ISS Muon Bunch Structure

Convenors: S Berg, BNL, and G H Rees, RAL
ISS 201 MHz Bunch Structure Options
(Separate comparison later for lower frequencies)

Single train of 80 $\mu^+$ and 80 $\mu^-$ bunches/cycle? Or, $n$ trains of 80 $\mu^+$ and 80 $\mu^-$ bunches/cycle?

Factor of 50/3 in $\mu^\pm$ currents proposed to date:
$n = 1$ at $F = 15$ Hz, $I/nF = 50/750$ (USA)
$n = 5$ at $F = 50$ Hz, $I/nF = 3/750$ (RAL)

What are the criteria for choosing $n$ and $F$?
Factors involved in choice of $n$ and $F$

Preferred values

Target thermal shock effect: high $F$ and energy
Design of the proton driver: $n = 3, 4$ or $5$; $F = 50$ Hz
Beam loading in $\mu^\pm$ stages: high $n$, $F$ and radius
Switch-on power for $\mu^\pm$ RF: low $F$ ($P$ scales with $F$)
Design of $\mu^+$, $\mu^-$ decay rings: $n$ (inj) < 6; high $F$ (RF)
Target Effects (SB & RB)

(SB) $\pi^\pm$ and $\mu^\pm$ yields: 10 GeV a good proton energy (except for a carbon target?)

(RB) Solid Targets: Thermal shocks reduced at higher $F$, & by 50 $\mu$s delays of $n$ bunches.

Liquid Target: The total duration of the proton pulse/cycle must be $< 60 \mu$s.
Proton Drivers for the Two Cases

\[(n, F(Hz), T(GeV)) = (1, 15, 26) \quad (5, 50, 10)\]

- Protons per bunch = \(6.4 \times 10^{13}\) \(10^{13}\)
- Booster bunch \(L_b\) = \(6.4 \times L_b\) \(L_b\)
- Bunch A (eV sec) = \(6.4 \times A\) \(A (= 0.66)\)
- Booster harmonic = 1 \(6\)
- Driver harmonics = \(\sim 6, 36, 216\) \(36, 216\)
- Final bunch \(\Delta p/p\) = \(\pm 2.0\%\) \(\pm 0.8\%\)
- Bunch \(\Delta t\) (ns, rms) = < 3 ? \(2 \rightarrow 1\)
- \(\Delta t\) compression = problematic \(\text{adiabatic}\)
**Proton Driver Longitudinal Bunch Area**

The bunch area to be compressed (in eV sec) is:

\[ A = \left( \frac{8R\alpha}{(ch)} \right) \left( \frac{2 V(I-\eta_{sc})E_0\gamma}{h\eta\pi} \right)^{\frac{1}{2}} \]

Choose low linac energy & booster radius for \( A < 0.7 \).

Choose 200 MeV linac & 63.777 m, booster radius. Choose \( n = 5, h = 6, \) and \( 10^{13} \) protons per bunch. (These allow room for the RF and ease extraction). Choose \( h = 36 & 216, \) and \( 2 \times \) radius for an NFFAG.
Proton Driver Longitudinal Space Charge

\( \eta_{sc} \) is the ratio of the longitudinal space charge forces to the focusing forces of RF system.

\( \eta_{sc} = 1 \) corresponds to an RF bucket collapse.
\( \eta_{sc} = 0.4 \) gives onset of a microwave instability.

\( \eta_{sc} = 0.21 \) for \( V = 93 \text{ kV at 0.20 GeV} \), and
\( \eta_{sc} = 0.39 \) for \( V = 476 \text{ kV at 3 GeV in booster} \).

\( \eta_{sc} = 0.11 \) for \( V = 469 \text{ kV at 3 GeV in NFFAG} \) and
\( \eta_{sc} \) cancels the inductive wall fields at 10 GeV.

\( V = 1.0, 2.5 \text{ MV (h=36, 216) at 10 GeV (1 ns rms)} \).
Proton Driver Transverse Space Charge

Assume a 2-D elliptic beam density distribution.

$$\Delta Q_v = - N r_p G F / [ \pi \varepsilon_v (1 + a/b) \beta \gamma^2 B_f ]$$

$$\varepsilon_v = \text{normalised, } 2\sigma \text{ vertical beam emittance}$$

$$G = 1.2 \text{ and } F = \text{image force enhancement}.$$  

$$\varepsilon_v = 122 (\pi) \text{ mm mrad (2}\sigma, \text{ normalised)}$$

$$\Delta Q_v = - 0.35 \text{ in booster after } H^- \text{ injection}$$

$$\Delta Q_v = - 0.25 \text{ in driver at 3 GeV injection}$$

$$\Delta Q_v = - 0.14 \text{ at 10 GeV after compression.}$$
10 GeV, 50 Hz, 4 MW Proton Drivers

180 MeV $H^- \text{ linac} + 50 \text{ Hz boosters} + 2, 25 \text{ Hz RCS}$
180 MeV $H^- \text{ linac} + 50 \text{ Hz booster} + 50 \text{ Hz NFFAG(I)}$
A $H^- \text{ linac}$ feeding a chain of 50 Hz FFAGs in series

For 1, a slower RCS needs more difficult boosters.
For 2, electron models are needed for both options
For 3, injection of $H^-$ into the first FFAG is difficult.

Typical number of bunches are $n = 4, 5, \ldots \ldots$ or 1

8 GeV, 50 Hz, $H^- \text{ linac} + \text{ accumulator} + \text{ compressor}$?
NFFAGI Proton Driver

0.18 GeV H⁻ Linac

0.18 GeV H⁻ Achromat

3 GeV, 50 Hz, \(h=5\), RCS
(one at 50 Hz or two at 25 Hz)

10 GeV, 50 Hz, \(N=5\), NFFAGI
with \(10^{13}\) protons per bunch

Target, Muon Cooling and Muon Acceleration

Alternative is a larger,
\(h=6, n=5\) RCS & NFFAG
Proton and Muon, 50 Hz Bunch Trains

Proton booster \((n=5, h=6)\)

Proton driver \((n=5, h=36)\)

Proton bunches at target
Pion bunches after target
Muon, 400 ns bunch trains

\((n-1)T < 60 \, \mu s \) (liquid target)
\(T > 60 \, \mu s \) (for solid targets)

20 GeV \(\mu^+ \) & \(\mu^-\) accelerator

20/50 GeV \(\mu^+\) decay ring
C>1500 m circumference
20/50 GeV \(\mu^-\) decay ring

400 ns bunch trains & 600(+) ns gaps
Box-car Stacking for Decay Rings

Driver has protons, while muons are to be stacked. So, a revised method of box-car stacking is needed.

Sequential delays for proton bunches ~ 30 - 70 µs, and an unchanging delay through the muon stages.

Times insufficient to adjust 201.25 MHz RF phases. Make 201.25 MHz a harmonic of driver at 10 GeV.
$n = 5$, Muon Bunch Pattern in Decay Rings

2 of 5 interleaved 80 $\mu^-$ bunch trains of the adjacent 2nd ring

$80 \mu^+$

Solid/liquid

$148(136)$

$>100 \text{ ns intervals}$

$127(130)$

$80 \mu^+$

$80 \mu^+$

$127(130)$

$80 \mu^+$

$80 \mu^+$

$127(130)$

$127(130)$

$80 \mu^+$

80 full and 127 (or 130) empty RF buckets
## Ring RF Harmonic Numbers

<table>
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<tr>
<th>Rings</th>
<th>Beta</th>
<th>Circ (m)</th>
<th>h</th>
<th>RF (MHz)</th>
<th>N_b /Ring</th>
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<tr>
<td>50 GeV µ Decay</td>
<td>0.9999977</td>
<td>1573.0691</td>
<td>1056</td>
<td>201.250</td>
<td>5x80</td>
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<td>? GeV µ± Acc</td>
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<tr>
<td>3-10 GeV P Driver</td>
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<td>13.079-13.417</td>
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<td></td>
<td></td>
<td></td>
<td>216</td>
<td>80.500</td>
<td>5</td>
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<td></td>
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<td>540</td>
<td>201.250</td>
<td>5</td>
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<tr>
<td>0.18-3 GeV Booster</td>
<td>0.9712057</td>
<td>400.72372</td>
<td>6</td>
<td>2.5413-4.3595</td>
<td>5</td>
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</table>
Box-car Transfer of $\mu^+ & \mu^-$ to Decay Rings

The 20 GeV decay rings, 20 GeV $\mu^+$ acc and P driver, of periods $T_d$, $T_a$, $T_p$, all have a harmonic at 201.25 MHz. The integers $p (=1,2,3,4)$, $n$ and $m$ are chosen so the proton bunch delays are a good approximation to:

$$(n \pm p/5) \ T_d \approx (m \pm 1/12) \ T_p$$

$T_d$, $T_a$, $T_p$ = 5.2472044, 3.7863345, 2.6832296 μs, $(T_d/T_p) = 1.9555554$

Target | $m$ | $n$ | $p$ | $(m \pm 1/12)$ | $(n \pm p/5) \ (T_d/T_p)$ | Difference |
--- | --- | --- | --- | --- | --- | --- |
solid | 23 | 12 | -1 | 23 + 0.083333 | 23.075553 | 0.007780 |
liquid | 5 | 3 | -2 | 5 + 0.083333 | 5.084444 | 0.001111 |

For solid target: \( (m + 1/12) \ T_p = n \ T_d - 207 \ T_b \) (RF period $T_b$)

For liquid target: \( (m + 1/12) \ T_p = n \ T_d - 423 \ T_b \)
Summary of Proton Driver for 201.25 MHz Muon Stages (n = 4 considered later)

Compression harder if n < 5, F < 50 Hz or T < 10 GeV.
The muon decay rings limit n to a maximum at n = 5.
Limit F to 50 Hz because of muon RF switch-on costs.
It does not appear necessary to increase T > 10 GeV.
Final bunch structure depends on accel. & target sites.
201.25 MHz Muon Stages

1. Initial Bunch Rotation Stage (Neuffer / Iwashita)
   Division into 80 bunches is needed to reduce the longitudinal bunch areas and later beam losses.

2. Transverse Cooling Stage (45 → 30 → ? (mm rad))
   Helps to reduce losses during muon acceleration. 
   Lowers apertures in $\mu^\pm$ rings & transfer lines ($1/\sqrt{\varepsilon}$).
   Lowers $\mu / \nu$ divergence ratio in decay rings ($1/\sqrt{\varepsilon}$).
   Eases downstream kickers (power scales as $\varepsilon^2$).

3. Linac + Ring Options: 1. RLAs, Dog-bone, DRLAs.
   2. Linear, Non-scaling, Near-Isochronous FFAGs.
   3. Non-linear, Not-scaling, Isochronous IFFAG(I)s.
201.25 MHz Muon Acceleration

- No allowance for emittance growth in acceleration
  Beam loss collectors needed for high power levels.
  Long collimators for the counter-rotating $\mu^\pm$ beams.
  This infers long straights or insertions for the rings.

- Beam loading power for the rapid acceleration
  This scales as $1/nFR$, where $R$ is the ring radius.
  Factor of $\sim 50$ higher for (1, 15Hz, low $R$) scheme.
  20 GeV ring: 1000 cf 20 units, for 2 MW couplers.

- Injection and Extraction Fast Kicker Systems
  Large systems needed for the two decay rings.
  Kickers for low $R$, FFAGs may not be feasible.
Aspects of 201.25 MHz Options

**D/RLAs:** Kicker magnet systems not necessary. RF systems in zero dispersion straights. Beam loss collectors in some of the arcs?

**FFAGs:** Long.-transv. coupling at large amplitudes. Is there coherent trans. motion or $\epsilon$ growth? How large does the final $\Delta p/p$ become?

**IFFAGIs:** Beam losses at $Q_h = 1/3$ cell resonances. New 9.5-20 GeV design avoids this feature. Tracking studies haven’t yet re-commenced.
µ⁺ and µ⁻ Decay Rings

Separate rings are required to allow both fast injection and the time separations for the n (= 5) bunch trains.

For a single detector, racetrack rings are preferred. For 2 detectors, two may be used, in own tunnels.

For two distant detectors, triangular, side by side rings in vertical or near vertical plane, have higher efficiency.

For detectors at 7500 & 3500 km, rings need to be in a near vertical plane & to have an apex angle of ~ 50°.
Features of Decay Rings

- The RF containing fields have to scale as $(\Delta p/p)^2$
- 3, 10 MV systems needed/ring for $\Delta p/p = \pm 1\%$
- Reactive beam loading compensation is needed
- 16, 50 kV PFN, 5 kA pulsers & 10 $\Omega$ feeders/ring
- 8, shorted, 3m, 10 $\Omega$ delay line, push pull kickers
- The kicker rise and fall times have to be < 600 ns
- Collimators in short straight of the isosceles $\Delta$
- Use of radiation hard quadrupoles is proposed
Effect of $n = 4$ in smaller Decay Rings

Benefits are smaller depth, cheaper tunnels for decay rings.

Efficiency of the two racetracks is reduced from 38 to 35%.
Efficiency of two, 50° apex rings is reduced from 48 to 43%.

Options (last is favoured) for changes needed to Proton Driver:
1. $F = 62.5$ Hz; RF costs up in both Proton Driver and $\mu^\pm$ rings.
2. $T = 10 \rightarrow 12.5$ GeV (4 -12.5 or 3-8 and 8-12.5, GeV FFAGs).
3. $N = 1.0 \rightarrow 1.25 \times 10^{13}$/bunch; RF costs up in Driver & $\mu^\pm$ rings.
   Lower frequency, longer cavity, RF systems are required.

$N = 1.66 \times 10^{13}$/bunch for $n = 3$ is also feasible (bunches longer).
Lower Frequency Muon RF Systems

Examples: Scaling FFAG schemes (KEK), 44/88 MHz RF systems (CERN).

KEK:

A low repetition rate, 3-50 GeV, Proton Synchrotron. A chain of variable low frequency, scaling FFAGS.

RF systems compensate for cavity and beam power.

No transverse cooling & no separate bunch division. Apertures are enhanced in scaling FFAG magnets.
Issues for Low Frequency Muon RF

RF systems & power costs are key considerations. More space & switch-on power needed for cavities?

Issues little changed for the Proton Driver (n = 7?). Keep F at 50 Hz to limit beam loading in $\mu^\pm$ rings.

How to provide transverse cooling at low frequency? Possibility of NFFAGs or IFFAGs instead of FFAGs?
Bunch Structure Issues

1. Change from 1 to \(\leq 5\) bunch trains per cycle?
2. Use 50 Hz, 4 MW, 10 GeV Proton Driver, \(n=5, 4, 3\)?
3. Use proton bunch delays for low \(\mu\) beam loading?
4. Compare low & high frequencies for muon stages.
5. Delay decision on \(\mu^\pm\) acceleration for further R&D?
6. Use 201.25 MHz for \(\mu^\pm\) rotation, cooling & acceln?
7. Create trains of 80 \(\mu^+\) & \(\mu^-\) bunches while rotating.
8. Accelerate the bunch trains singly in the \(\mu^\pm\) rings.
9. Provide transverse cooling to give \(\varepsilon \leq 30\) mm rad?
10. Two rings (racetracks) needed for single detector.
11. Use two vert. \(\Delta\) rings for best \(\xi\) for two detectors.