Muon Acceleration by Scaling FFAG with Harmonic Number Jump

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FFAG chains

- **Non-scaling FFAG**
- **Advantages**
  - small aperture
  - const. rf frequency (high frequency & field)
- **Problems (issues)**
  - time of flight (path length) for large amplitude: cascade rings
Scaling FFAG with HNJ

Scaling FFAG with HNJ for low energy (5-10GeV) ring

- Higher frequency (~200MHz) rf cavity: good matching -> Phase Rotation & non-scaling FFAG
- Harmonic Number Jump (HNJ) acceleration
  \[ \text{const. rf frequency} \rightarrow \text{high frequency & high field} \]

Scaling FFAG with HNJ for high energy (10-20GeV) ring

- T. Uesugi

Scaling FFAG proton driver with HNJ

- T. Planche
Harmonic Number Jump (HNJ) acceleration

**HNJ-acceleration**  
(Kolomenski, Fujisawa, Ruggiero)

- Difference of revolution period between n-th and (n-1)-th turn equals $m$ (integer) times $r_f$ period.

\[ T_n - T_{n-1} = r_f \times m \]

- $T_n$: revolution period for n-turn
- $r_f$: rf period
- $m$: integer ($<0$: before, $>0$: after transition)
HNJ Acceleration

**Revolution period for n-th turn**

\[
\left( \frac{T_n}{T_1} \right) = \left( \frac{C_n}{C_1} \right) \left( \frac{v_n}{v_1} \right)
\]

C: circumference, v: particle velocity

**Scaling FFAG**

\[
\frac{C_n}{C_1} = \left( \frac{p_n}{p_1} \right)^{k+1}
\]

**For muon acceleration (\(v \sim c\))**

When \(k\) increases, or ring size decreases,

- No. of turns decreases.
- Energy gain/turn increases.

Need optimization!

\[
\frac{C_n}{C_1} = \frac{h_n}{h_1}, \quad p_n = p_1 \left( \frac{h_n}{h_1} \right)^{k+1}, \quad h_n = h_1 + n \times m
\]
Issues of HNJ

**Phase acceptance**

- Smaller for HNJ cf. synchronized acceleration
- Because energy gain/turn is so large for HNJ that phase slip/turn should be $2\pi$. If stable phase is away from $\pi/2$, phase slip/turn should be much less than $2\pi$. 
Phase acceptance
Non-linear behavior

Non-linear source dynamic aperture problems in longitudinal direction

- Sinusoidal rf field contains non-linear components.
- Synchroton tune is high enough to see non-linear resonances. \( mQs=n \)
Scaling FFAG

**Types**
- Spiral sector
  - Focusing: body + edge
  - Small ring size
  - Rather large edge angle > 60 degree
- Radial sector
  - Negative bend
  - Doublet, triplet (DFD, FDF)

**Muon acceleration with Scaling FFAG**
- Energy $P=5-10\text{GeV}$
- $B_{\text{max}} < 2\text{T}$ (Iron magnet: NC or super ferric)
- Orbit excursion < 0.5m
- Beam size: full aperture @ 10 GeV < ~15 cm
- RF frequency: 200-400 MHz
- RF field: ~30 MV/m, Energy gain/m > 1.5 MeV/m

\[ B = B_0 \left( \frac{r}{r_0} \right)^k \left( \theta - \varsigma \ln \frac{r}{r_0} \right) \]
5-10 GeV scaling FFAG spiral sector

Ring parameters
- $r = 25\text{m}$
- $N = 20\text{cells}$
- Spiral angle: 72 degree
- $B_{\text{max}} \sim 2.2\text{T} (p.f. = 0.6)$
- $k = 20$
- Orbit excursion: 67.5 cm
- Beam size (half) at 10 GeV:
  - $H: 3.8\text{cm} + 1.2\text{cm} = 5.0\text{cm}, V = 5.0\text{cm} \text{ @s.s.}$
  - $H: 5.2\text{cm} + 1.5\text{cm} = 6.7\text{cm}, V = 6.9\text{cm} \text{ @magnet}$
Spiral FFAG
5-10GeV

rf parameters

- $h=200$
- $f=400\text{MHz}$
- $f\text{ai}_s=2\pi/3$
- 4-cell cavity 45MV/cavity(30MV/m)
Spiral FFAG
5-10GeV

Lattice
- almost satisfied but need more optimization
  - k-value: lower, Bmax: lower, packing factor: smaller, circumference: larger

HNJ acceleration
- seems to have enough acceptance
- frequency of rf cavity
  - 400MHz --> 200MHz (depend on lattice design)
- No. of turns: should be larger >10 turns (now 7 turns)
  - reduce rf voltage 30MV/m --> 20MV/m
- Increase ring radius!
5-10GeV Scaling FFAG
radial sector - FDF lattice

**Ring parameters**
- \( r = 200 \text{m} \)
- \( N = 70 \text{cells} \)
- \( B_{\text{max}} \approx \text{F}:1.6\text{T} , \text{D}:1.5\text{T} \)
- \( k = 150 \)
- Orbit excursion \( \approx 62\text{cm} \)
- Beam size (half) at 10GeV
  - \( H: 7.4\text{cm} + 1.2\text{cm} = 8.6\text{cm}, V:6.1\text{cm} \) @s.s.
  - \( H: 7.4\text{cm} + 1.2\text{cm} = 8.6\text{cm}, V:6.1\text{cm} \) @F-magnet
  - \( H: 4.0\text{cm} + 1.2\text{cm} = 5.2\text{cm}, V:7.0\text{cm} \) @D-magnet

![Graph showing beta function and Lorentz factor over the ring parameters.](image)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>FD ratio @ BL</td>
<td>F/D</td>
<td>1.3</td>
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<tr>
<td>number of cell</td>
<td>N</td>
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<tr>
<td>field index (k-value)</td>
<td>k</td>
<td>150</td>
</tr>
<tr>
<td>opening angle of F with respect to machine center</td>
<td>( \beta_F )</td>
<td>0.02 [rad]</td>
</tr>
<tr>
<td>opening angle of D/2 with respect to machine center</td>
<td>( \beta_D )</td>
<td>0.016 [rad]</td>
</tr>
<tr>
<td>bending angle of F</td>
<td>( \theta_F )</td>
<td>11.14285714 [deg]</td>
</tr>
<tr>
<td>bending angle of D/2</td>
<td>( \theta_D )</td>
<td>8.571428571 [deg]</td>
</tr>
<tr>
<td>orbit radius of D center (@ ext.)</td>
<td>( r_0 )</td>
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<tr>
<td>kinetic energy (@ ext.)</td>
<td>( E_k )</td>
<td>9.10E+09 [eV]</td>
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<tr>
<td>momentum (@ ext.)</td>
<td>( p )</td>
<td>9.994E+09 [eV/c]</td>
</tr>
<tr>
<td>( B ) field of F (@ ext.)</td>
<td>( B_F )</td>
<td>1.606E+00 [T]</td>
</tr>
<tr>
<td>( B ) field of D (@ ext.)</td>
<td>( B_D )</td>
<td>1.547E+00 [T]</td>
</tr>
<tr>
<td>drift length (half)</td>
<td>( L_s )</td>
<td>1.783871159 [m]</td>
</tr>
<tr>
<td>drift length (full size)</td>
<td></td>
<td>3.567742318 [m]</td>
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<tr>
<td>bending radius of F</td>
<td>( \rho_F )</td>
<td>20.73903955 [m]</td>
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<tr>
<td>bendign radius of D</td>
<td>( \rho_D )</td>
<td>21.531353 [m]</td>
</tr>
<tr>
<td>path length of F/2</td>
<td>( L_{F/2} )</td>
<td>4.03331898 [m]</td>
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<tr>
<td>path length of F</td>
<td>( L_F )</td>
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<tr>
<td>path length of D</td>
<td>( L_D )</td>
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<tr>
<td>pulse drift length (half) [for race track FFAG]</td>
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<td>total drift length (half)</td>
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<td>betatron tune</td>
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<td>phase advance per cell</td>
<td>( \phi_h )</td>
<td>94.84687648 [deg]</td>
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</table>
Radial FFAG
5-10GeV

**rf parameters**
- \( h = 1200 \)
- \( f = 400 \text{MHz} \)
- \( \text{fai}_s = 2\pi/3 \)
- 1-cell cavity \( 15\text{MV/cavity} (15\text{MV/m}) \)
Summary

Scaling FFAG with HNJ acceleration for Muon 5-10GeV (10-20GeV) seems promising

- B field <2T: NC magnet cost reduction
- Ring size
- Spiral: r=25m, Radial(FDF): r=200m: Spiral is more compact.
- rf frequency: 400MHz need optimization for lower frequency: ex. increase of ring size
- Longitudinal acceptance: large enough

Flight time problem of non-scaling FFAG may be cured by scaling FFAG

Subjects

- asymmetric rf cavity