Time of Flight Dependence on Transverse Amplitude for Non-Scaling FFAG Lattices

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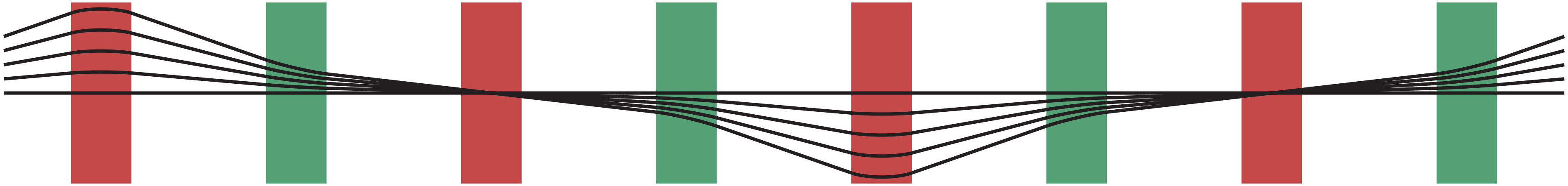
ISS Machine Working Group Meeting
26 July 2006

What is the Problem?

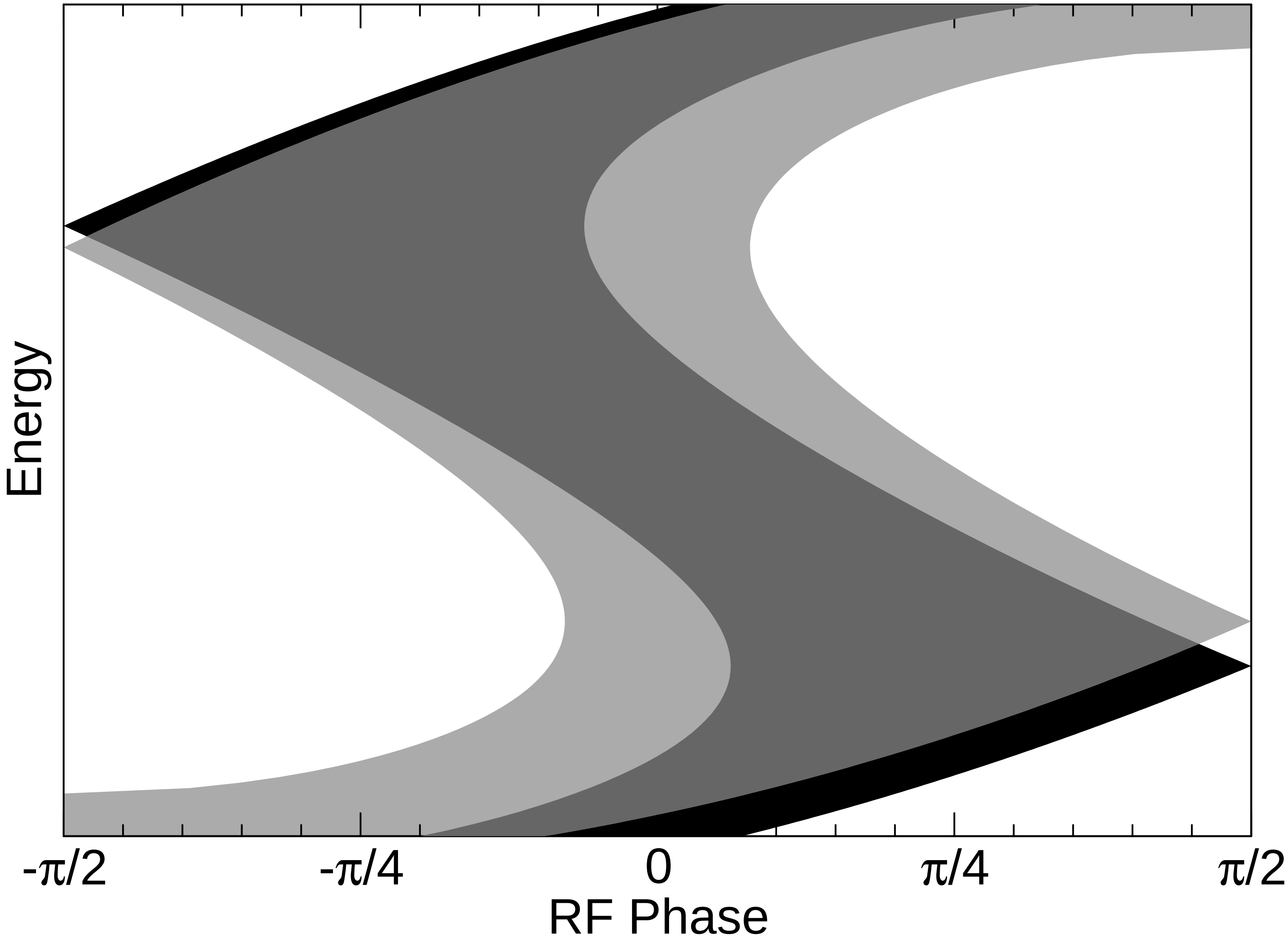
- Time of flight depends on transverse amplitude
- Reason: larger amplitudes, angles make longer path length



- Different times of flight for different amplitudes create acceleration problems in FFAGs
- Time of flight dependence on amplitude related to chromaticity

$$\frac{d\bar{t}}{ds} = -\partial_E H_T - \frac{2\pi(\partial_E \boldsymbol{\nu}) \cdot \boldsymbol{J}_n}{L} + O(\boldsymbol{J}_n^{3/2}).$$

Acceleration Channels in FFAGs



Plan for Addressing Time of Flight Problem

- Time of flight difference at end for uniform acceleration

$$-2\pi\Delta\boldsymbol{\nu}\cdot\boldsymbol{J}_n/(\Delta E)$$

  $\Delta\nu_v$ is tune difference from beginning to end per cell, ΔE is energy gain per cell

- Increase energy gain per cell (expensive)
- Use third harmonic RF to make phase space more forgiving (kind of expensive)
- Correct chromaticity (free!) in FFAG
- Put positive chromaticity in transfer lines

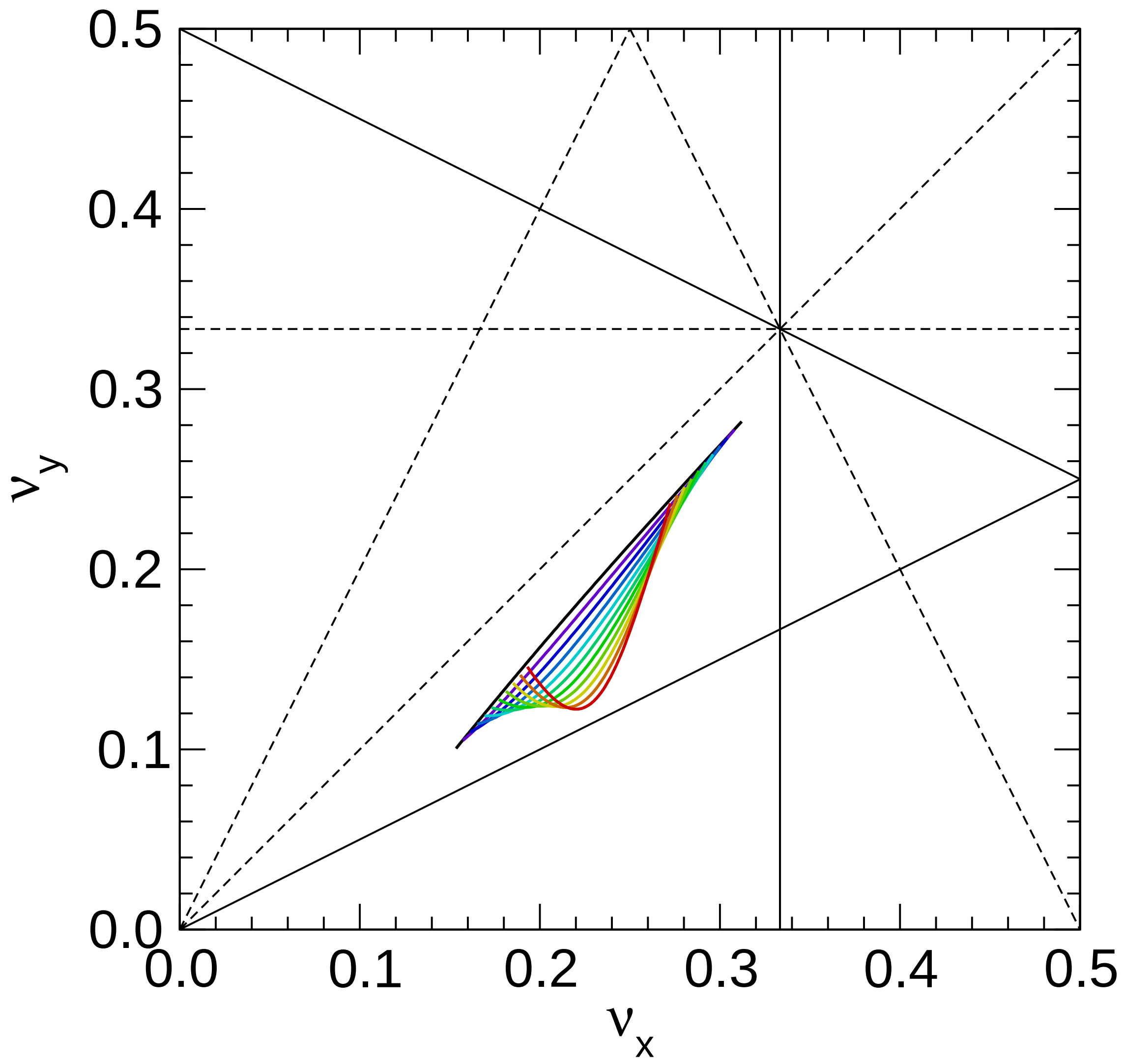
Chromaticity Correction Method

- Correct chromaticity with a sextupole component to magnets as follows
  - Construct a linear lattice where
    - Magnet lengths, drift lengths, and the number of cells are fixed
    - Time of flight is the same at low and high energy
    - The following three distances in the tune plane are equal
      - Low energy tune ($\boldsymbol{\nu}_{\mathrm{lo},0}$) to $3\nu_x = 1$ line
      - Low energy tune to $\nu_x - \nu_y = 0$ line
      - High energy tune ($\boldsymbol{\nu}_{\mathrm{hi},0}$) to $\nu_x - 2\nu_y = 0$ line

Chromaticity Correction Method

- Chromaticity correction procedure (cont.)
  - Add sextupole components, and modify dipole and gradient components so that
    - ⋆ Magnet lengths, drift lengths, and the number of cells are fixed
    - ⋆ Time of flight is the same at low and high energy
    - ⋆ If x is the fraction of chromatic correction
      - ➢ $\boldsymbol{\nu}_{\mathrm{lo}} = (1 - x/2)\boldsymbol{\nu}_{\mathrm{lo},0} + (x/2)\boldsymbol{\nu}_{\mathrm{hi},0}$
      - ➢ $\boldsymbol{\nu}_{\mathrm{hi}} = (x/2)\boldsymbol{\nu}_{\mathrm{lo},0} + (1 - x/2)\boldsymbol{\nu}_{\mathrm{hi},0}$
- Choice of tune range to avoid third order resonances which sextupole will drive
- Plot shows to $x = 0.5$

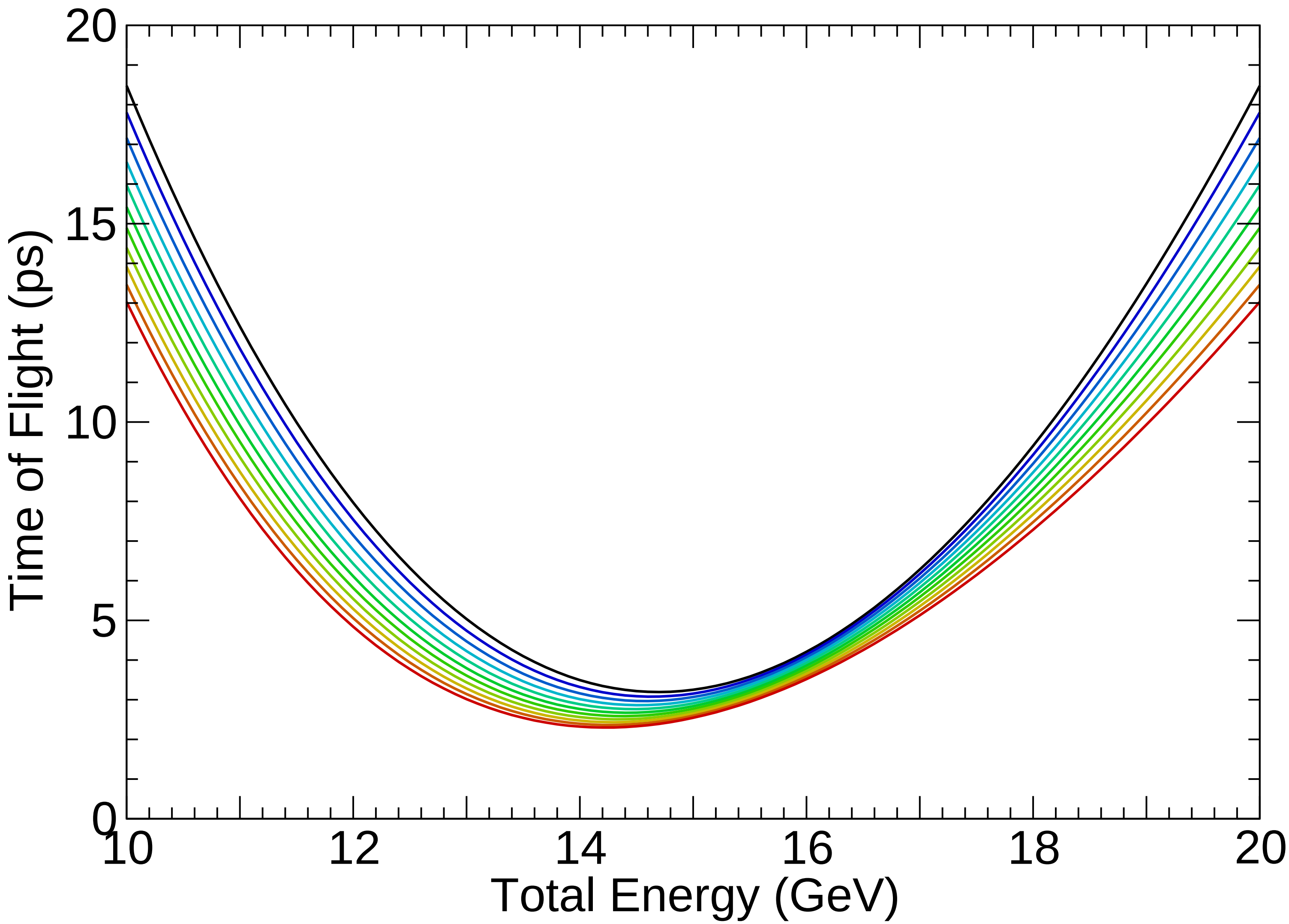
Tune Range with Chromaticity Correction



Observations

- Note chromaticity is locally higher!
- However, for uniform acceleration, what matters is the total change in tune
  - However, increased chromaticity may affect phase space locally!
- Time of flight range actually improves with more sextupole
- Must determine if dynamic aperture is sufficient
  - Losses likely on $4\nu_x = 1$ resonance
  - Should ascertain if we have decent dynamic aperture except for that

Time of Flight Variation with Chromaticity Correction



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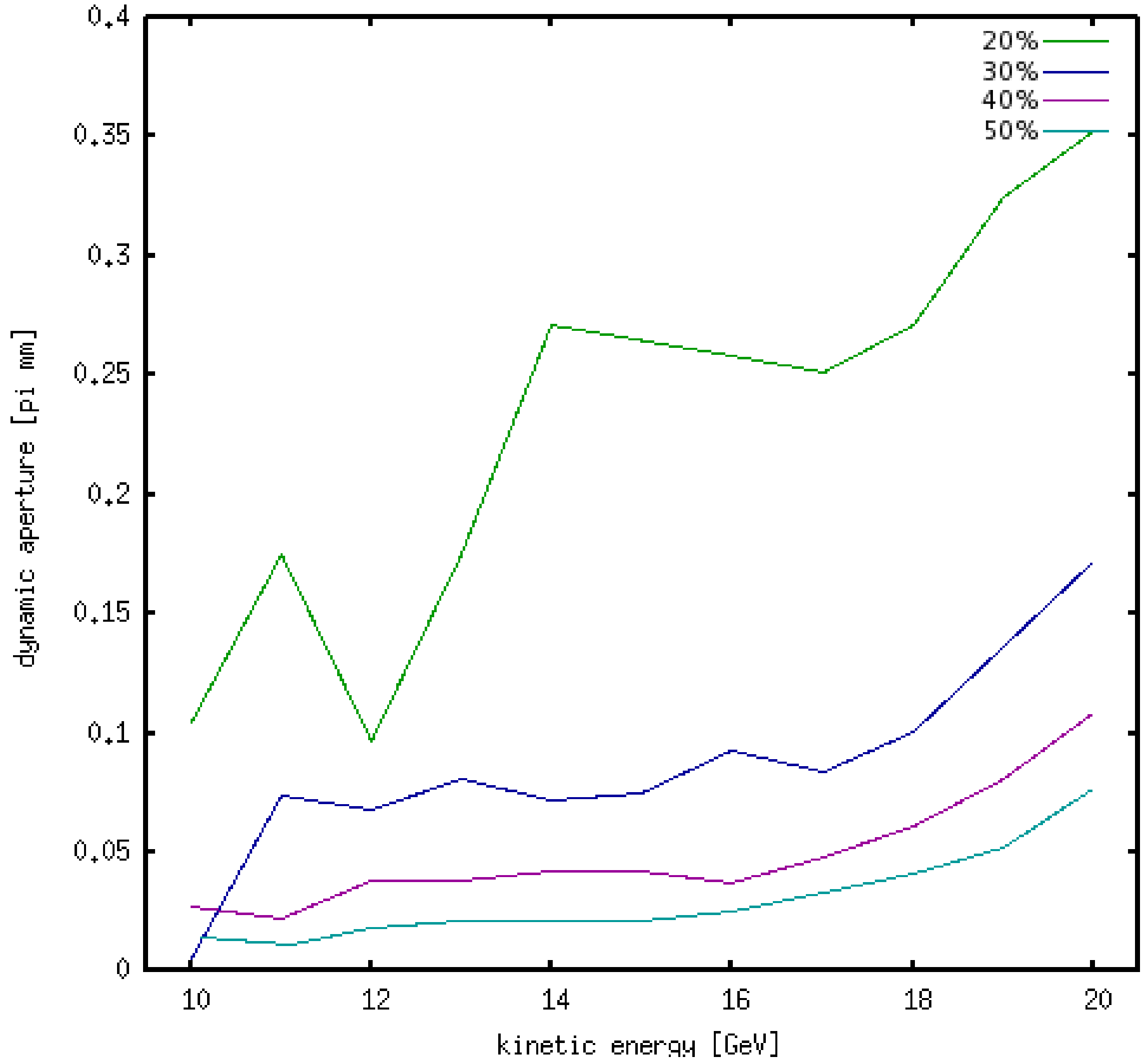
Dynamic Aperture (Machida)

- Dynamic aperture less for higher chromaticity
- Some dynamic aperture reduction on $4\nu_x = 1$ or $4\nu_y = 1$
- 20–30% may be tolerable



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What About Initial Linac?

- Chromaticity is uncorrected in linac
- Time of flight deviation is approximately

$$-\frac{2\pi}{\Delta E}\ln\left(\frac{p_f}{p_i}\right)\boldsymbol{\xi}\cdot\boldsymbol{J}_n,$$

  - Initial and final momenta p_i and p_f, chromaticity $\boldsymbol{\xi}$ and energy gain ΔE per cell, normalized transverse action $\boldsymbol{J}_n$ in eV-s
- Synchrotron oscillations alleviate the problem somewhat
  - Don't occur in higher energy part of linac
- About 30° of phase slip in 500–1500 MeV linac

What to do

- Need to do tracking in linac to ascertain the effect
  - Tracking code needs to include **everything**: avoid approximations
- Could we add occasional chicanes with positive chromaticity?
  - Dynamic aperture or beam blowup
- Shorten linac, go into small RLA sooner
- RLA may see this issue also
  - Alleviated by synchrotron oscillations somewhat
    - Turns into energy shift
  - Can we over-correct chromaticity in arcs?
    - Geometric aberrations