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MUON COLLIDER TARGET SIMULATION: EFFECTS OF THE PROTON ENERGY DEPOSITION AND MAGNETIC FIELD

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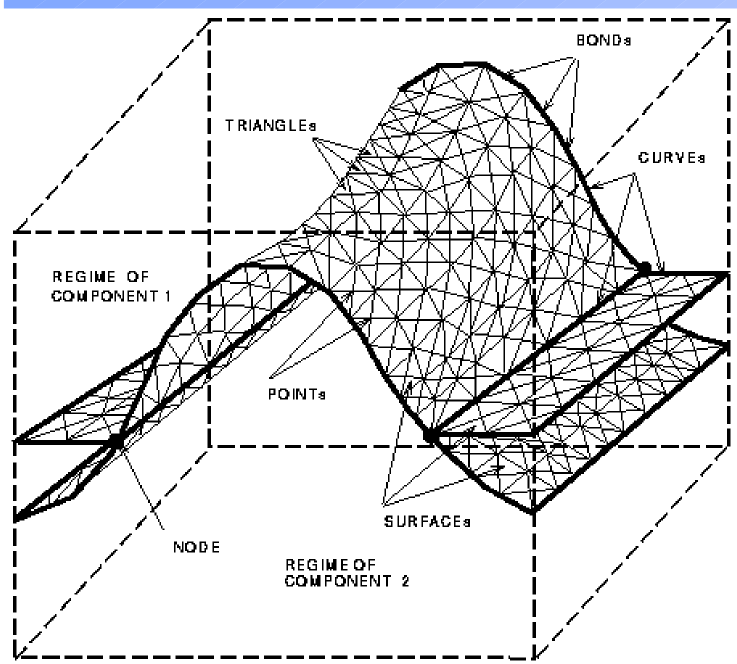
The *FronTier* Code

- The FronTier code is based on front tracking, a numerical method for solving systems of conservation laws in which the evolution of discontinuities is determined through the solution of the Riemann problem.

- The method often doesn't require highly refined grid and has no numerical diffusion. The method is ideal for problems in which discontinuities are an important feature.

3D FronTier Structures

Interfaces model different types of discontinuities in a medium such as shock waves in gas dynamics, boundaries between fluid-gas states, different fluids or their phases in fluid dynamics, component boundaries in solid dynamics etc.



Front tracking represents interfaces as lower dimensional meshes moving through a volume filling grid. The traditional volume filling finite difference grid supports smooth solutions located in the region between interfaces and the lower dimensional grid or interface defines the location of the discontinuity and the jump in the solution variables across it.

The *FronTier* code: capabilities of the interior solvers

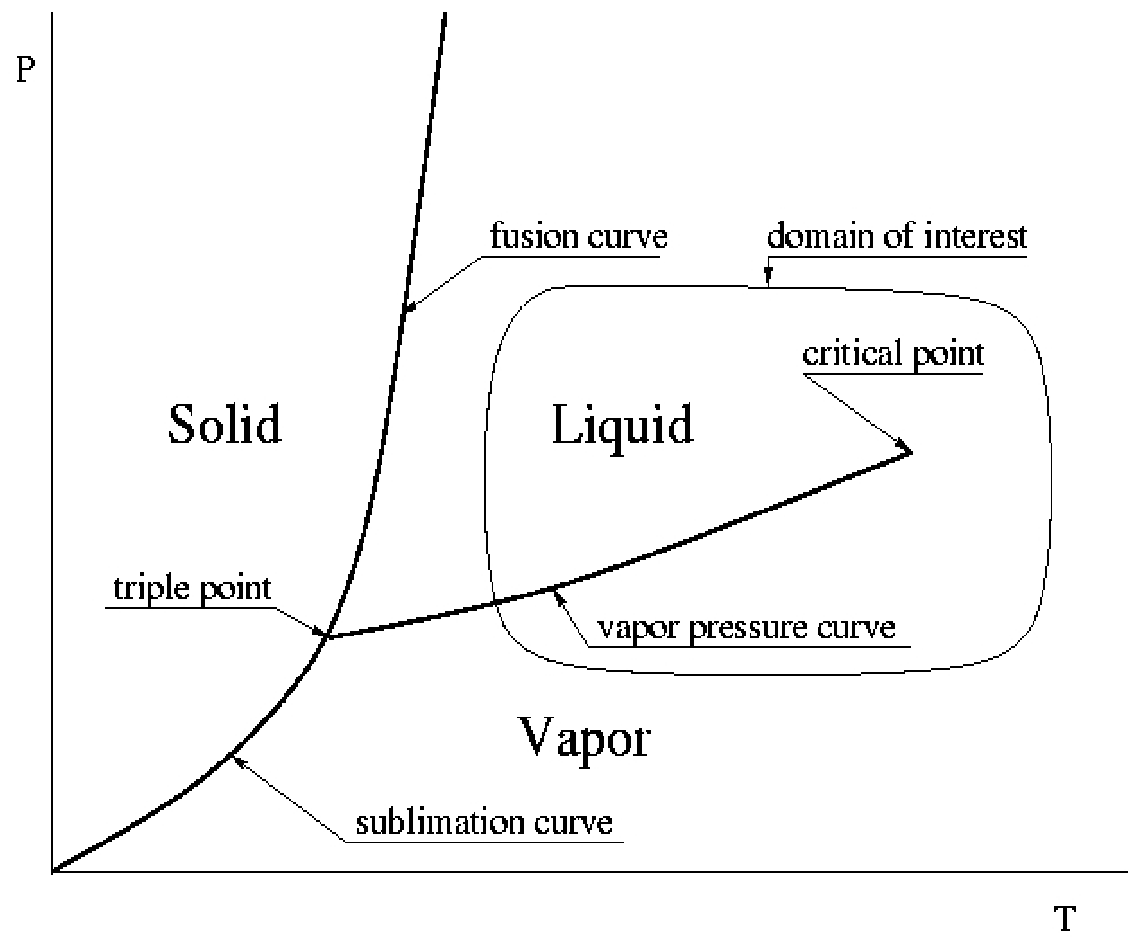
FronTier uses high resolution methods for the interior hyperbolic solvers such as Lax-Wendroff, Godunov and MUSCL and the following Riemann solvers:

- Exact Riemann solver
- Colella-Glaz approximate Riemann solver
- Linear US/UP fit (Dukowicz) Riemann solver
- Gamma law fit

FronTier uses realistic models for the equation of state:

- Polytropic Equation of State
- Stiffened Polytropic Equation of State
- Gruneisen Equation of State
- SESAME Tabular Equation of State

Mercury phase diagram



Critical point: $T_c = 1750\text{K}$,

$P_c = 172\,\text{MPa}$, $V_c = 43\text{cm}^3\text{mol}^{-1}$

Boiling point: $T_b = 629.84\text{K}$,

$P_b = 0.1\text{MPa}, \rho = 13.546\text{g} \cdot \text{cm}^{-3}$

Analytical expression for the vapor pressure curve:

$P = 133.3\exp(18.41 - 7318/T)$,

$[P] = \text{Pa}, \quad [T] = \text{K}$

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SESAME tabular equation of state

- The SESAME EOS is a tabular equation of state which gives the thermodynamic functions for a large number of materials, including gases, metals and minerals, in a computerized database.

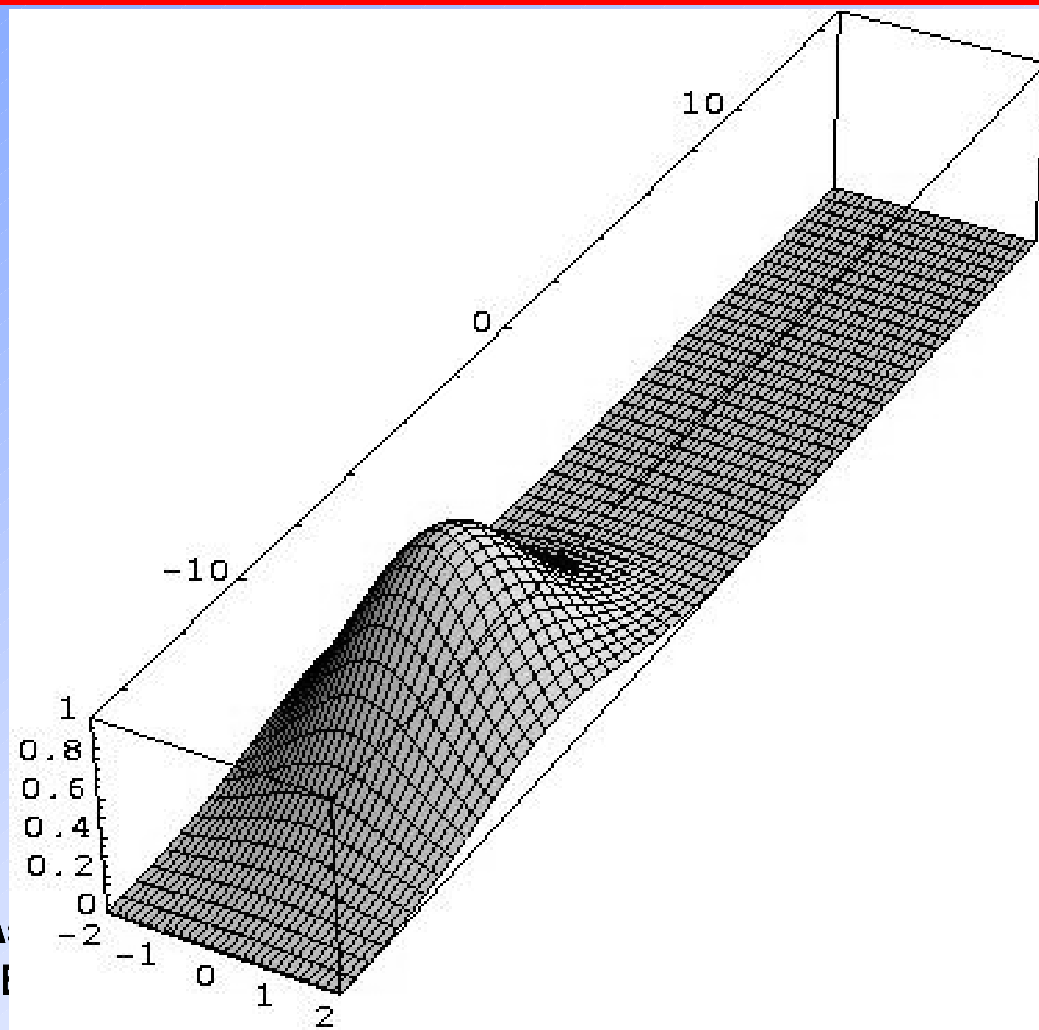
- The SESAME EOS includes the following tables:

201 Tables. The 201 tables contains 5 floats which are the *atomic number, atomic weight, density, pressure* and *internal energy* at the normal conditions.

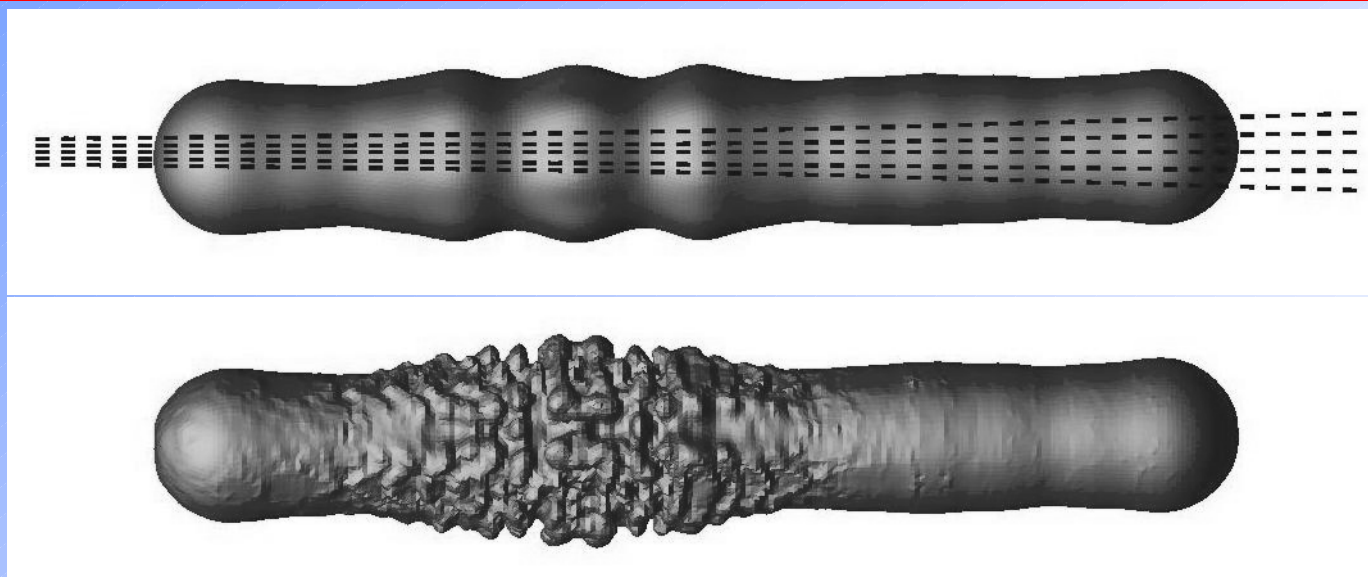
301 Tables. The 301 tables is a database for the *pressure, internal energy* and, in some cases, the *free energy* as functions of the *temperature* and *density*.

401 Tables. The 401 tables contain data for the thermodynamic functions at the liquid/vapor phase transition.

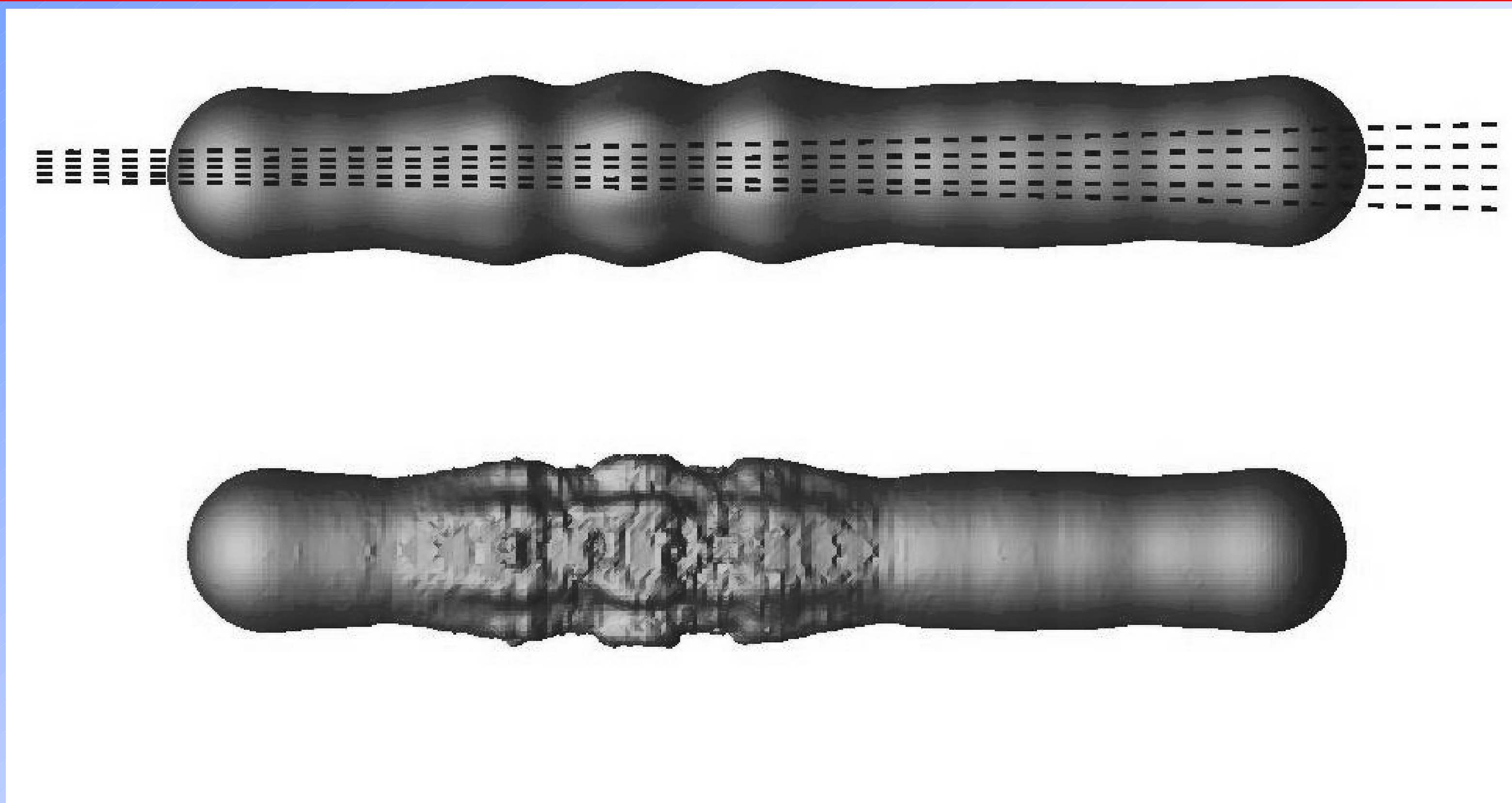
Energy deposition profile



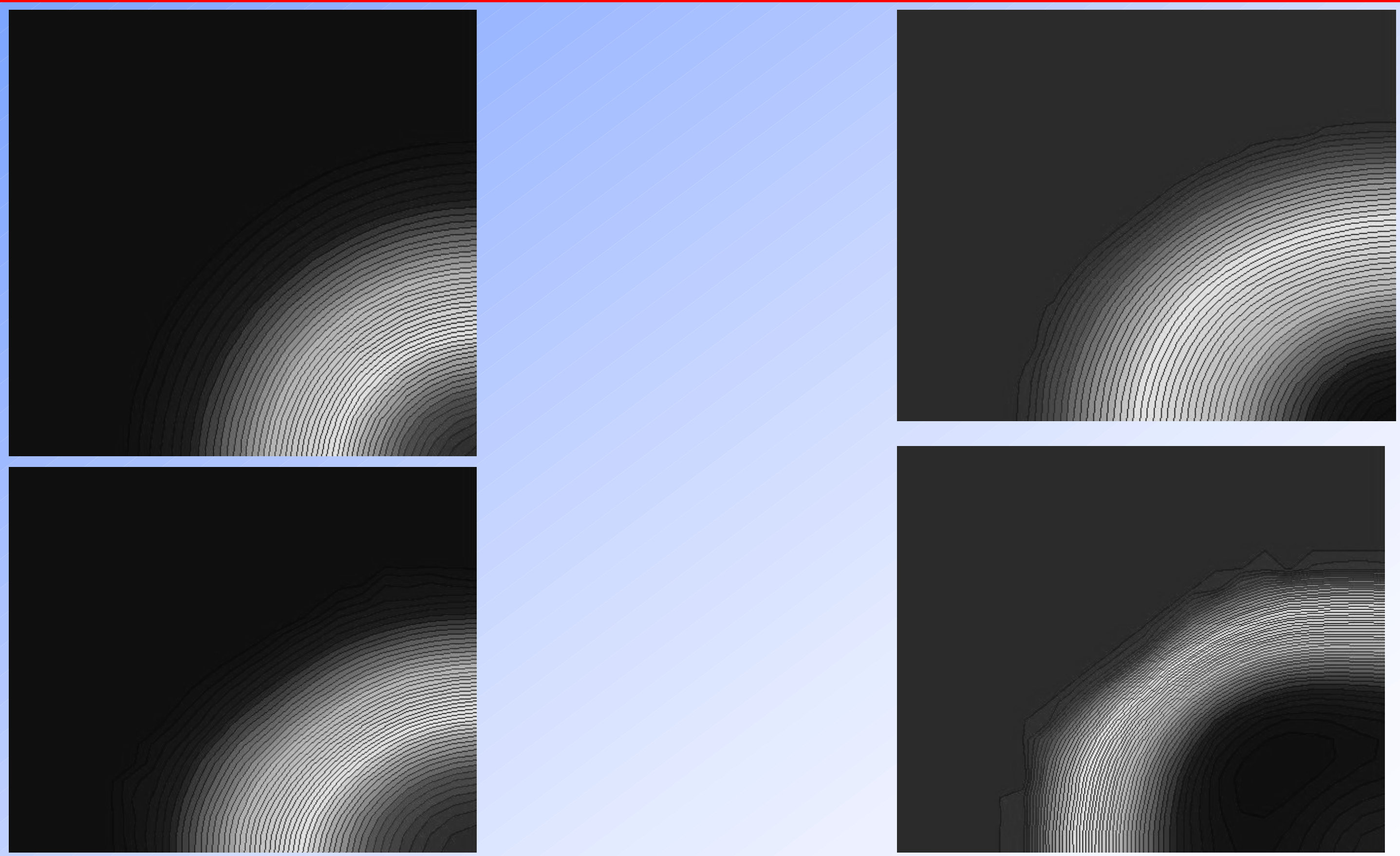
Mercury target evolution due to three pulses of the proton energy deposition.



Mercury target evolution due to a single pulse of the proton energy deposition.



Pressure waves in the mercury target due to the proton energy deposition



The system of equation of compressible magnetohydrodynamics

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho u$$

$$\frac{d\rho \mathbf{u}}{dt} = -\nabla P + \rho \mathbf{X} + \frac{\mu}{4\pi}(\nabla \times \mathbf{H}) \times \mathbf{H}$$

$$\frac{dU}{dt} = -\nabla \cdot (U + P)\mathbf{u} + \frac{1}{\sigma}\mathbf{J}^2 - \frac{1}{c}\mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) - \frac{1}{c}\mathbf{J} \cdot (\mathbf{u} \times \mathbf{B})$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times \left(\frac{c^2}{4\pi\sigma} \nabla \times \mathbf{H} \right)$$

$$\nabla \cdot \mathbf{B} = 0$$

The interface condition

Interface conditions for the normal and tangential components of the magnetic field

$$\mathbf{n}\cdot\left(\mathbf{B}_2-\mathbf{B}_1\right)=0$$

$$\mathbf{n}\times\left(\mathbf{H}_2-\mathbf{H}_1\right)=\frac{4\pi}{c}\mathbf{K}$$

Different levels of approximation

- General approach: the magnetic field is not constant in time.

  Time scales: acoustic waves time scale = 7 microseconds;

  magnetic diffusion time scale = 33 microseconds

  Alfven waves time scale = 70 microseconds

- The magnetic field is constant in time. The distribution of currents can be found by solving Poisson's equation:

$$\Delta\phi = -\frac{1}{c}\nabla(\mathbf{u}\times\mathbf{B}), \qquad \mathbf{j} = \sigma\left(\nabla\phi + \frac{1}{c}\mathbf{u}\times\mathbf{B}\right)$$

- Approximate asymptotic solution for the azimuthal current density distribution:

$$j_\phi = \frac{\sigma r v_z}{2c}\frac{\partial B_z}{\partial z} - \frac{\sigma v_r B_z}{c}$$

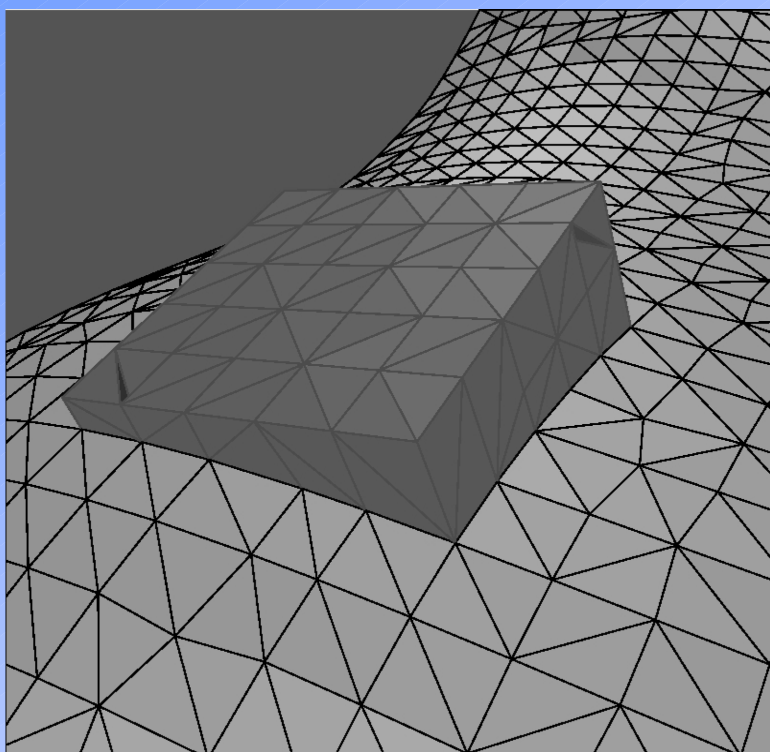
Magnetohydrodynamics of Multi Fluid Systems: Numerical Approach

- The system of MHD equations contains the hyperbolic subsystem (the mass, momentum and energy conservation equations) and the parabolic (the magnetic field evolution equation) or elliptic (Poisson's equation for the current density distribution) subsystems.

- The hyperbolic subsystem is solved on a finite difference grid in both domains separated by the free surface using FronTier's interface tracking numerical techniques. The evolution of the free fluid surface is obtained through the solution of the Riemann problem for compressible fluids.

- The parabolic subsystem or elliptic subsystems is solved using a vector finite elements method based on Whitney elements. The grid is rebuilt at every time step and conformed to the evolving interface.

Finite Element Mesh Generation with interface constraints



Triangulated tracked surface and tetrahedralized hexahedra conforming to the surface. For clarity, only a limited number of hexahedra have been displayed.

Elliptic/Parabolic Solvers

- 3D version of the Chavent - Jaffre mixed-hybrid finite element formulation.

- Utilization of the Glowinski-Wheeler domain decomposition and parallelization of the (global) wire basket problem, introducing, a two-level multi-grid structure into the algorithm. This method will reduce the iterative portion of the algorithm to the coarsest level of granularity, where the condition numbers are favorable.

- Optimized iterative methods, starting with the vector probing preconditioner for the wire basked problem.

Mixed element formulation

We introduce $\mathbf{J}$ as an unknown function since the numerical differentiation of $\mathbf{B}$ would lead to one order of accuracy loss:

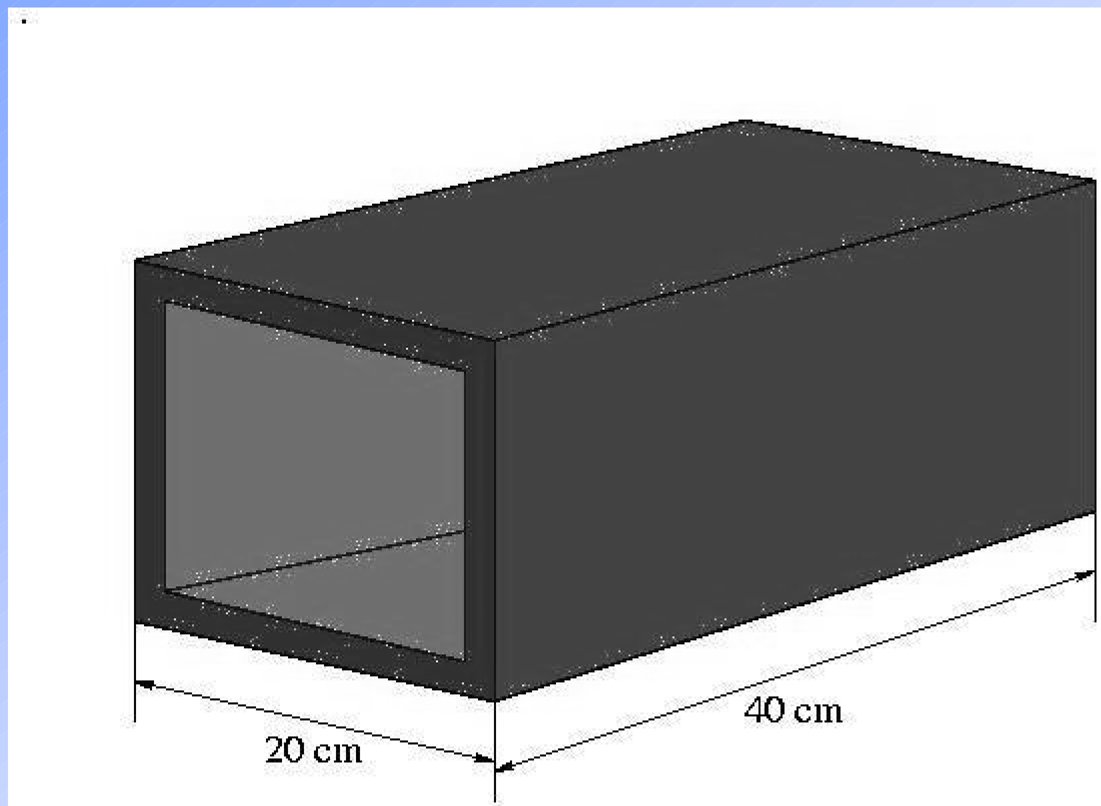
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{u} \times \mathbf{B} - \frac{c}{\sigma} \mathbf{J} \right)$$

$$\mathbf{J} = \frac{c}{4\pi} \nabla \times \frac{\mathbf{B}}{\mu}, \quad \text{with} \quad \nabla \cdot \mathbf{B} = 0$$

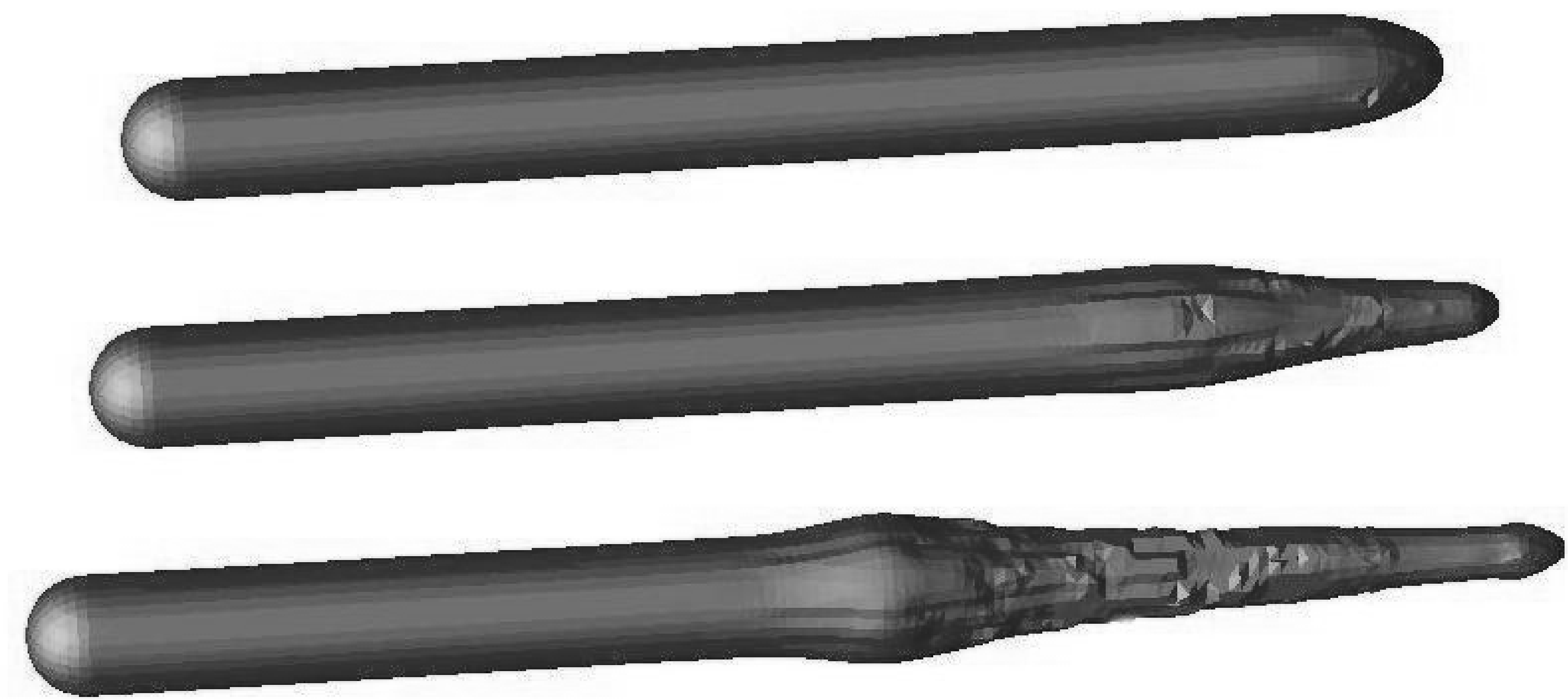
We seek finite element solutions in the following form:

$$\mathbf{B} = \sum b_e w^e, \quad \mathbf{J} = \sum j_f w^f$$

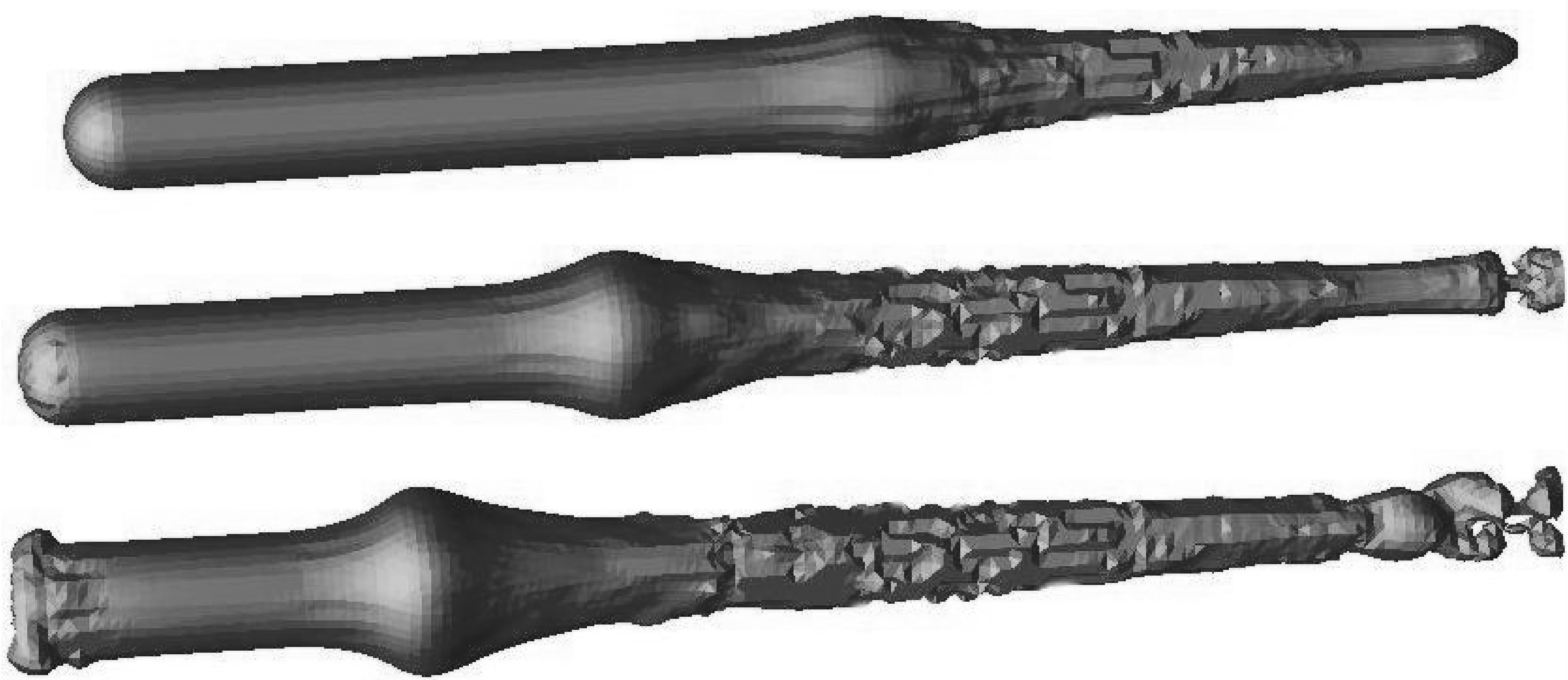
20 T magnetic coil



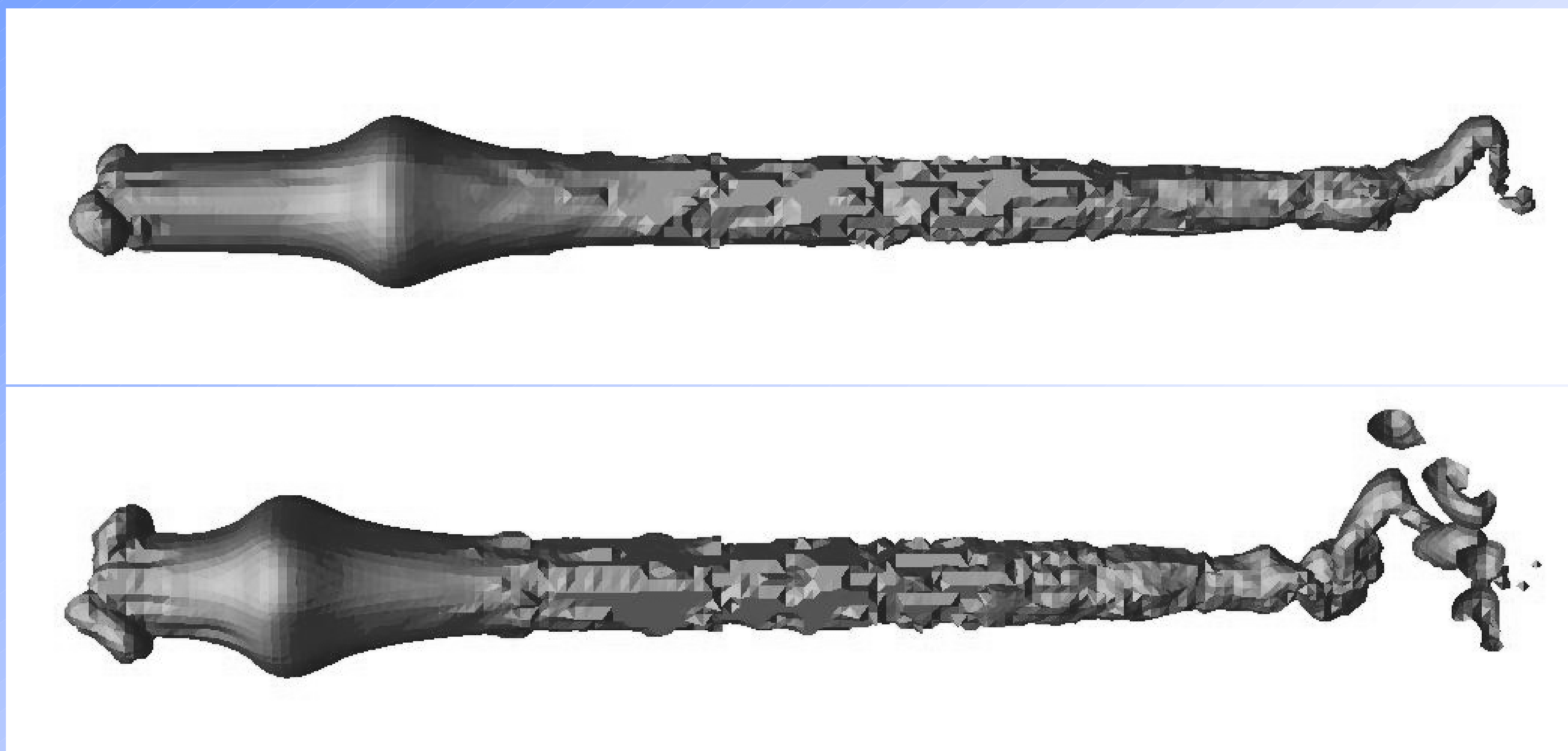
Liquid metal jet entering 20 T solenoid



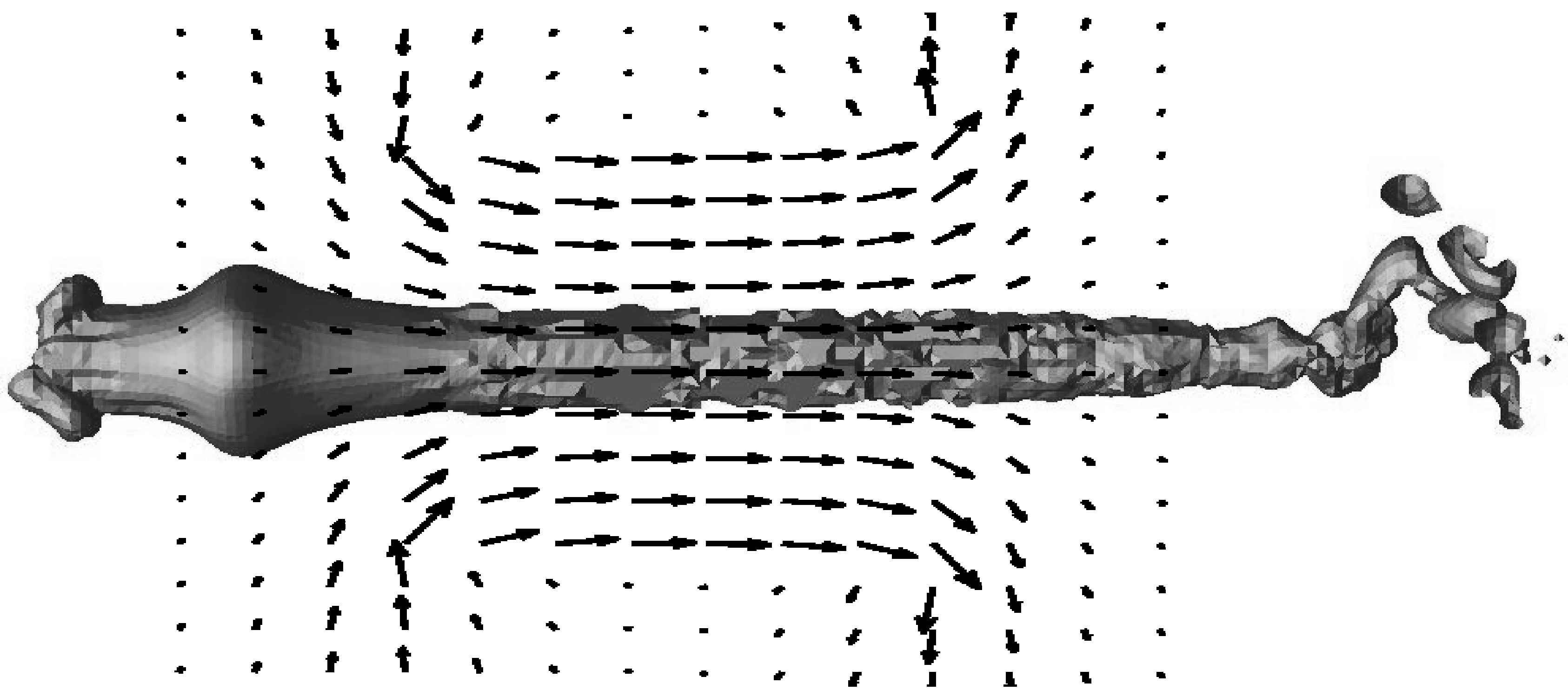
Liquid metal jet leaving 20 T solenoid



Liquid metal jet flying through a 20T solenoid

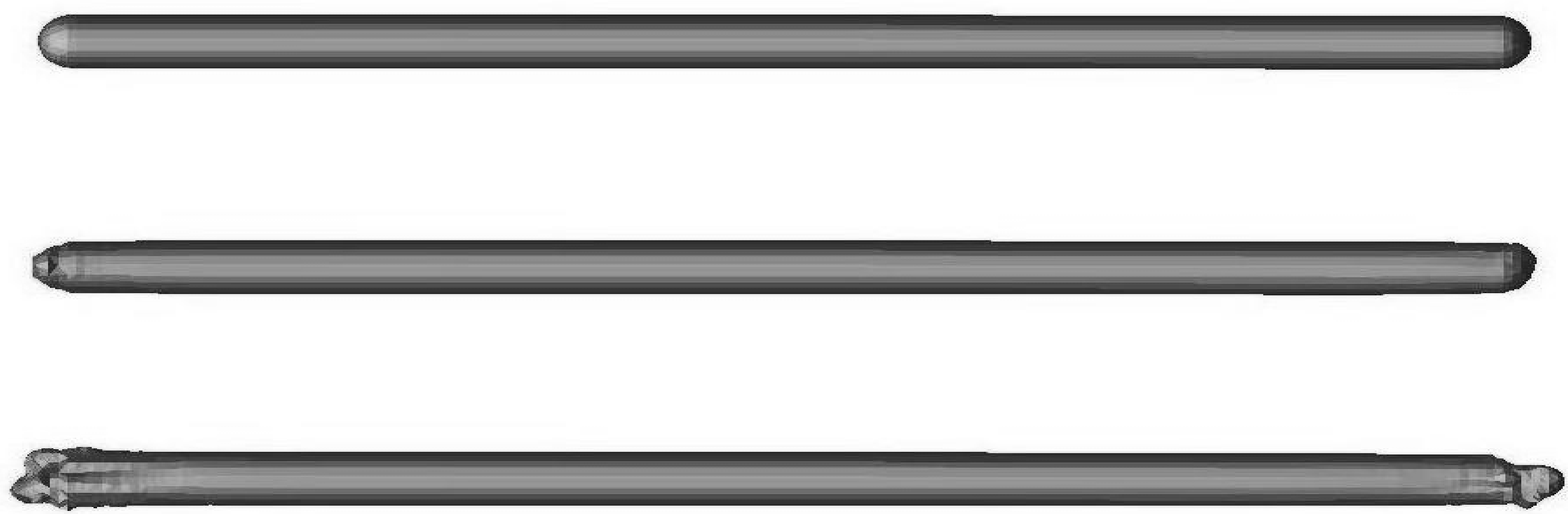


Liquid metal jet flying through a 20T solenoid



Initial state of the mercury target moving in the solenoid (d = 1 cm, L = 30 cm)

Surface deformation of the target moving in the magnetic field



Mercury target moving in the magnetic field: surface instabilities and droplet formation