Dependence of Transmission on Proton Bunch Length: Theory and Simplified Simulation

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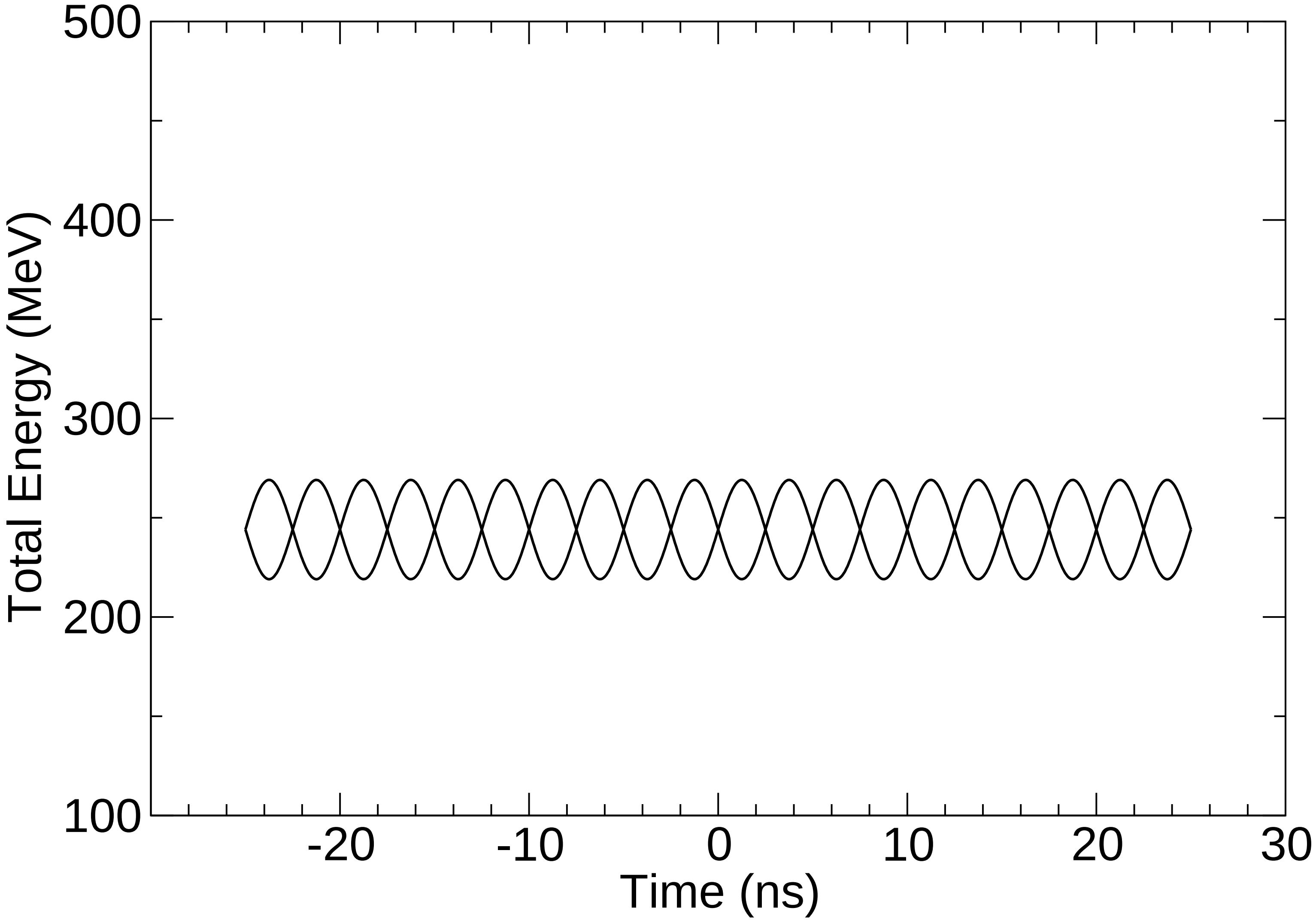
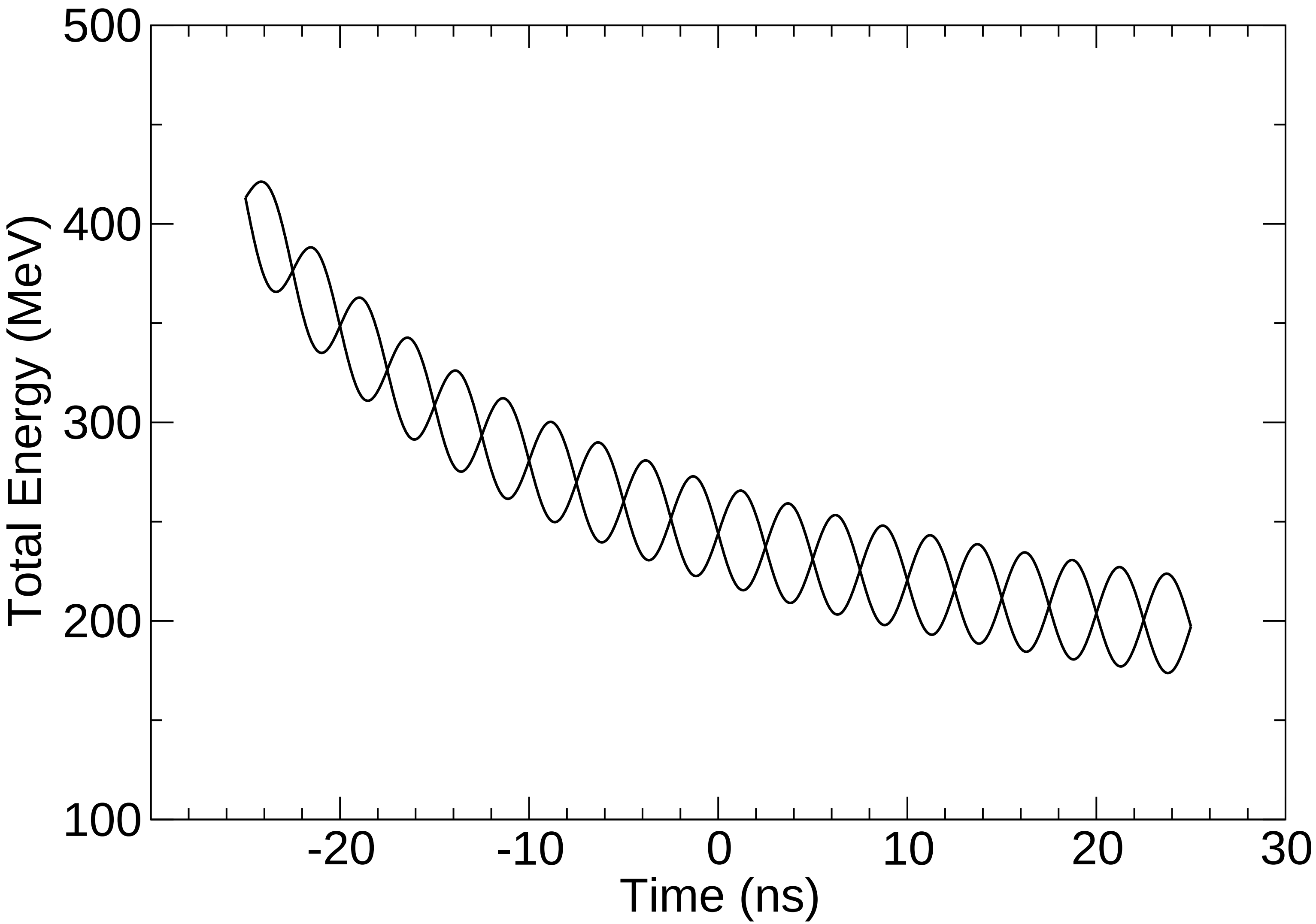
What Happens in the Phase Rotation and Bunching Channel?

- Look at the process in reverse
- Particles are captured into a series of buckets in the buncher section
- The phase rotation channel changed energies to put particles into bucket energy range
- A drift spread the particles out in time
- Longer proton (thus muon) bunch length: fewer particles fit in buckets
  - Note: more bunch length important for high-energy muons

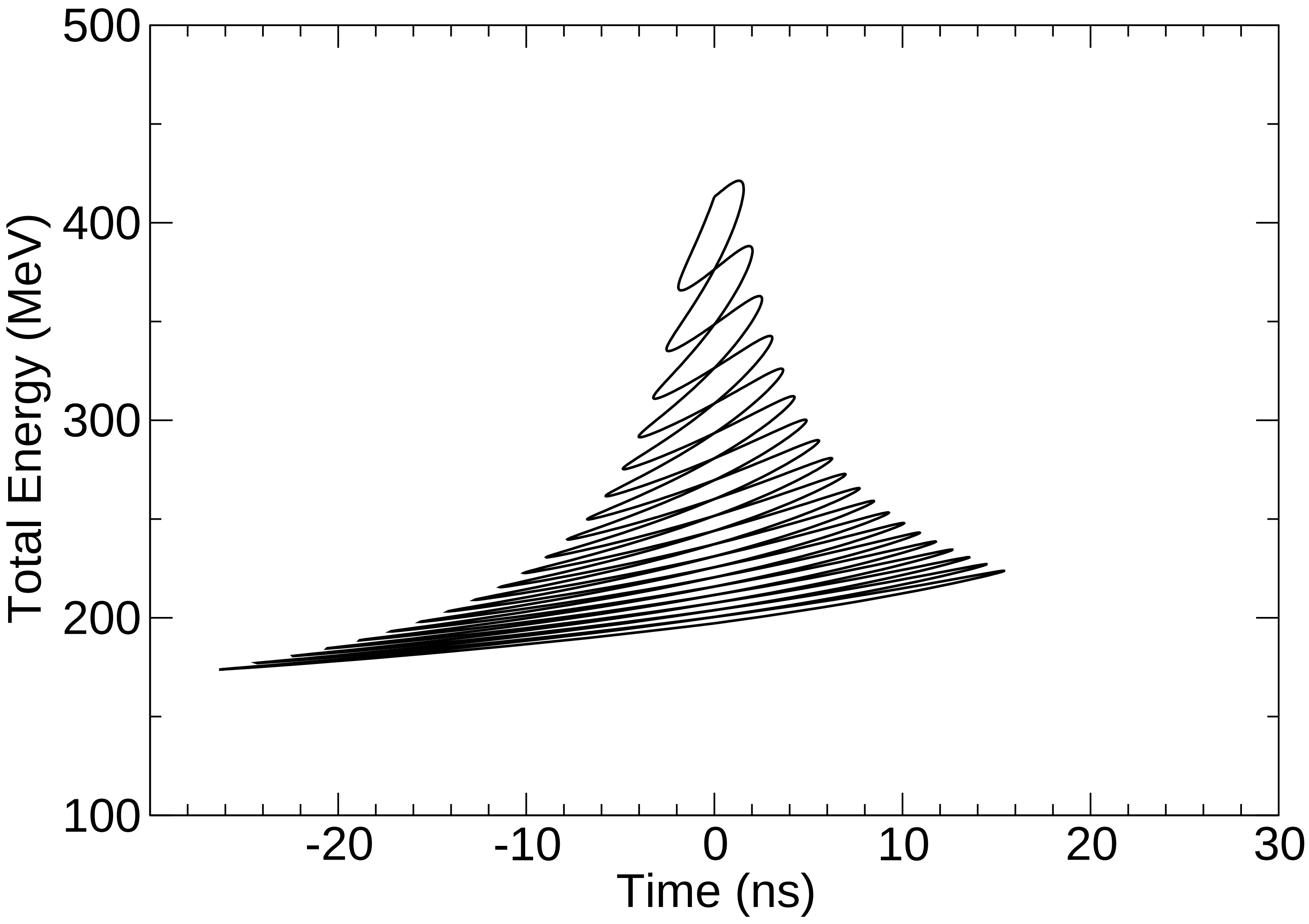
Buckets in Bunching Channel



Buckets Mapped Back to Before Phase Rotation



Buckets Mapped Back to Before Phase Rotation



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Analyze with Model

- Start with a distribution of pions in energy, $\rho(E_\pi)$
- Assume zero transverse momentum
- Assume equal probability of decay forward and backward in COM frame
- Look at distribution in energy-time of muons at distance L

Results from Model

- Can compute range for RMS bunch length at fixed energy

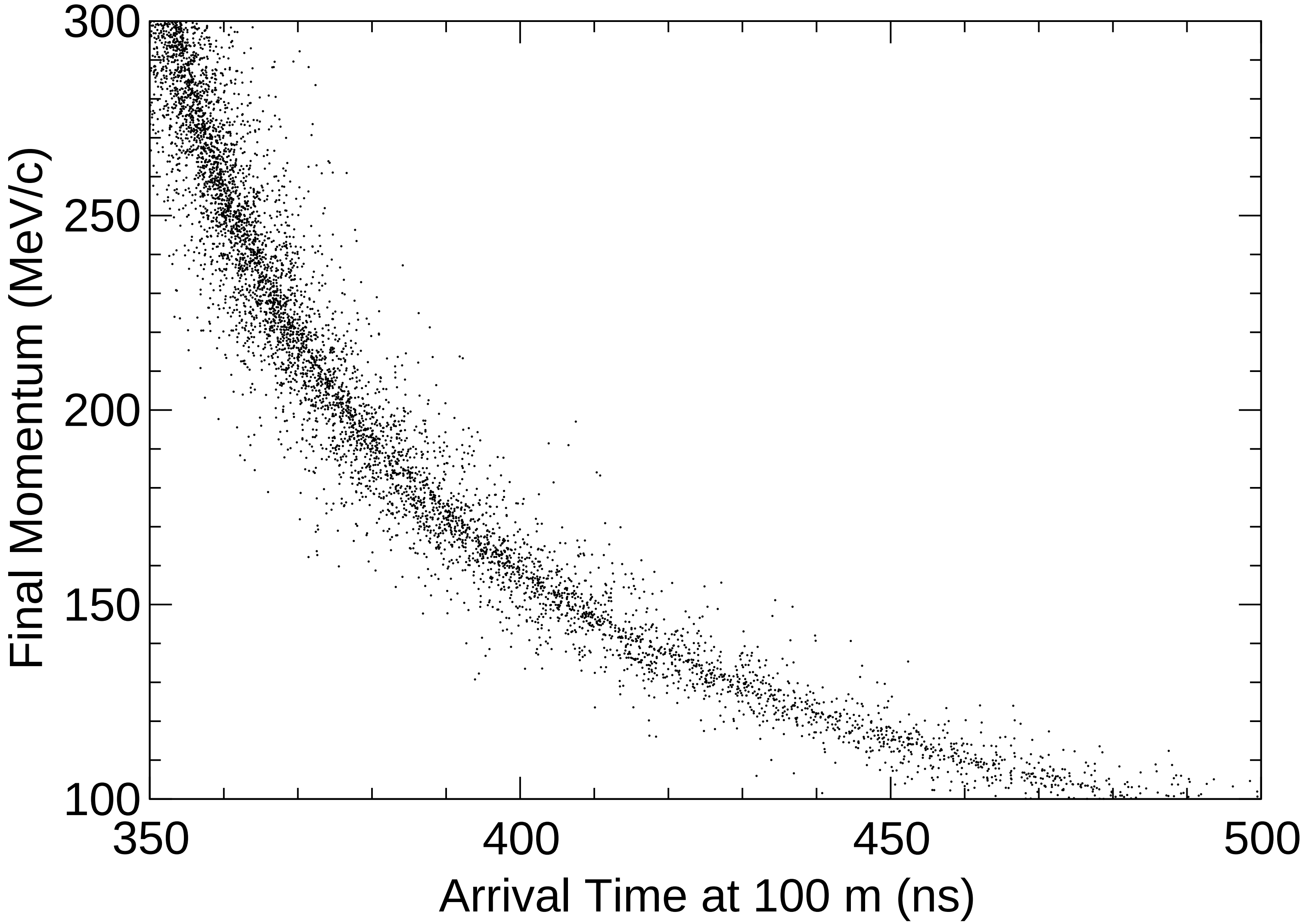
$$\tau_\pi \frac{m_\pi^2 - m_\mu^2}{2m_\pi p_\mu} < \sigma_T(E_\mu) < \tau_\pi \frac{m_\pi^2 - m_\mu^2}{\sqrt{2}m_\pi p_\mu}$$

- Add in quadrature to get bunch length σ_T
- Assume bunch length much longer than length captured
  - Then density at core determines amount captured
  - Integrate σ_T over energy to get inverse of core density

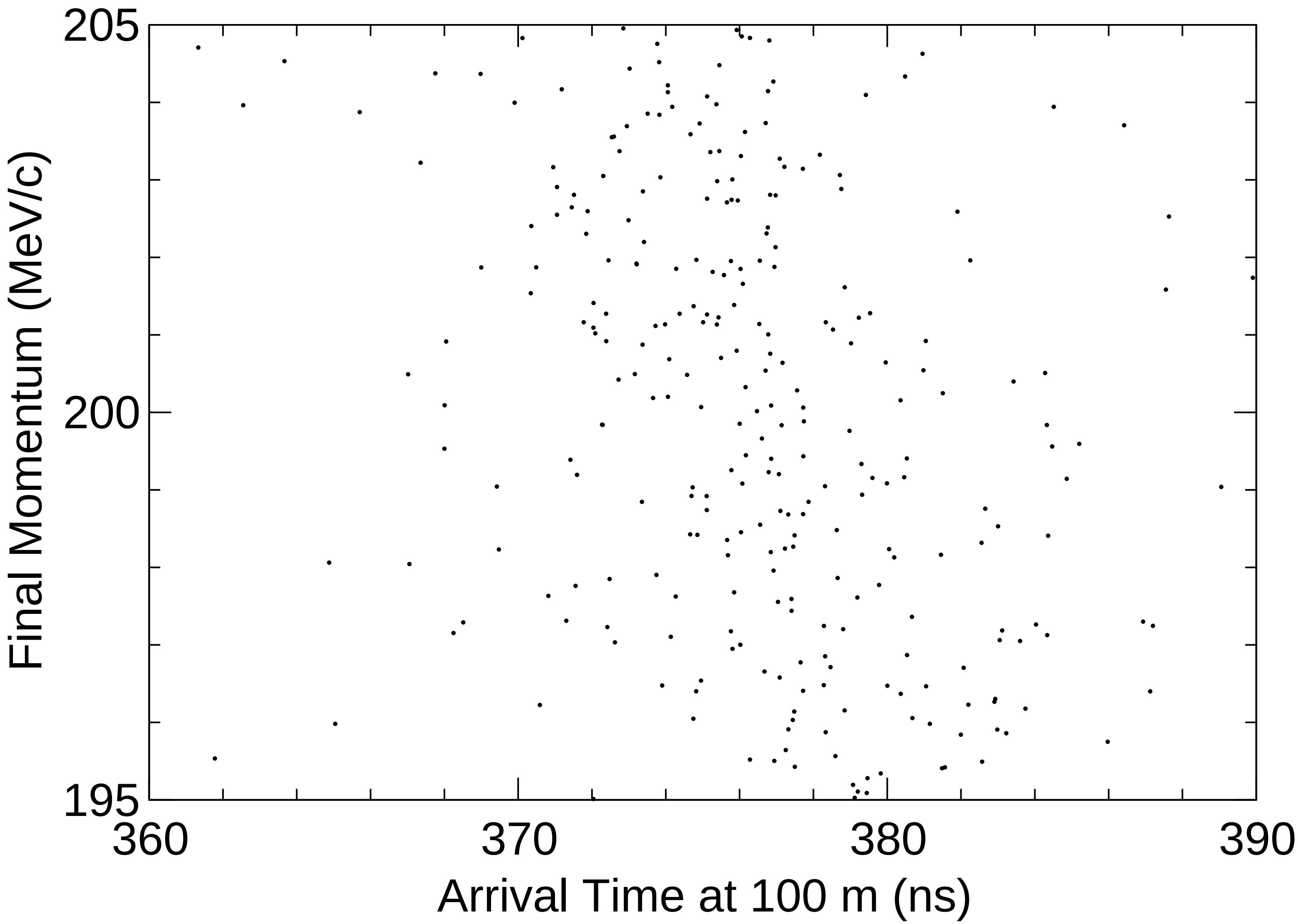
Simulation

- Distribute initial times according to Gaussian
- Uniform initial energy distribution
- Do random decay times (exponential)
- Do random decay direction (forward/backward only) in COM frame
- Look only at final momentum 100 to 300 MeV/c

Distribution in Final Phase Space



Distribution in Final Phase Space
Zoom In Near 200 MeV/c



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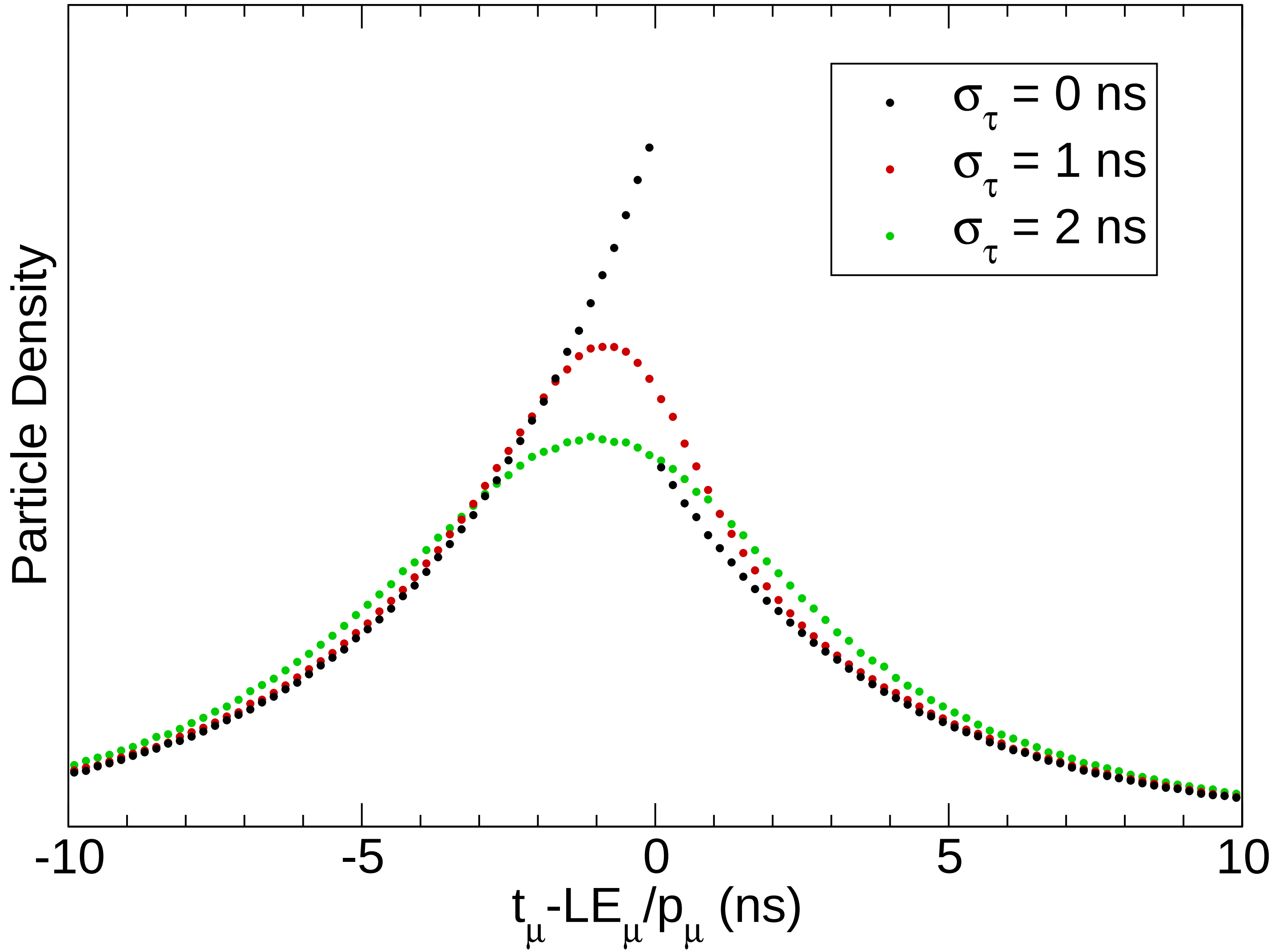
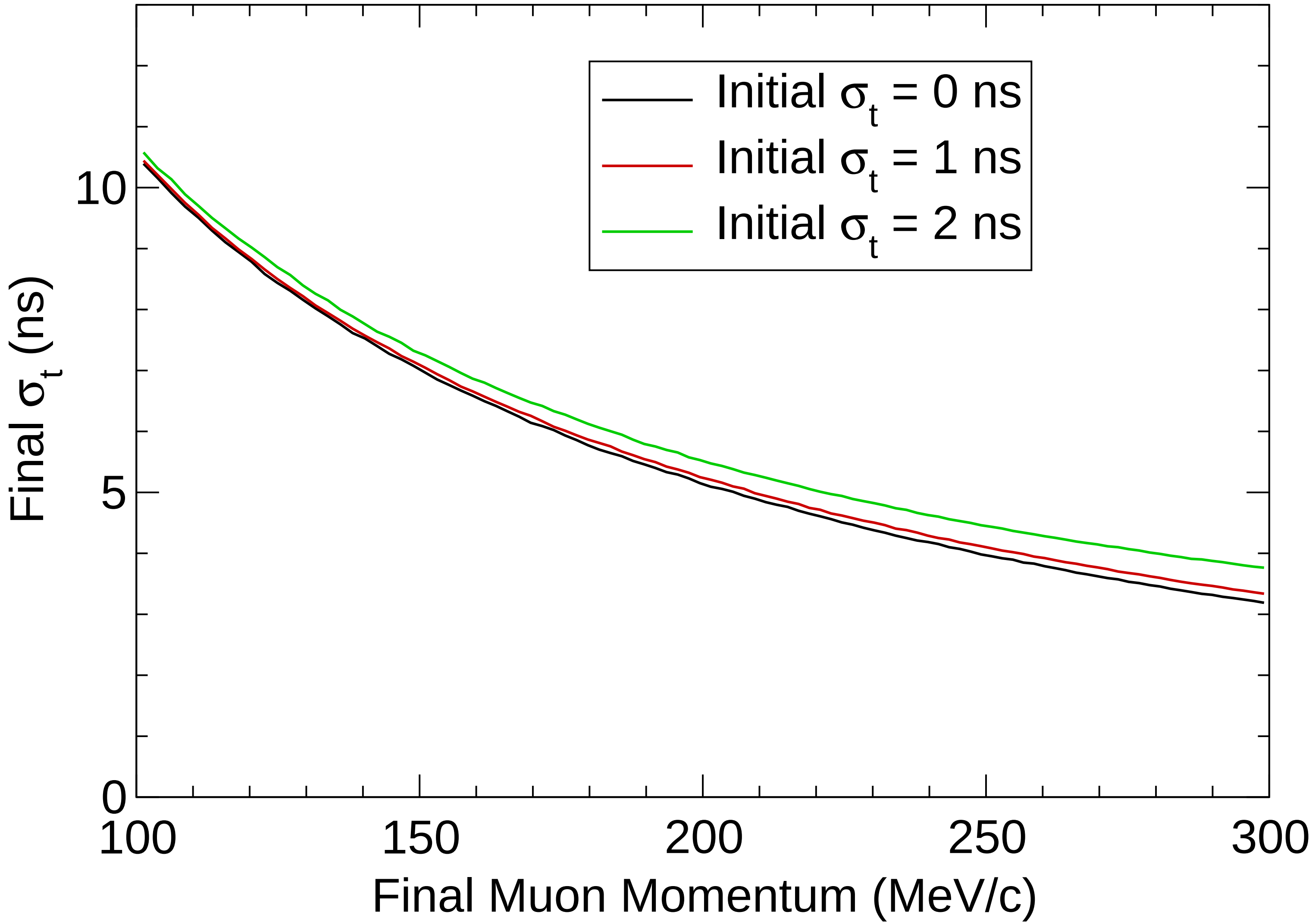
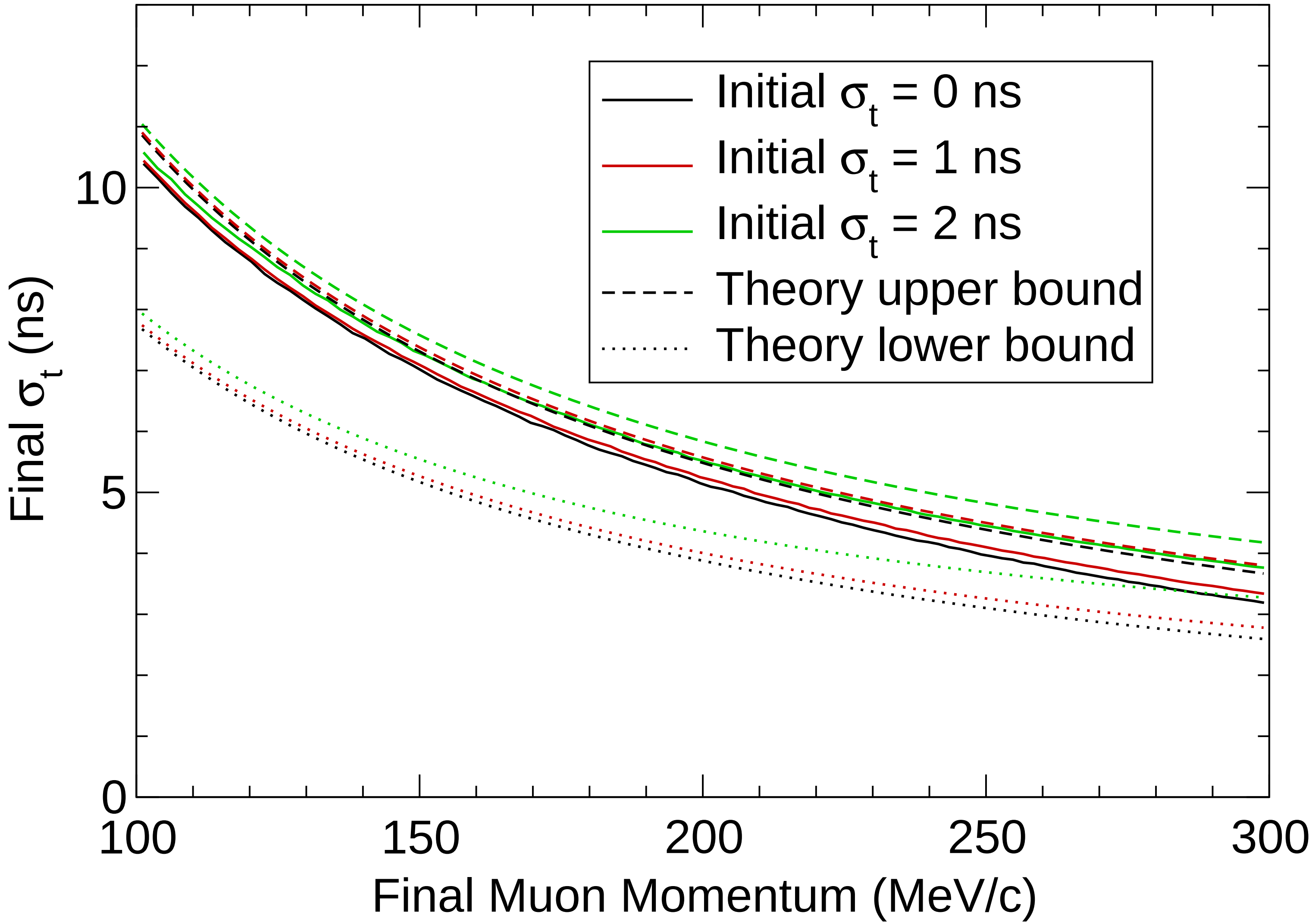
Distribution in Time



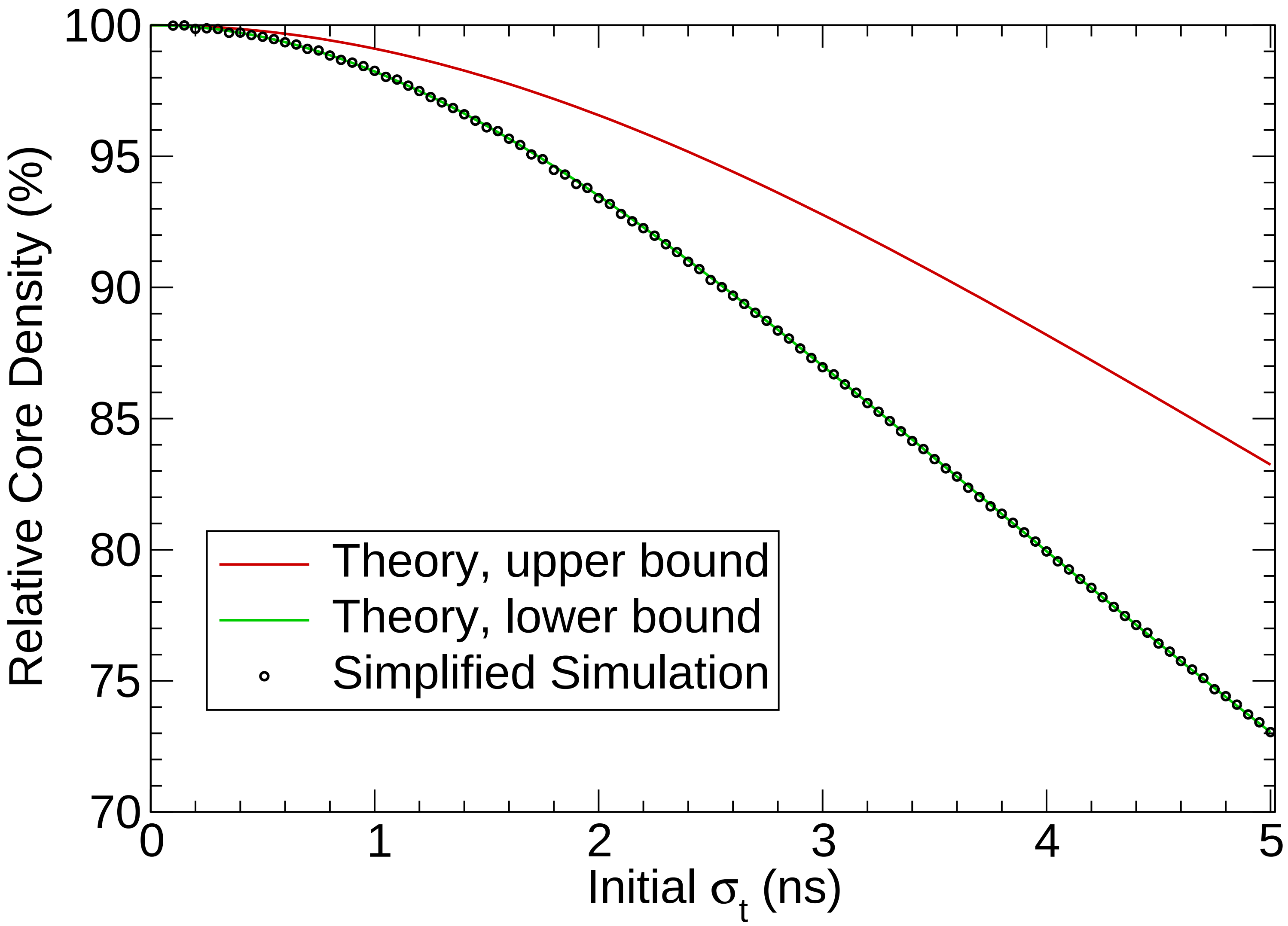
σ_T vs. Energy



σ_T vs. Energy
Theory Included



Estimated Capture Efficiency vs. σ_0



Discussion

- Simulation close to lower bound
  - Asymmetry of $dE_{\pi\pm}/dE_{\mu}$
  - Adding RMS in quadrature not quite right
- Can potentially get different effect from phase rotation
  - Phase rotation gives largest energy spread on early times and high initial energy
  - Energy cut at capture will affect different pieces differently
- Could also try using real initial distribution of pions
  - However, since simulation matches theory lower bound, don't expect much

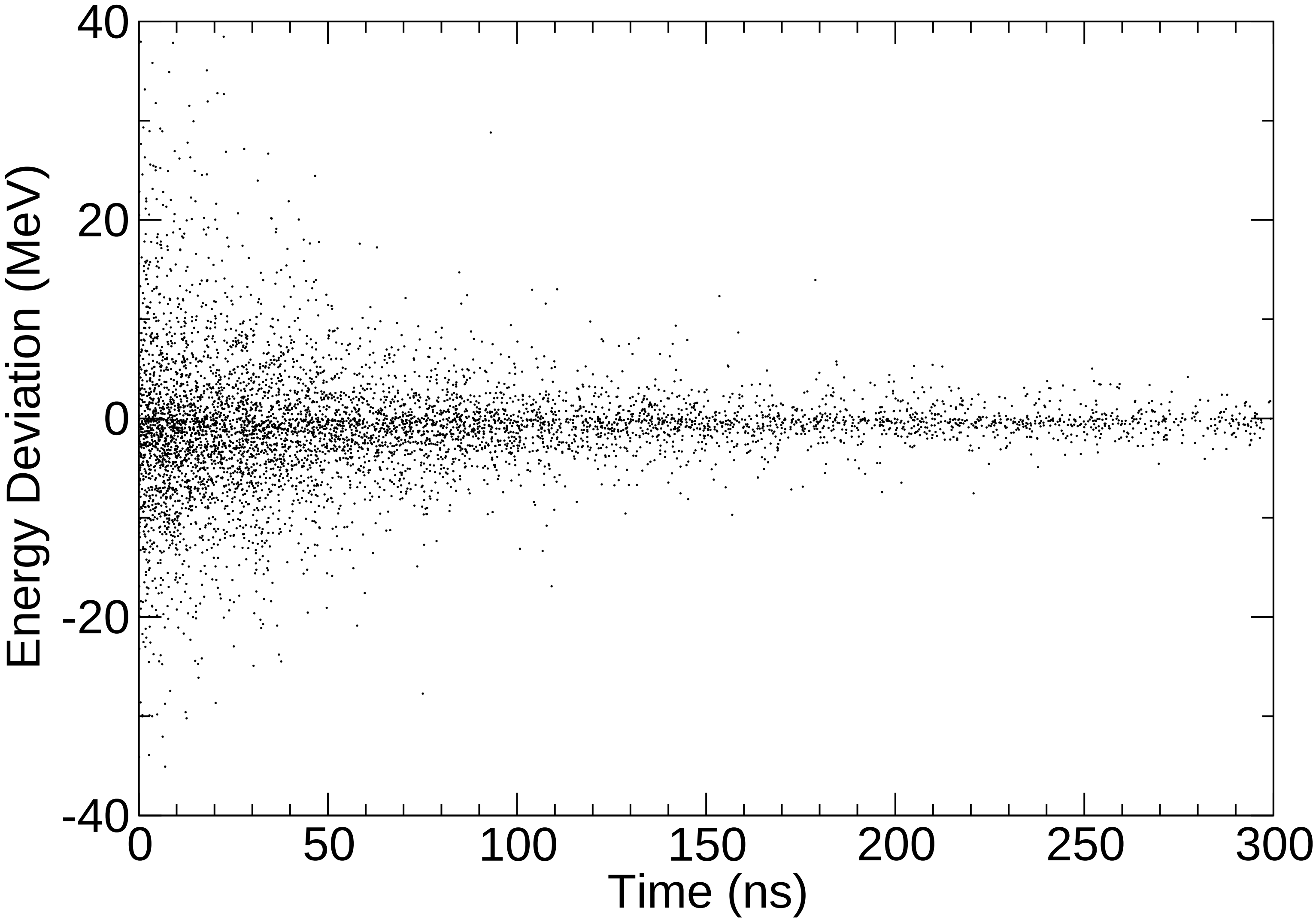
Perfect Phase Rotation

- Reduce E_μ by

$$\frac{m_\mu}{\sqrt{1-(L/t_\mu)^2}}$$

- Not really “perfect”
  - Should first leave high energy part with spread
  - Allow to stretch out more in time
  - Then rotate it down
  - Makes energy spread uniform in time

Phase Space After Perfect Rotation



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