THEORETICAL ASPECTS
OF (CHARGED-)LEPTON
FLAVOR VIOLATION [CLFV]

• LEPTON FLAVOR VIOLATION IN THE STANDARD MODEL

• "NEUTRAL" X "CHARGED" LEPTON FLAVOR VIOLATION

• PHENOMENOLOGY OF CLFV

• A FEW MODELS...
  - SUSY + SEESAW
  - SUSY + R-PARITY VIOLATION
  - EXTRADIMENSIONS

• SUMMARY, CONCLUDING MESSAGES
Lepton Flavor Violation in the SM

- In the old S.M., individual lepton number (lepton flavor) is exactly conserved.

\[ L_e (e^-) = L_e (e^+) = 1 \]
\[ L_\mu (\mu^-) = L_\mu (\mu^+) = 1 \]
\[ L_\tau (\tau^-) = L_\tau (\tau^+) = 1 \]

Indeed, we have never been able to observe lepton flavor violation with charged leptons as the "in" or "out" states...

Even though we have looked hard for it.
Searches for Lepton Number Violation

UL Branching Ratio (Conversion Probability)

- $\mu \rightarrow e \gamma$
- $\mu^{-}N \rightarrow e^{-}N$
- $\mu^{+}e^{-} \rightarrow \mu^{-}e^{+}$
- $\mu \rightarrow eee$
- $K_{L} \rightarrow \pi^{+} \mu e$
- $K_{L} \rightarrow \mu e$
- $K_{L} \rightarrow \pi^{0} \mu e$

Year
In the **NEW SM**, lepton flavor is no longer conserved.

* OLD SM + massive neutrinos.

→ There is a mismatch between the flavor and mass eigenstates ⇒ MNS matrix (like CKM for quarks)

In this case, there are FCNC-like effects

\[
\begin{align*}
\mu & \rightarrow e \gamma \\
\mu^- & \rightarrow e^- \nu \bar{\nu}
\end{align*}
\]

**Technical difficulty**: 

\[
B(\mu \rightarrow e\gamma) = \frac{3\alpha}{32\pi} \sum_i U_{\mu i} U_{ei}^* \frac{m_{\nu_i}^2}{M_W^2} e^{-\frac{m_{\mu}^2}{2g W^2}} \approx 10^{-60}\left(\frac{m_{\mu}}{10^2 eV}\right)^4
\]

(Γν suppression)

We are not there yet...
We can only see these tiny effects in interference phenomena: Neutrino oscillations (this is where the large distances come in...)

Theoretically, however, we have "strong" reason to believe that CLFV is indeed "observable"...

... and it may turn out that \( \nu \)-masses + CLFV are deeply related?

(More on this later---)
ON THE PHENOMENOLOGY OF CLFV

IN PARTICULAR RARE MUON PROCESSES

EFFECTIVE OPERATORS THAT MEDIATE CLFV:

\[ \mu \rightarrow e\gamma \]

\[ \lambda = \frac{m_\mu}{\Lambda^2} \bar{\mu}_R \gamma^\mu \mu^e F_{\mu}\nu + h.c + ... \]

\[ B(\mu \rightarrow e\gamma) = \frac{3(4\pi)^2}{G_F \Lambda^4} \]

THIS SAME LAGRANGIAN ALSO GENERATES \( \mu \rightarrow eee \)

\[ \frac{B(\mu \rightarrow 3e)}{B(\mu \rightarrow e\gamma)} = 6 \times 10^{-3} \]

\[ \frac{B(\mu \rightarrow e\gamma)}{B(\mu \rightarrow e\gamma)} \sim 2 \times 10^{-3} \]

\[ \Rightarrow \mu \rightarrow e\gamma \text{ DOMINATES} \]
But there might be extra contributions to...

\[ \mu \rightarrow eee \]

\[ \lambda = \lambda_{\mu \rightarrow ee} + \frac{1}{\Lambda^2} \mu L \gamma \mu e_L \bar{e}_L \gamma \mu e_L \text{the...} \]

\[ \begin{array}{c}
\left[ \begin{array}{c}
\mu \\
\mu \\
e \\
e \\
e \\
e \\
e \\
\end{array} \right] \\
\end{array} \]

On which case the story might be very different.

---

\[ \mu N \rightarrow eN \]

\[ \lambda = \lambda_{\mu \rightarrow eN} + \frac{1}{\Lambda^2} \mu L \gamma \mu e_L \bar{g} L \gamma \mu g_L + \text{...} \]

\[ \begin{array}{c}
\left[ \begin{array}{c}
e \\
\mu \\
g \\
g \\
g \\
g \\
g \\
\end{array} \right] \\
\end{array} \]

Same here.
$B(\mu N \rightarrow eN)/B(\mu \rightarrow e\gamma)$

$(\Lambda/\Lambda_F)^2$
PHENOMENOLOGY

MODEL INDEPENDENT LAGRANGIANS (DIM-5 AND 6 OPERATORS)

\[ L_{\mu-eX} = -\frac{4G_F m_{\mu\nu}}{\sqrt{2}} \left[ A_R \bar{\mu}_R \gamma^\mu e_L F_{\mu\nu} + A_L \bar{\mu}_L \gamma^\mu e_R F_{\mu\nu} + h.c. \right] \]

\[ B(\mu^+\to e^+X) = 384\pi^2 \left( |A_R|^2 + |A_L|^2 \right) \]

\[ L_{\mu-e\text{ee}} = \alpha_{\mu-e\text{ee}} - \frac{4G_F}{\sqrt{2}} \left[ g_1 (\bar{\mu}_R e_L)(\bar{e}_R e_L) + g_2 (\bar{\mu}_L e_R)(\bar{e}_L e_R) + g_3 (\bar{\mu}_R e_{\mu R})(\bar{e}_R \gamma^\mu e_L) + g_4 (\bar{\mu}_L e_{\mu L})(\bar{e}_L \gamma^\mu e_R) + g_5 (\bar{\mu}_R e_{\mu R})(\bar{e}_L \gamma^\mu e_L) + g_6 (\bar{\mu}_L e_{\mu L})(\bar{e}_R \gamma^\mu e_R) \right] + h.c. \]

\[ B(\mu^+\to e^+e^-e^-) = 2(C_1 + C_2) + (C_3 + C_4) + 16(C_7 + C_8) + 8(C_9 + C_{10}) + 32 \left[ \ln \left( \frac{m_\mu^2}{m_e^2} \right) - \frac{11}{4} \right] \left( C_5 + C_6 \right) \]

WHERE

\[ C_1 = \frac{|g_1|^2 + |g_3|^2}{16} \]
\[ C_2 = \frac{|g_2|^2 + |g_4|^2}{16} \]
\[ C_3 = |g_5|^2 \]
\[ C_4 = |g_6|^2 \]
\[ C_5 = |e A_R e^L|^2 \]
\[ C_6 = |e A_L e^R|^2 \]
\[ C_7 = \text{Re} (e A_R g_1^* e^L) \]
\[ C_8 = \text{Re} (e A_L g_2^* e^R) \]
\[ C_9 = \text{Re} (e A_R g_3^* e^L) \]
\[ C_{10} = \text{Re} (e A_L g_4^* e^R) \]
\[ C_{11} = \text{Im} (e A_R g_1^* e^L + e A_L g_2^* e^R) \]
\( \mu^+ \to e^+ e^- e^- \) and CP-Violation

\[
\mu^+ (\delta_{\mu\mu}) \to e^+ (\vec{p}_1) \quad e^+ (\vec{p}_2) \quad e^- (\vec{p}_3)
\]

Todd Combination: \( \delta_{\mu\mu} \cdot (\vec{p}_1 \times \vec{p}_2) \)

OBSERVABLE EVENTS WITH \( \phi \in [0, \pi] \) - EVENTS WITH \( \phi \in [\pi/2, \pi] \)

**FIG. 27.** Kinematics of the \( \mu^+ \to e^+ e^- e^- \) decay in the muon center-of-mass system, in which \( \vec{p}_1, \vec{p}_2 \) are the momentum vectors of the two \( e^+ \)'s and \( \vec{p}_3 \) is that of the \( e^- \), respectively. The plane-I is the decay plane on which \( \vec{p}_1, \vec{p}_2, \) and \( \vec{p}_3 \) lie. The plane-II is the plane in which the muon polarization vectors, \( \vec{P} \) and \( \vec{p}_3 \), are located (after Okada, et al., 1999).

\[
A_T (\mu^+ e^+ e^-) = \frac{64}{35} \left( \frac{3C_{11} - 2C_{12}}{B(\mu^+ e^+ e^-)} \right)
\]

**CHALLENGE:** \( A_T^{\max} \leq 15\% \) (MODEL INDEPENDENT)

INTERFERENCE BETWEEN ON-SHELL PHOTON AND 4 FERMION INTERACTION
Some Models

See Saw

\[ \lambda e - \chi^3 L N H_u \sim \frac{M_R N N}{2} + h.c. \]

At energies below M_R

\[ \lambda \sim \chi^3 (L^i H_u)(L^i H_u) \]

\[ \chi^3 \sim \left( \frac{1}{M_R} \right)^4 \lambda \]

+ Low Energy SUSY

\[ \mathcal{B}(\mu \rightarrow e\gamma) \propto \left( \frac{m_e}{\mu} \right)^4 \left( \frac{1}{M_{\text{susy}}} \right)^4 \]

• Potentially huge = problem
• Not suppressed by \chi masses

\( \Rightarrow \) forced to impose \( \tilde{m}_e = 0 \) at high energy scale...
The off-diagonal terms are reserected by renormalization group running...

\[
(m^2_{\nu i j}) \propto 3 \frac{m_0^2}{8\pi^2} (y_{\nu j}^*)^k_i (y_{\nu j}^*)_i^k \ln \left( \frac{M_{Pl}}{M_R} \right)
\]

Proportional to $\nu$-Yukawa couplings

Remember

\[
m_{\nu} \propto (y_{\nu j})^i_k \left( \frac{1}{M_R} \right)^{k l} (y_{\nu j})_k^l \nu^2
\]

Different dependencies on $y_{\nu}$

Could be important for piecing together high-energy

See saw lagrangian

Test of leptogenesis?
\[ \mathcal{B}(\mu \to e\gamma) \sim M_R^2 \left( \ln \left( \frac{M_{\tilde{H}_1}}{M_R} \right) \right)^2 \]

**MSSM with right-handed neutrinos**

- \( M_R = 10^{13} \text{ GeV} \)
- \( \tan \beta = 10 \)
- \( M_1 = 100 \text{ GeV} \)
- \( A_0 = 0 \)
- \( \mu > 0 \)

- \( U_{e3} = 0 \)
- \( U_{e3} = 0.05 \)

**Left-handed selectron mass (GeV)**

- **LMA**
- **LOW & SMA**
- **LOW**
- **SMA**
\[ W_R = \mu L^i H_u + \frac{\lambda_{ijk}}{2} L^i L^j E^k + \lambda'_{ijk} L^i Q^j D^k + \lambda''^i U^i D^j D^k \]

**Lepton Number Violating**

**3-violating**

**Impose**: "Baryon-Parity" \( \lambda''_{ijk} = 0 \) [Prevents Proton Decay]

**Ignore**: \( \mu \) (For simplicity)

\( \lambda, \lambda' \) couplings will both generate \( \nu \)-mass...

\[ m_\nu \propto \frac{m_f^2 (m_\nu^2)_{LR}}{m_f^2} \frac{\lambda^2}{16\pi^2} \]

**AND CLFV**

[Diagrams showing fermion decay processes involving muon and electron]
TABLE I. The ratios of branching ratios $Br(\mu^+\rightarrow e^+\gamma)/Br(\mu^+\rightarrow e^-e^-\gamma)$ and $R(\mu^+\rightarrow e^+\gamma)$ in Ti)/Br(\mu^+\rightarrow e^-e^-\gamma), P-odd asymmetries $A_P$ for $\mu^+\rightarrow e^+\gamma$, $A_{P_1}$ and $A_{P_2}$ for $\mu^+\rightarrow e^+e^-e^-$ are shown when the listed pair of Yukawa couplings is dominant. Cases (1), (2), and (3) refer to the representative classes of models discussed in Secs. IV A, IV B, and IV C, respectively. Here, we assume $m_{\tilde{e}_L}=100$ GeV and no mixing in the charged slepton mass matrix, and $m_{\tilde{e}_R}=300$ GeV. We also show a typical result obtained for the MSSM with heavy right-handed neutrinos and $R$-parity conservation [7].

<table>
<thead>
<tr>
<th>Case (1)</th>
<th>Br($\mu\rightarrow e\gamma$)</th>
<th>Br($\mu\rightarrow 3e$)</th>
<th>$A_P$</th>
<th>$A_{P_1}$</th>
<th>$A_{P_2}$</th>
<th>$A_{P_1}/A_{P_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{131}\lambda_{231}$</td>
<td>$1 \times 10^{-4}$</td>
<td>$2 \times 10^{-3}$</td>
<td>$-100%$</td>
<td>$+19%$</td>
<td>$-15%$</td>
<td>$-1.3$</td>
</tr>
<tr>
<td>$\lambda_{121}\lambda_{122}$</td>
<td>$8 \times 10^{-4}$</td>
<td>$7 \times 10^{-3}$</td>
<td>$+100%$</td>
<td>$-19%$</td>
<td>$+15%$</td>
<td>$-1.3$</td>
</tr>
<tr>
<td>$\lambda_{131}\lambda_{132}$</td>
<td>$8 \times 10^{-4}$</td>
<td>$5 \times 10^{-3}$</td>
<td>$+100%$</td>
<td>$-19%$</td>
<td>$+15%$</td>
<td>$-1.3$</td>
</tr>
<tr>
<td>Case (2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_{132}\lambda_{232}$</td>
<td>$1.2$</td>
<td>$18$</td>
<td>$-100%$</td>
<td>$-25%$</td>
<td>$-5%$</td>
<td>$5.6$</td>
</tr>
<tr>
<td>$\lambda_{133}\lambda_{233}$</td>
<td>$3.7$</td>
<td>$18$</td>
<td>$-100%$</td>
<td>$-25%$</td>
<td>$-4%$</td>
<td>$6.2$</td>
</tr>
<tr>
<td>$\lambda_{231}\lambda_{232}$</td>
<td>$3.6$</td>
<td>$18$</td>
<td>$+100%$</td>
<td>$+25%$</td>
<td>$+4%$</td>
<td>$6.2$</td>
</tr>
<tr>
<td>$\lambda_{122}\lambda_{122}'$</td>
<td>$1.4$</td>
<td>$18$</td>
<td>$-100%$</td>
<td>$-25%$</td>
<td>$-4%$</td>
<td>$5.7$</td>
</tr>
<tr>
<td>$\lambda_{133}\lambda_{233}'$</td>
<td>$2.2$</td>
<td>$18$</td>
<td>$-100%$</td>
<td>$-25%$</td>
<td>$-4%$</td>
<td>$5.9$</td>
</tr>
<tr>
<td>Case (3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_{111}\lambda_{211}$</td>
<td>$0.4$</td>
<td>$3 \times 10^3$</td>
<td>$-100%$</td>
<td>$-26%$</td>
<td>$-5%$</td>
<td>$5.4$</td>
</tr>
<tr>
<td>$\lambda_{112}\lambda_{212}'$</td>
<td>$0.5$</td>
<td>$8 \times 10^4$</td>
<td>$-100%$</td>
<td>$-26%$</td>
<td>$-5%$</td>
<td>$5.4$</td>
</tr>
<tr>
<td>$\lambda_{123}\lambda_{213}'$</td>
<td>$0.7$</td>
<td>$1 \times 10^5$</td>
<td>$-100%$</td>
<td>$-26%$</td>
<td>$-5%$</td>
<td>$5.5$</td>
</tr>
<tr>
<td>$\lambda_{221}\lambda_{221}'$</td>
<td>$1.1$</td>
<td>$2 \times 10^1$</td>
<td>$-100%$</td>
<td>$-26%$</td>
<td>$-5%$</td>
<td>$5.6$</td>
</tr>
<tr>
<td>MSSM with $\nu_R$</td>
<td>$1.6 \times 10^3$</td>
<td>$0.92$</td>
<td>$-100%$</td>
<td>$10%$</td>
<td>$17%$</td>
<td>$0.6$</td>
</tr>
</tbody>
</table>

\[ \text{\"USUAL SUSY\"} \]

\[ \text{\"MAY BE DOMINANT WITH SUSY\"} \]
Table 1: Upper limits on products of R-parity-violating couplings from the current limits for muon-number-violating processes, taking slepton masses equal to 100 GeV and squark masses equal to 300 GeV. The values in parenthesis indicate the sensitivity which could be achieved at a neutrino factory complex, i.e. resulting from improvements in experimental sensitivities to rare μ decays by 4-6 orders of magnitude. The coupling constants λ and λ' refer to the interactions \( \mathcal{L}_{\text{int}} = \lambda_{ijk} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k \), where i, j, k are generation indices. Tree-level constraints are indicated by \([\text{tree}]\).

<table>
<thead>
<tr>
<th></th>
<th>(\mu \rightarrow e\gamma)</th>
<th>(\mu \rightarrow eee)</th>
<th>(\mu \rightarrow e) conversion in nuclei</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda_{131}\lambda_{231})</td>
<td>(2.3 \times 10^{-4} (2 \times 10^{-6}))</td>
<td>(6.7 \times 10^{-7} (7 \times 10^{-9}) [\text{tree}])</td>
<td>(1.1 \times 10^{-5} (1 \times 10^{-8}))</td>
</tr>
<tr>
<td>(\lambda_{132}\lambda_{232})</td>
<td>(2.3 \times 10^{-4} (2 \times 10^{-6}))</td>
<td>(7.1 \times 10^{-5} (7 \times 10^{-7}))</td>
<td>(1.3 \times 10^{-5} (2 \times 10^{-8}))</td>
</tr>
<tr>
<td>(\lambda_{133}\lambda_{233})</td>
<td>(2.3 \times 10^{-4} (2 \times 10^{-6}))</td>
<td>(1.2 \times 10^{-4} (1 \times 10^{-6}))</td>
<td>(2.3 \times 10^{-5} (3 \times 10^{-8}))</td>
</tr>
<tr>
<td>(\lambda_{121}\lambda_{221})</td>
<td>(8.2 \times 10^{-5} (7 \times 10^{-7}))</td>
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</tr>
<tr>
<td>(\lambda_{231}\lambda_{232})</td>
<td>(8.2 \times 10^{-5} (7 \times 10^{-7}))</td>
<td>(4.5 \times 10^{-5} (5 \times 10^{-7}))</td>
<td>(8.3 \times 10^{-6} (1 \times 10^{-8}))</td>
</tr>
<tr>
<td>(\lambda'<em>{111}\lambda'</em>{211})</td>
<td>(6.8 \times 10^{-4} (6 \times 10^{-6}))</td>
<td>(1.3 \times 10^{-4} (1 \times 10^{-6}))</td>
<td>(5.4 \times 10^{-6} (7 \times 10^{-9}) [\text{tree}])</td>
</tr>
<tr>
<td>(\lambda'<em>{122}\lambda'</em>{222})</td>
<td>(6.8 \times 10^{-4} (6 \times 10^{-6}))</td>
<td>(1.4 \times 10^{-4} (1 \times 10^{-6}))</td>
<td>(3.9 \times 10^{-7} (5 \times 10^{-10}) [\text{tree}])</td>
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<td>(\lambda'<em>{133}\lambda'</em>{233})</td>
<td>(6.8 \times 10^{-4} (6 \times 10^{-6}))</td>
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</tr>
<tr>
<td>(\lambda'<em>{221}\lambda'</em>{222})</td>
<td>(6.8 \times 10^{-4} (6 \times 10^{-6}))</td>
<td>(2.0 \times 10^{-4} (2 \times 10^{-6}))</td>
<td>(3.6 \times 10^{-7} (5 \times 10^{-10}) [\text{tree}])</td>
</tr>
<tr>
<td>(\lambda'<em>{232}\lambda'</em>{233})</td>
<td>(6.8 \times 10^{-4} (6 \times 10^{-6}))</td>
<td>(2.3 \times 10^{-4} (2 \times 10^{-6}))</td>
<td>(4.3 \times 10^{-5} (6 \times 10^{-8}))</td>
</tr>
<tr>
<td>(\lambda'<em>{231}\lambda'</em>{233})</td>
<td>(6.9 \times 10^{-4} (6 \times 10^{-6}))</td>
<td>(2.9 \times 10^{-4} (3 \times 10^{-6}))</td>
<td>(5.4 \times 10^{-5} (7 \times 10^{-8}))</td>
</tr>
</tbody>
</table>
Figure 1: $m_\nu / \lambda$ (left) and $\epsilon$ (right) as a function of $M_D$ for different values of $\delta/d$, assuming $\Lambda = M_D$. The horizontal band in the left panel shows the mass range selected by atmospheric neutrino data for $\lambda = 1$.

$$\epsilon = \frac{\rho_\delta}{\delta - 2} \frac{\Delta m_{\text{ATM}}^2}{\lambda^2} \sqrt{\delta} \lambda^\delta$$
Figure 2: The most interesting leptonic observables as a function of $|U_{e3}|$, for $\delta = 5, \epsilon = 0.003$ (top) or 0.0003 (bottom), and $\Lambda = 1$ TeV (left) or 10 TeV (right). We assume $\Delta m_{sol}^2/\Delta m_{atm}^2 = 10^{-2}$, $|U_{\mu 3}/U_{\tau 3}|^2 = 1$, $|U_{e2}/U_{e1}|^2 = 2/3$ (i.e., maximal mixing in the atmospheric sector and the LMA solution to the solar neutrino puzzle), no CP-violation in the neutrino mixing matrix, and hierarchical neutrino masses ($m_1^2 \ll m_2^2 \ll m_3^2$). Note that $\tilde{\lambda}$ is the largest Yukawa coupling, $P(\nu_i \to \nu_j)$ is the neutrino conversion probability at very short baselines, and the $\mu \to e$ conversion rate is computed for $^{27}$Al.
CONCLUSIONS

- We know that lepton flavor is not a good symmetry of the new S.M.

- If there is new physics just above the electroweak scale, there is no reason to believe that CLFV is not within reach of future experiments (why haven't we seen it yet?)

- In "most cases" (?) $\mu\rightarrow e\gamma$ is largest. However, this is certainly not guaranteed. We need to probe all channels (if possible...).
CLFV might be intimately related to neutrino mixing. In that case, CLFV will offer crucial pieces for understanding the new physics behind neutrino masses.