Beam transmission in isochronous FFAG lattices (\(\mu\), here)

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1 Isochronous cell, ring parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>8 - 20 GeV</td>
</tr>
<tr>
<td>C</td>
<td>1254.6 m</td>
</tr>
<tr>
<td>num. turns</td>
<td>16</td>
</tr>
<tr>
<td>num. cells</td>
<td>123</td>
</tr>
<tr>
<td>cell length</td>
<td>10.2 m</td>
</tr>
<tr>
<td>cell type</td>
<td>$\overline{d}FBD\overline{d}$ at low E</td>
</tr>
<tr>
<td></td>
<td>$\overline{b}FDF\overline{b}$ at highest E</td>
</tr>
<tr>
<td>$f_{RF}$</td>
<td>201.20 Hz</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0</td>
</tr>
<tr>
<td>E gain</td>
<td>750 MeV/turn</td>
</tr>
<tr>
<td>$\xi$</td>
<td>&gt; 0</td>
</tr>
</tbody>
</table>

3 × 41, provision for resonant $D_x$ excitation (?)
2 Magnet fields

Strength (m$^{-2}$) in bd and BF multipoles:

Field (T) in BF sector magnet:

Polynomial representation (and modeling):

**Straight multipoles, bd and BD**: get strengths that ensure tunes on closed orbits sticking to design data:

\[
K_{bd}(x) = -1.16096 + 52.7949 \, x + 3349.85 \, x^2 + 24707.2 \, x^3 - 5.7728410^6 \, x^4
\]

\[
K_{BD}(x) = -1.92981 - 168.352 \, x - 10056.6 \, x^2 - 2.108410^6 \, x^3 - 1.7228210^8 \, x^4
\]

then, integration yields the series expansions

\[
B_{bd}(x) = b_{bd0} - 1.16096 \, x + 26.3975 \, x^2 + 1116.62 \, x^3 + 6176.79 \, x^4 - 1.154710^6 \, x^5
\]

(1)

\[
B_{BD}(x) = b_{BD0} - 1.92981 \, x - 84.1762 \, x^2 - 3352.2 \, x^3 - 527101. \, x^4 - 3.4456510^6 \, x^5
\]

(2)

**BF sector magnet**:

\[
B_{BF}(r) = b_{BF0} + 16.5655 \, r + 12.612 \, r^2 + 86.4359 \, r^3 + 2987.43 \, r^4 + 13647.1 \, r^5
\]
3  Ring parameters

closed orbits along cell:

fields:

Bzo (T) vs. s (m)

tunes:

Cell Tunes vs. Energy (GeV)
4 Stability limits
Goal 1. Check symplecticity of the motion over the all energy span, at largest transverse amplitudes. This will guarantee correct 6-D simulation of 8-20 GeV acceleration.
Goal 2. Find the maximum stable amplitudes.

Figure 1: 1000-cell, stability limits of pure horizontal motion, in presence of paraxial z component. Precision beyond 0.1 cm.

Figure 2: 1000-cell or more, vertical motion stability limits. Precision 0.1 cm.
5 Amplitude detuning

Figure 3: Amplitude detuning.
Left: pure radial motion, with for each energy, $x_0$ varied from closed orbit position to maximum stable amplitude.
Right: axial motion, with for each energy, $z_0$ varied from zero to maximum stable amplitude while $x_{initial} \equiv x_{c.o.}$ always.
6 Beam transmission

A 10000 particles beam is launched for 16 turn acceleration (123 cells/turn), from 8 to 20 GeV. 18.3 MV per cavity, no synchrotron motion. Cavities are put every three cells at the center of the long drift.

Figure 4: Initial phase spaces

Figure 5: Left : Envelopes. Right : transmission.

Figure 6: Initial phase spaces of transmitted particles after the acceleration cycle.
Beam transmission, after iterating

A 10000 particles beam is launched for 16 turn acceleration, from 8 to 20 GeV. Initial coordinates fill the previous acceptance.

![Graph showing beam transmission and trajectory in tune diagram]

Figure 7: Left: transmission. Right: beam trajectory in tune diagram.

Acceptance, for 8-17 GeV acceleration range:

\[ \epsilon_x = 0.08 \, \pi \, \text{cm (norm.)}, \quad \epsilon_z = 0.12 \, \pi \, \text{cm (norm.)}. \]
7 Don’t give up! : lattice with insertions

- It is possible to design matched cell insertions over the full energy range.
- Similar mumulet cells may be considered for both the arcs and insertions, but with shorter straight sections and changed non-linear fields in the arc cells.

As an example:
- ring composed of 4 superperiods, = 4 × (21 arc cells + 9 insertion cells),
- total of 120 cells (compared to the 123 identical cells of the 1254.6 m ring).

Assuming cells of length 10.2 m (as before) for the insertion, but cells of length 6.4 m in the arcs, the circumference reduces to 904.8 m.

There are the benefits of the reduced ring size and of localising the rf systems, but the possible disadvantage of the reduced ring periodicity with more dangerous resonance crossing.

<table>
<thead>
<tr>
<th>Tunes</th>
<th>$\mu_z$</th>
<th>$\mu_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\forall$ E</td>
<td>$9.5 \text{ GeV} \rightarrow 20 \text{ GeV}$</td>
<td></td>
</tr>
<tr>
<td>Insertion cell</td>
<td>0.16</td>
<td>0.12 $\rightarrow$ 0.32</td>
</tr>
<tr>
<td>Normal cell</td>
<td>0.10</td>
<td>0.09 $\rightarrow$ 0.25</td>
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$\nu_x$ will not reach 1/3