NUFACT 03
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COLUMBIA U.

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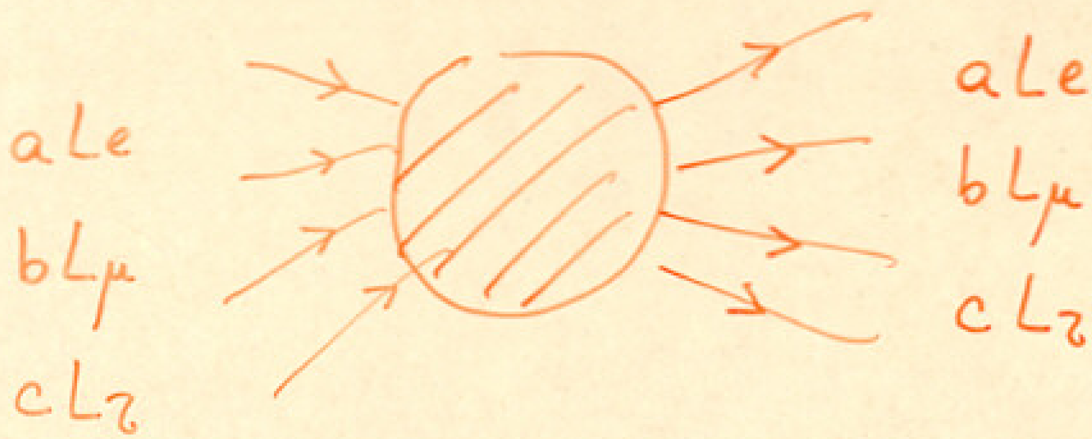
FERMILAB -TH

THEORETICAL ASPECTS OF (CHARGED-)LEPTON FLAVOR VIOLATION [CLFV]

- LEPTON FLAVOR VIOLATION IN THE STANDARD MODEL
- "NEUTRAL" x "CHARGED" LEPTON FLAVOR VIOLATION
- PHENOMENOLOGY OF CLFV
- A FEW MODELS...
  - SUSY + SEESAW
  - SUSY + R-PARITY VIOLATION
  - EXTRADIMENSIONS
- SUMMARY, CONCLUDING MESSAGES

LEPTON FLAVOR VIOLATION IN THE SM

- IN THE <u>OLD S.M.</u>, INDIVIDUAL LEPTON NUMBER (LEPTON FLAVOR) IS EXACTLY CONSERVED.



$L_e(e^-) = L_e(\nu_e) = 1$

$L_\mu(\mu^-) = L_\mu(\nu_\mu) = 1$

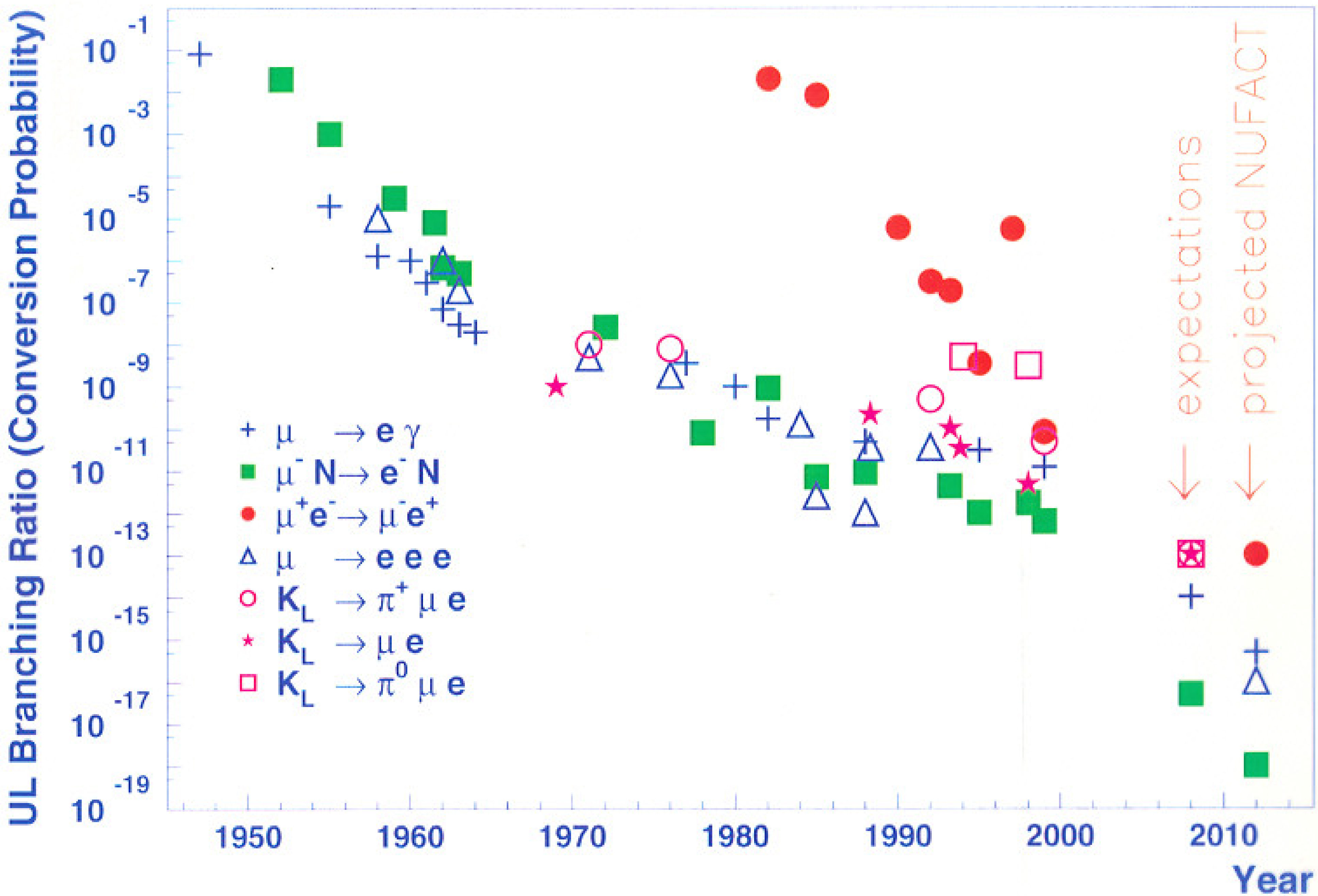
$L_\tau(\tau^-) = L_\tau(\nu_\tau) = 1$

INDEED, WE HAVE NEVER BEEN ABLE TO OBSERVE LEPTON FLAVOR VIOLATION WITH <u>CHARGED LEPTONS</u> AS THE "IN" ~~AND~~/OR "OUT" STATES...

[PLOT] $\rightarrow$

EVEN THOUGH WE HAVE LOOKED HARD FOR IT!

Searches for Lepton Number Violation

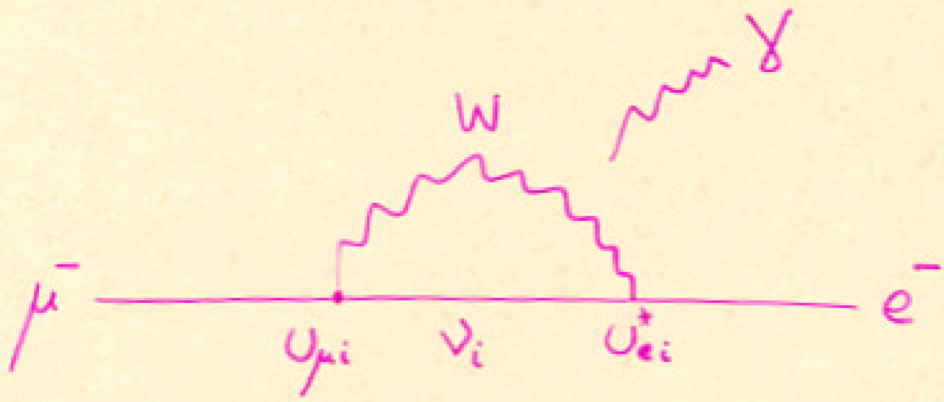


IN THE NEW SM* LEPTON FLAVOR IS NO LONGER CONSERVED!

* OLD SM + MASSIVE NEUTRINOS.

$\Rightarrow$ THERE IS A MISMATCH BETWEEN THE FLAVOR AND MASS EIGENSTATES $\Rightarrow$ MNS MATRIX (LIKE CKM FOR QUARKS)

IN THIS CASE, THERE ARE FCNC-LIKE EFFECTS



TECHNICAL DIFFICULTY (GIM SUPPRESSION):

$$B(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \sum_i \left| U_{\mu i} U^*_{ei} \frac{m^2_{\nu_i}}{M^2_W} \right|^2 \simeq 10^{-60} \left(\frac{m_\nu}{10^{-2}\,\mathrm{eV}} \right)^4$$

WE ARE NOT THERE YET...

WE CAN ONLY SEE THESE TINY EFFECTS IN INTERFERENCE PHENOMENA: NEUTRINO OSCILLATIONS (THIS IS WHERE THE LARGE DISTANCES COME IN...)

• THEORETICALLY, HOWEVER, WE HAVE "STRONG" REASON TO BELIEVE THAT CLFV IS INDEED "OBSERVABLE"...

... AND IT MAY TURN OUT THAT ν-MASSES + CLFV ARE DEEPLY RELATED!

(MORE ON THIS LATER...)

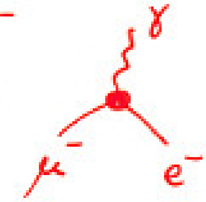
ON THE PHENOMENOLOGY OF CLFV,

IN PARTICULAR RARE MUON PROCESSES

EFFECTIVE OPERATORS THAT MEDIATE CLFV:

$\mu \to e\gamma$

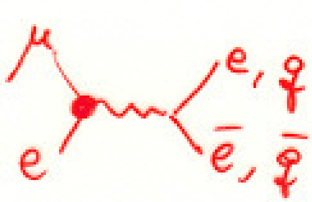
$$\mathcal{L}_{\mu\to e\gamma} = \frac{m_\mu}{\Lambda^2}\, \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + \text{h.c} + \dots$$

[]

$$B(\mu\to e\gamma) = \frac{3(4\pi)^2}{G_F \Lambda^4}$$

THIS SAME LAGRANGIAN ALSO GENERATES $\begin{cases} \mu \to eee \\ \mu N \to eN \end{cases}$

$$\frac{B(\mu\to 3e)}{B(\mu\to e\gamma)} = 6\times 10^{-3} \quad ; \quad \frac{B(\mu N\to eN)}{B(\mu\to e\gamma)} \sim 2\times 10^{-3}$$

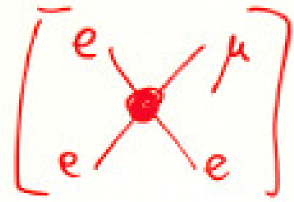


$\Rightarrow$ $\mu \to e\gamma$ DOMINATES !

BUT THERE MIGHT BE EXTRA CONTRIBUTIONS TO ...

$\mu \to eee$

$$\mathcal{L} = \mathcal{L}_{\mu \to e\gamma} + \frac{1}{\Lambda_F^2} \bar{\mu}_L \gamma^\mu e_L \, \bar{e}_L \gamma_\mu e_L + h.c. + \dots$$

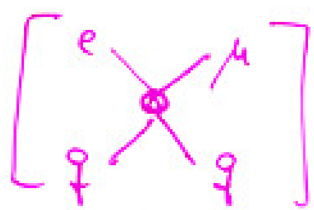


ON WHICH CASE THE STORY MIGHT BE VERY DIFFERENT

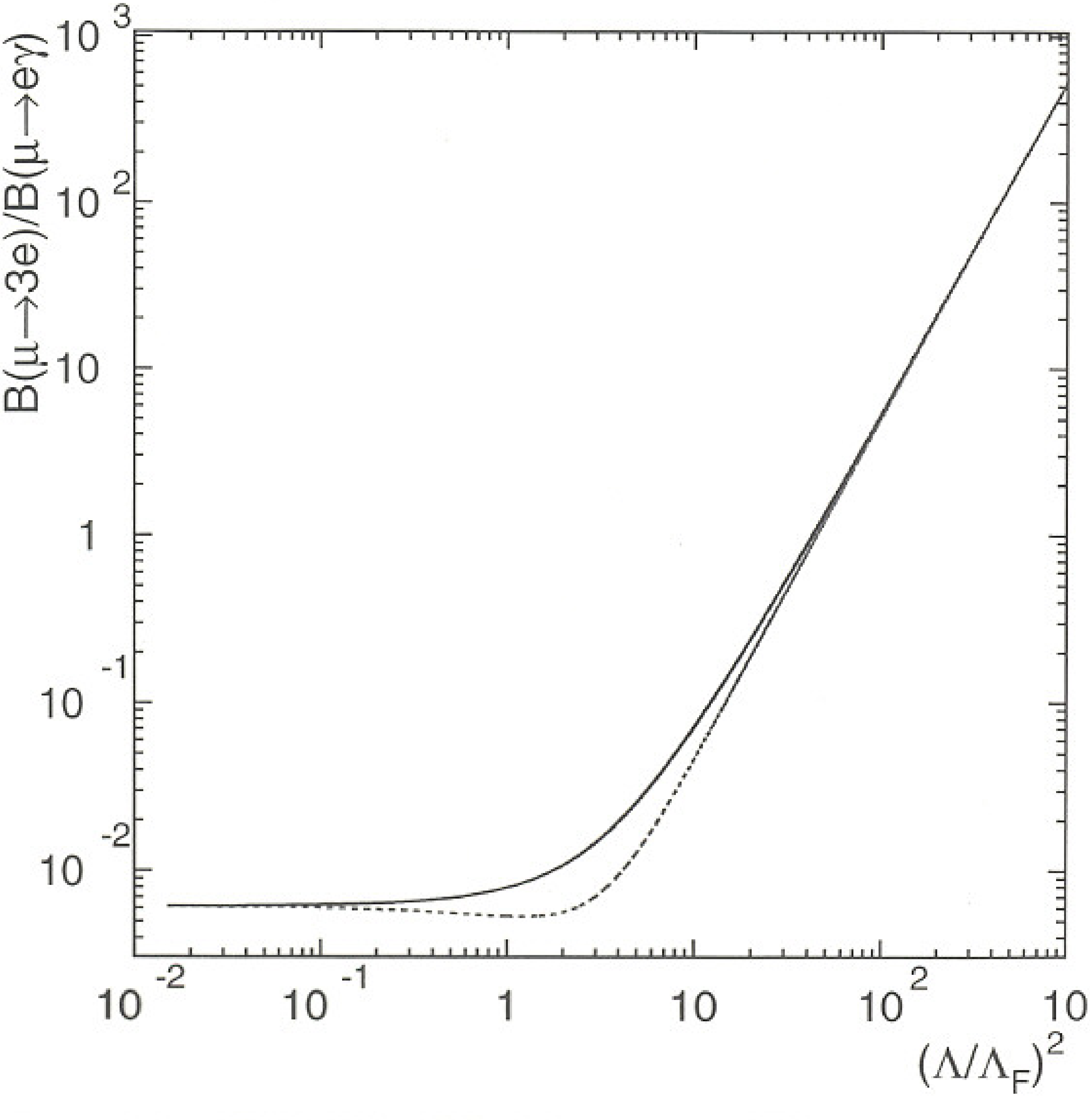
$\longrightarrow$

$\mu N \to e N$

$$\mathcal{L} = \mathcal{L}_{\mu \to e\gamma} + \frac{1}{\Lambda_F^2} \bar{\mu}_L \gamma^\mu e_L \, \bar{q}_L \gamma_\mu q_L + \dots$$

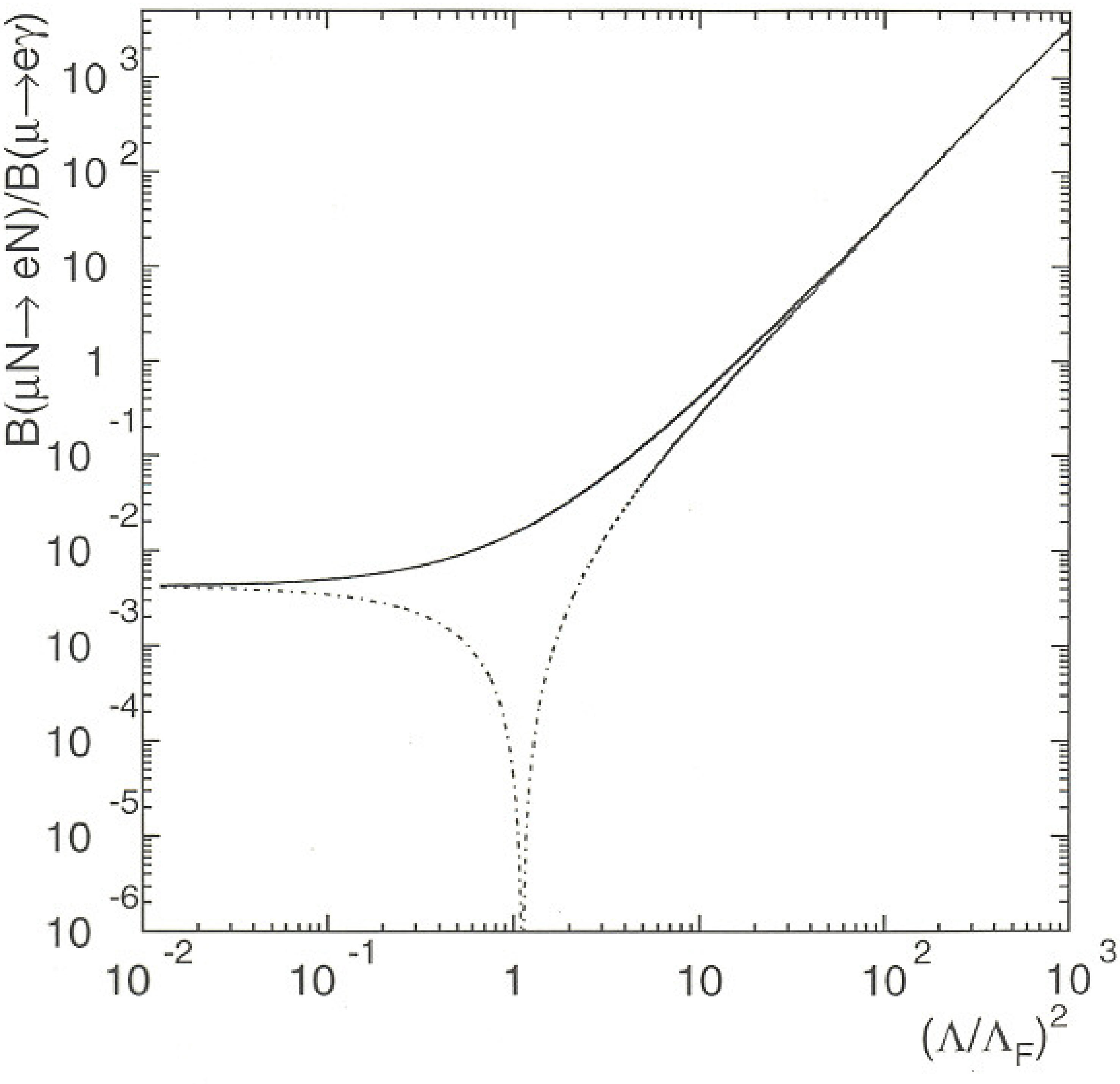


SAME HERE $\longrightarrow$



B(μ→3e)/B(μ→eγ)
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-2}
10^{-1}
1
10
10^2
10^3
$(\Lambda/\Lambda_F)^2$

(in Ti)



B(μN→ eN)/B(μ→eγ)
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-2}
10^{-1}
1
10
10^2
10^3
$(\Lambda/\Lambda_F)^2$

PHENOMENOLOGY

MODEL INDEPENDENT LAGRANGIANS (DIM-5 AND 6 OPERATORS)

$$\mathcal{L}_{\mu\to e\gamma} = -\frac{4G_F m_\mu}{\sqrt{2}}\left[A_R\, \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + A_L\, \bar{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu} + \text{H.c.}\right]$$

$$B(\mu^+\to e^+\gamma) = 384\pi^2\left(|A_R|^2 + |A_L|^2\right)$$

$$\mathcal{L}_{\mu\to eee} = \mathcal{L}_{\mu\to e\gamma} - \frac{4G_F}{\sqrt{2}}\Big[g_1\,(\bar{\mu}_R e_L)(\bar{e}_R e_L) + g_2\,(\bar{\mu}_L e_R)(\bar{e}_L e_R)$$

$$+ g_3\,(\bar{\mu}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R) + g_4\,(\bar{\mu}_L\gamma_\mu e_L)(\bar{e}_L\gamma^\mu e_L) +$$

$$+ g_5\,(\bar{\mu}_R\gamma_\mu e_R)(\bar{e}_L\gamma^\mu e_L) + g_6\,(\bar{\mu}_L\gamma_\mu e_L)\,\bar{e}_R\gamma^\mu e_R) + \text{H.c.}$$

$$B(\mu^+\to e^+e^+e^-) = 2(C_1+C_2) + (C_3+C_4) + 16(C_7+C_8)$$

$$+ 8(C_9+C_{10}) + 32\left[\underbrace{\ln\left(\frac{m_\mu^2}{m_e^2}\right)}_{} - \frac{11}{4}\right](C_5+C_6)$$

WHERE

$$C_1 = \frac{|g_1|^2}{16} + |g_3|^2 \;; \qquad C_2 = \frac{|g_2|^2}{16} + |g_4|^2$$

$$C_3 = |g_5|^2 \;;\; C_4 = |g_6|^2 \;;\; C_5 = |eA_R|^2 \;;\; C_6 = |eA_L|^2$$

$$C_7 = \mathrm{Re}(eA_R g_4^*) \;;\; C_8 = \mathrm{Re}(eA_L g_3^*) \;;\; C_9 = \mathrm{Re}(eA_R g_6^*)$$

$$C_{10} = \mathrm{Re}(eA_L g_5^*) \qquad C_{11} = \mathrm{Im}(eA_R g_4^* + eA_L g_3^*)$$

$\mu^+ \to e^+ e^- e^+$ AND CP- VIOLATION

$$\mu^+(\vec{\sigma}_\mu) \to e^+(\vec{p}_1)\, e^+(\vec{p}_2)\, e^-(\vec{p}_3)$$

TODD COMBINATION : $\vec{\sigma}_\mu \cdot (\vec{p}_1 \times \vec{p}_2)$

OBSERVABLE $\propto$ events with $\phi \in [0, \pi]$ - events with $\phi \in [\pi, 2\pi]$

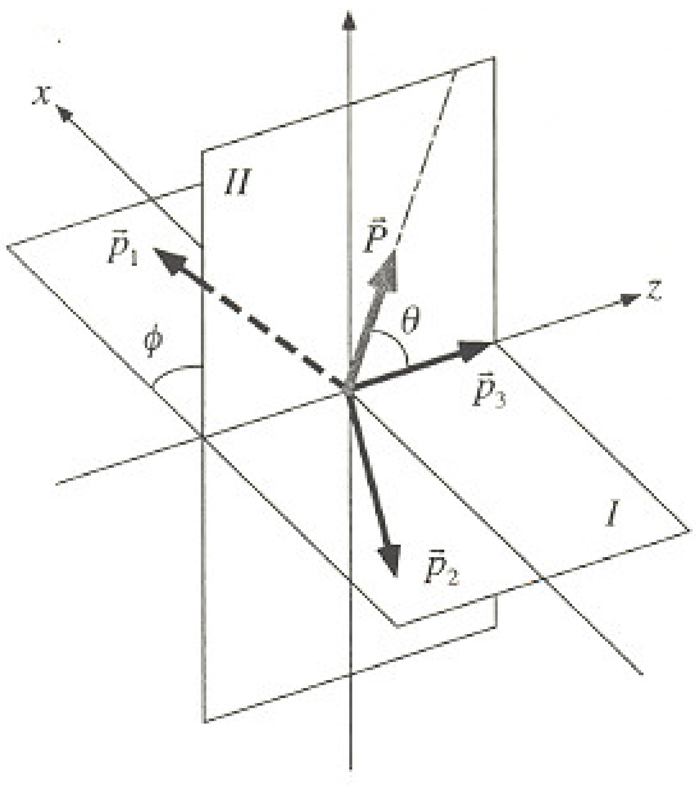


FIG. 27. Kinematics of the $\mu^+ \to e^+e^+e^-$ decay in the muon center-of-mass system, in which $\vec{p}_1$, $\vec{p}_2$ are the momentum vectors of the two e$^+$s and $\vec{p}_3$ is that of the e^-, respectively. The plane-I is the decay plane on which $\vec{p}_1$, $\vec{p}_2$, and $\vec{p}_3$ lie. The plane-II is the plane in which the muon polarization vectors, $\vec{P}$ and $\vec{p}_3$, are located (after Okada, *et al.*, (1999)).

$$A_T(\mu^+ e^+ e^+ e^-) = \frac{64}{35} \frac{(3C_{11} - 2C_{12})}{B(\mu^+ \to e^+ e^+ e^-)}$$

INTERFERENCE BETWEEN ON-SHELL PHOTON AND 4 FERMION INTERACTION

CHALLENGE $A_T^{MAX} \lesssim 15\%$ (MODEL INDEPENDENT !)

SOME MODELS

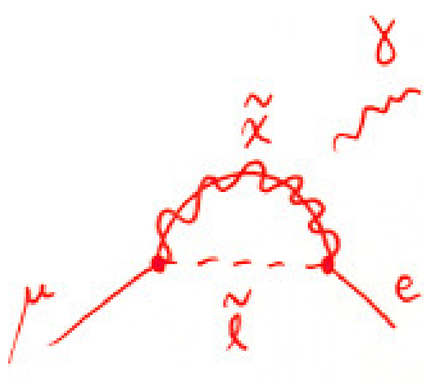
SEE SAW

$$\mathcal{L} \ni -\lambda_\nu^{ij} L^i N^j H_u - \frac{M_R^{ij}}{2} N^i N^j + h.c.$$

AT ENERGIES BELOW M_R $\Longrightarrow$ $\mathcal{L} \ni K^{ij} (L^i H_u)(L^j H_u)$

$$K^{ij} \to \lambda^{ij} \left(\frac{1}{M_R}\right)^{jk} \lambda^{kj}$$

+ LOW ENERGY SUSY



$$B(\mu \to e\gamma) \propto (\tilde{m}_\ell)^4_{e\mu} \left(\frac{1}{M_{SUSY}}\right)^4$$

$$\Downarrow$$

- POTENTIALLY HUGE $\Rightarrow$ PROBLEM!
- NOT SUPPRESSED BY ν-MASSES

$\Rightarrow$ FORCED TO IMPOSE $\tilde{m}_{e\mu} = 0$ AT HIGH ENERGY SCALE...

THE OFF-DIAGONAL TERMS ARE RESURECTED BY RENORMALIZATION GROUP RUNNING...

$$\left(m^2_{\tilde{\ell}}\right)_{ij} \propto \frac{3\, m_0^2}{8\pi^2} \, (y_\nu)^*_{ki} (y_\nu)_{kj} \, \ln\left(\frac{M_{Pl}}{M_R}\right)$$

$\Downarrow$

PROPORTIONAL TO ν-YUKAWA COUPLINGS

REMEMBER $\quad m_\nu^{ij} \propto (y_\nu)^{ik} \left(\frac{1}{M_R}\right)^{k\ell} (y_\nu)^{kj} \, v^2$

DIFFERENT DEPENDENCIES ON y_ν

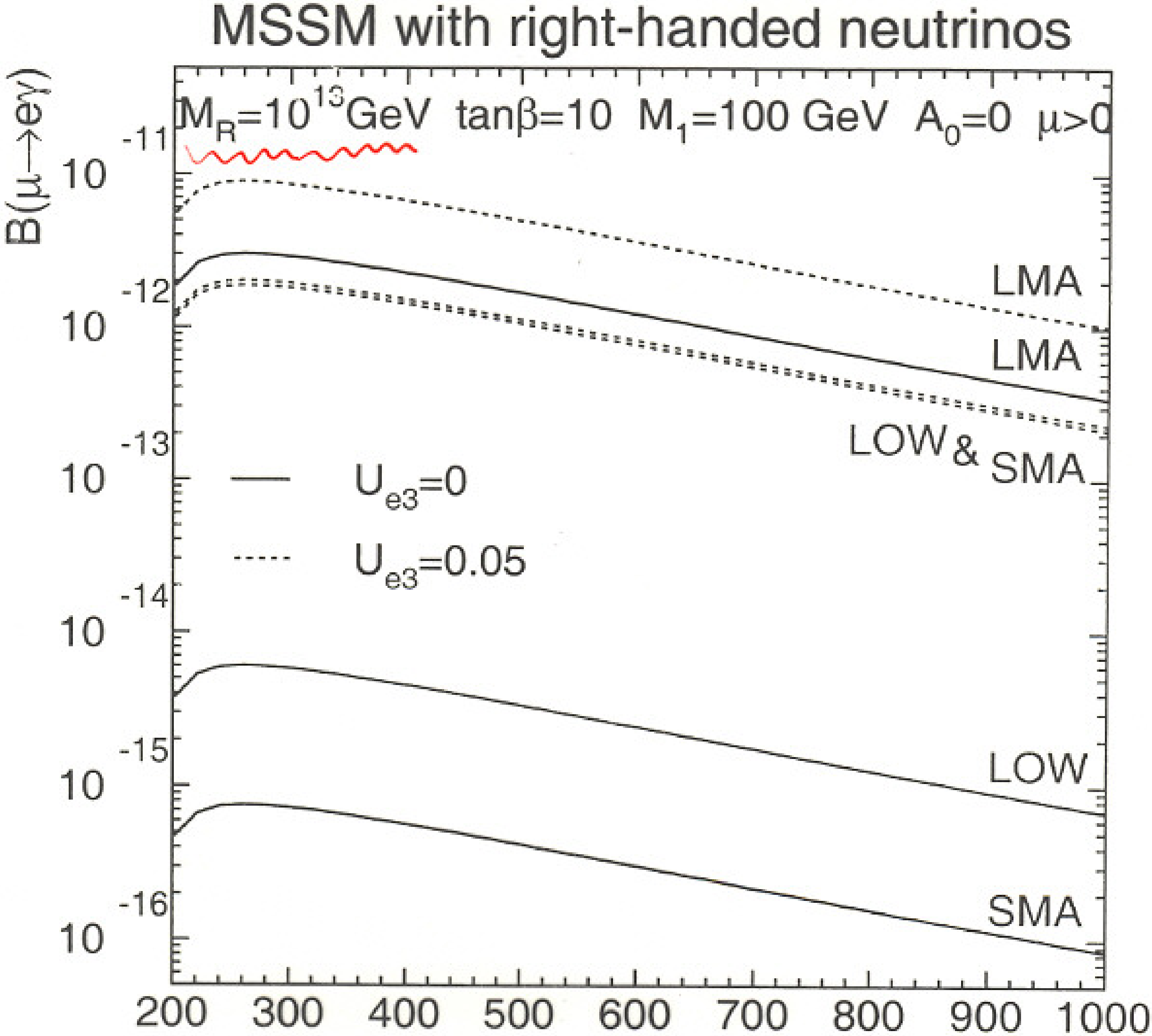
$\Downarrow$

COULD BE IMPORTANT FOR PIECING TOGETHER HIGH-ENERGY SEESAW LAGRANGIAN $\Rightarrow$ SUBJECT OF INTENSE TH. RESEARCH!

$\Downarrow$

TEST OF LEPTOGENESIS ?

$$B(\mu \to e\gamma) \propto M_R^2 \left(\ln\left[\frac{M_{Pl}}{M_R}\right]\right)^2$$



left-handed selectron mass (GeV)

SUSY + TRILINEAR R_{PARITY} - VIOLATION (AdG, LOLA, TOBE PRD63 03500...

$$W_{\not R} = \mu^i L^i H_u + \frac{\lambda_{ijk}}{2} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \lambda'' \bar{U}_i \bar{D}_j \bar{D}_k$$

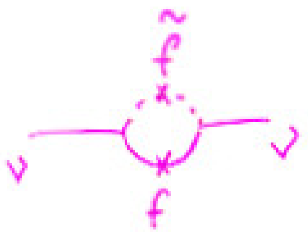
LEPTON NUMBER VIOLATING

B-VIOLATING

IMPOSE → "BARYON-PARITY" $\lambda''_{ijk} = 0$ [PREVENTS PROTON DECAY]

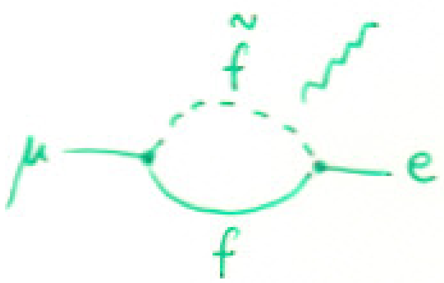
IGNORE → μ^i (FOR SIMPLICITY)

λ, λ' COUPLINGS WILL BOTH GENERATE ν-MASS...

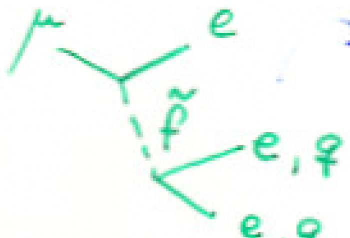


$$m_\nu \propto \frac{m_f^2 (m_{\tilde{f}}^2)_{LR}}{m_{\tilde{f}}^2} \frac{\lambda^2}{16\pi^2}$$

AND CLFV



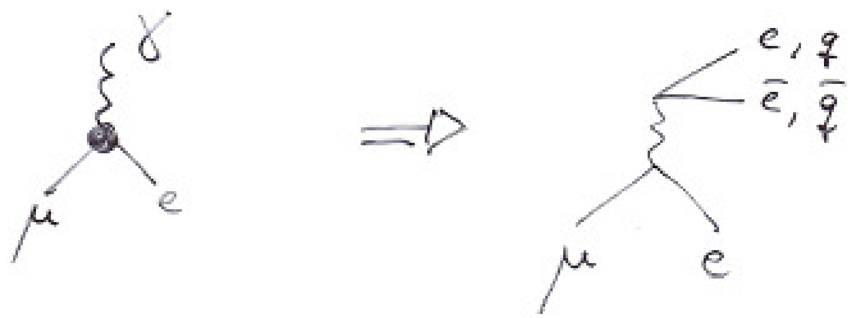
OR



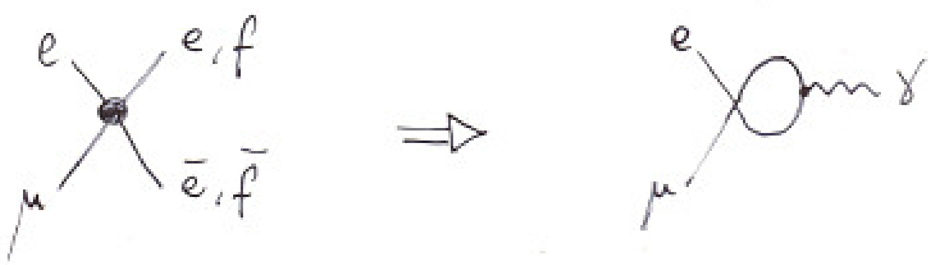
↳ USUALLY DOMINANT [TREE × 1-LOOP]

TABLE I. The ratios of branching ratios $\mathrm{Br}(\mu^+\to e^+\gamma)/\mathrm{Br}(\mu^+\to e^+e^-e^+)$ and $\mathrm{R}(\mu^-\to e^-$ in Ti$)/\mathrm{Br}(\mu^+\to e^+e^-e^+)$, P-odd asymmetries A_P for $\mu^+\to e^+\gamma$, A_{P_1} and A_{P_2} for $\mu^+\to e^+e^-e^+$ are shown when the listed pair of Yukawa couplings is dominant. Cases (1), (2), and (3) refer to the representative classes of models discussed in Secs. IV A, IV B, and IV C, respectively. Here, we assume $m_{\tilde{\nu},\tilde{l}_R}=100$ GeV and no mixing in the charged slepton mass matrix, and $m_{\tilde{q}}=300$ GeV. We also show a typical result obtained for the MSSM with heavy right-handed neutrinos and R-parity conservation [7].

	$\frac{\mathrm{Br}(\mu\to e\gamma)}{\mathrm{Br}(\mu\to 3e)}$	$\frac{\mathrm{R}(\mu\to e \text{ in Ti})}{\mathrm{Br}(\mu\to 3e)}$	A_P	A_{P_1}	A_{P_2}	A_{P_1}/A_{P_2}
Case (1)						
$\lambda_{131}\lambda_{231}$	1×10^{-4}	2×10^{-3}	-100%	$+19\%$	-15%	-1.3
$\lambda_{121}\lambda_{122}$	8×10^{-4}	7×10^{-3}	$+100\%$	-19%	$+15\%$	-1.3
$\lambda_{131}\lambda_{132}$	8×10^{-4}	5×10^{-3}	$+100\%$	-19%	$+15\%$	-1.3
Case (2)						
$\lambda_{132}\lambda_{232}$	1.2	18	-100%	-25%	-5%	5.6
$\lambda_{133}\lambda_{233}$	3.7	18	-100%	-25%	-4%	6.2
$\lambda_{231}\lambda_{232}$	3.6	18	$+100\%$	$+25\%$	$+4\%$	6.2
$\lambda'_{122}\lambda'_{222}$	1.4	18	-100%	-25%	-4%	5.7
$\lambda'_{123}\lambda'_{223}$	2.2	18	-100%	-25%	-4%	5.9
Case (3)						
$\lambda'_{111}\lambda'_{211}$	0.4	3×10^{2}	-100%	-26%	-5%	5.4
$\lambda'_{112}\lambda'_{212}$	0.5	8×10^{4}	-100%	-26%	-5%	5.4
$\lambda'_{113}\lambda'_{213}$	0.7	1×10^{5}	-100%	-26%	-5%	5.5
$\lambda'_{121}\lambda'_{221}$	1.1	2×10^{5}	-100%	-26%	-5%	5.6
MSSM with ν_R	1.6×10^{2}	0.92	-100%	10%	17%	0.6



"USUAL SUSY"



MAY BE DOMINANT WITH $\cancel{R_p}$

Table 1: *Upper limits on products of R-parity-violating couplings from the current limits for muon-number-violating processes, taking slepton masses equal to 100 GeV and squark masses equal to 300 GeV. The values in parenthesis indicate the sensitivity which could be achieved at a neutrino factory complex, i.e. resulting from improvements in experimental sensitivities to rare μ decays by 4-6 orders of magnitude. The coupling constants λ and λ' refer to the interactions $W_{\not R} \supset \lambda_{ijk} L^i L^j E^k + \lambda'_{ijk} L^i Q^j D^k$, where i, j, k are generation indices. Tree-level constraints are indicated by [tree].*

	$\mu \to e\gamma$	$\mu \to eee$	$\mu \to e$ conversion in nuclei
$\lvert\lambda_{131}\lambda_{231}\rvert$	2.3×10^{-4} (2×10^{-6})	6.7×10^{-7} (7×10^{-9})[tree]	1.1×10^{-5} (1×10^{-8})
$\lvert\lambda_{132}\lambda_{232}\rvert$	2.3×10^{-4} (2×10^{-6})	7.1×10^{-5} (7×10^{-7})	1.3×10^{-5} (2×10^{-8})
$\lvert\lambda_{133}\lambda_{233}\rvert$	2.3×10^{-4} (2×10^{-6})	1.2×10^{-4} (1×10^{-6})	2.3×10^{-5} (3×10^{-8})
$\lvert\lambda_{121}\lambda_{122}\rvert$	8.2×10^{-5} (7×10^{-7})	6.7×10^{-7} (7×10^{-9})[tree]	6.1×10^{-6} (8×10^{-9})
$\lvert\lambda_{131}\lambda_{132}\rvert$	8.2×10^{-5} (7×10^{-7})	6.7×10^{-7} (7×10^{-9})[tree]	7.6×10^{-6} (1×10^{-8})
$\lvert\lambda_{231}\lambda_{232}\rvert$	8.2×10^{-5} (7×10^{-7})	4.5×10^{-5} (5×10^{-7})	8.3×10^{-6} (1×10^{-8})
$\lvert\lambda'_{111}\lambda'_{211}\rvert$	6.8×10^{-4} (6×10^{-6})	1.3×10^{-4} (1×10^{-6})	5.4×10^{-6} (7×10^{-9}) [tree]
$\lvert\lambda'_{112}\lambda'_{212}\rvert$	6.8×10^{-4} (6×10^{-6})	1.4×10^{-4} (1×10^{-6})	3.9×10^{-7} (5×10^{-10}) [tree]
$\lvert\lambda'_{113}\lambda'_{213}\rvert$	6.8×10^{-4} (6×10^{-6})	1.6×10^{-4} (2×10^{-6})	3.9×10^{-7} (5×10^{-10}) [tree]
$\lvert\lambda'_{121}\lambda'_{221}\rvert$	6.8×10^{-4} (6×10^{-6})	2.0×10^{-4} (2×10^{-6})	3.6×10^{-7} (5×10^{-10}) [tree]
$\lvert\lambda'_{122}\lambda'_{222}\rvert$	6.8×10^{-4} (6×10^{-6})	2.3×10^{-4} (2×10^{-6})	4.3×10^{-5} (6×10^{-8})
$\lvert\lambda'_{123}\lambda'_{223}\rvert$	6.9×10^{-4} (6×10^{-6})	2.9×10^{-4} (3×10^{-6})	5.4×10^{-5} (7×10^{-8})

AdG, GIUDICE, STRUMIA, TOBE, NPB 623, 395 (2002)

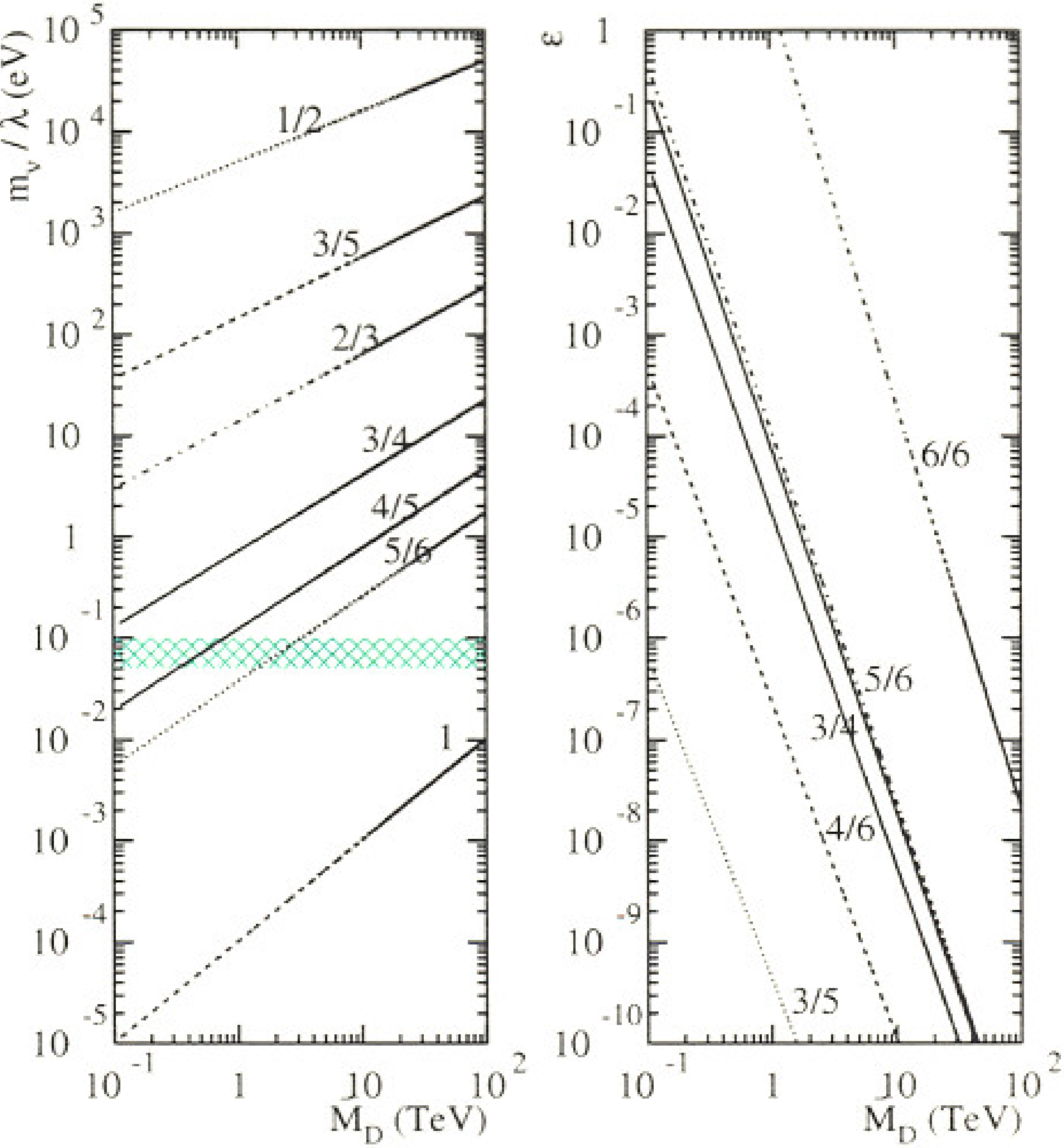


Figure 1: $m_\nu/\hat{\lambda}$ *(left) and* ϵ *(right) as a function of* $\bar{M}_D$ *for different values of* δ/d*, assuming* $\Lambda = \bar{M}_D$*. The horizontal band in the left panel shows the mass range selected by atmospheric neutrino data for* $\hat{\lambda} = 1$.

$$\boxed{\varepsilon \equiv \frac{l_\delta}{\delta-2}\,\frac{\Delta m^2_{ATM}}{\Lambda^2}\,V_\delta\Lambda^\delta}$$

AdG, Giudice, Strumia, Tobe
NPB 623, 375 (2002)

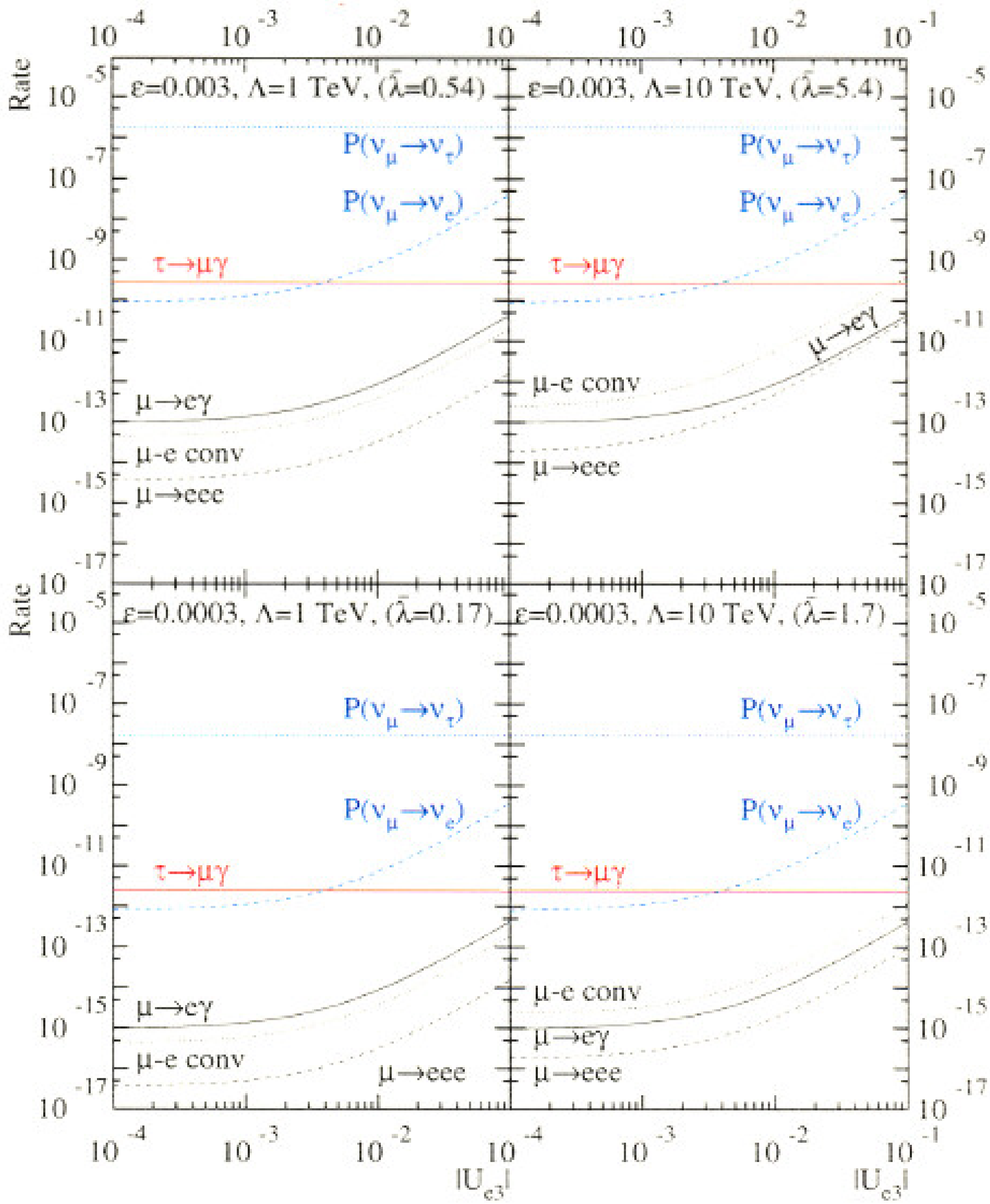


Figure 2: *The most interesting leptonic observables as a function of* $|U_{e3}|$, *for* $\delta = 5$, $\epsilon = 0.003$ *(top) or* 0.0003 *(bottom), and* $\Lambda = 1$ *TeV (left) or 10 TeV (right). We assume* $\Delta m^2_{\rm sun}/\Delta m^2_{\rm atm} = 10^{-2}$, $|U_{\mu 3}/U_{\tau 3}|^2 = 1$, $|U_{e2}/U_{e1}|^2 = 2/3$ *(i.e., maximal mixing in the atmospheric sector and the LMA solution to the solar neutrino puzzle), no CP-violation in the neutrino mixing matrix, and hierarchical neutrino masses* ($m_1^2 \ll m_2^2 \ll m_3^2$). *Note that* $\bar\lambda$ *is the largest Yukawa coupling,* $P(\nu_i \to \nu_j)$ *is the neutrino conversion probability at very short baselines, and the* $\mu \to e$ *conversion rate is computed for* ^{27}Al.

CONCLUSIONS

- WE KNOW THAT LEPTON FLAVOR IS NOT A GOOD SYMMETRY OF THE NEW S.M.

- IF THERE IS NEW PHYSICS JUST ABOVE THE ELECTROWEAK SCALE, THERE IS NO REASON TO BELIEVE THAT CLFV IS NOT WITHIN REACH OF FUTURE EXPERIMENTS.

(WHY HAVEN'T WE SEEN IT YET?)

- IN "MOST CASES" (?) $\mu \to e\gamma$ IS LARGEST. HOWEVER, THIS IS CERTAINLY NOT GUARANTEE

$\boxed{\text{WE NEED TO PROBE ALL CHANNELS}}$

(IF POSSIBLE...)

- CLFV MIGHT BE INTIMATELY RELATED TO NEUTRINO MIXING. IN THAT CASE, CLFV WILL OFFER CRUCIAL PIECES FOR UNDERSTANDI[NG] THE NEW PHYSICS BEHIND NEUTRINO MASSES.