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Neutrino Masses in Theories with Dynamical Electroweak Symmetry Breaking

work with T. Appelquist

Outline

1. Motivations and theoretical framework of (extended) technicolor; challenge of explaining light neutrino masses without a GUT-scale seesaw
2. Some general features of TC, ETC models
3. Specific ETC model
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5. General structure of the neutrino mass matrix
6. Dynamical symmetry breaking
7. Calculations and Results
8. Models with extended gauge groups
9. Conclusions

refs: T. Appelquist and R. Shrock, Phys. Lett. B **548**, 204 (2002); Phys. Rev. Lett. **90**, 201801 (2003)

General question: What do we infer from the discovery of neutrino masses and lepton mixing about physics beyond the standard model? One possibility: a seesaw involving a (SUSY) GUT mass scale, with $m_\nu \sim m_D^2/M_{GUT}$, where m_D is a typical Dirac mass and $M_{GUT} \sim 10^{16}$ GeV.

Here we shall consider how one can explain light neutrino masses in models with dynamical electroweak symmetry breaking (EWSB). This has been a longstanding problem, since in these theories one cannot use a conventional type of seesaw, because they do not have nearly a large enough scale. In ETC, typically the largest scale is $\sim 10^6$ GeV.

Here we will propose an explanation and show how it is realized in an explicit ETC model. This model involves a seesaw, but one with mass scales well within the ETC range.

Some Possible Motivations for Dynamical Electroweak Symmetry Breaking

- This solves the hierarchy problem, i.e., the quadratic sensitivity of the Higgs potential of the SM to high-scale physics and associated fine-tuning (as does SUSY, if one solves the μ problem)

It is important to recall that in both of two major previous cases of symmetry breaking in which one used a scalar field as part of a phenomenological model of the phenomenon, the microscopic physics actually involved bilinear fermion condensates:

- Ginzburg-Landau approach to superconductivity using complex scalar field; microscopic BCS theory involved dynamical formation of Cooper pair condensate
- Gell-Mann Levy σ model for spontaneous chiral symmetry breaking, in which the SχSB was manifested via $\langle\sigma\rangle \neq 0$; in the actual underlying QCD theory, the SχSB is due to the dynamical formation of a $\langle\bar{q}q\rangle$ condensate

Could these previous examples be teaching us something about what we might expect for electroweak symmetry breaking?

Motivations and Theoretical Framework

Understanding the fermion mass spectrum remains a challenge.

The Standard Model (SM) accomodates quark and charged lepton masses by the mechanism of Yukawa couplings to a postulated Higgs boson, but this does not provide insight into these masses, especially since it requires Yukawa couplings of order $10^{-6} - 10^{-5}$ for e, u, d. SM Higgs sector is unstable to large loop corrections.

SM predicts zero neutrino masses and no lepton mixing, and hence must be modified to take account of current experimental evidence for neutrino masses and mixing. solar: (Cl, Kam., GALLEX, SAGE, SuperK, SNO)
atm.: (IMB, Kam., Soudan-2, SuperK, MACRO)
accel. - K2K
reactor - KamLAND

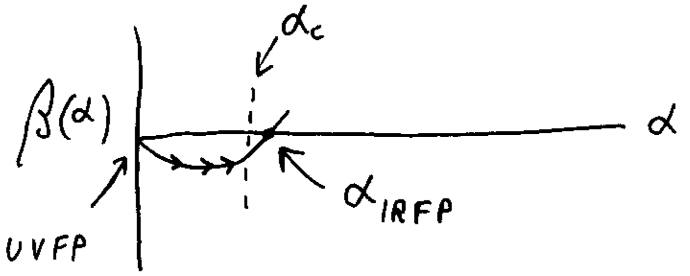
Since masses for the quarks, charged leptons, and observed neutrinos break the SM gauge symmetry, an explanation of these masses requires a model for electroweak symmetry breaking (EWSB). Here we assume dynamical EWSB driven by a strongly coupled gauge interaction, associated with an exact gauge symmetry, technicolor (TC) embedded in an extended technicolor (ETC) theory (Weinberg, Susskind, Dimopoulos, Eichten, Lane...).

slowly running gauge coupling over certain range

The TC theory is designed to have "walking" behavior (Holdom, Yamawaki et al., Appelquist et al.), which can produce realistically large quark and charged lepton masses and sufficiently small TC electroweak corrections. Further ingredients are likely needed to explain m_t and, in some cases, to avoid problems with Nambu-Goldstone bosons.

TC gauge coupling = α

Walking TC: $\beta(\alpha) = \mu \dfrac{d\alpha}{d\mu} \propto -\alpha^2(\beta_0 + \beta_1 \alpha)$

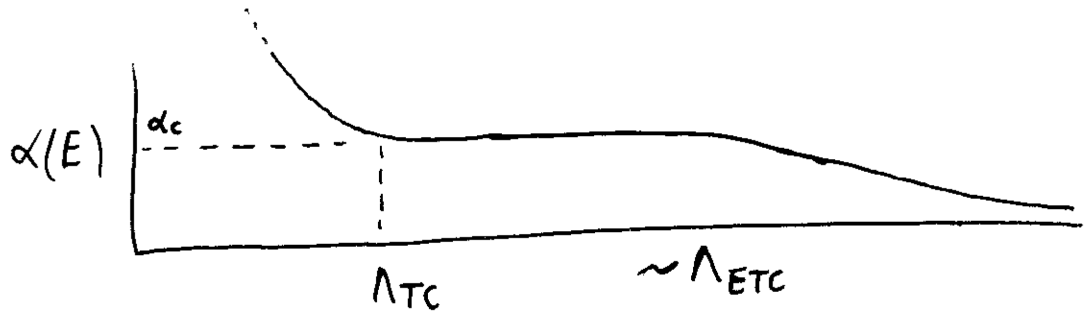


$\beta_0 > 0$: asymptotic freedom

$\beta_1 < 0 \Rightarrow$

$\exists \; \alpha_{IRFP} = -\dfrac{\beta_0}{\beta_1}$

so $\beta(\alpha_{IRFP}) = 0$



$E = \mu$

As E decreases, $\alpha(E)$ increases, but $|\beta|$ decreases, so α only evolves slowly for an extended interval in E. Eventually α exceeds α_c, the value for condensation (at $E \sim \Lambda_{TC}$ here, $\to \langle \bar{F}F \rangle$ formation) - then technifermions pick up dynamical masses $\sim \Lambda_{TC}$, TC theory confines. Effect of (approx.) IRFP.

Some General Features of TC, ETC models

We will take the technicolor group to be $SU(2)_{TC}$. The technifermions include, as a subset, one family,

$i = 4,5$: TC
a : color

$$Q_L^{i,a} = \begin{pmatrix} U^{i,a} \\ D^{i,a} \end{pmatrix}_L , \quad L^i_{TC,L} = \begin{pmatrix} N^i \\ E^i \end{pmatrix}_L , \quad U_R^{i,a} , \; D_R^{i,a} , \; N_R^i , \; E_R$$

transforming according to the fundamental rep. of $SU(2)_{TC}$ and usual reps. of $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$. These comprise $N_f = 2(N_c + 1) = 8$ fermions with vectorial TC couplings.

Choice of $N_{TC} = 2$ has appeal that with $N_f \simeq 8$, studies suggest that the TC theory could have an (approximate) infrared fixed point (IRFP) in the confining phase with spontaneous chiral symmetry breaking but near to the phase transition (as a function of N_f) beyond which it would go over into a nonabelian Coulomb phase.

Beta function calculation suggests that for $N_{TC} = 2$, IRFP exists if $N_f \gtrsim 5$ or 6 and nonabelian Coulomb phase occurs if $N_f \gtrsim 8$ (with $N_f < 11$ for asymptotic freedom). This has two valuable consequences: (i) walking behavior, which enhances condensates and fermion masses, (ii) reduction of TC contributions to S parameter (Appelquist and Sannino). Some uncertainty in N_f for onset of nonabelian Coulomb phase; could be larger.

$\langle \bar{F}F \rangle$: $I = \frac{1}{2}$ $|Y| = 1$ operator, same as Higgs vev in SM
$\Rightarrow$ usual m_W-m_Z mass relation. $m_F = 0$ initially so $\langle \bar{F}F \rangle \neq 0$
breaks associated global technichiral sym. $\rightarrow$ NGB's.

At a scale Λ_{TC}, the TC coupling gets strong enough to produce a bilinear technifermion condensate $\langle \bar{F}F \rangle$ and corresponding dynamical masses for the technifermions. Some would-be Nambu-Goldstone bosons become longitudinal modes of W and Z, giving them masses

$$m_W^2 = m_Z^2 \cos^2 \theta_W = \frac{g^2}{4}(N_c f_Q^2 + f_L^2) \simeq \frac{g^2}{4}(N_c + 1) f_F^2$$

where it suffices to take the TC pseudoscalar decay constants $f_L \simeq f_Q \equiv f_F$. Hence, $f_F \simeq 130$ GeV. In QCD, $f_\pi = 93$ MeV and $\Lambda_{QCD} \sim 170$ MeV, so $\Lambda_{QCD}/f_\pi \sim 2$; hence, we take $\Lambda_{TC} \sim 260$ GeV.

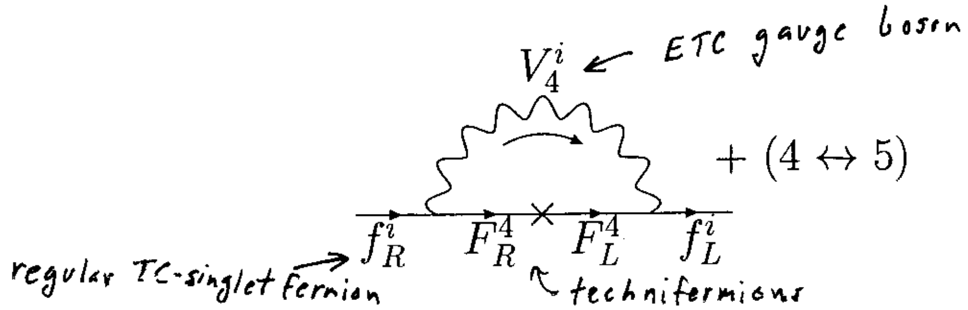
To generate fermion masses, embed TC in ETC theory; role of ETC gauge bosons is to connect TC-singlet and TC-nonsinglet fermions and communicate EWSB in TC sector to the TC-singlet fermions.

To satisfy constraints on flavor-changing neutral-current processes, ETC vector bosons must have large masses. These can arise from self-breaking of the ETC gauge symmetry, which in turn requires that ETC be a strongly coupled, chiral gauge theory.

The ETC self-breaking occurs in stages, e.g., $\Lambda_1 \sim 10^3$ TeV, $\Lambda_2 \sim 50$ TeV, and $\Lambda_3 \sim 3$ TeV, leaving as an exact residual invariance group SU(N_{TC}). This entails the relation $N_{ETC} = N_{TC} + N_{gen}$. Hence, with $N_{gen} = 3$ and $N_{TC} = 2$, we have $N_{ETC} = 5$.

Mass Generation for Quarks and Charged Leptons

Recall dynamical ETC mass generation mechanism for quarks and charged leptons. For rough estimate, consider one-loop diagram shown.



$\to \bar{f}_{L i} f_R^i$

One-loop graphs contributing to the mass term $\bar{f}_{i,L} f_R^i$ where $1 \le i \le 3$.

This yields

$$m_{fi} \sim \frac{g_{ETC}^2 \eta_i \Lambda_{TC}^3}{2\pi^2 M_i^2}$$

where

$$M_i \sim g_{ETC} \Lambda_i$$

is the mass of the ETC gauge bosons that gain mass at scale Λ_i and g_{ETC} is the running ETC gauge coupling at this scale. The factor η_i is a possible enhancement factor incorporating walking:

$$\eta_i = \exp\left[\int_{f_F}^{\Lambda_i} \frac{d\mu}{\mu} \gamma(\alpha(\mu))\right] ,$$

In walking TC, the anomalous dimension $\gamma \simeq 1$ so

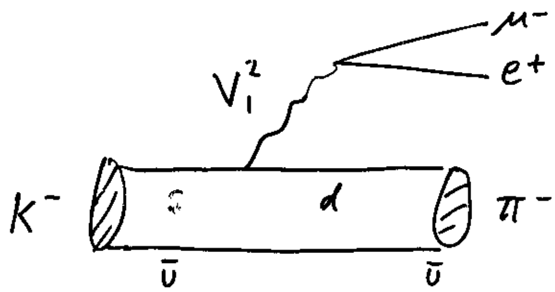
$$\eta_i \simeq \frac{\Lambda_i}{f_F}$$

whence

$$m_{fi} \sim \frac{\Lambda_{TC}^2}{2\pi^2 \Lambda_i}$$

– gives larger masses since $\Lambda_{TC} < \Lambda_{ETC}$

ETC leads, e.g., to FCNC processes such as



$K^- \to \pi^- \mu^- e^+$

$K^+ \to \pi^+ \mu^+ e^-$

Current limit (BNL E865): $BR(K^+ \to \pi^+ \mu^+ e^-) < 2.8 \times 10^{-11}$

$\to m(V_1^2) \gtrsim 200$ TeV $\quad m(V_1^2) \simeq \Lambda_1$

Lower bounds on Λ_i - ETC scales $\Rightarrow$ upper bounds on fermion masses.

QCD-like variants of TC were ruled out by their inability to satisfy the constraints of

- not producing excessively large FCNC processes
- producing large enough fermion masses

The enhancement factor in walking TC played a crucial role in ameliorating this problem, and modern TC models use walking

- still a challenge to get a large enough top quark mass

Constraints from precision electroweak data

$$S \simeq \frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2}$$

Naive perturbative evaluation: each extra EW doublet gives contribution

$$\Delta S \simeq \frac{N_{TC}}{6\pi}\Big[1 - Y \ln\Big(\frac{m_{1/2}^2}{m_{-1/2}^2}\Big)\Big]$$

where subscript is weak T_3. In one-family TC, have

$$\begin{pmatrix} U^{ia} \\ D^{ia} \end{pmatrix} \qquad \begin{pmatrix} N^i \\ E^i \end{pmatrix}$$

where $\{i\}$ = TC indices, a = color indices

Nondegeneracy of U, D and of N, E masses can reduce TC contribution to S, as constrained by T parameter ($\rho = \alpha T$).

Since technifermions are strongly interacting at scale $E \sim m_Z$, one cannot use perturbative estimates, and one does not have reliable methods to calculate TC contributions to S and T. Non-QCD-like behavior (walking) means that one cannot reliably try to scale up from QCD.

N.B.: Efforts to perform precision electroweak fits with oblique corrections have been complicated considerably by the NuTeV anomaly

General Structure of Neutrino Mass Matrix

In general, (E)TC models have a set of n_s electroweak-singlet neutrinos, denoted $\chi_R = (\chi_1, ..., \chi_{n_s})_R$, including both TC-singlets and TC-nonsinglets.

There are three types of contributions to the total neutrino mass matrix; here these are generated by condensates arising at the TC and ETC scales:

1. left-handed Majorana ($I = 1$, $Y = 2$)
2. Dirac ($I = 1/2$, $Y = 1$)
3. right-handed Majorana ($I = 0$, $Y = 0$)

The LH Majorana mass terms, which violate total lepton number L as $\Delta L = 2$ (and violate the EW gauge symmetry) have the form

$$\sum_{i,j=1}^{N_{ETC}} [n_{iL}^T C (M_L)_{ij} n_{jL}] + h.c.$$

where $n_L = (\{\nu_\ell\}, \{N\})_L$ and $C = i\gamma_2\gamma_0$.

Dirac mass terms (which also violate EW symmetry):

$$\sum_{i=1}^{N_{ETC}} \sum_{s=1}^{n_s} \bar{n}_{iL} (M_D)_{is} \chi_{sR} + h.c.$$

Majorana mass terms with SM-singlet (RH) neutrinos:

$$\sum_{s,s'=1}^{n_s} \chi_{sR}^T C (M_R)_{ss'} \chi_{s'R} \ ,$$

Entries in M_L, M_D, and M_R that would violate TC are zero since TC is exact; e.g., $(M_L)_{ij} = 0$ for $4 \le i, j \le 5$.

Full neutrino mass term:

$$-\mathcal{L}_m = \frac{1}{2}(\bar{n}_L \ \overline{\chi^c}_L) \begin{pmatrix} M_L & M_D \\ (M_D)^T & M_R \end{pmatrix} \begin{pmatrix} n_R^c \\ \chi_R \end{pmatrix} + h.c.$$

The diagonalization of the full $(N_{ETC} + n_s) \times (N_{ETC} + n_s)$ dimensional neutrino mass matrix yields the neutrino masses and mass eigenstates; combining the corresponding unitary transformation with the unitary transformation diagonalizing the charged lepton mass matrix then yields the observed lepton mixing matrix.

Specific ETC Model

Our first model has the gauge group

$$G = \mathrm{SU}(5)_{ETC} \times \mathrm{SU}(2)_{HC} \times \overbrace{\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y}^{G_{SM}}$$

In addition to extended technicolor, this has another strongly coupled interaction, hypercolor (HC), which helps to produce the desired symmetry breaking pattern.

The fermions transform according to the representations

$$(5,1,3,2)_{1/3,L}\ , \qquad (5,1,3,1)_{4/3,R}\ , \quad (5,1,3,1)_{-2/3,R}$$
$$(5,1,1,2)_{-1,L}\ , \qquad (5,1,1,1)_{-2,R}\ ,$$
$$(\overline{10},1,1,1)_{0,R} = \psi_{ij,R}\ , \qquad (10,2,1,1)_{0,R} = \zeta_R^{ij,\alpha}$$
$$2 \times (1,2,1,1)_{0,R} = \omega^{\alpha}_{p,R}, \quad p = 1,2$$

where $1 \le i,j \le 5$ are $\mathrm{SU}(5)_{ETC}$ indices and $\alpha = 1,2$ is a $\mathrm{SU}(2)_{HC}$ index. Line 1 contains quarks and techniquarks; line 2 contains LH leptons, technileptons and RH charged leptons and technileptons; line 3 contains SM-singlet fields with quantum numbers of (electroweak-singlet) neutrinos, taken by convention to be right-handed. Explicitly,

$$(5,1,3,2)_{1/3,L} = \begin{pmatrix} \overbrace{u^{1,a}, & u^{2,a}, & u^{3,a},}^{N_{gen}=3} & \overbrace{u^{4,a}, & u^{5,a}}^{N_{TC}=2} \\ d^{1,a}, & d^{2,a}, & d^{3,a}, & d^{4,a}, & d^{5,a} \end{pmatrix}_L$$
$$= \begin{pmatrix} u^a, & c^a, & t^a, & U^{4,a}, & U^{5,a} \\ d^a, & s^a, & b^a, & D^{4,a}, & D^{5,a} \end{pmatrix}_L$$

generations are gauged

$$N_{ETC} = N_{gen.} + N_{TC}$$
$$5 = 3 + 2$$

$$(5,1,1,2)_{-1,L} = \begin{pmatrix} n^1, & n^2, & n^3, & n^4, & n^5 \\ \ell^1, & \ell^2, & \ell^3, & \ell^4, & \ell^5 \end{pmatrix}_L$$
$$= \begin{pmatrix} \nu_e, & \nu_\mu, & \nu_\tau, & N^4, & N^5 \\ e, & \mu, & \tau, & E^4, & E^5 \end{pmatrix}_L$$

$$(5,1,3,1)_{4/3,R} = (u^{1,a}, u^{2,a}, u^{3,a}, u^{4,a}, u^{5,a})_R = (u^a, c^a, t^a, U^{4,a}, U^{5,a})_R$$

$$(5,1,3,1)_{-2/3,R} = (d^{1,a}, d^{2,a}, d^{3,a}, d^{4,a}, d^{5,a})_R = (d^a, s^a, b^a, D^{4,a}, D^{5,a})_R$$

$$(5,1,1,1)_{-2,R} = (\ell^1, \ell^2, \ell^3, \ell^4, \ell^5)_R = (e, \mu, \tau, E^4, E^5)_R$$

where $a = 1, 2, 3$ is color index, and $1 \le i \le 5$ is the ETC index, with $i = 1, 2, 3$ indexing the three generations of technisinglet fermions and $i = 4, 5$ indexing the technidoublet fermions.

The $\psi_{ij,R}$ and $\zeta_R^{ij,\alpha}$ are antisymmetric rank-2 tensor representations of $SU(5)_{ETC}$. Assign lepton number $L = 1$ to $\psi_{ij,R}$ so that, as usual, the Dirac terms $\bar{n}_{i,L}\psi_{jk,R}$ conserve L. (L left arbitrary for $\zeta_R^{ij,\alpha}$ since it has no Dirac mass terms with EW-doublet neutrinos.)

The $SU(5)_{ETC}$ group thus has vectorial couplings to (techni)quarks and charged (techni)leptons, but the SM-singlet fermion (i.e., electroweak-singlet neutrino) content makes the full $SU(5)_{ETC}$ a chiral gauge theory.

There are no bilinear fermion operators invariant under G and hence there are no bare fermion mass terms.

Each of the nonabelian factor groups in G is asymptotically free and G has no gauge or global anomalies.

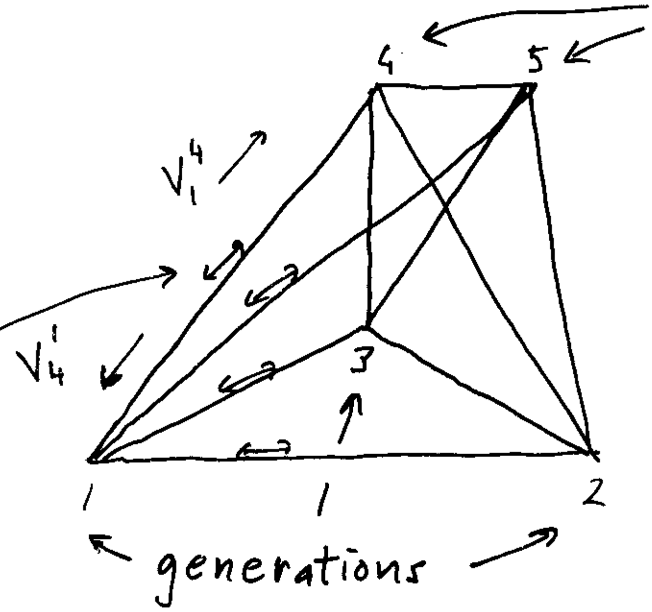
The $SU(2)_{HC}$ and $SU(2)_{TC}$ subsectors of $SU(5)_{ETC}$ are vectorial.

This model has some features in common with a previous SU(5) ETC model (Appelquist-Terning, 1994), but has different total gauge group and fermion content.

$SU(5)_{ETC}$ structure :

$SU(2)_{TC}$ residual unbroken subgroup of $SU(5)_{ETC}$

$V^i_j = (V^j_i)^\dagger$



9 ETC gauge bosons in coset $SU(5)/SU(4)$ get masses $\sim \Lambda_1$ (4 shift operators, their adjoints, and a diagonal op. in Cartan subalgebra, T_{24})

etc. for subsequent stages of ETC symmetry breaking

Dynamical Symmetry Breaking

Next consider the dynamical symmetry breaking in this model. To identify plausible channels for formation of bilinear fermion condensates, we use a generalized most attractive channel (GMAC) analysis that takes account of the attractiveness due to each strong gauge interaction and the cost incurred in producing vector boson masses when gauge symmetries are broken.

Recall use of GMAC (in the special case of vacuum alignment in the presence of additional perturbative gauge interactions) explains why TC produces $\langle \bar{F}F \rangle$ condensates, where $F = U^a, D^a, E, N$, but not, e.g., $\langle \bar{U}_a E \rangle$, $\langle \bar{D}_a E \rangle$, $\langle \bar{U}_a N \rangle$, $\langle \bar{D}_a N \rangle$, which would break color and electric charge and give masses to gluons and the photon.

An approximate measure of the attractiveness of a condensation channel $R_1 \times R_2 \to R_{cond.}$ is

e.g. for $\langle \bar{q}\, q \rangle$ in QCD
$C_2 = 4/3$; $\Delta C_2 = \frac{8}{3}$

$$\Delta C_2 = C_2(R_1) + C_2(R_2) - C_2(R_{cond.})$$

where R_j denotes the fermion representation under a relevant gauge interaction and $C_2(R)$ is the quadratic Casimir,

$$\sum_{a=1}^{o(G)} [D_R(T_a)]^i_j [D_R(T_a)]^j_k = C_2(R)\delta^i_k$$

with $C_2(R) = C_2(R^*)$. For SU(N), $C_2(\psi^i) = (N^2-1)/(2N)$,

$$C_2(\psi^{[ij]}) = \frac{(N-2)(N+1)}{N} \quad \text{etc.}$$

We envision that as the energy E decreases from high values, α_{ETC} and α_{HC} get large; at $E \sim \Lambda_1 \sim 10^3$ TeV, α_{ETC} is large enough to produce the condensation

$$(\overline{10},1,1,1)_{0,R} \times (\overline{10},1,1,1)_{0,R} \to (5,1,1,1)_0$$

with $\Delta C_2 = 24/5$, breaking $SU(5)_{ETC} \to SU(4)_{ETC}$.

With no loss of generality, take breaking direction in $SU(5)_{ETC}$ as $i=1$; this entails the separation of the first generation from the ETC fermions with $2 \le i \le 5$.

With respect to the unbroken $SU(4)_{ETC}$, we have the decomposition

$$(\overline{10},1,1,1)_{0,R} = (\bar{4},1,1,1)_{0,R} + (\bar{6},1,1,1)_{0,R}$$

Denote

$$(\bar{4},1,1,1)_{0,R} = \psi_{1i,R} \equiv \alpha_{1iR}$$
$$(\bar{6},1,1,1)_{0,R} = \psi_{ij,R} \equiv \xi_{ij,R}, 2 \le i,j \le 5$$

The associated $SU(5)_{ETC}$-breaking, $SU(4)_{ETC}$-invariant condensate is

$$\langle \epsilon^{1ijk\ell} \xi^T_{ij,R} C \xi_{k\ell,R} \rangle = 8 \langle \xi^T_{23,R} C \xi_{45,R} - \xi^T_{24,R} C \xi_{35,R} + \xi^T_{25,R} C \xi_{34,R} \rangle$$

The six fields $\xi_{ij,R}$, $2 \le i,j \le 5$ gain dynamical masses $\sim \Lambda_1$

With our lepton number assignments, this and the resultant dynamical Majorana mass terms for ξ violate L as $|\Delta L| = 2$.

At lower scales, depending on relative strengths of gauge couplings, different symmetry-breaking sequences can occur. We have studied two such sequences and discuss one here (denoted G_b in the PLB):

As E decreases to $\Lambda_{BHC} \lesssim \Lambda_1$ (BHC = broken HC), the $\mathrm{SU}(4)_{ETC}$ interaction produces a condensation

$$(6,2,1,1)_{0,R} \times (6,2,1,1)_{0,R} \to (1,3,1,1)_0$$

With respect to ETC, this channel has $\Delta C_2 = 5$; it occurs at a lower scale than Λ_1 because it is repulsive with respect to HC ($\Delta C_2 = -1/4$). The condensate is given by

$$\langle \epsilon_{1ijk\ell} \zeta_R^{ij,1\ T} C \zeta_R^{k\ell,2} \rangle + (1 \leftrightarrow 2)\ .$$

This is an adjoint rep. of hypercolor and breaks $\mathrm{SU}(2)_{HC} \to \mathrm{U}(1)_{HC}$. Let $\alpha = 1,2$ correspond to $Q_{HC} = \pm 1$ under the $\mathrm{U}(1)_{HC}$. The twelve $\zeta_R^{ij,\alpha}$ fields involved gain dynamical masses $\sim \Lambda_{BHC}$. $2 \le i,j \le 5$

At the lower scale, Λ_{23}, the $\mathrm{SU}(4)_{ETC}$ and $\mathrm{U}(1)_{HC}$ interactions produce the condensation $4 \times 4 \to 6$ with $\Delta C_2 = 5/4$ and condensate

$$\langle \epsilon_{\alpha\beta} \zeta_R^{12,\alpha\ T} C \zeta_R^{13,\beta} \rangle \qquad SU(2)_{ETC} \equiv SU(2)_{TC}$$

which breaks $\mathrm{SU}(4)_{ETC} \to \mathrm{SU}(2)_{ETC}$ and is $\mathrm{U}(1)_{HC}$-invariant. We take $\Lambda_{23} \sim 10$ TeV. The $\mathrm{U}(1)_{HC}$ interaction does not couple directly to SM particles. Consider other condensates also.

Finally, at $E \sim \Lambda_{TC}$, technifermion condensation occurs, breaking $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{em}$.

The strong $\mathrm{U}(1)_{HC}$ interaction naturally also produces a number of additional condensates between fermions with opposite $\mathrm{U}(1)_{HC}$ charges at a scale $\sim \Lambda_{23}$:

$$\langle \zeta_R^{12,\alpha\ T} C \omega_{p,R}^{\beta} \rangle\ , \quad \langle \zeta_R^{13,\alpha\ T} C \omega_{p,R}^{\beta} \rangle\ , \quad p = 1, 2\ ; \quad \alpha \neq \beta$$

$$\langle \omega_{p,R}^{\alpha\ T} C \omega_{p',R}^{\beta} \rangle\ , \quad 1 \leq p, p' \leq 2\ ; \alpha \neq \beta$$

Calculations and Results

The full neutrino mass matrix M is 39×39 dimensional, since $N_{ETC} = 5$ and the number of electroweak-singlet neutrinos is

dims. $n_s = 34$; $\chi = (\alpha_{ij}, \xi_{ij}, \zeta^{k\ell,\alpha}, \omega_p^{\beta})_R$. Because the hypercolored fields do not form bilinear condensates and resultant mass terms with hypercolor singlets, M is block-diagonal,

$$M = \begin{pmatrix} M_{HCS} & 0 \\ 0 & M_{HC} \end{pmatrix}$$

where the 15×15 block M_{HCS} involves hypercolor-singlet neutrinos and the 24×24 block M_{HC} involves the hypercolored fermions.

The nonzero entries of M arise in two different ways: (i) directly, as dynamical masses associated with various condensates, and (ii) via dynamical masses insertions on internal loop diagrams involving dynamical mass insertions on internal fermion lines and, in most cases, also mixings among ETC gauge bosons on internal lines.

M_{HC} involves dynamical masses for the $\zeta_R^{ij,\alpha}$ resulting from HC condensates.

M_{HCS} is defined by the operator product

$$-\mathcal{L}_{HCS} = \frac{1}{2}(\bar{n}_L,\ \overline{\alpha^c}_L,\ \overline{\xi^c}_L) M_{HCS} \begin{pmatrix} n_R^c \\ \alpha_R \\ \xi_R \end{pmatrix} + h.c.$$

so that

$$M_{HCS} = \begin{pmatrix} M_L & (M_D)_{\bar{n}\alpha} & (M_D)_{\bar{n}\xi} \\ (M_D)^T_{\bar{n}\alpha} & (M_R)_{\alpha\alpha} & (M_R)_{\alpha\xi} \\ (M_D)^T_{\bar{n}\xi} & (M_R)^T_{\alpha\xi} & (M_R)_{\xi\xi} \end{pmatrix}$$

M_L : 5 × 5

$(M_D)_{\bar{n}\alpha}$: 5 × 4

$(M_D)_{\bar{n}\xi}$: 5 × [illegible]

The states $\alpha_{1j,R}$ with $j = 2,3$ play the role of the RH EW-singlet neutrinos that get induced Dirac neutrino mass terms connecting with $(n^1, n^2, n^3)_L = (\nu_e, \nu_\mu, \nu_\tau)_L$. Because these $\alpha_{1j,R}$ transform as part of a $\bar{4}$ rather than a 4 of $SU(4)_{ETC}$, the resultant masses cannot be generated by the usual one-loop ETC graph that produces quark and charged lepton masses and are strongly suppressed, as in AT94.
(similar to Sikivie, Shifman, Voloshin, Zakharov mechanism for m_D suppression)
The Dirac submatrix $(M_D)_{\bar{n}\alpha}$ is defined by the operator product

$$\bar{n}_{i,L}[(M_D)_{\bar{n}\alpha}]_{ij}\alpha_{1j,R}$$

and has the form

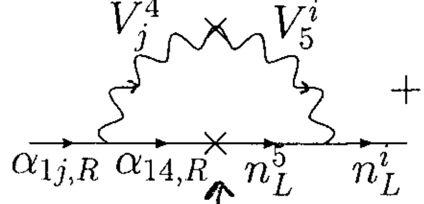
$$(M_D)_{\bar{n}\alpha} = \begin{array}{c} \\ \bar{n}_{1L} \\ \bar{n}_{2L} \\ \bar{n}_{3L} \\ \bar{n}_{4L} \\ \bar{n}_{5L} \end{array} \begin{array}{c} \begin{array}{cccc} \alpha_{12R} & \alpha_{13R} & \alpha_{14R} & \alpha_{15R} \end{array} \\ \left(\begin{array}{cc|cc} b_{12} & b_{13} & 0 & 0 \\ b_{22} & b_{23} & 0 & 0 \\ b_{32} & b_{33} & 0 & 0 \\ \hline 0 & 0 & 0 & c_1 \\ 0 & 0 & -c_1 & 0 \end{array}\right) \end{array}$$

where the 0's are exact and due to exact TC gauge invariance.

The entry c_1 has magnitude $|c_1| \sim \Lambda_{TC}$ and represents a TC dynamical mass term $\sum_{i,j=4,5} \epsilon^{ij}\bar{n}_{i,L}\alpha_{1j,R}$.

b_{ij}'s produced by lowest-order graphs $\longrightarrow$

ETC gauge boson mixing required because $\alpha_{1j} \sim \bar{4}$ for $j = 2,3,4,5$ rather than a 4 of $SU(4)_{ETC}$

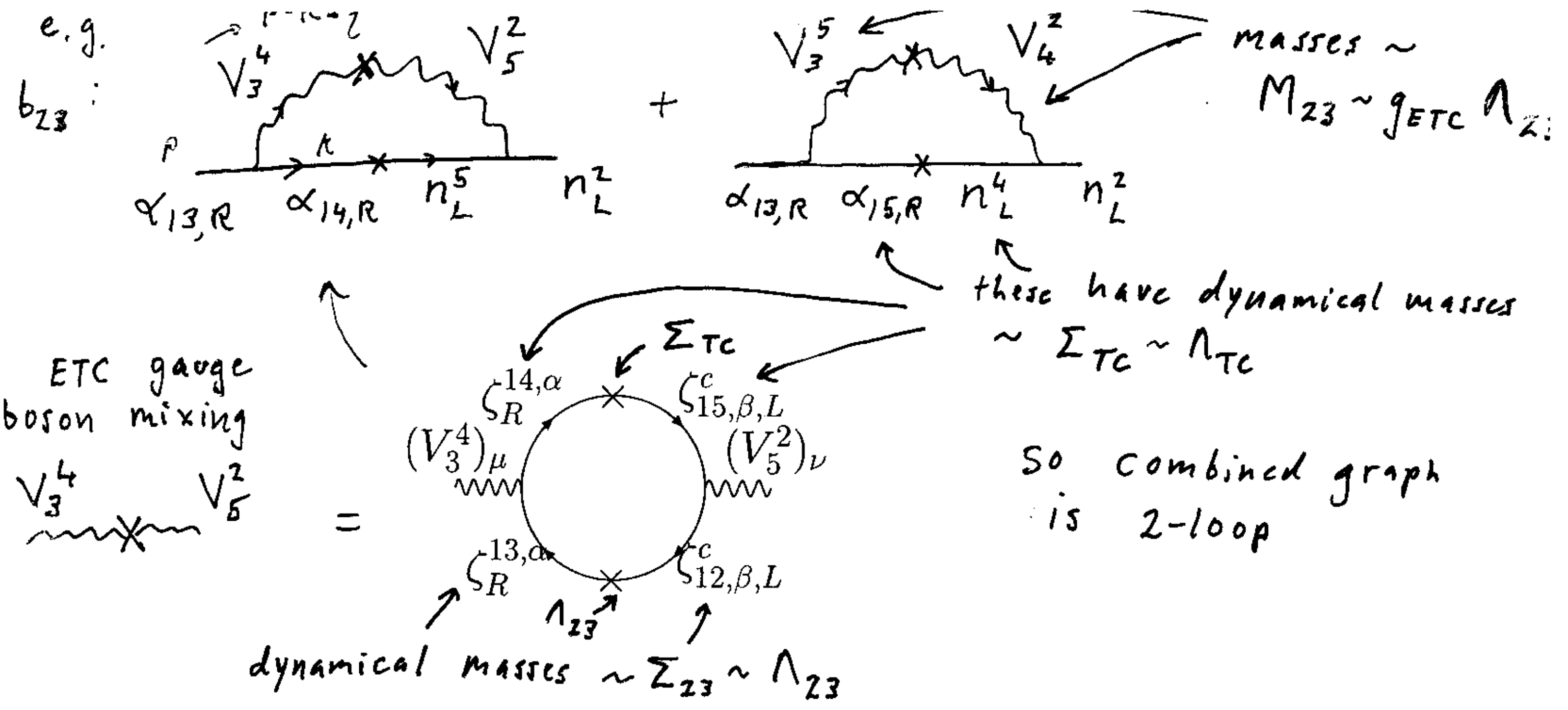


$+ (4 \leftrightarrow 5)$

dynamical TC mass $\Sigma_{TC}(k) \sim c_1$ corresponding to TC condensate $\langle \sum_{ij=4,5} \epsilon^{ij}\bar{n}_{iL}\alpha_{1jR} \rangle$

Graphs generating $\bar{n}_{i,L}b_{ij}\alpha_{1j,R}$ for $i = 1,2,3$ and $j = 2,3$

NB : The ϵ^{ij} TC contraction contrasts with the usual δ_i^j TC contraction in $\langle \bar{F}F \rangle$ and relies on use of $SU(2)$ for TC group



One-loop graph contributing to the gauge boson mixing $V_3^4 \leftrightarrow V_5^2$.

We show graphs that contribute to the b_{ij}'s. The necessary ETC gauge boson mixings occur to leading (one-loop) order for b_{23} and b_{32}, which involve $V_3^4 \leftrightarrow V_5^2$ and $V_3^5 \leftrightarrow V_4^2$. The one-loop graph that produces this ETC gauge boson mixing is shown above. Other b_{ij}'s can be produced by higher-loop diagrams.

We next estimate the leading b_{ij} entries. Denote the ETC gauge boson 2-point function as

$$ {}^k_n\Pi^i_j(q)_{\mu\lambda} = \int \frac{d^4x}{(2\pi)^4} e^{iq\cdot x} \langle T\left[(V^k_n)_\mu(x/2)(V^i_j)_\lambda(-x/2)\right]\rangle_0 \ . $$

The above graph yields, after Wick rotation, with $q = p - k$

$$ g^2_{ETC}[\bar{n}_{i,L}(p)\gamma_\mu\gamma_\lambda\alpha_{1j,R}(p)] \int \frac{d^4k}{(2\pi)^4} \frac{k^2\Sigma_{TC}(k)[{}^i_5\Pi^4_j((p-k)^2)]^{\mu\lambda}}{(k^2+\Sigma_{TC}(k)^2)^2[(p-k)^2+M_j^2][(p-k)^2+M_i^2]} $$

here $M_i, M_j \sim M_{23}$

where $\Sigma_{TC}(k)$ is the dynamical technicolor mass associated with the transition $\alpha_{14,R} \to n_L^5$, behaving as

$$\Sigma_{TC}(k) \sim \Lambda_{TC} \quad \text{for} \;\; k^2 << \Lambda_{TC}^2$$

and, for walking TC,

$$\Sigma_{TC}(k) \sim \frac{\Lambda_{TC}^2}{k} \quad \text{for} \;\; k^2 >> \Lambda_{TC}^2$$

We need ${}^k_n\Pi^i_j((p-k)^2)_{\mu\lambda}$ only for $(p-k)^2/\Lambda_1^2 << 1$, since the loop momenta in the graph are cut off far below Λ_1. Here we estimate

$$[{}^2_5\Pi^4_3(q)]_{\mu\lambda} \sim [{}^2_4\Pi^5_3(q)]_{\mu\lambda} \sim \frac{g^2_{ETC}\Lambda^2_{TC}}{(2\pi^2)} g_{\mu\lambda}$$

For $i,j = 2,3$ and $3,2$, adding the other graph with $4 \leftrightarrow 5$, we get

$$|b_{23}| = |b_{32}| \sim \frac{g^4_{ETC}\Lambda^4_{TC}\Lambda_{23}}{2\pi^4 M^4_{23}} \sim \frac{\Lambda^4_{TC}}{2\pi^4\Lambda^3_{23}}$$

Numerically, $|b_{23}| = |b_{32}| \sim O(1)$ KeV. Elements $b_{22} = b_{33}$ generated similarly; b_{1j} smaller. (These are rough estimates, owing to the strongly coupled nature of the ETC and TC theories.)

This shows the heavy suppression of Dirac neutrino masses, by a factor of $\sim 10^5 - 10^6$ relative to usual 2nd and 3rd generation lepton masses. Although specific results for the various b_{ij} are dependent on the symmetry breaking pattern, this suppression is a general feature of this type of ETC model.

The other Dirac submatrix, $(M_D)_{\bar{n}\xi}$, associated with the operator product

$$\bar{n}_{i,L}[(M_D)_{\bar{n}\xi}]_{i,kn}\xi_{kn,R}$$

can be analyzed in a similar manner; it does not play as important a role as $(M_D)_{\bar{n}\alpha}$.

In $(M_R)_{HCS}$ the 6×6 submatrix $(M_R)_{\xi\xi}$ contains entries $\sim \Lambda_1$ from the highest-scale condensation. The 4×4 submatrix $(M_R)_{\alpha\alpha}$ associated with the operator product

$$\overline{\alpha^c}_{1i,L} r_{ij} \alpha_{1j,R} = \alpha^T_{1i,R} C r_{ij} \alpha_{1j,R} \ .$$

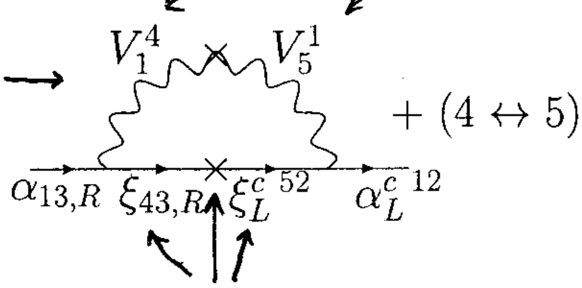
has the form

$$(M_R)_{\alpha\alpha} = \begin{array}{c} \begin{array}{cccc} \alpha_{12} & \alpha_{13} & \alpha_{14} & \alpha_{15} \end{array} \\ \left(\begin{array}{cc|cc} r_{22} & r_{23} & 0 & 0 \\ r_{23} & r_{33} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \end{array}$$

where again the 0's are exact and follow from TC invariance. If the 2×2 r_{ij} submatrix has maximal rank, this can provide a seesaw which, in conjunction with the suppression of the Dirac entries b_{ij} discussed above, can yield adequate suppression of neutrino masses. The submatrix r_{ij}, $2 \le i, j \le 3$, produces this seesaw because $\alpha_{12,R}$ and $\alpha_{13,R}$ are the electroweak-singlet techni-singlet neutrinos that remain as part of the low-energy effective field theory at and below the electroweak scale.

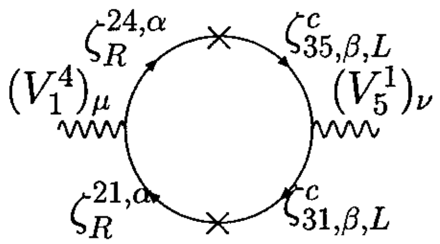
masses $M_1 \sim g_{ETC}\Lambda_1$

This again requires ETC gauge boson mixing



Graphs for $\alpha^T_{12,R} C r_{23} \alpha_{13,R}$.

dynamical mass $\Sigma_1 \sim \Lambda_1$ corresponding to $\langle \epsilon^{ijkl} \xi^T_{ij,R} C \xi_{kl,R} \rangle$ condensate



One-loop graph for the ETC gauge boson mixing $V_1^4 \leftrightarrow V_5^1$. The graph with indices 2 and 3 interchanged on the internal ζ lines also contributes to $V_1^4 \leftrightarrow V_5^1$.

We calculate

$$r_{23} \sim \frac{\Lambda_{BHC}^2 \Lambda_{23}^2}{2\pi^4 \Lambda_1^3}$$

Numerically, $|r_{23}| \sim O(0.1)$ GeV in one variant of model, but can be larger. The submatrix $(M_R)_{\alpha\xi}$ can be analyzed in a similar manner but does not play as important a role.

Carrying out the diagonalization of M, we obtain the following results. The EW-nonsinglet neutrinos are, to very good approximation, linear combinations of three mass eigenstates with normal hierarchy and ν_3 mass

$$m_{\nu_3} \sim \frac{(b_{23}+b_{22})^2}{|r_{23}|} \sim \frac{\Lambda_{TC}^8 \Lambda_1^3}{2\pi^4 \Lambda_{23}^8 \Lambda_{BHC}^2}$$

Since $|r_{23}| >> |b_{ij}|$, this is a seesaw, but quite different from the SUSY GUT seesaw. For $m(\nu_2)$:

$$\frac{m(\nu_2)}{m(\nu_3)} = \left(\frac{b_{23} - b_{22}}{b_{23} + b_{22}}\right)^2$$

With the above-mentioned numerical values and $\Lambda_{BHC} \simeq 0.3\Lambda_1$, the model can fit $m_{\nu_3} \simeq 0.05$ eV and $m(\nu_2) \simeq 0.008$ eV, in agreement with experimental indications; atmospheric neutrino data yields $|\Delta m^2_{32}| \simeq 2.5 \times 10^{-3}$ eV^2, so that $m(\nu_3) \simeq 0.05$ eV, and solar neutrino data yields $\Delta m^2_{21} \simeq 0.7 \times 10^{-4}$ eV^2, whence $m(\nu_2) \simeq 0.008$ eV, in a hierarchical framework, as applies here. The lowest neutrino mass, $m(\nu_1) << m(\nu_2), m(\nu_3)$. These are Majorana mass eigenstates.

The model can also naturally yield large $\nu_\mu - \nu_\tau$ mixing, i.e., $\theta_{23} \simeq \pi/4$, because of the leading off-diagonal structure of the r_{ij}. However, because of the strong coupling nature of the physics, it is difficult to give precise predictions, e.g., for θ_{13}.

The model also yields the following mass eigenvalues and corresponding eigenvectors for the other neutrino-like states:

- linear combinations (LC's) the $\xi_{ij,R}$ with $2 \le i, j \le 5$ get masses $\sim \Lambda_1$
- LC's of the $\zeta_R^{ij,\alpha}$ with $2 \le i, j \le 5$ get masses $\sim \Lambda_{BHC}$
- LC's of the $\zeta_R^{1j,\alpha}$ with $j = 2, 3$ get masses $\sim \Lambda_{23}$
- LC's of the $\zeta_R^{1j,\alpha}$ with $j = 4, 5$ and LC's of $n^c_{i,R}$ and $\alpha_{1i,R}$ with $i = 4, 5$ get masses $\sim \Lambda_{TC}$
- LC's of $\alpha_{1i,R}$ with $i = 2, 3$ get masses $\sim r_{23}$

The r_{ij} entries in M_R responsible for the seesaw are not superheavy masses and are actually much smaller than the ETC scales Λ_i. The resultant EW-singlet neutrinos with masses $\sim r_{23}$ are unstable and appear to be consistent with astrophysical and cosmological constraints, and constraints from searches for heavy neutrino emission in $K^+_{\mu 2}$ and K^+_{e2} decays (RS, '80-'84).

Neutrino Masses in Models with Extended Gauge Symmetries

We have succeeded in constructing similar models explaining light neutrino masses in theories with dynamical symmetry breaking of extended strong-electroweak gauge groups that have appealing features going beyond those of the SM. The first such group is

$$G_{LR} = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$$

in which the usual fermions of each generation transform as

$$\begin{array}{cc} (3,2,1)_{1/3,L}\ , & (3,1,2)_{1/3,R} \\ (1,2,1)_{-1,L}\ , & (1,1,2)_{-1,R} \end{array}$$

The gauge couplings are defined via the covariant derivative

$$D_\mu = \partial_\mu - ig_3 \mathbf{T}_c \cdot \mathbf{A}_{c,\mu} - ig_{2L} \mathbf{T}_L \cdot \mathbf{A}_{L,\mu} - ig_{2R} \mathbf{T}_R \cdot \mathbf{A}_{R,\mu} - i(g_U/2)(B-L)\mathbf{U}_\mu$$

Here the electric charge is given by the elegant relation

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2}$$

where B = baryon no., L = lepton number. Given experimental limits on right-handed charged currents and an associated W_R, and on extra Z''s, $\mathrm{SU}(2)_R$ must be broken at a scale Λ_{LR} well above the electroweak scale. Similarly for $U(1)_{B-L}$;

$SU(2)_R \times U(1)_{B-L} \to U(1)_Y$ at Λ_{LR}

The second extended gauge group is

$$G_{422} = \mathrm{SU}(4)_{PS} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$$

(Pati, Salam, Mohapatra, Senjanovic..)

with the usual fermions transforming as

$$(4,2,1)_L \ , \quad (4,1,2)_R$$

G_{422} provides a higher degree of unification since

- It unifies quarks and leptons in the $(4,2,1)_L$ and $(4,1,2)_R$ representations for each generation; e.g., for the first-generation, these are

$$\begin{pmatrix} u^a & \nu_e \\ d^a & e \end{pmatrix}_{L,R}$$

- It combines $\mathrm{U}(1)_{B-L}$ and $\mathrm{SU}(3)_c$ (in a maximal subgroup) in the Pati-Salam group $\mathrm{SU}(4)_{PS}$ and hence relates g_U and g_3 Denoting the generators of $\mathrm{SU}(4)_{PS}$ as $T_{PS,i}$, $1 \le i \le 15$, with

$$T_{PS,15} = \frac{1}{2\sqrt{6}} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix}$$

  and setting $U_\mu = A_{PS,15,\mu}$, one has $(B-L)/2 = \sqrt{2/3}T_{PS,15}$ and hence

$$\frac{g_U^2}{g_{PS}^2} = \frac{3}{2}$$

- It quantizes electric charge:

$$Q = T_{3L} + T_{3R} + \sqrt{2/3}T_{PS,15} = T_{3L} + T_{3R} + (1/6)\mathrm{diag}(1,1,1,-3)$$

The quark-lepton unification in the G_{422} theory does not lead to proton decay, but it does lead to the decays such as $K_L \to \mu^{\pm}e^{\mp}$ and $K^+ \to \pi^+\mu^+e^-$; consistency with experimental upper limits on these decays implies that the scale Λ_{PS} at which $\mathrm{SU}(4)_{PS}$ breaks is $\Lambda_{PS} \gtrsim 300$ TeV.

The conventional approach to the gauge symmetry breaking in these models employs elementary Higgs fields and arranges for a hierarchy of breaking scales by making the vacuum expectation values (vev's) of the Higgs that break G_{LR} or G_{422} to G_{SM} much larger than the Higgs vev's that break $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{em}$.

Questions:

- Can one construct theories with the G_{LR} and/or G_{422} groups that exhibit dynamical breaking of all the gauge symmetries other than $SU(3)_c$ and $U(1)_{em}$?
- If so, can these theories naturally explain why the $\mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ symmetry of the G_{LR} model breaks to $\mathrm{U}(1)_Y$, and why the $\mathrm{SU}(4)_{PS} \times \mathrm{SU}(2)_R$ part of the G_{422} model breaks to $\mathrm{SU}(3)_c \times \mathrm{U}(1)_Y$, and why the associated breaking scales Λ_{LR} and Λ_{PS} are large wrt. the electroweak scale?
- Given that one has answered these two questions affirmatively, can one explain light neutrino masses in such dynamical symmetry breaking theories?

Here we present models in which the answers to these three questions are yes.

Note that technicolor itself could not provide a mechanism for all the breaking, because it is too weak at the the scales Λ_{LR} or Λ_{PS} and because the technifermion condensate $\langle \bar{F}F \rangle = \langle \bar{F}_L F_R \rangle + \langle \bar{F}_R F_L \rangle$ would break both $SU(2)_L$ and $SU(2)_R$ at the same scale (to the diagonal (vector) group $SU(2)_V$). Of course, TC must be enlarged anyway (here to ETC) in order to explain quark and lepton masses.

Our models are ETC-type theories, with the breaking of G_{LR} and G_{422} to G_{SM} being driven by the same interactions that break the ETC group and generate quark and lepton masses.
Different approach to dynamical breaking of G_{LR} : use NJL 4-fermion operators - Akhmedov, Lindner, Schnapka, Valle '96

Dynamical Symmetry Breaking of G_{LR}

Our DSB model for the theory with G_{LR} uses the gauge group

$$G = SU(5)_{ETC} \times SU(2)_{HC} \times G_{LR}$$

with fermion content

$$(5,1,3,2,1)_{1/3,L}\ , \quad (5,1,3,1,2)_{1/3,R}$$

$$(5,1,1,2,1)_{-1,L}\ , \quad (5,1,1,1,2)_{-1,R}$$

$$(\bar{5},1,1,1,1)_{0,R}\ , \quad (\overline{10},1,1,1,1)_{0,R}\ , \quad (10,2,1,1,1)_{0,R}$$

η_{iR} $\qquad \psi_{ij,R} = \{\alpha_{ij,R}, \xi_{ij}\}$ $\qquad \zeta^{ij,\alpha}_R$

$\alpha_{i1,R}$

$2 \le i,j \le 5$ $\qquad \omega^{\alpha}_{p,R} : (1,2,1,1,1)_{0,R}$

neglect ω's here

The $\mathrm{SU}(5)_{ETC}$ theory is an anomaly-free, chiral gauge theory and, like the TC and HC theories, is asymptotically free. There are no bilinear fermion operators invariant under G, and hence there are no bare fermion mass terms. The $\mathrm{SU}(2)_{HC}$ and $\mathrm{SU}(2)_{TC}$ subsectors of $\mathrm{SU}(5)_{ETC}$ are vectorial.

As the energy decreases from some high value, the $\mathrm{SU}(5)_{ETC}$ and $\mathrm{SU}(2)_{HC}$ couplings increase. We envision that at $E \sim \Lambda_{LR} \gtrsim 10^3$ TeV, α_{ETC} is sufficiently strong to produce condensation in the channel

$$(5,1,1,1,2)_{-1,R} \times (\bar{5},1,1,1,1)_{0,R} \to (1,1,1,1,2)_{-1}$$

with $\Delta C_2 = 24/5$, breaking G_{LR} to $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. The associated condensate is $\langle {n_R^i}^T C \mathcal{N}_{i,R} \rangle$. The n_R^i and $\mathcal{N}_{i,R}$ thus gain dynamical masses $\sim \Lambda_{LR}$.

part of $SU(2)_R$ doublet, $T_{3R} = \frac{1}{2}$
$L = 1$ so $(B-L)$-nonsinglet
so breaks $SU(2)_R \times U(1)_{B-L} \to$
$\to U(1)_Y$
($Y = 0$ for $n_R, \mathcal{N}_R$)

This condensation generates masses

$$m_{W_R} = \frac{g_{2R}}{2} \Lambda_{LR} \qquad m_{Z'} = \frac{g_{2u}}{2} \Lambda_{LR}$$

where $g_{2u} \equiv \sqrt{g_{2R}^2 + g_U^2}$, for the $W_{R,\mu}^{\pm} = A_{R,\mu}^{\pm}$ gauge bosons and the linear combination

$$Z'_\mu = \frac{g_{2R} A_{3,R,\mu} - g_U U_\mu}{g_{2u}}$$

This leaves the orthogonal combination

$$B_\mu = \frac{g_U A_{3,R,\mu} + g_{2R} U_\mu}{g_{2u}}$$

as the weak hypercharge $\mathrm{U}(1)_Y$ gauge boson, which is massless at this stage. The hypercharge coupling is then

$$g' = \frac{g_{2R} g_U}{g_{2u}}$$

so that, with $e^{-2} = g_{2L}^{-2} + (g')^{-2} = g_{2L}^{-2} + g_{2R}^{-2} + g_U^{-2}$, the weak mixing angle is given by

$$\sin^2\theta_W = \left[1 + \left(\frac{g_{2L}}{g_{2R}}\right)^2 + \left(\frac{g_{2L}}{g_U}\right)^2\right]^{-1}$$

at the scale Λ_{LR}. The experimental value of $\sin^2\theta_W$ at M_Z can be accommodated easily.

For $E < \Lambda_{LR}$, the effective theory has gauge symmetry $\mathrm{SU}(5)_{ETC} \times \mathrm{SU}(2)_{HC} \times G_{SM}$ and fermion content

$$(5,1,3,2)_{1/3,L}\ , \quad (5,1,3,1)_{4/3,R}\ , \quad (5,1,3,1)_{-2/3,R}$$
$$(5,1,1,2)_{-1,L}\ , \quad (5,1,1,1)_{-2,R}$$
$$(\overline{10},1,1,1)_{0,R}\ , \quad (10,2,1,1)_{0,R}$$

which is the same content as in our first ETC model. Hence, our discussion for that model can essentially be taken over and applied here.

In a model in which L is not gauged, it is a convention how one assigns the lepton number L to the SM-singlet fields. Here, since L is gauged, this assignment is not a convention; $L = 0$ for the fields that are singlets under G_{LR} or G_{422}, since they are singlets under $\mathrm{U}(1)_{B-L}$ and have $B = 0$.

$L = 1$ $\quad$ $L = 0$

Hence, the condensate $\langle n_R^{i\,T} C \mathcal{N}_{i,R} \rangle$ and induced Dirac neutrino mass terms like $\bar{n}_{iL} b_{ij} \alpha_{1j,R}$ violate L as $\Delta L = 1$, while the $\langle \xi_R^T C \xi_R \rangle$ and $\langle \zeta_R^T C \zeta_R \rangle$ condensates do not violate L. The resultant physical left-handed Majorana neutrino bilinears generated by the seesaw violate L as $\Delta L = 2$ operators, as before. Because L is gauged, there is no majoron. $\quad m_L \nu_L^T C \nu_L$: $\quad \underset{\Delta L = 2}{m_L} \sim \dfrac{m_D^2}{m_R}$ $\quad$ model 1: $\Delta L = 0$, $\Delta L = 2$ $\quad$ here: $(\Delta L = 1)^2$, $\Delta L = 0$

Dynamical Symmetry Breaking of G_{422}

Our model uses the gauge group

$$G = \mathrm{SU}(5)_{ETC} \times \mathrm{SU}(2)_{HC} \times G_{422}$$

with fermion content

$$(5, 1, 4, 2, 1)_L \ , \quad (5, 1, 4, 1, 2)_R$$

$$(\bar{5}, 1, 1, 1, 1)_R \ , \quad (\overline{10}, 1, 1, 1, 1)_R \ , \quad (10, 2, 1, 1, 1)_R$$

$n_{i,R}$ $\qquad$ $\psi_{ij,R}$ $\qquad$ $\zeta_R^{ij,\alpha}$

As E decreases through the scale Λ_{PS}, the $\mathrm{SU}(5)_{ETC}$ interaction is large enough to produce condensation in the channel

$$(5, 1, 4, 1, 2)_R \times (\bar{5}, 1, 1, 1, 1)_R \to (1, 1, 4, 1, 2)$$

This breaks $SU(4)_{PS} \times SU(2)_R$ directly to $SU(3)_c \times U(1)_Y$. The associated condensate is again $\langle n^{i\,T}_R C \mathcal{N}_{i,R} \rangle$, and the n^i_R and $\mathcal{N}_{i,R}$ gain masses $\sim \Lambda_{PS}$. The above results for m_{W_R}, $m_{Z'}$, and $\sin^2\theta_W$ apply with $g_U^2/g_{PS}^2 = 3/2$ at Λ_{PS}.

Further breaking at lower scales proceeds as in the first ETC model and in the G_{LR} model.

The experimental value of $\sin^2\theta_W$ can again be accommodated. We use $\alpha_3(m_Z) = 0.118$, $\alpha_{em}(m_Z)^{-1} = 129$, and $(\sin^2\theta_W)_{\overline{MS}}(m_Z) = 0.231$. With $\Lambda_{PS} = 10^6$ GeV, we calculate the values $\alpha_3 = 0.064$, $\alpha_{2L} = 0.032$, and $\alpha_1 = 0.012$ at Λ_{PS}, and hence to fit $\sin^2\theta_W$ in this model, we find $\alpha_{2R}(\Lambda_{PS}) \simeq 0.013$ so $g_{2R}/g_{2L} \simeq 0.64$ at Λ_{PS}.

Given that the effective low-energy theories in the ETC models with strong-electroweak G_{LR} and G_{422} are the same as in our first ETC model with the standard strong-electroweak group, it follows that our explanation for the generation of light neutrino masses also applies to these theories with extended gauge symmetries.

Some topics that we are investigating at present (T. Appelquist, M. Piai, RS, to appear):

- quark masses and mixing
- different assignments of fermion representations and symmetry-breaking patterns
- global symmetries
- further phenomenological implications

Conclusions

We have addressed the question of what one infers about physics beyond the standard model from the discovery of neutrino masses. As an alternative to the usual SUSY GUT seesaw, we have explored a different approach based on theories with dynamical electroweak symmetry breaking. We have shown that it is possible to get light neutrino masses in this class of theories, thereby providing a plausible answer to a longstanding question of whether this could be done.

Our mechanism uses a seesaw but one with strong suppression of both Dirac and Majorana mass entries, which does not involve any superheavy mass scales such as GUT scales. We have shown the robustness of the mechanism by demonstrating that it works with strong-EW groups G_{SM}, G_{LR}, and G_{422}.

The model still has a number of phenomenological challenges characteristic of dynamical EWSB theories, but we believe that it motivates further work with this approach.