Extrinsic CPT Violation in Neutrino Oscillations

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In collaboration with: Magnus Jacobson

Based on: `hep-ph/0305064`

Introduction

Several studies on CPT violation in order to incorporate the results of the LSND experiment.
$\Rightarrow$ A third mass squared difference required, not compatible with three neutrino flavors.

Phenomenological studies of CPT violation: Different mass squared differences and mixing parameters for neutrinos and antineutrinos.
$\Rightarrow$ Four mass squared differences and eight mixing parameters.
$\therefore$ It is possible to include the results of the LSND experiment.

The results of the LSND experiment will be tested by the MiniBooNE experiment (September 2002 $\rightarrow \sim$ 2005).

Another possible description: sterile neutrinos
However, excluded by the SNO experiment.

The first KamLAND data are consistent with the LMA solution.
$\therefore$ No need for fundamental CPT violation.

Eccentric or *extrinsic* CPT violation?

Neutrino oscillation transition probabilities: $P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta)$

Intrinsic (eccentric) CPT violation:

Fundamental (genuine) CPT violation = due to violation of the CPT invariance theorem

Extrinsic CPT violation:

Matter-induced (fake) CPT violation = due to presence of ordinary matter

Here: The CPT invariance theorem is valid. $\Rightarrow$

Violation	Intrinsic	Extrinsic	Probability differences
CP	$\times$	$\times$	$\Delta P_{\alpha\beta}^{\mathrm{CP}} \equiv P_{\alpha\beta} - P_{\bar{\alpha}\bar{\beta}}$
T	$\times$	$\times$	$\Delta P_{\alpha\beta}^{\mathrm{T}} \equiv P_{\alpha\beta} - P_{\beta\alpha}$
CPT	-	$\times$	$\Delta P_{\alpha\beta}^{\mathrm{CPT}} \equiv P_{\alpha\beta} - P_{\bar{\beta}\bar{\alpha}}$

CP & T prob. diff.'s: intrinsic and extrinsic effects are mixed
CPT prob. diff.'s: extrinsic effects only

$\therefore$ Non-zero CPT prob. diff.'s show matter effects.

(Extrinsic) CPT-violating effects

Conservation of probability $\Rightarrow$

$$\sum_{\alpha=e,\mu,\tau,\ldots} \Delta P_{\alpha\beta}^{\rm CPT} = \sum_{\beta=e,\mu,\tau,\ldots} \Delta P_{\alpha\beta}^{\rm CPT} = 0.$$

Note! Not all of these equations are linearly independent. For three neutrino flavors, we have nine CPT probability differences for neutrinos (only four are linearly independent). Choosing, *e.g.*, $\Delta P_{ee}^{\rm CPT}$, $\Delta P_{e\mu}^{\rm CPT}$, $\Delta P_{\mu e}^{\rm CPT}$, and $\Delta P_{\mu\mu}^{\rm CPT}$ as the known ones, the other five can be expressed in terms of these.

Furthermore, we have

$$\Delta P_{\alpha\beta}^{\rm CPT} = -\Delta P_{\bar\beta\bar\alpha}^{\rm CPT},$$

i.e., the CPT probability differences for antineutrinos do not give any further information.

The CPT probability differences

The (extrinsic) CPT probability differences are one way of quantifying matter effects.

In vacuum:

$$\Delta P_{\alpha\beta}^{\mathrm{CPT}} = P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\beta \to \bar{\nu}_\alpha) = 0, \quad \alpha, \beta = e, \mu, \tau.$$

In matter:

$$\Delta P_{\alpha\beta}^{\mathrm{CPT}} = \left| \left[S_f(t, t_0) \right]_{\beta\alpha} \right|^2 - \left| \left[\bar{S}_f(t, t_0) \right]_{\alpha\beta} \right|^2,$$

where $S_f \equiv S_f(t, t_0)$ and $\bar{S}_f \equiv \bar{S}_f(t, t_0)$ are the evolution operators for neutrinos and antineutrinos, respectively.

We have calculated S_f and $\bar{S}_f$ explicitly using first order perturbation theory in the small leptonic mixing angle $\theta_1{}^3$.

The CPT probability differences

The Schrödinger equation for neutrinos in flavor basis:

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}S_f(t,t_0) = O_{23}\underbrace{\left[U'\begin{pmatrix} 0 & 0 & 0 \\ 0 & \delta & 0 \\ 0 & 0 & \Delta \end{pmatrix}U'^{\dagger} + \begin{pmatrix} V(t) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]}_{H(t)=H_0(t)+H_1(\theta_{13})+H_2(\theta_{13}^2)}O_{23}^T S_f(t,t_0),$$

where

$$U' = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}\mathrm{e}^{-\mathrm{i}\delta_{\mathrm{CP}}} \\ -s_{12} & c_{12} & 0 \\ -s_{13}c_{12}\mathrm{e}^{\mathrm{i}\delta_{\mathrm{CP}}} & -s_{13}s_{12}\mathrm{e}^{\mathrm{i}\delta_{\mathrm{CP}}} & c_{13} \end{pmatrix} \quad \text{and} \quad O_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}.$$

Here:

$$\delta \equiv \frac{\Delta m_2^{21}}{2E_\nu} \quad \text{and} \quad \Delta \equiv \frac{\Delta m_{31}^2}{2E_\nu} \simeq \frac{\Delta m_{32}^2}{2E_\nu}.$$

The CPT probability differences

First order perturbation theory (in θ_{13}):

$$S(t,t_0) \simeq S_0(t,t_0) - \mathrm{i} S_0(t,t_0) \int_{t_0}^{t} S_0^{-1}(t',t_0) H_1 S_0(t',t_0) \mathrm{d}t',$$

where

$$S_0(t,t_0) = \begin{pmatrix} \alpha(t,t_0) & \beta(t,t_0) & 0 \\ -\beta^*(t,t_0) & \alpha^*(t,t_0) & 0 \\ 0 & 0 & f(t,t_0) \end{pmatrix}$$

and

$$H_1 = \begin{pmatrix} 0 & 0 & \theta_{13}\left(\Delta - s_{12}^2 \delta\right) \mathrm{e}^{-\mathrm{i}\delta_{\mathrm{CP}}} \\ 0 & 0 & -\theta_{13} c_{12} s_{12} \mathrm{e}^{-\mathrm{i}\delta_{\mathrm{CP}}} \delta \\ \theta_{13}\left(\Delta - s_{12}^2 \delta\right) \mathrm{e}^{\mathrm{i}\delta_{\mathrm{CP}}} & -\theta_{13} c_{12} s_{12} \mathrm{e}^{\mathrm{i}\delta_{\mathrm{CP}}} \delta & 0 \end{pmatrix} \equiv \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ a^* & b^* & 0 \end{pmatrix}.$$

The CPT probability differences

The evolution operator for neutrinos in flavor basis:

$$
\begin{aligned}
S_f(t,t_0) &= O_{23}^T S(t,t_0) O_{23} \\
&\simeq \begin{pmatrix} \alpha & c_{23}\beta - \mathrm{i} s_{23} f A & -s_{23}\beta - \mathrm{i} c_{23} f A \\ -c_{23}\beta^* - \mathrm{i} s_{23} f C & S_{22} & S_{23} \\ s_{23}\beta^* - \mathrm{i} c_{23} f C & S_{32} & S_{33} \end{pmatrix},
\end{aligned}
$$

where A, B, C, and D are complicated functions.

The CPT probability differences (arbitrary matter density profile):

$$
\begin{aligned}
\Delta P_{ee}^{\mathrm{CPT}} &\simeq |\bar{\beta}|^2 - |\beta|^2, \\
\Delta P_{e\mu}^{\mathrm{CPT}} &\simeq c_{23}^2 \left(|\beta|^2 - |\bar{\beta}|^2\right) - 2 c_{23} s_{23} \Im\left(\beta f C - \bar{\beta} \bar{f}^* \bar{A}^*\right), \\
\Delta P_{e\tau}^{\mathrm{CPT}} &\simeq s_{23}^2 \left(|\beta|^2 - |\bar{\beta}|^2\right) + 2 c_{23} s_{23} \Im\left(\beta f C - \bar{\beta} \bar{f}^* \bar{A}^*\right), \\
\Delta P_{\mu e}^{\mathrm{CPT}} &\simeq c_{23}^2 \left(|\beta|^2 - |\bar{\beta}|^2\right) - 2 c_{23} s_{23} \Im\left(\beta f^* A^* - \bar{\beta} \bar{f} \bar{C}\right), \\
\Delta P_{\tau e}^{\mathrm{CPT}} &\simeq s_{23}^2 \left(|\beta|^2 - |\bar{\beta}|^2\right) + 2 c_{23} s_{23} \Im\left(\beta f^* A^* - \bar{\beta} \bar{f} \bar{C}\right).
\end{aligned}
$$

The CPT probability differences

The CPT probability differences in matter of constant density:

$$
\begin{aligned}
\Delta P_{ee}^{\rm CPT} &\simeq -s_{12}^2 c_{12}^2 \delta^2 \left(\frac{\sin^2 \omega L}{\omega^2} - \frac{\sin^2 \bar{\omega} L}{\bar{\omega}^2} \right), \\
\Delta P_{\mu e}^{\rm CPT} &\simeq s_{12}^2 c_{12}^2 c_{23}^2 \delta^2 \left(\frac{\sin^2 \omega L}{\omega^2} - \frac{\sin^2 \bar{\omega} L}{\bar{\omega}^2} \right) - 2 s_{12} c_{12} s_{13} s_{23} c_{23} \delta \left(\Delta - s_{12}^2 \delta \right) \\
&\times \left\{ \left(\frac{\sin^2 \bar{\omega} L}{\bar{\omega}^2} - \frac{\sin^2 \omega L}{\omega^2} \right) \cos \delta_{\rm CP} \right. \\
&+ \left(\Delta - c_{12}^2 \delta \right) \left[\frac{(\bar{\tilde{\Delta}} \bar{s} - \bar{\omega} \sin \bar{\tilde{\Delta}} L) \bar{s}}{\bar{\omega}^2 (\bar{\omega}^2 - \bar{\tilde{\Delta}}^2)} - \frac{(\tilde{\Delta} s - \omega \sin \tilde{\Delta} L) s}{\omega^2 (\omega^2 - \tilde{\Delta}^2)} \right] \cos \delta_{\rm CP} \\
&- \left. \left(\Delta - c_{12}^2 \delta \right) \left[\frac{(\cos \bar{\tilde{\Delta}} L - \bar{c}) \bar{s}}{\bar{\omega} (\bar{\omega}^2 - \bar{\tilde{\Delta}}^2)} - \frac{(\cos \tilde{\Delta} L - c) s}{\omega (\omega^2 - \tilde{\Delta}^2)} \right] \sin \delta_{\rm CP} \right\},
\end{aligned}
$$

Note! If one makes the replacement $\delta_{\rm CP} \to -\delta_{\rm CP}$, then $\Delta P_{e\mu}^{\rm CPT} \to \Delta P_{\mu e}^{\rm CPT}$ and $\Delta P_{e\tau}^{\rm CPT} \to \Delta P_{\tau e}^{\rm CPT}$ and in the case that $\delta_{\rm CP} = 0$ one has $\Delta P_{e\mu}^{\rm CPT} = \Delta P_{\mu e}^{\rm CPT}$ and $\Delta P_{e\tau}^{\rm CPT} = \Delta P_{\tau e}^{\rm CPT}$.

The CPT probability differences

Low-energy approximations ($V \lesssim \delta \ll \Delta$): Constant matter density:

$$\Delta P_{ee}^{\rm CPT} \simeq 8 s_{12}^2 c_{12}^2 \cos 2\theta_{12} \left(\delta L \cos \frac{\delta L}{2} - 2 \sin \frac{\delta L}{2} \right) \sin \frac{\delta L}{2} \frac{V}{\delta} + \mathcal{O}\left((V/\delta)^3 \right),$$

$$\begin{aligned}
\Delta P_{\mu e}^{\rm CPT} &\simeq -8 s_{12}^2 c_{12}^2 c_{23}^2 \cos 2\theta_{12} \left(\delta L \cos \frac{\delta L}{2} - 2 \sin \frac{\delta L}{2} \right) \sin \frac{\delta L}{2} \frac{V}{\delta} \\
&- 16 s_{12} c_{12}^3 s_{13} s_{23} c_{23} \cos \delta_{\rm CP} \cos 2\theta_{12} \left(\delta L \cos \frac{\delta L}{2} - 2 \sin \frac{\delta L}{2} \right) \sin \frac{\delta L}{2} \frac{V}{\delta} \\
&+ 16 s_{12} c_{12} s_{13} s_{23} c_{23} \sin \delta_{\rm CP} \left\{ \cos 2\theta_{12} \left[\delta L \cos \delta L - \cos \Delta L \right. \right. \\
&\times \left. \left. \left(\delta L \cos \frac{\delta L}{2} - 2 \sin \frac{\delta L}{2} \right) - \sin \delta L \right] + \delta L \sin \frac{\delta L}{2} \sin \Delta L \right\} \frac{V}{\delta} + \mathcal{O}\left((V/\delta)^3 \right).
\end{aligned}$$

$\Delta P_{ee}^{\rm CPT}$: No $\delta_{\rm CP}$ terms $\quad$ $\Delta P_{\mu e}^{\rm CPT}$: Both $\sin \delta_{\rm CP}$ and $\cos \delta_{\rm CP}$ terms

$\therefore$ Possible extraction of $\delta_{\rm CP}$ from $\Delta P_{\mu e}^{\rm CPT}$.

The CPT probability differences

Properties:

- Useful for "shorter" ($L \lesssim 3\,000\,\mathrm{km}$) and "longer" ($3\,000\,\mathrm{km} \lesssim L \lesssim 10\,670\,\mathrm{km}$) long baseline experiments.
- Note that there are, of course, no terms in the CPT probability differences that are constant in the matter potential V, since in the limit $V \to 0$, *i.e.*, in vacuum, the CPT probability differences must vanish, because in vacuum they are equal to zero.
- We observe that the leading order terms in the CPT probability differences are linear in the matter potential V, whereas the next-to-leading order terms are cubic, *i.e.*, there are no second order terms.
- Actually, for symmetric matter density profiles it holds that the oscillation transition probabilities in matter for neutrinos and antineutrinos, $P(\nu_\alpha \to \nu_\beta; V)$ and $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta; V)$, respectively, are related by $P(\nu_\alpha \to \nu_\beta; V) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha; -V)$. Hence, in this case, the CPT probability differences $\Delta P_{\alpha\beta}^{\mathrm{CPT}}(V) = P(\nu_\alpha \to \nu_\beta; V) - P(\bar{\nu}_\beta \to \bar{\nu}_\alpha; V) = P(\nu_\alpha \to \nu_\beta; V) - P(\nu_\alpha \to \nu_\beta; -V) \equiv f(V) - f(-V)$ are always odd functions with respect to the (symmetric) matter potential V, since $\Delta P_{\alpha\beta}^{\mathrm{CPT}}(-V) = f(-V) - f(V) = -[f(V) - f(-V)] = -\Delta P_{\alpha\beta}^{\mathrm{CPT}}(V)$.

Thanks to Hisakazu Minakata.

The CPT probability differences

Introducing the Jarlskog invariant

$$J \equiv s_{12}c_{12}s_{13}c_{13}^2 s_{23}c_{23} \sin\delta_{\mathrm{CP}} \simeq s_{12}c_{12}s_{13}s_{23}c_{23}\sin\delta_{\mathrm{CP}},$$

we can, *e.g.*, write the CPT probability difference $\Delta P_{e\mu}^{\mathrm{CPT}}$ as

$$\begin{aligned}
\Delta P_{e\mu}^{\mathrm{CPT}} &\simeq -c_{23}^2 \Delta P_{ee}^{\mathrm{CPT}} \\
&- 16 c_{12}^2 \cos 2\theta_{12} J \cot\delta_{\mathrm{CP}} \left(\delta L \cos\frac{\delta L}{2} - 2\sin\frac{\delta L}{2} \right) \sin\frac{\delta L}{2}\frac{V}{\delta} \\
&- 16 J \left\{ \cos 2\theta_{12} \left[\delta L \cos\delta L - \cos\Delta L \left(\delta L \cos\frac{\delta L}{2} - 2\sin\frac{\delta L}{2} \right) \right.\right. \\
&- \left.\left. \sin\delta L \right] + \delta L \sin\frac{\delta L}{2} \sin\Delta L \right\} \frac{V}{\delta} + \mathcal{O}\left((V/\delta)^3 \right).
\end{aligned}$$

In the case of maximal solar mixing, *i.e.*, if the solar mixing angle $\theta_{12} = \frac{\pi}{4}$, then we have

$$\Delta P_{ee}^{\mathrm{CPT}} \simeq 0 \quad \text{and} \quad \Delta P_{e\mu}^{\mathrm{CPT}} \simeq -16 J \delta L \sin\frac{\delta L}{2} \sin\Delta L \frac{V}{\delta} \simeq -\Delta P_{\mu e}^{\mathrm{CPT}}.$$

The CPT probability differences

The T probability difference

(E. Akhmedov, P. Huber, M. Lindner, and T. Ohlsson, hep-ph/0105029)

In the low-energy regime ($\delta = \Delta m^2_{21}/(2E_\nu) \gtrsim V_{1,2}$) for constant matter density:

$$\begin{aligned}
\Delta P^{\mathrm{T}}_{\alpha\beta} &\simeq \cos\delta_{\mathrm{CP}} \cdot 8 \underbrace{s_{12}c_{12}s_{13}s_{23}c_{23}\frac{\sin(2\theta_1 - 2\theta_2)}{\sin 2\theta_{12}}}_{J_{\mathrm{eff}}} \\
&\times \left\{ s_1 s_2 \left[Y - \cos\left(\Delta_1 L_1 + \Delta_2 L_2\right)\right]\right\} \\
&+ \sin\delta_{\mathrm{CP}} \cdot 4 s_{13}s_{23}c_{23} \\
&\times X_1 \left[Y - \cos\left(\Delta_1 L_1 + \Delta_2 L_2\right)\right]
\end{aligned}$$

$\cos\delta_{CP}$ term: matter-induced T violation

$\sin\delta_{CP}$ term: fundamental T violation

The CPT probability differences

Low-energy approximation of $\Delta P_{ee}^{\mathrm{CPT}}$ ($V_{1,2} \lesssim \delta \ll \Delta$):
Step-function matter density:

$$\begin{aligned}
\Delta P_{ee}^{\mathrm{CPT}} &\simeq 8 s_{12}^2 c_{12}^2 \cos 2\theta_{12} \left[\delta \left(L_1 \frac{V_1}{\delta} + L_2 \frac{V_2}{\delta} \right) \cos \frac{\delta (L_1 + L_2)}{2} \right. \\
&- \left. 2 \left(\frac{V_1}{\delta} \sin \frac{\delta L_1}{2} \cos \frac{\delta L_2}{2} + \frac{V_2}{\delta} \sin \frac{\delta L_2}{2} \cos \frac{\delta L_1}{2} \right) \right] \sin \frac{\delta (L_1 + L_2)}{2} \\
&+ \mathcal{O}\left((V_1/\delta)^2, (V_2/\delta)^2, V_1 V_2/\delta^2 \right).
\end{aligned}$$

- Completely symmetric with respect to interchange of layers 1 and 2.
- In the limit $V_{1,2} \to V$ and $L_{1,2} \to L/2$, one has

$$\Delta P_{ee}^{\mathrm{CPT}}(\text{step-function}) \to \Delta P_{ee}^{\mathrm{CPT}}(\text{constant}).$$

- Only useful for very long baseline experiments ($L \gtrsim 10\,670\,\mathrm{km}$).

Numerical calculations and implications

Present values of the fundamental neutrino parameters:

Parameter	Best-fit value	Range
Δm^2_{21}	$7.1 \cdot 10^{-5}\,\mathrm{eV}^2$	$\sim (6 \div 9) \cdot 10^{-5}\,\mathrm{eV}^2$ (99.73 % C.L.)
$\lvert\Delta m^2_{31}\rvert$	$2.5 \cdot 10^{-3}\,\mathrm{eV}^2$	$(1.6 \div 3.9) \cdot 10^{-3}\,\mathrm{eV}^2$ (90 % C.L.)
θ_{12}	34°	$27^\circ \div 44^\circ$ (99.73 % C.L.)
θ_{13}	-	$0 \div 9.2^\circ$ (90 % C.L.)
θ_{23}	45°	$37^\circ \div 45^\circ$ (90 % C.L.)
δ_{CP}	-	$[0, 2\pi)$

In addition, we have chosen:

- A normal mass hierarchy spectrum ($\mathrm{sign}(\Delta m^2_{31}) = 1$)
- $\theta_{13} = 9.2^\circ$
- $\delta_{\mathrm{CP}} = 0$

Numerical calculations and implications

Experiment	CPT probability differences	
	Quantities	Numerical value
BNL NWG	$\Delta P_{\mu e}^{\mathrm{CPT}}$	0.010
BNL NWG	$\Delta P_{\mu e}^{\mathrm{CPT}}$	0.032
BooNE	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$6.6 \cdot 10^{-13}$
MiniBooNE	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$4.1 \cdot 10^{-14}$
CHOOZ	$\Delta P_{ee}^{\mathrm{CPT}}$	$-3.6 \cdot 10^{-5}$
ICARUS	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$4.0 \cdot 10^{-5}$
	$\Delta P_{\mu \tau}^{\mathrm{CPT}}$	$-3.8 \cdot 10^{-5}$
JHF-Kamioka	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$3.8 \cdot 10^{-3}$
	$\Delta P_{\mu \mu}^{\mathrm{CPT}}$	$-1.3 \cdot 10^{-4}$
K2K	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$1.0 \cdot 10^{-3}$
	$\Delta P_{\mu \mu}^{\mathrm{CPT}}$	$-5.3 \cdot 10^{-5}$

Numerical calculations and implications

Cont'd:

Experiment	CPT probability differences	
	Quantities	Numerical value
KamLAND	$\Delta P_{ee}^{\rm CPT}$	-0.033
LSND	$\Delta P_{\mu e}^{\rm CPT}$	$4.8 \cdot 10^{-15}$
MINOS	$\Delta P_{\mu e}^{\rm CPT}$	$1.9 \cdot 10^{-4}$
	$\Delta P_{\mu\mu}^{\rm CPT}$	$-1.1 \cdot 10^{-5}$
NuMI I	$\Delta P_{\mu e}^{\rm CPT}$	0.026
NuMI II	$\Delta P_{\mu e}^{\rm CPT}$	$2.6 \cdot 10^{-3}$
NuTeV	$\Delta P_{\mu e}^{\rm CPT}$	$1.6 \cdot 10^{-18}$
NuTeV	$\Delta P_{\mu e}^{\rm CPT}$	$8.2 \cdot 10^{-20}$
OPERA	$\Delta P_{\mu\tau}^{\rm CPT}$	$-3.8 \cdot 10^{-5}$
Palo Verde	$\Delta P_{ee}^{\rm CPT}$	$-1.2 \cdot 10^{-5}$
Palo Verde	$\Delta P_{ee}^{\rm CPT}$	$-2.2 \cdot 10^{-5}$

Numerical calculations and implications

Neutrino factory:

The CPT probability difference $\Delta P_{\mu e}^{\mathrm{CPT}}$ for two potential neutrino factory setups:

(Parameters: $\rho = \rho_{\mathrm{mantle}} \simeq 4.5\,\mathrm{g/cm^3}$, $E_\nu = 50\,\mathrm{GeV}$, $L \in \{3\,000, 7\,000\}\,\mathrm{km}$)

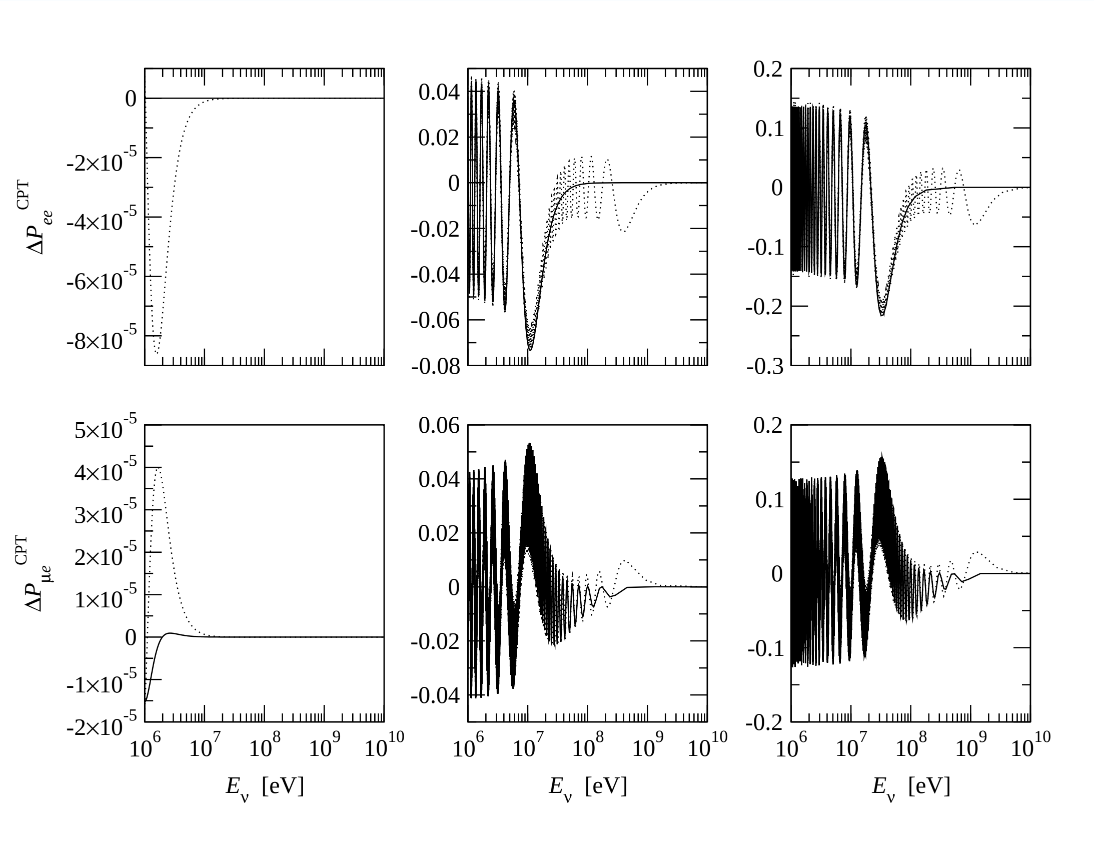
$$L = 3\,000\,\mathrm{km} \quad \Rightarrow \quad \Delta P_{\mu e}^{\mathrm{CPT}} \simeq 3.0 \cdot 10^{-5}$$

$$L = 7\,000\,\mathrm{km} \quad \Rightarrow \quad \Delta P_{\mu e}^{\mathrm{CPT}} \simeq 1.8 \cdot 10^{-5}$$

$\therefore$ The extrinsic CPT violation is practically negligible for a future neutrino factory.

Numerical calculations and implications

The CPT probability differences $\Delta P_{ee}^{\mathrm{CPT}}$ and $\Delta P_{\mu e}^{\mathrm{CPT}}$ plotted as functions of the neutrino energy E_ν for $L \in \{1, 250, 750\}\,\mathrm{km}$:



solid curve = analytical

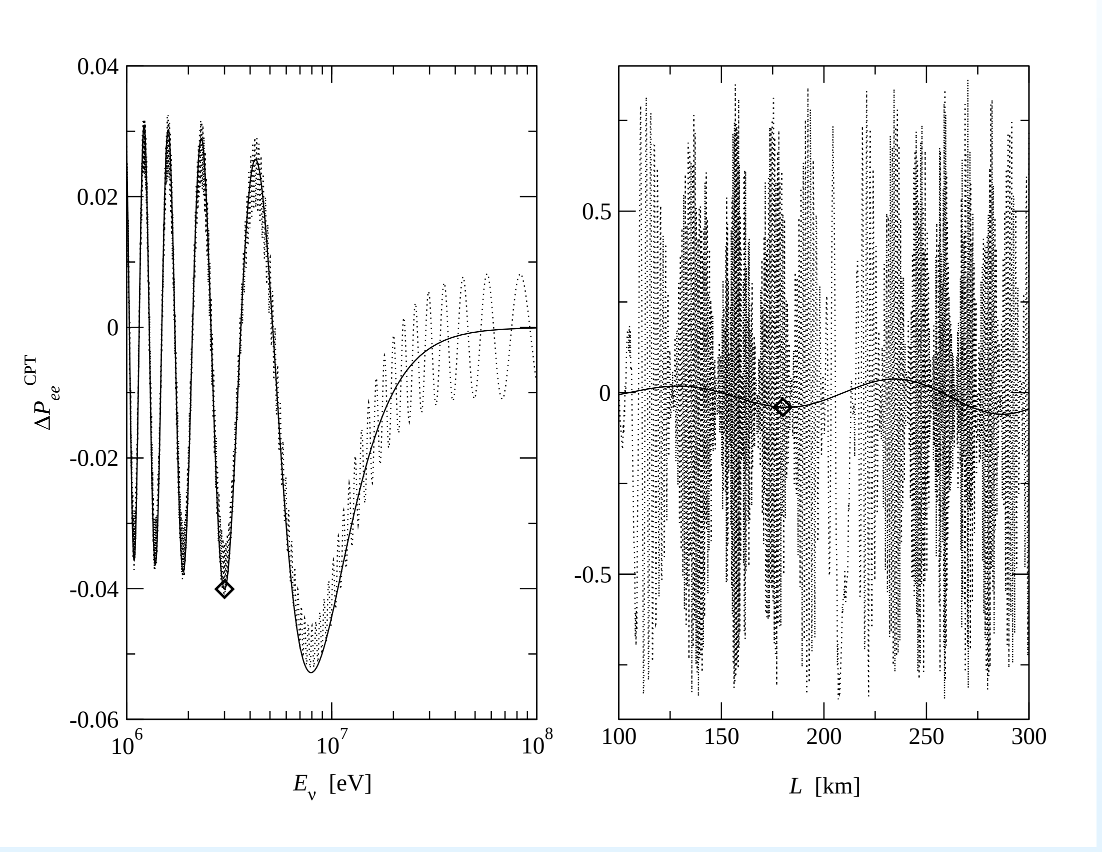
dotted curve = numerical

Fast oscillations averaged out in analytical calculations.

Note! $L \uparrow \Rightarrow \Delta P_{\alpha\beta}^{\mathrm{CPT}} \uparrow$ and $E_\nu \to \infty \Rightarrow \Delta P_{\alpha\beta}^{\mathrm{CPT}} \to 0$

Numerical calculations and implications

The CPT probability difference $\Delta P_{ee}^{\mathrm{CPT}}$ for the KamLAND experiment:



solid curve = analytical

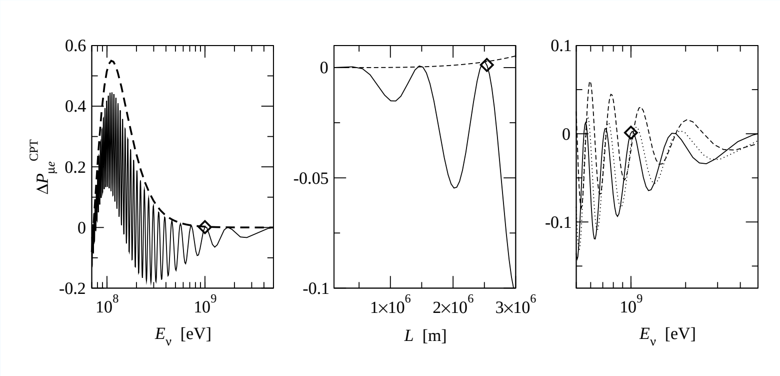
dotted curve = numerical

Again, fast oscillations averaged out in analytical calculations.

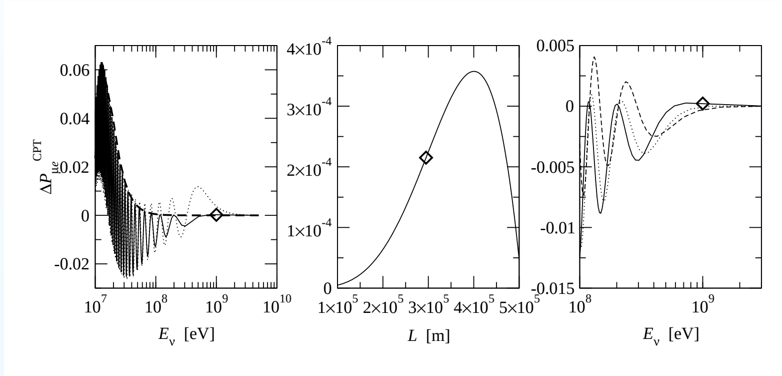
$$|\Delta P_{ee}^{\mathrm{CPT}}| \sim 3\ \% - 5\ \% \quad \Rightarrow \quad \text{CPT violation non-negligible}$$

Numerical calculations and implications

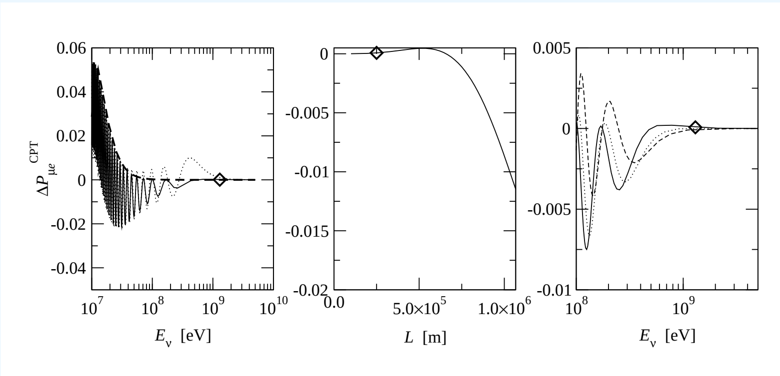
The CPT probability difference $\Delta P_{\mu e}^{\mathrm{CPT}}$ for different experiments:



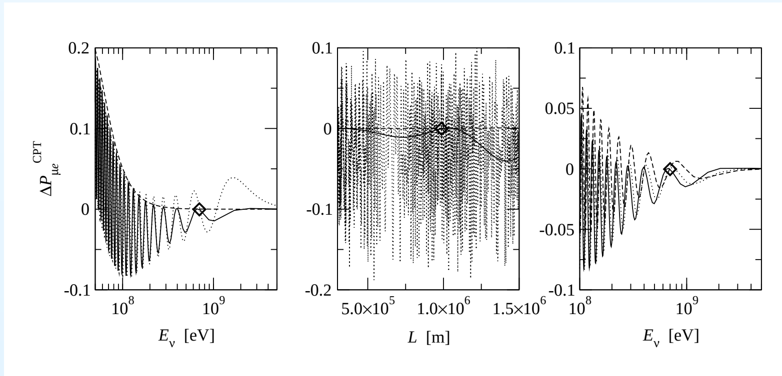
BNL NWG $L = 2\ 540\,\mathrm{km}$



JHF-Kamioka



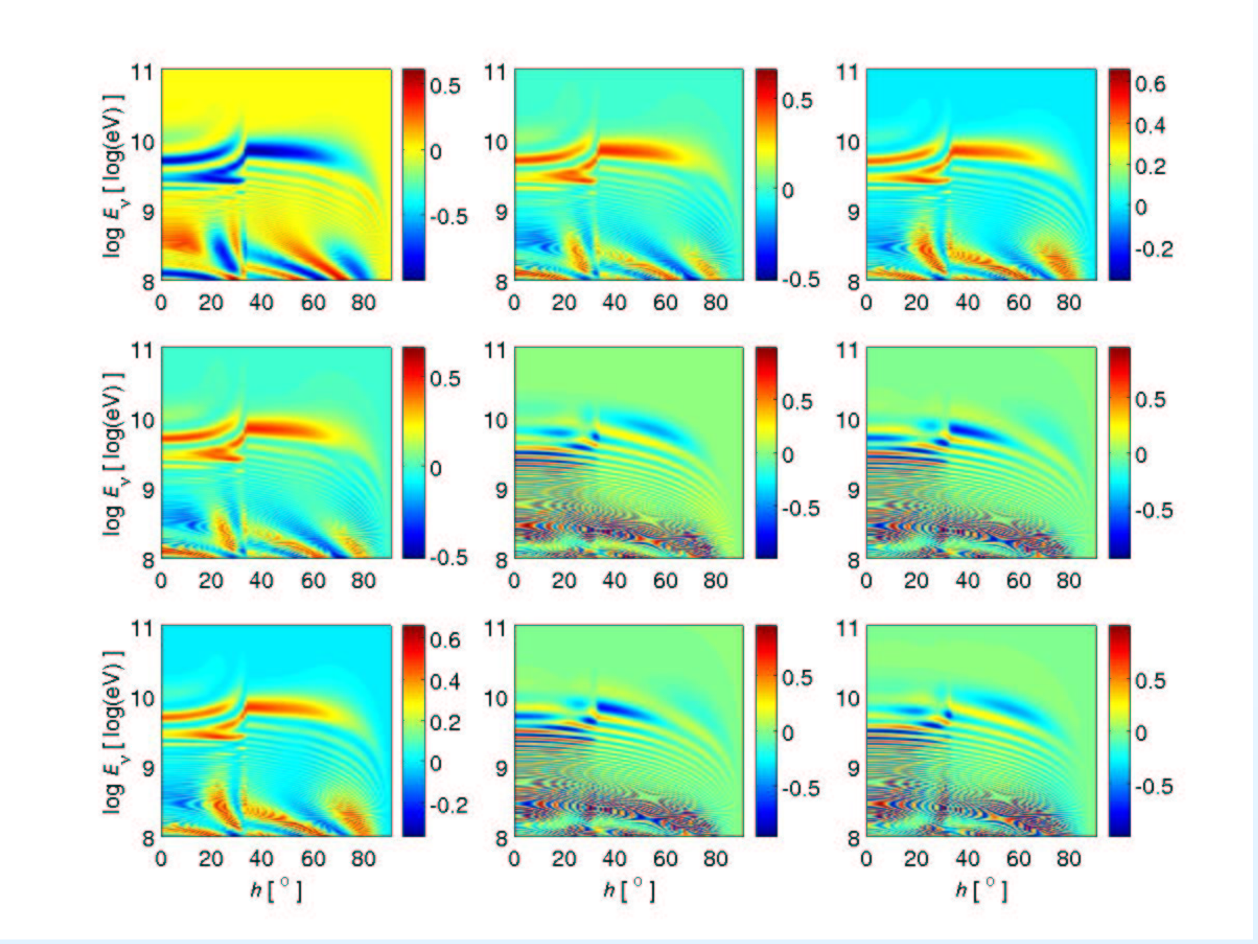
K2K



NuMI Phase II

Numerical calculations and implications

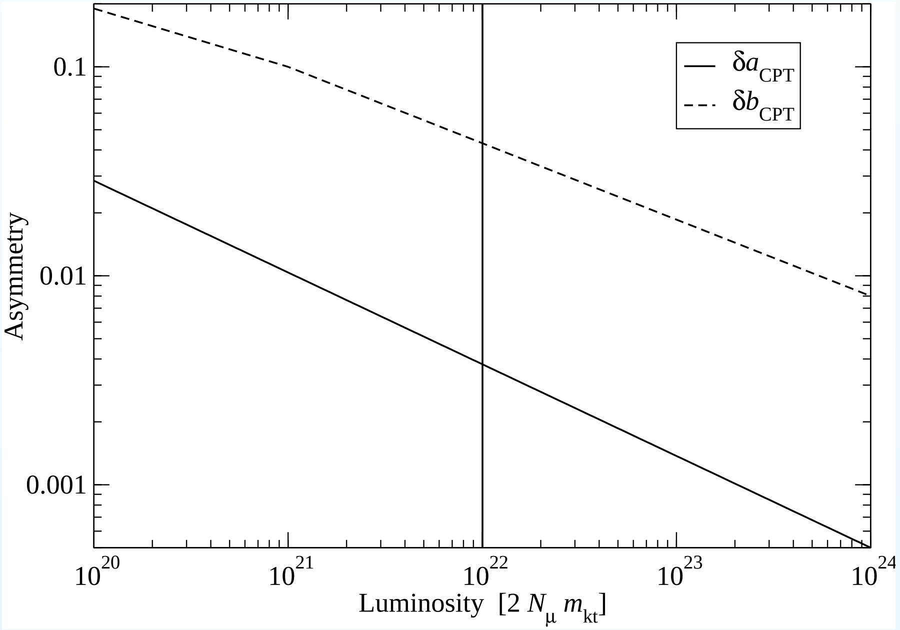
Density plots of the CPT probability differences for neutrinos traversing the Earth: $\Delta P_{\alpha\beta}^{\mathrm{CPT}} = \Delta P_{\alpha\beta}^{\mathrm{CPT}}(h, E_\nu)$



Numerical calculations and implications

CPT invariance test at NuFact

(S.M. Bilenky, M. Freund, M. Lindner, T. Ohlsson, and W. Winter, `hep-ph/0112226`)



NuFact: $E_\nu = 50\,\mathrm{GeV}$, $L = 3000\,\mathrm{km}$ (Δm^2_{23}) / $L = 7000\,\mathrm{km}$ (θ_{23}), 10^{20} muons/year, 5 years, 10kt

$\Rightarrow$ Upper bounds: $|m_3 - \bar{m}_3| \lesssim 1.9 \cdot 10^{-4}\ \mathrm{eV}$ and $|\theta_{23} - \bar{\theta}_{23}| \lesssim 2^\circ$

Summary & Conclusions

- ✔ We have studied extrinsic CPT violation in three flavor neutrino oscillations assuming the CPT invariance theorem.
- ✔ The (extrinsic) CPT probability differences for an arbitrary matter density profile have been derived.
- ✔ First order perturbation theory formulas for constant and step-function matter density profiles have been calculated as well as low-energy approximations.
- ✔ Implications for accelerator and reactor long baseline experiments as well as neutrino factory setups have been presented.
- ✔ For certain experiments: $|\Delta P_{\alpha\beta}^{\rm CPT}| \sim 5~\%$.
- ✔ In general, the CPT probability differences increase with increasing baseline length and decrease with increasing neutrino energy.

References

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