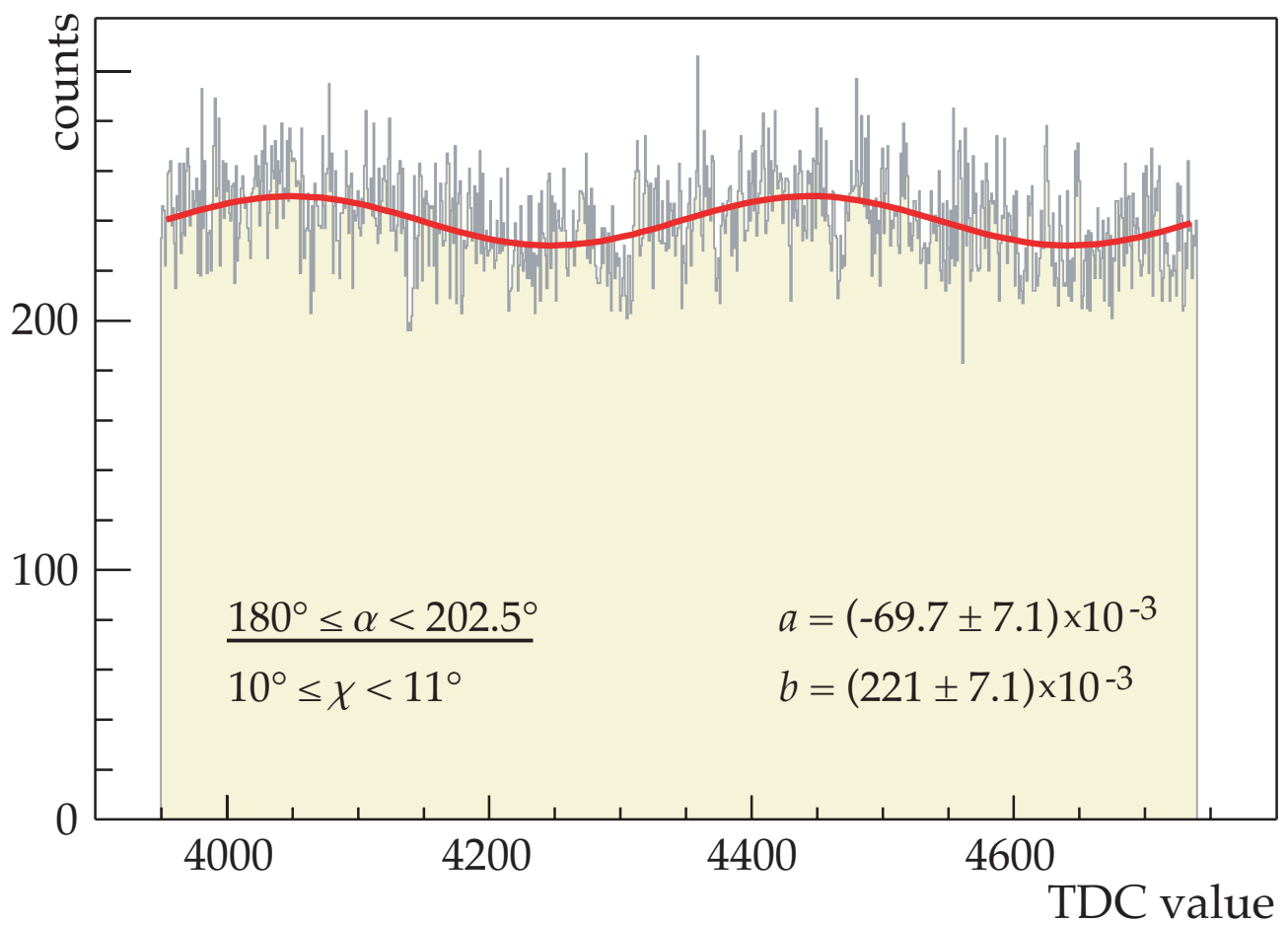
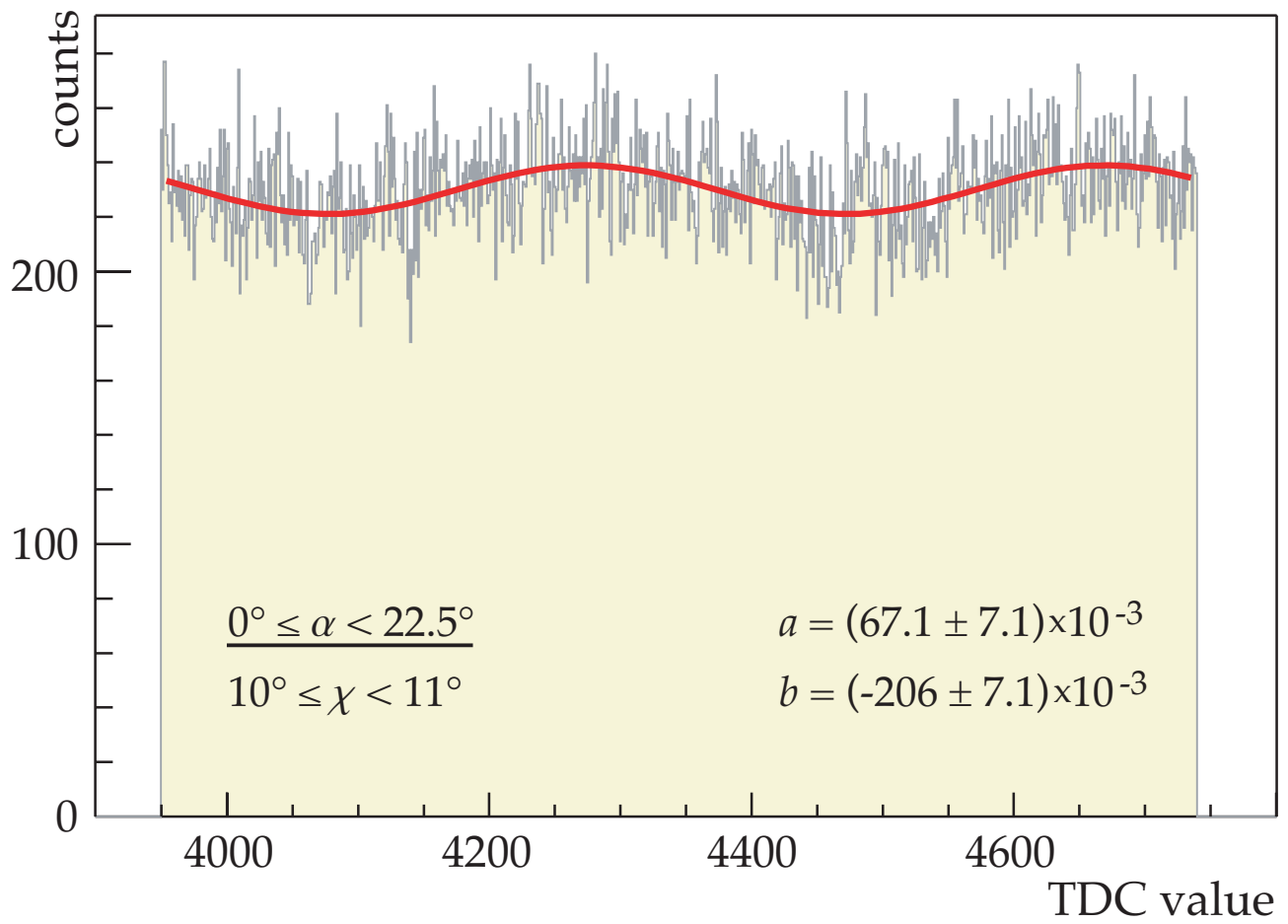
Make use of the various inherent symmetries:

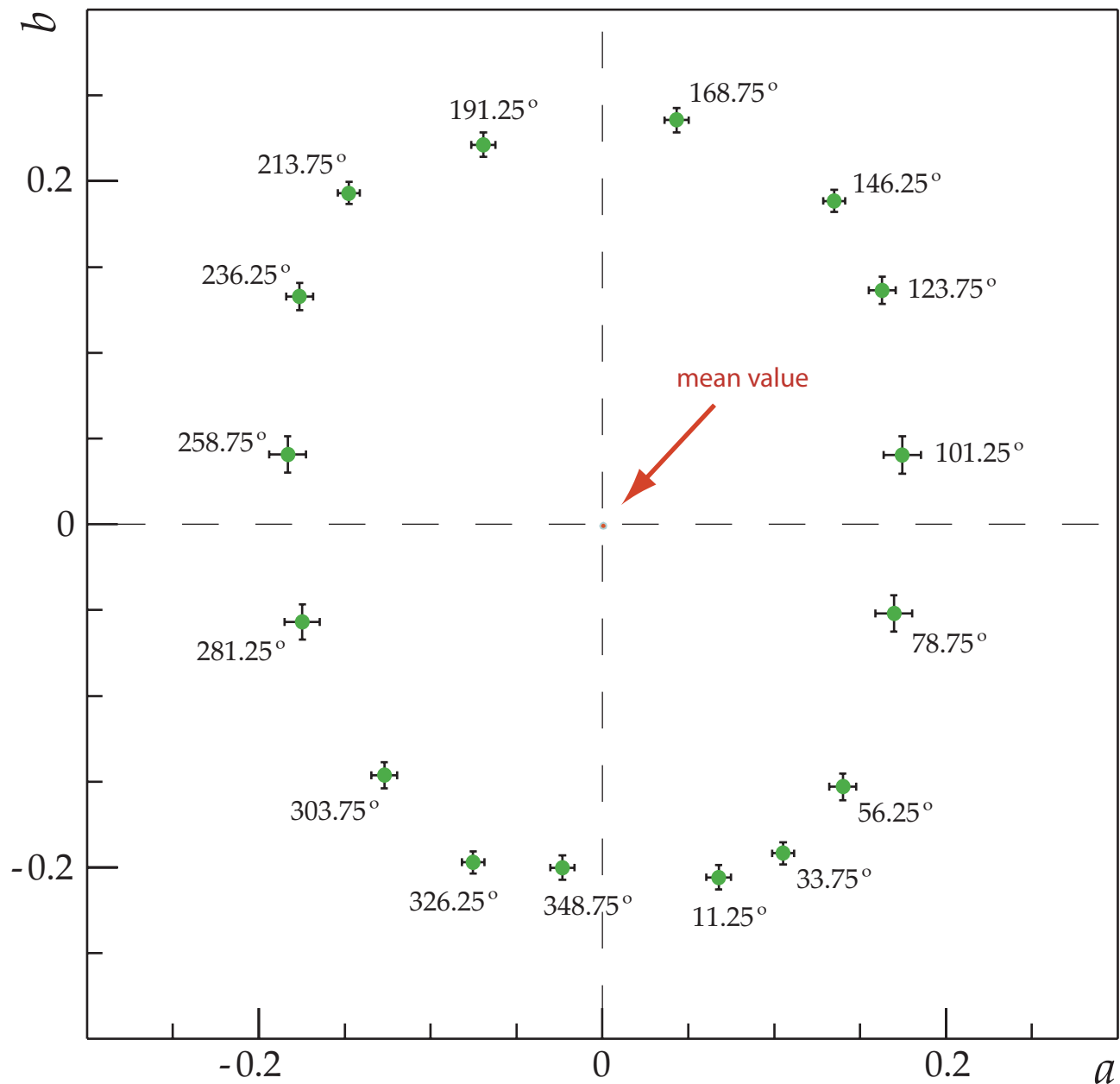
1: Integrate over time, sum over P_{e-} , integrate over positron direction (α, χ) :

$$\Rightarrow a_{\text{res}}, b_{\text{res}}$$

2: Sum over P_{e-} , determine as a function of the positron direction (α, χ) , subtract $a_{\text{res}}, b_{\text{res}}$ and finally, divide by $P_\mu \sin \chi$:

$$\Rightarrow a_{\mu\text{SR}}(\alpha), b_{\mu\text{SR}}(\alpha), \text{Phase } \delta_0$$



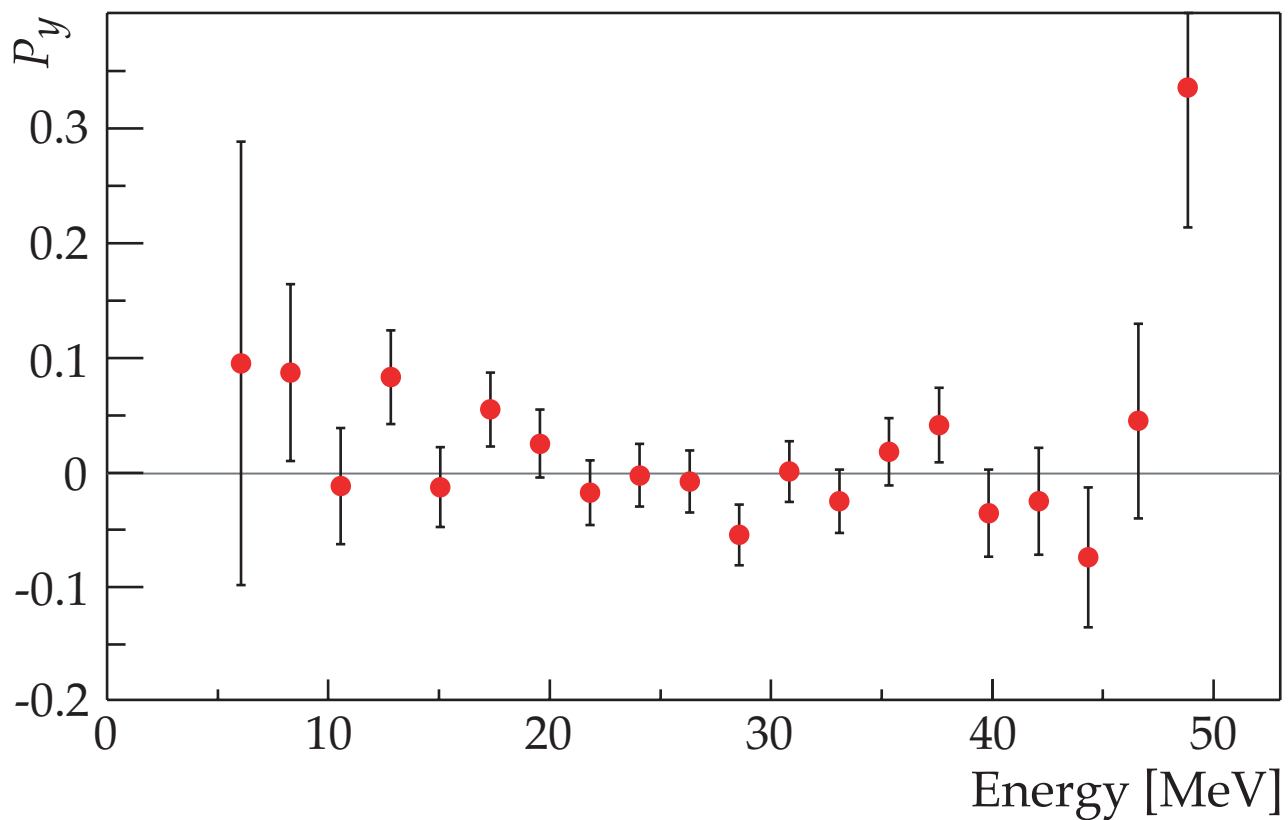
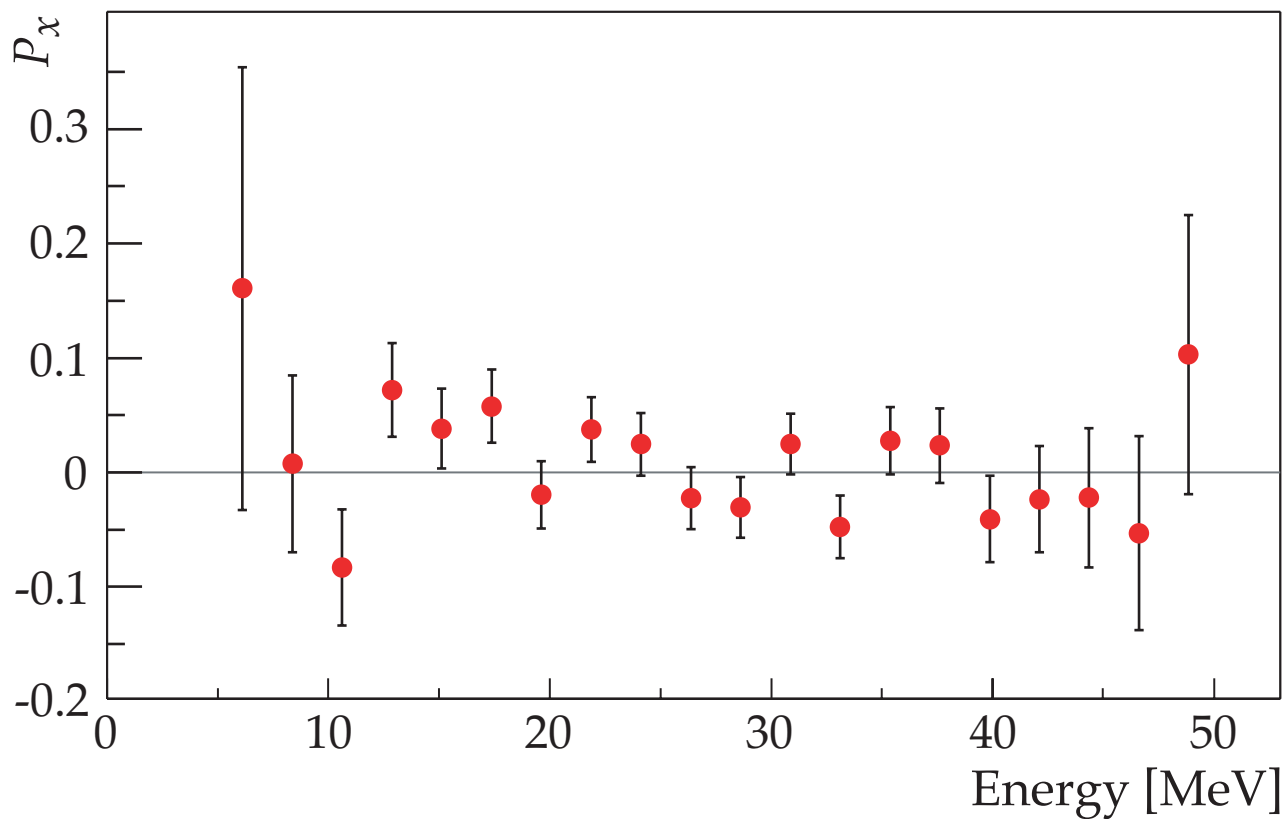


Fourier coefficients $a_{\mu\text{SR}}(\alpha), b_{\mu\text{SR}}(\alpha)$
Determine $P_{\mu}(t)$!

3: Integrate over positron direction (α, χ) , subtract coefficients for $P_{e^-} = -1$ from coefficients for $P_{e^-} = +1$.

$$\Rightarrow (P_x, P_y)$$

Transverse e^+ polarization components
at the moment of annihilation

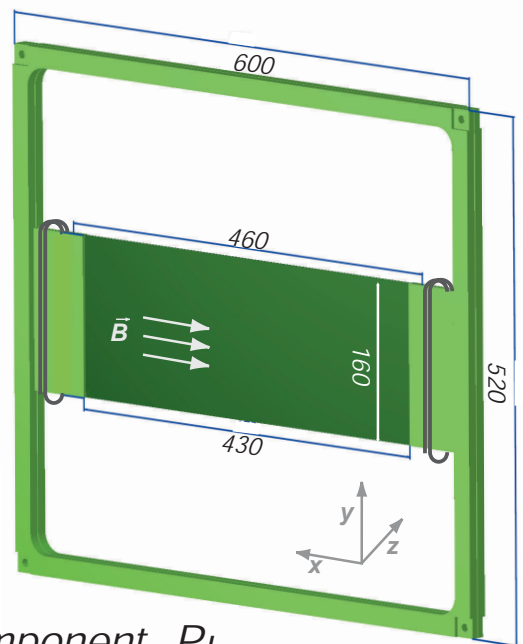


Measurement of the Longitudinal Polarization

using information about position
on magnetized Vacoflux foil
(determined by tracks reconstructed
from drift-chamber data)
where annihilations take place

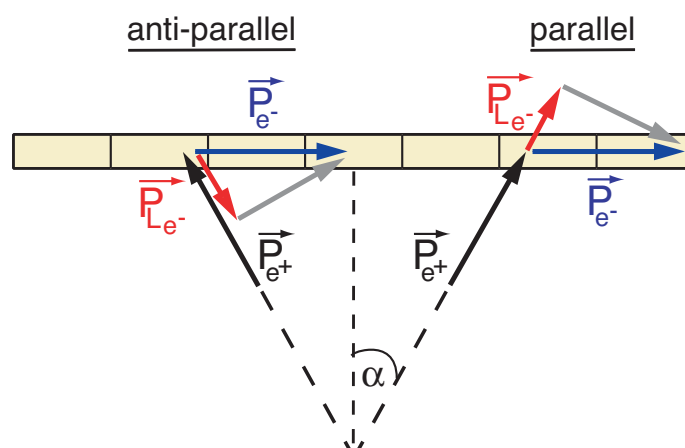
area on foil taken into account: 140^2 mm^2

area divided into rectangular bins (ij),
17 bins in x- and y-direction, respectively



*Tracks that do not hit the center
of the foil 'see' a longitudinal component P_{Le^-}
of the polarization of the electrons in the foil.*

*This P_{Le^-} can either be parallel or anti-parallel
to the positron polarization :*



Longitudinal Polarization P_L of the Positrons

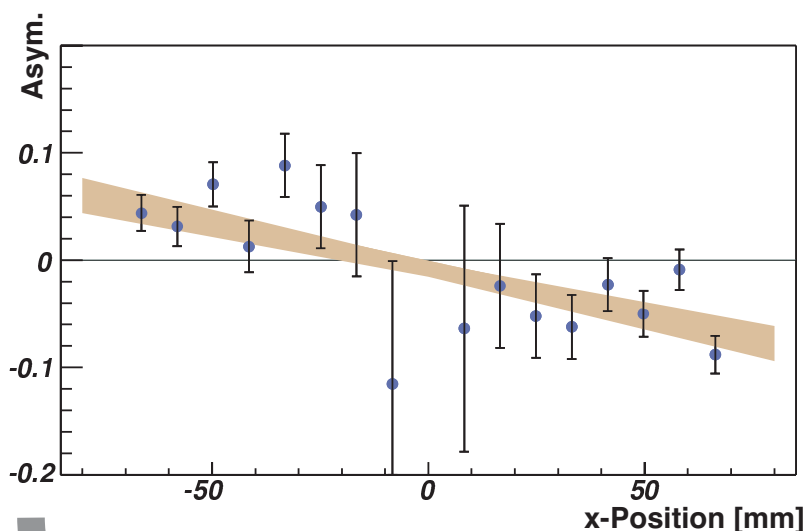
annihilation cross section depends
on relative orientations of spins;
it is larger if both spins
are anti-parallel

Asymmetry:
$$A_{ij} = \frac{\frac{n_{ij}^-}{N^-} - \frac{n_{ij}^+}{N^+}}{\frac{n_{ij}^-}{N^-} + \frac{n_{ij}^+}{N^+}}$$

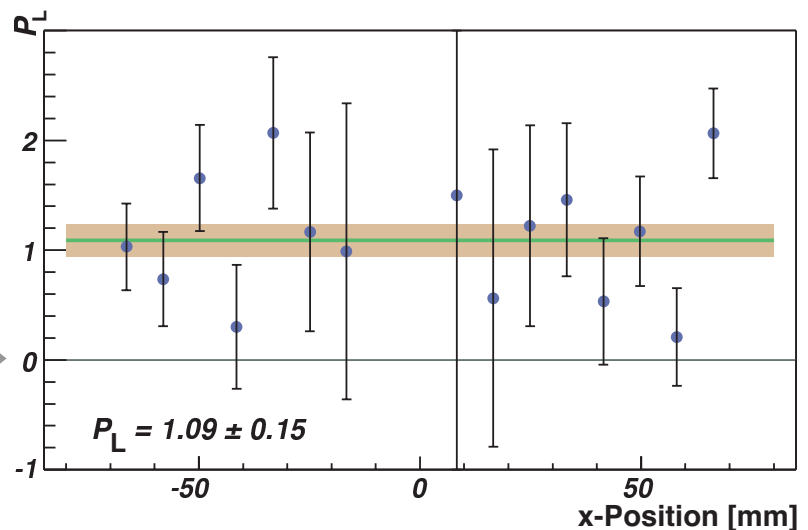
where n_{ij}^+ : number of annihilations in bin ij
for positive foil polarization

N^+ : total number of annihilations
for positive polarization

n_{ij}^-, N^- : same for negative polarization



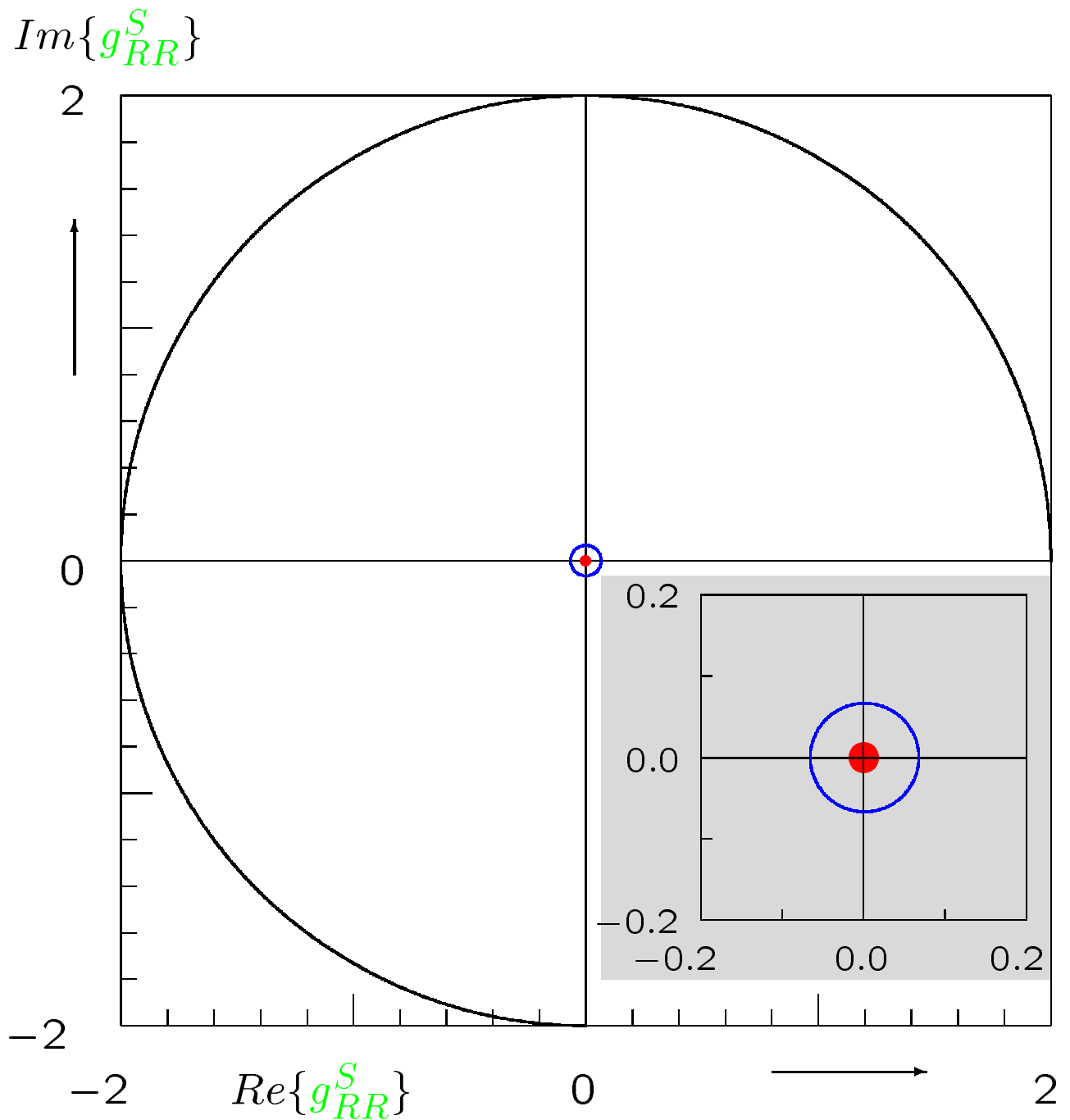
- angle α
- elektron polarization in foil ($P_{e^-} = 7.2\%$)
- analysing power of 0.79
- background factor of 0.75 (backgr. ratio 25 %, mainly due to bremsstrahlung)



8. Preliminary Results

All results are compatible with the standard model. Analysis is in progress. The **experimental errors** are given below:

	General analysis	V - A + g_{RR}^S
$10^3 \times \Delta \langle P_{T_1}(90^\circ) \rangle_E$	14	14
$10^3 \times \Delta \langle P_{T_2}(90^\circ) \rangle_E$	14	14
$10^3 \times \Delta \eta$	53	10
$10^3 \times \Delta \eta''$	75	10
$10^3 \times \alpha'/A$	31	-
$10^3 \times \Delta \beta'/A$	11	5
$10^3 \times \Delta \text{Re } g_{RR}^S$	—	20
$10^3 \times \Delta \text{Im } g_{RR}^S$	—	20
$10^3 \times \Delta g_{RR}^S $	—	20



Limits for scalar coupling g_{RR}^S .

Black (outer) circle: by definition.

Blue circle: previous measurement [1]

Inner red circle: this measurement.

Position of red circle: not yet determined.

9. Outlook

Improve precision of previous experiment [1] by almost one order in magnitude to:

$$\begin{aligned}\Delta\langle P_{T_1}(90^0)\rangle &= 0.008 \\ \Delta\langle P_{T_2}(90^0)\rangle &= 0.008\end{aligned}$$

Assuming $V - A$ and one additional coupling, this will reduce the limits for η and g_{RR}^S to

$$\begin{aligned}\Delta\eta &= 0.006 \\ \Delta Re\{g_{RR}^S\} &= 0.011 \\ \Delta Im\{g_{RR}^S\} &= 0.011\end{aligned}$$

[1] H. Burkard, F. Corriveau, J. Egger, W. Fetscher, H.-J. Gerber, K.F. Johnson, H. Kaspar, H-J. Mahler, M. Salzmann, F. Scheck

Phys. Lett. **160B** (1985) 343.