Recent Results from Optimization Studies of Linear Non-Scaling FFAGs for Muon Acceleration

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Abstract. Because of the highly repetitive nature and simple cell structure of FFAG lattices, it is possible to automatically design these lattices. In designing an FFAG lattice, one will try to meet certain constraints and then minimize some cost function by varying any remaining free parameters. I will first review previously published work on optimized FFAG design. Then I will describe recent advances in the understanding of linear non-scaling FFAG design that have come from these optimization techniques. I will describe how the lattice designs depend on some input parameters to the design. Finally, I will present a set of FFAG lattices that are optimized for muon acceleration using these techniques.

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FFAG OPTIMIZATION TECHNIQUE

Linear non-scaling FFAGs have a relatively simple structure: they have a small number of different magnets (generally only two different kinds) that only have dipole and quadrupole fields. These magnets are arranged into a simple cell structure (generally a doublet, triplet, or FODO) that is repeated some number of times in the ring. They have no magnets (with the exception of injection and extraction kickers) other than in these identical cells. Thus, there are a relatively small number of parameters that can be adjusted in designing these cells: two dipole fields, two quadrupole fields, two magnet lengths, and two drift lengths. Since these rings accelerate particles, there are also a number of RF cavities in the machine; thus, one additional free parameter is the amount of RF voltage installed, assuming that the RF frequency is fixed. In principle, one could think of varying the phase of the RF cavities, but that is not considered adjustable in these designs for reasons that will be discussed shortly.

The core of the optimization technique described here is to find the closed orbit in the machine as a function of energy and the linear map about those closed orbits. The closed orbit is needed at the lowest and highest energies, as well as at the energy where the time of flight along the closed orbit is a minimum (which must be found iteratively). Using these closed orbits and linear maps, the optimization algorithm will vary some of the free parameters to force the design to meet any constraints that are given (the "inner loop"). The inner loop is executed repeatedly, varying the remaining parameters, to minimize some cost function which is computed from the input parameters, the computed closed orbits, and the linear maps about the closed orbits.

The FFAGs must meet certain design specifications:

- The machines must transmit certain phase space areas with minimal distortion.
- Since superconducting RF is used for muon acceleration, the fields at the RF cavities must be below about 0.1 T, and thus sufficient space must be left between the magnets and the RF cavities.
- Sufficient space must be left between magnets.

To ensure that the necessary transverse phase space area is transmitted, ellipses are formed about the closed orbit with semi-axes $a_{x,y}$ given by $a_{x,y}^2 = \beta_{x,y} mcA_{x,y}/p$, where $\beta_{x,y}$ are the Courant-Snyder beta functions (computed from the one-cell linear matrix about the closed orbit), m is the particle mass, c is the speed of light, $A_{x,y}$ are what are called normalized transverse acceptances, and p is the momentum of the closed orbit in question. A circle is computed which just fits around the ellipses at all energies and all positions in the magnet, as described in [1]. An additional 30% is added to the radius of this circle, and this is taken to be the magnet aperture.

The main difficulty with muon acceleration in an FFAG accelerator is the variation of the time-of-flight with energy. Since muon acceleration must be rapid, there is insufficient time to synchronize the phase of the RF cavities to the muon bunches (doing so would require excessive amounts of RF power). Thus, the RF cavity phases are fixed. One might expect that the bunches will stay on-crest longer if the range of times-of-flight over the energy range of the

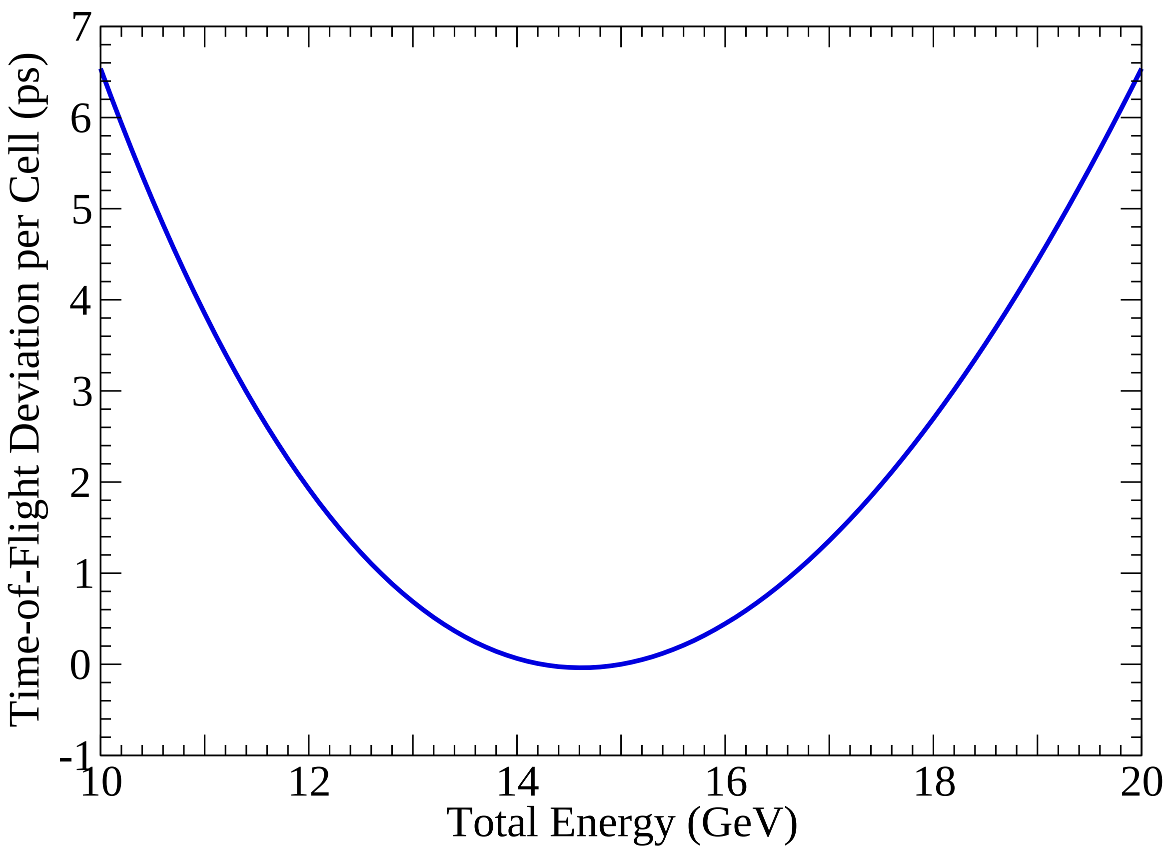

FIGURE 1. Time-of-flight as a function of energy in an FFAG cell.

accelerator is minimized. The optimal configuration, if the time-of-flight is a quadratic function of energy, is to have the times-of-flight be the same at the minimum and maximum energies, with the derivative being zero in-between. Since time-of-flight is not precisely a quadratic function of energy, we nonetheless specify as a constraint in the inner loop that the time-of-flight at the minimum and maximum energies be the same, as shown in Fig. 1.

The previous paragraph's discussion of time-of-flight is an oversimplification. However, one can demonstrate that there is a direct relationship between the amount of longitudinal phase space transmitted in the normalized phase space $\left(\omega\tau, (E-E_{\text{min}})/\Delta E\right)$ and the quantity $a = V/(\omega\Delta T\Delta E)$ [2, 3, 4], where ΔT is the difference between the maximum and minimum times-of-flight for the entire ring (i.e., the difference between the height of the ends of the parabola and the minimum of the parabola, as shown in Fig. 1), ω is 2π times the RF frequency, τ the difference between the time of flight and the time of flight of a reference particle arriving at the crest of the RF, V is the energy a particle would gain if it passed through every cavity on-crest, and one is trying to accelerate the total energy E from an energy E_{min} to an energy $E_{\text{max}} = E_{\text{min}} + \Delta E$. Thus, to satisfy the requirement that a sufficient longitudinal phase space is transmitted (for our case, 50 meV-s is the longitudinal equivalent of $A_{x,y}$ above), the inner loop requires a certain value of a. This value of a will decrease with increasing energy since the normalized variables are scaled by ΔE, and ΔE is generally higher for higher energy FFAG rings.

To ensure that there is sufficient space between magnets and that there is sufficient space between RF cavities and magnets (so that the magnetic fields don't quench the superconducting cavities), the drift lengths in the FFAG lattice will be specified. The distances between any two magnets that are placed close to each other will be set to 0.5 m. The length of any drift that can hold an RF cavity (there is at least one such drift per cell) is 2 m.

For this study, the RF frequency is 201.25 MHz, corresponding to the frequency of the bunch train that is accelerated in the US neutrino factory designs [5]. Three FFAGs will be considered, accelerating from 2.5–5 GeV, 5–10 GeV, and 10–20 GeV. The values of a for these designs were heuristically chosen to be $1/6$, $1/8$, and $1/12$ respectively.

PREVIOUS RESULTS

Palmer had constructed a model for the costs of an accelerator which included magnets, RF cavities, RF power supplies, and other costs which were considered to be linear in the length of the machine [6, 7]. This model was used to construct the cost function that was minimized by this algorithm.

At the previous FFAG workshop at TRIUMF, results using this optimization technique had been presented [8]. It had been well understood prior to that workshop that the single-cell tunes for these lattices needed to be below 0.5 to prevent beam loss due to linear resonances [9]. It was found that the optimum tune profiles seemed to have the horizontal tune well above the vertical tune over the entire energy range. This was due to the fact that a high horizontal tune resulted in a lower time-of-flight, and that the vertical tune gave the minimum beam pipe size in the defocusing magnets (the horizontal tune profile also kept the beam pipe size down). Edge effects in the magnets were included correctly for all energies, which gave an important correction to the vertical focusing.

Some simple arguments gave approximate dependencies of some parameters on others. For a given value of a, the voltage required per turn is approximately proportional to $1/n$, where n is the number of cells. Furthermore, the voltage required is proportional to cell length for a given type of cell [10]. It was assumed that magnet costs would increase with the number of cells, approximately linearly. Lattices were always designed with every cell containing RF cavities, except for a few which were left empty for injection and extraction. This was assumed to be optimal from a cost point of view, since leaving more empty cells would increase magnet costs for a given RF cost.

When comparing three types of lattices, doublet, triplet, and FODO, the doublet lattice was appearing to result in the lowest cost. The doublet lattice had higher RF costs, but lower magnet and linear costs, largely due to fewer magnets per cell. Furthermore, the cost per GeV increased with decreasing energy of the FFAGs, and it appeared that a 2.5–5 GeV FFAG (and lower energy FFAGs) might not be competitive with other methods for accelerating muons, such as a recirculating linear accelerator (RLA). The pole tip fields in the cost-optimized lattices were relatively low, due to the rapidly increasing cost of magnets with their pole tip field. At the time of the TRIUMF workshop, these costs were computed with either the low-energy tunes fixed at the same values, or with pole tip fields fixed.

RECENT RESULTS

The cost model used previously was improved to take into account the fact that magnet costs will not fall to zero as the field in the magnet goes to zero, as well as other less important improvements [11]. This new cost model was used for the cost function of this algorithm, both tunes and pole tip fields were allowed to be free, the requirement that every possible cell contain an RF cavity was removed, and cost optimum lattices were computed.

A surprising result was that the minimum cost lattices were extremely long, and had only a small fraction of the ring filled with RF cavities. These long rings were unacceptable for muon acceleration due to the excessive amounts of decay that would occur. As shown in Fig. 2, the cost of magnets is decreasing with an increasing number of cells, even for a very large number of cells. As the number of cells increases, the dispersion decreases. The magnet aperture in the focusing magnets is determined largely by the variation of the closed orbit position with energy, and thus the aperture (and therefore the cost) of the magnets decreases with an increasing number of cells. While this cannot continue indefinitely (at some point the magnet aperture will be determined by the finite transverse beam size), Fig. 2 illustrates that it is still true even for a very large number of cells.

Since the problem with the ring being long is the excessive muon decay, some kind of cost must be assigned to the muons that are lost to decay. For a fixed performance, any lost muons must be made up for by making the detector correspondingly larger. Taking an approximate detector cost of 500 PB (the "Palmer Buck" (PB) is my cost unit, designed to correspond to 1 million dollars in [11]) [12], every 1% muon decay is assigned a cost of 5 PB.

The cost optimized lattices have tune profiles which are nearly independent of the energy of the lattice, as shown in Fig. 3. They only depend on the type of lattice (doublet, triplet, etc.) and the factor of energy gain in the lattice.

PARAMETRIC DEPENDENCIES

This optimization technique can be used to find how some parameter of an FFAG ring, such as its cost or number of cells, depends on some input parameter to the optimization. I will illustrate some of the dependencies that are most interesting for the machine design.

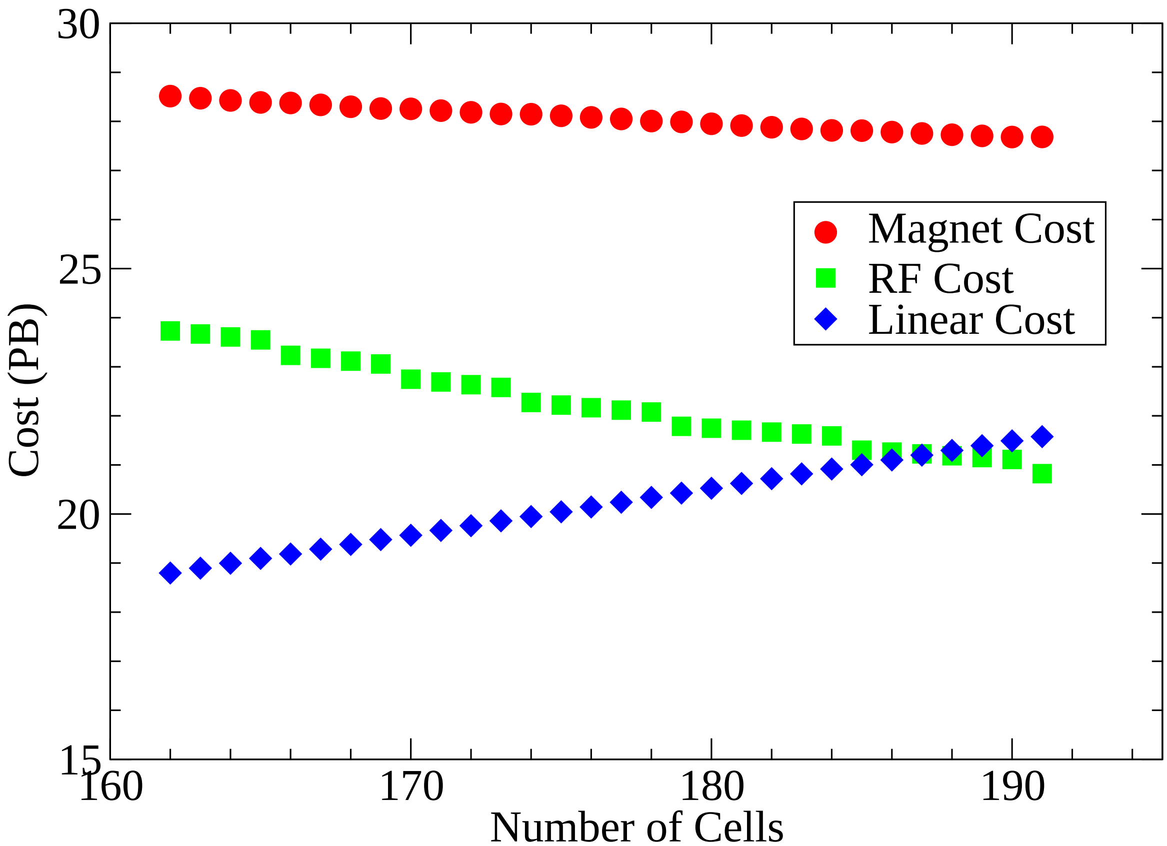

FIGURE 2. Lattice cost as function of the number of cells in the ring for a 10–20 GeV FFAG.

The detector cost is largely unknown at this point, and that cost may have a strong impact on the FFAG design. Figure 4 shows the FFAG cost (extra detector cost is not included) as a function of the muon cost, using two different RF gradients. The values for the 10–20 GeV ring with 10 MV/m RF gradient are a good illustration of how muon decay costs affect the FFAG design. As the cost of muons increases, the optimal cost solution is to reduce the decays produced by the acceleration. To reduce decays, the ring is shortened and the number of RF cavities in the ring is increased (the latter is required by the constraint on a). This can only continue until every cell of the ring has an RF cavity (except for 8 cells dedicated to injection and extraction). The decays in the FFAG ring can only be further reduced by increasing the magnet pole tip fields so as to decrease the ring circumference while reducing the RF voltage requirement. Increasing magnet pole tips is extremely expensive and produces very little reduction in decay; thus it becomes more cost effective at that point to increase the size of the detector to maintain a given performance. Thus, beyond a certain point, Fig. 4 shows an FFAG cost that doesn't increase very rapidly with increasing muon cost. At a higher gradient, fewer cavities must be installed to achieve a given voltage, so the point where no more cavities can be added to the ring occurs for a much larger cost per muon.

Since the case where all of the cells (except for the injection/extraction cells) are filled with RF cavities has one more constraint than the case where the number of cells containing cavities is free, the inner loop must be different in the two cases. The most straightforward way of finding the cost optimum is to minimize for the two types of constraints separately, and then take the lowest-cost solution.

The FFAG designs being examined here use 201.25 MHz superconducting RF cavities. Such cavities have been built and tested at Cornell [13]. Gradients of 10 MV/m were achieved at 4.2 K, and 11 MV/m was attained at 2.5 K. The cavity surface is being improved to try to attain higher gradients [14]. Due to the low power requirements for superconducting RF, a relatively high gradient should be optimal. Figure 5 shows the cost of the FFAG and the decay cost as a function of the gradient. The reduction in cost with increasing gradient is relatively modest. This is because the cost reduction per unit acceleration with higher gradient is relatively small above 10 MV/m, and RF costs are only

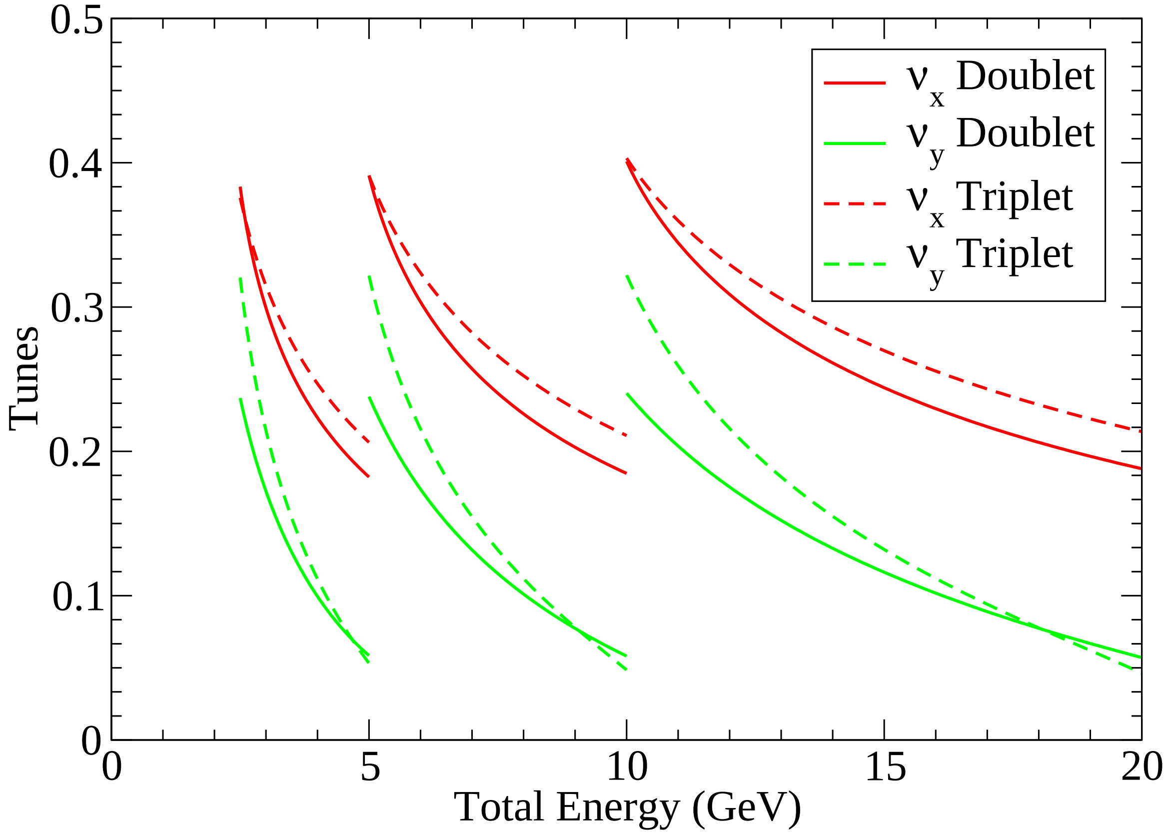

FIGURE 3. Tune as a function of energy in doublet and triplet FFAG lattices, for various lattice energies.

a fraction of the total machine cost. The cost model assumes that the cost of the cavity structure is independent of the cavity gradient, but this is unlikely to be completely true. At higher gradients, the requirements on the cavity surface will be more stringent, and the input coupler design will be more challenging. Furthermore, running at higher gradients may require a lower temperature cryogenic system and a greater cryogenic capacity. Thus, considering that 10 MV/m is what has been achieved thus far at 4.2 K (although that was only limited by the relatively low-power input coupler used), I believe that 10 MV/m may be a better choice for the gradient than the 15 to 17 MV/m that has been discussed to this point [5].

A significant fraction of the cost of a neutrino factory is in ionization cooling. One would like to use less cooling, but to have the same number of neutrino events at the detector, the transverse normalized acceptances $A_{x,y}$ of the acceleration stages and storage ring must be increased. The increased acceptance will increase the cost of the acceleration, and for a total system optimization, the acceleration cost as a function of acceptance must be computed. Figure 6 shows the acceleration cost, including the associated decay costs, as a function of $A_{x,y}$. The acceleration cost depends strongly on the acceptance, primarily due to the increased magnet apertures required at a larger acceptance. If the cooling and storage ring costs can be similarly found as a function of aperture, one can find cost-optimum aperture for the entire neutrino factory.

CURRENT BASELINE LATTICES

Based on the understanding described above, the lattice designs that result from this optimization process are presented in Tab. 1. The cavity gradients are 10 MV/m, $A_{x,y} = 30$ mm, and the decay cost is 5 PB/%. The lattices are all doublet lattices. Compared to earlier optimized designs, the pole tip fields are relatively high, because the magnet lengths have been reduced to reduce decay. The cost per GeV for the 2.5–5 GeV FFAG is comparable to that of an RLA (but one with a larger energy range). Since one would make about 6 turns in that FFAG, one is making relatively effective use

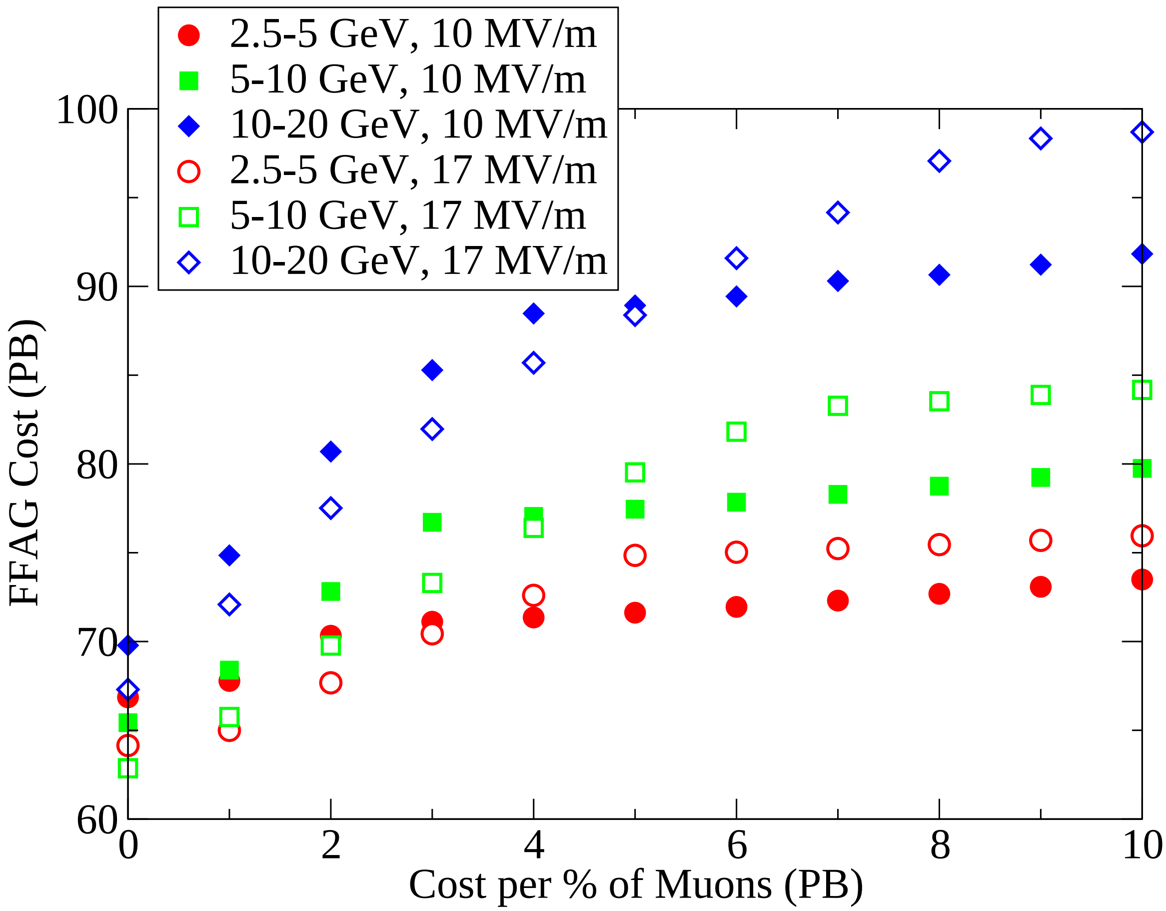

FIGURE 4. FFAG cost as a function of the muon decay cost.

of the RF (one can typically only do 4 to 5 turns in an RLA). It is thus unclear whether an FFAG would be preferred at this lower energy range.

CONCLUSIONS AND FUTURE WORK

I have presented a technique for automatically producing designs of FFAG lattices. These techniques have been applied to the design of muon acceleration rings. As a result, we have a better understanding of the design of such machines, and we are able to determine how the resulting designs depend on input parameters. I have produced cost-optimized designs for muon acceleration FFAGs.

There is still more work to do on this subject. The choice of a was not very systematic, and there is currently no precise way to relate a to the longitudinal acceptance. I have developed a method for determining that relationship, and the completion of that work will be incorporated into this optimization. The choice of energy ranges for the FFAG rings was also highly arbitrary. We need to find a way to optimize the energies of these devisions, including the cost of transport lines between machines. The cost of the cooling system and storage ring as a function of aperture should be found, and an optimum value for the aperture found as described above. The drifts between the magnets and between the magnets and cavities should be specified more precisely, possibly as a function of the magnet aperture and field. Finally, this technique should be applied to FFAGs designed for other applications.

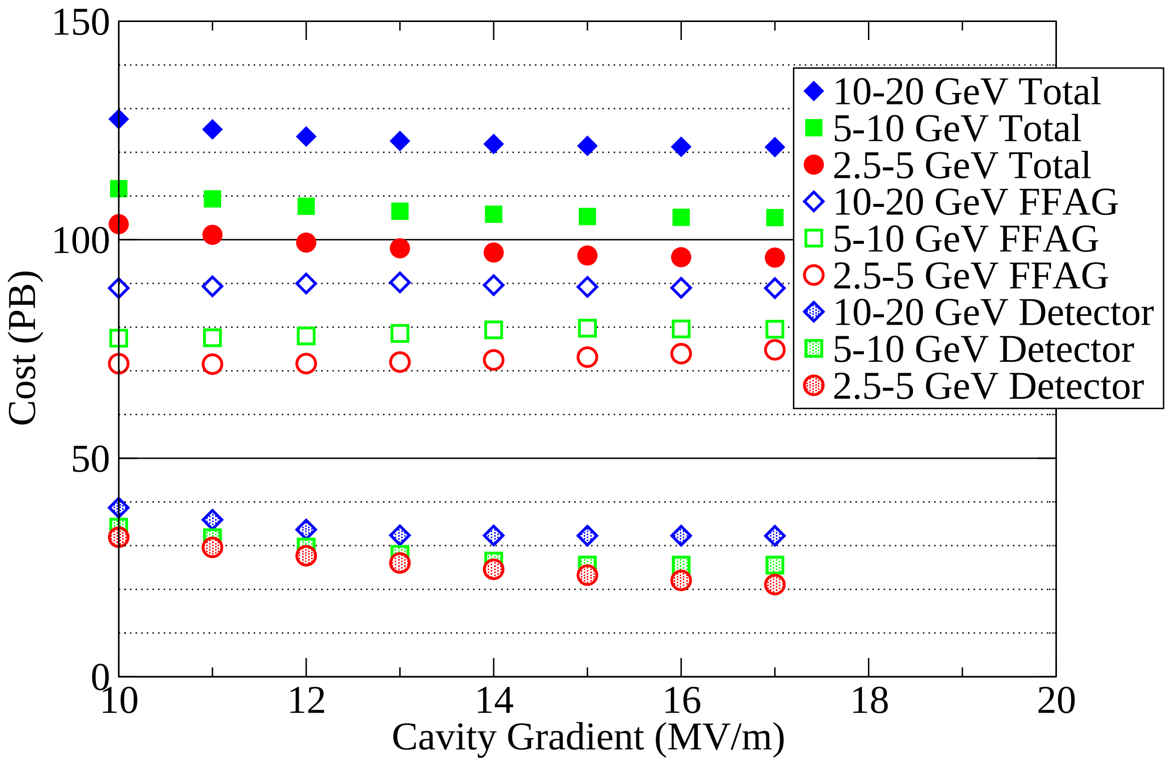

FIGURE 5. FFAG and marginal detector cost as a function of cavity gradient.

TABLE 1. Optimized lattice designs.

Minimum total energy (GeV)	2.5	5	10
Maximum total energy (GeV)	5	10	20
$V/(\omega \Delta T \Delta E)$	1/6	1/8	1/12
No. of cells	64	77	91
D length (cm)	54	69	91
D radius (cm)	13.0	9.7	7.3
D pole tip field (T)	4.4	5.6	6.9
F length (cm)	80	99	127
F radius (cm)	18.3	14.5	12.1
F pole tip field (T)	2.8	3.6	4.4
No. of cavities	56	69	83
RF voltage (MV)	419	516	621
Turns	6.0	9.9	17.0
Circumference (m)	246	322	426
Decay (%)	6.4	6.8	7.7
Magnet cost (PB)	38.4	36.0	38.1
RF cost (PB)	27.1	33.4	40.2
Linear cost (PB)	6.1	8.0	10.6
Total cost (PB)	71.6	77.5	88.9
Cost per GeV (PB/GeV)	28.7	15.5	8.9

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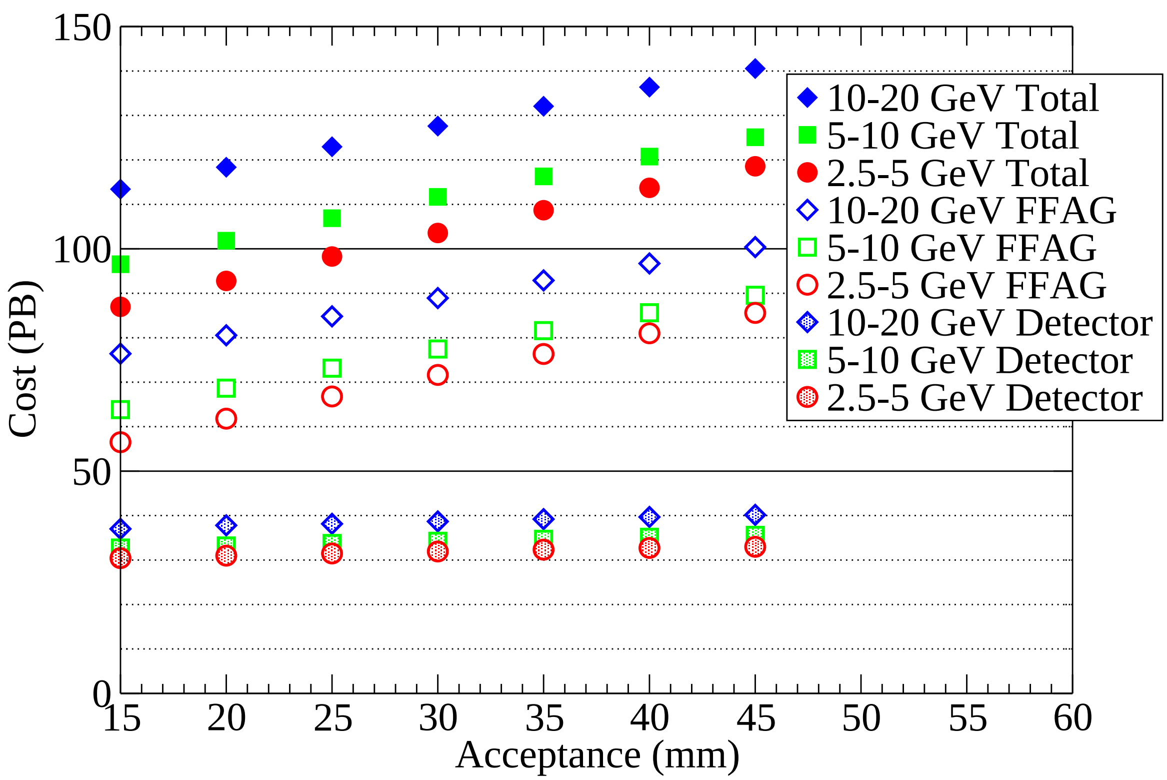

FIGURE 6. Acceleration cost as a function of normalized acceptance $A_{x,y}$. Decay (detector) costs are based on the assumption that the same number of muons enter each acceleration stage for every acceptance.

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