‘Dogbone’ RLA Lattice Optimization Including Errors

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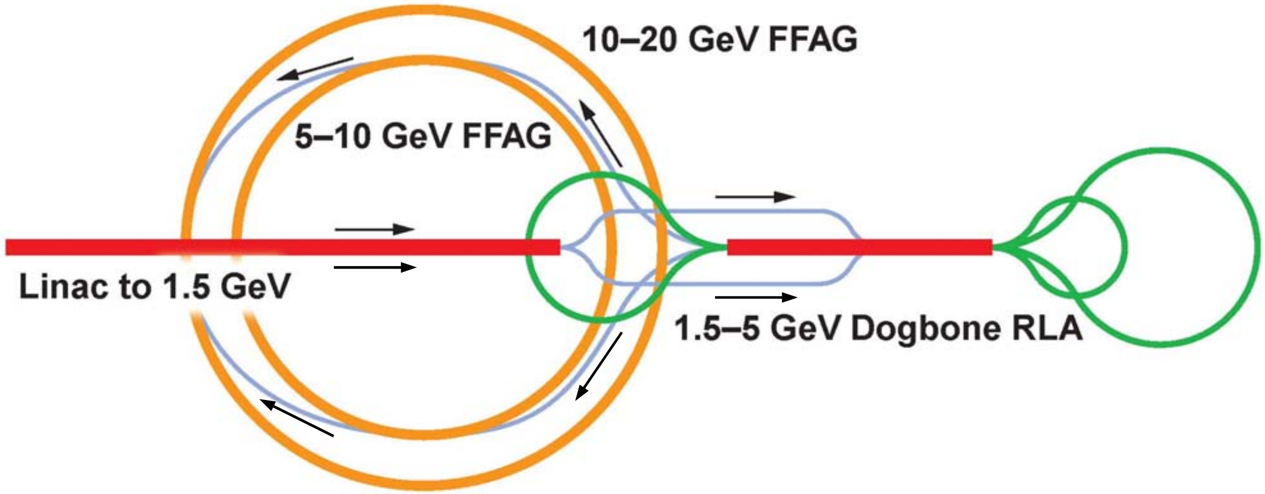
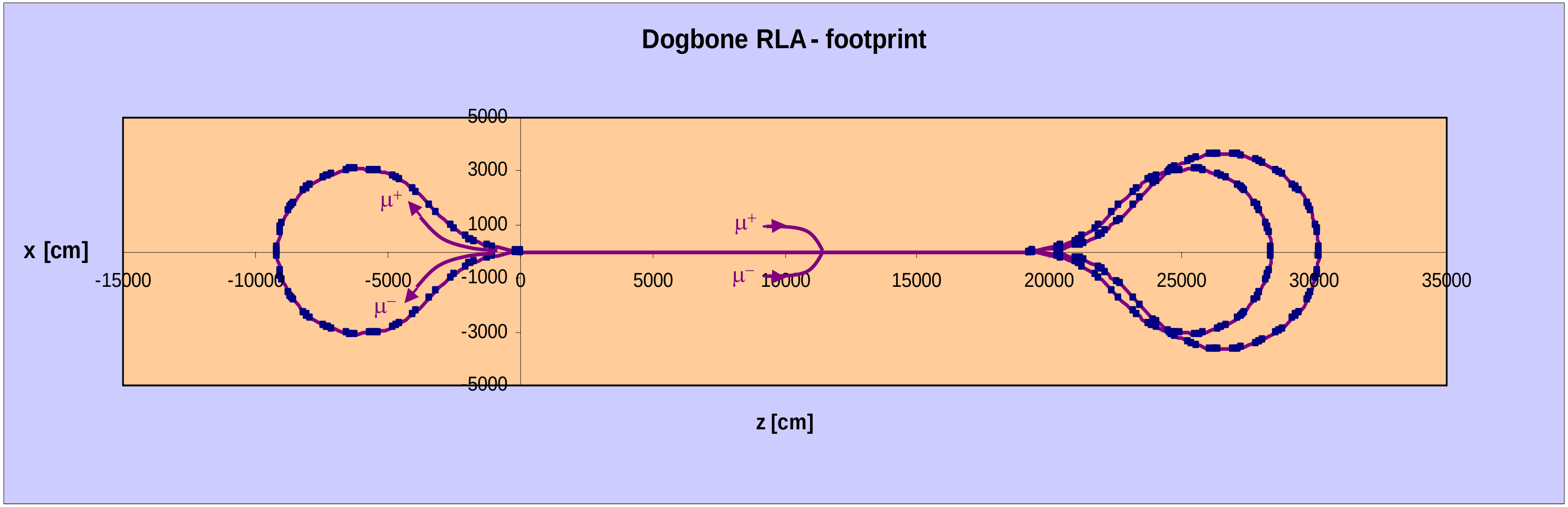
- Symmetric 5 GeV ‘Dogbone’ RLA – Linear Optics
  - Optimized multi-pass linac focusing (1 GeV per pass)
  - Mirror-symmetric ‘Droplet’ Arcs (2, 3 and 4 Gev) – lattices with geometric ‘closure’
  - Arc-to-Linacs betatron matching
- Magnet Misalignment Errors – DIMAD Monte Carlo Simulation
- Focusing Errors Tolerance – Betatron Mismatch Sensitivity

Symmetric Muon Acceleration Complex



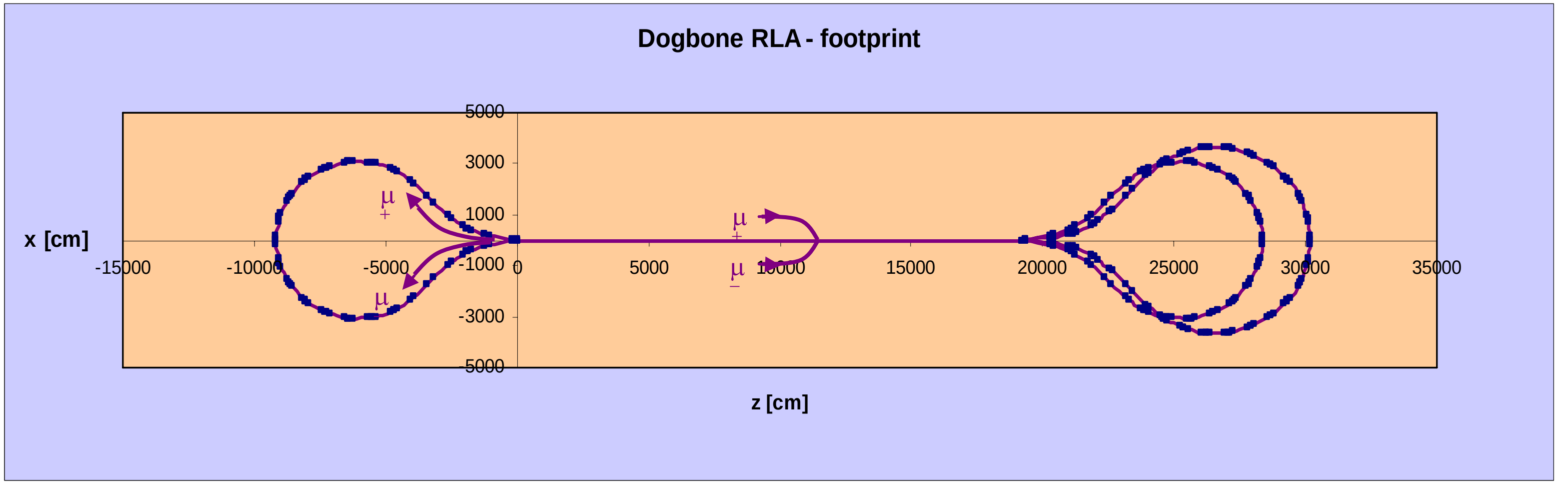
- Linear pre-accelerator (273 MeV/c – 1.5 GeV)
- Symmetric ‘Dogbone’ RLA (allowing to accelerate both μ^+ and μ^- species), 3.5-pass (1.5 – 5 GeV)
- 5 – 10 GeV FFAG
- 10 – 20 GeV FFAG

Symmetric 'Dogbone' RLA (3.5-pass) Scheme



- Main Linac (1 GeV/pass) - triplet focusing
- 3 'droplet' Arcs based on periodic triplet cells (90^0 betatron phase advance per cell)

Linac Optics - Beam Transport Choices

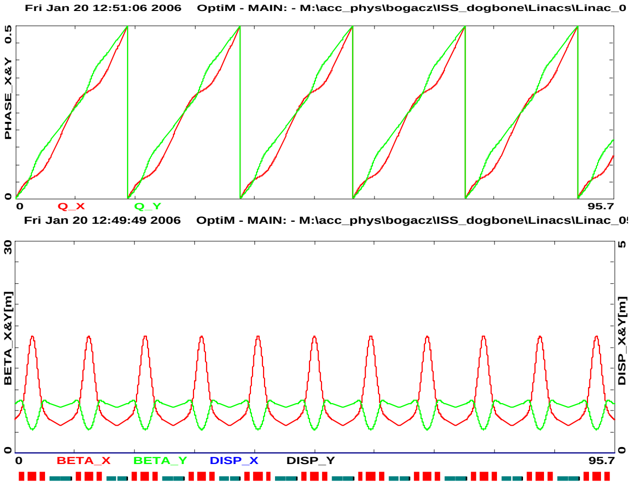


- Multi-pass linac focusing scheme guarantees (by design) mirror symmetry of the droplet arcs
  - at the exit/entrance from/to the previous/next linac: the betas are equal and the alphas are of the opposite sign
- Optimized 'bisected’ linac was chosen as follows:
  - 90^0 phase advance/cell is set for the 'half pass’ linac (1.5-2GeV).
  - as a consequence linac phase advance/cell in the first part of 1-pass drops to about 45^0.
  - to avoid large 'beta beating' one chooses to keep 45^0 phase advance/cell throughout the second part of the linac (Bob Palmer).
  - the phase advance at the end of 2-pass linac drops by another factor of two (22.5^0).
  - the 'beta beating' is rather small on higher passes (2 and 3)

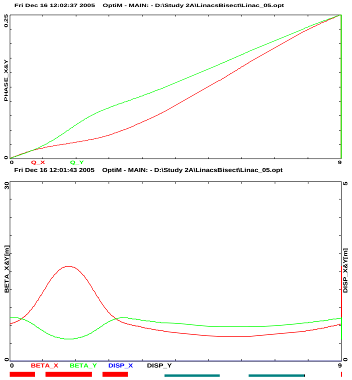
Multi-pass Linac Optics

mirror symmetry cond. ($\beta_{out}^{n} = \beta_{in}^{n+1}$,and $\alpha_{out}^{n} = -\alpha_{in}^{n+1}$, n = 0,1 pass index)

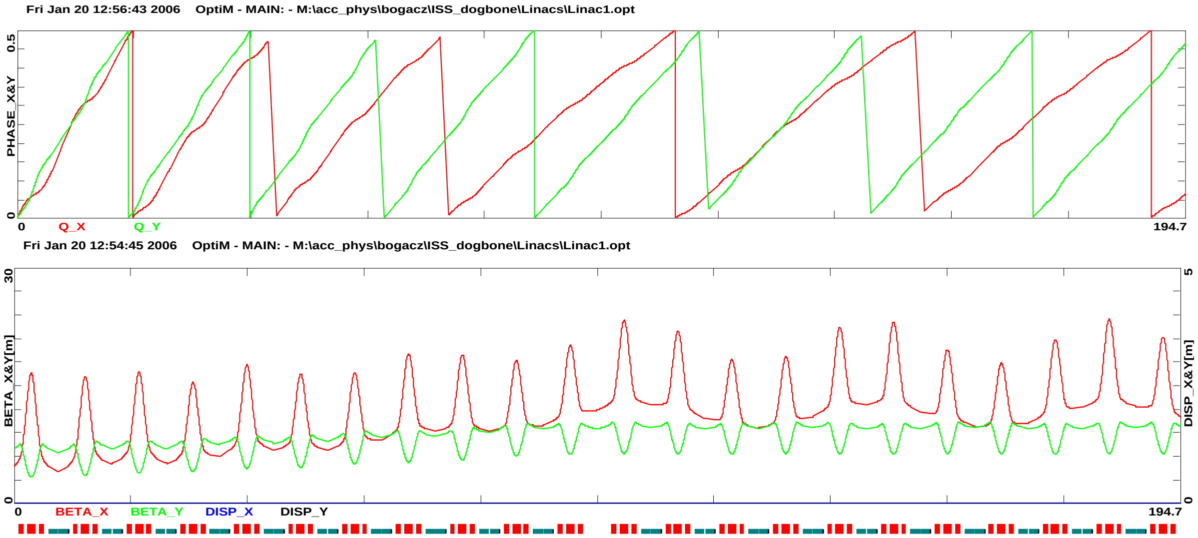
‘half pass’ (n = 0), 1.5-2GeV



uniform phase adv/cell ($\Delta\nu_x$=0.25, $\Delta\nu_y$=0.25)



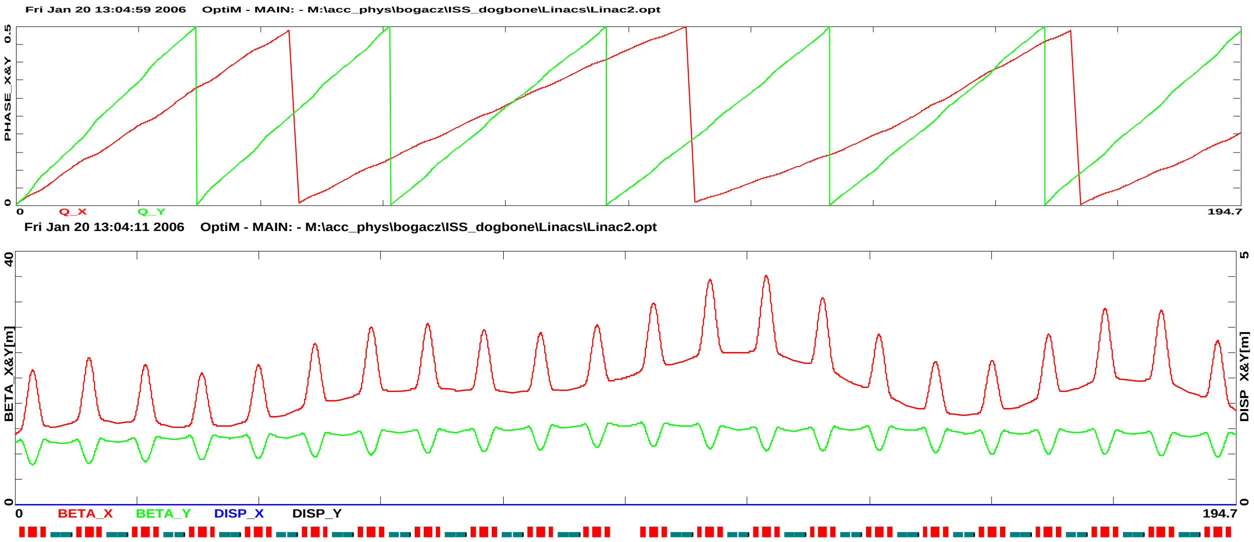
1-pass (n = 1), 2-3GeV



last cell phase adv. ($\Delta\nu_x$=0.11, $\Delta\nu_y$=0.16)

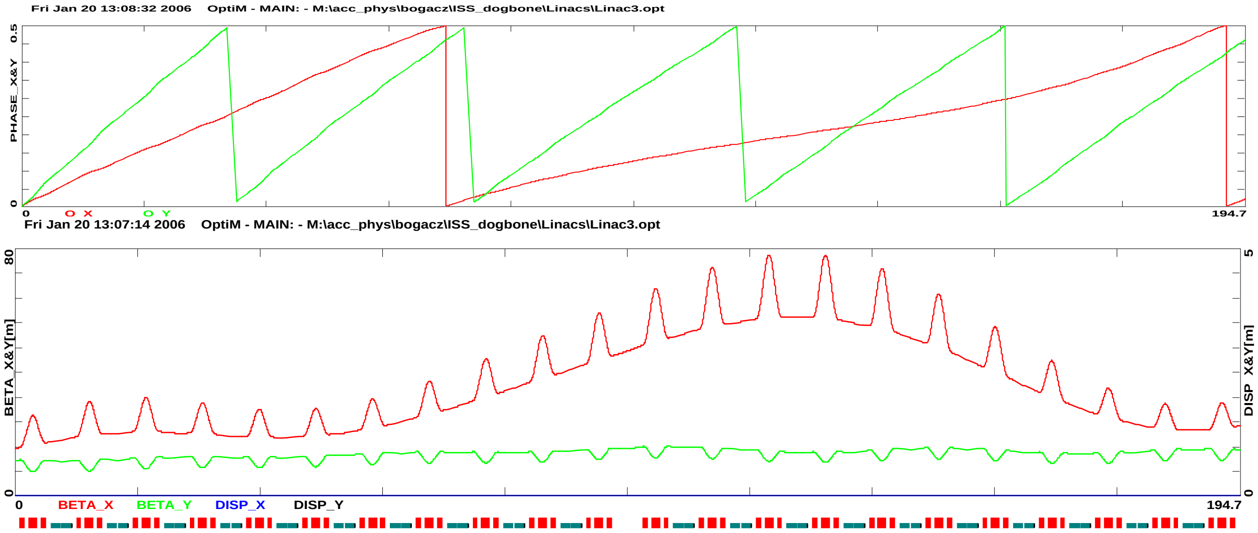
Multi-pass Linac Optics

mirror symmetry cond. ($\beta_{out}^{n} = \beta_{in}^{n+1}$, and $\alpha_{out}^{n} = -\alpha_{in}^{n+1}$, n = 2, 3 pass index)



2-pass (n = 2), 3-4GeV

last cell phase adv. ($\Delta\nu_x$=0.07, $\Delta\nu_y$=0.14)

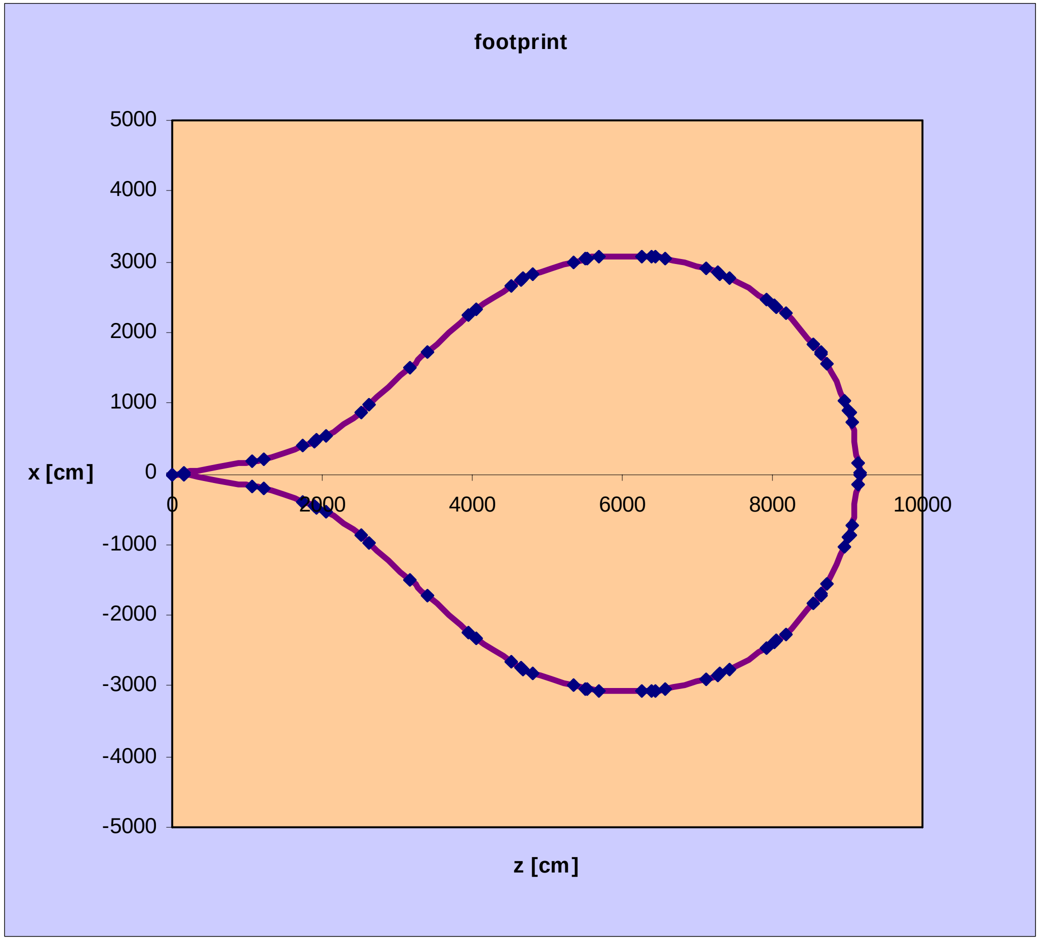


3-pass (n = 3), 4-5GeV

Arc Optics - Beam Transport Choices

- There is great advantage to have 90^0 phase advance/cell in the arcs – cancellation of chromatic effects.
- Phase advance 'mismatch' from/to the linacs is not detrimental - it induces larger 'beta beating' in Spr/Rec regions, but we match betas in this region anyway
- Dipole (horizontal) separation of multi-pass beams in RLA – a pair of dipoles (linac ends)
  - No need to maintain achromatic Spreaders/Recombiners
  - Compact Spreaders/Recombiners – minimized uniform focusing breakdown
- Arc1 and Arc 2 - scaled optics:
  - Keep the same number of cells in each arc (Spr + 2 out + 16 in + 2 out + Rec)
  - Scale Arc1 (bends and quads) by factor of 3/2 to get Arc2
- Arc 3 optics
  - Increase number of cells (Spr + 3 out + 20 in + 3 out + Rec)
  - Maintain the same 90^0 phase advance /cell.

Arc 1 – Layout



Arc dipoles

$Lb=150; =>   150 cm

$ang0=10.3283; =>   10.328 deg

$Nin=16; =>   16

$Nout=2; =>   2

$ang=(90+$ang0)/($Nin-2*$Nout); => 8.36 deg.

#

$Ang_out=$ang0+2*$Nout*$ang; => 43.77 deg.

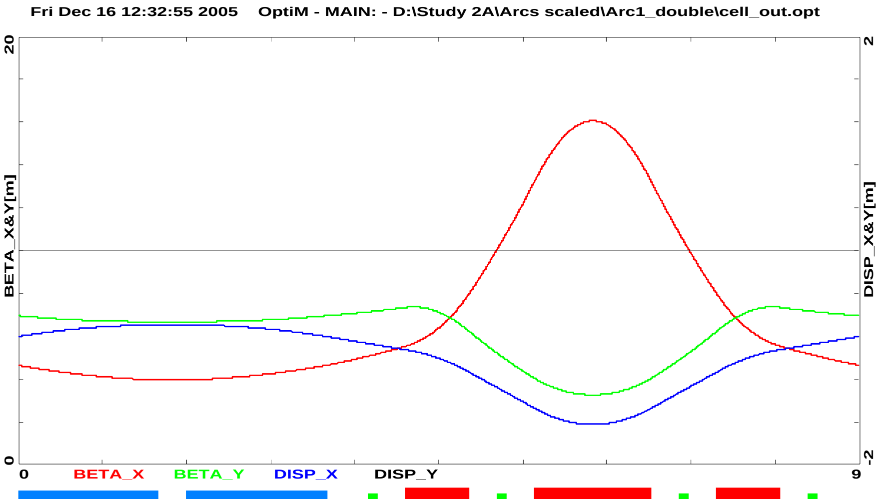
$Ang_in=2*$Nin*$ang; => 267.54 deg.

$BP=$PI*$Hr*$ang/(180*$Lb); => 6.537 kGauss

$Lring=227.3 m

Periodic Triplet Cell - Arc1 Optics

‘outward’ cell’ at 2GeV (T = 1911.64 MeV)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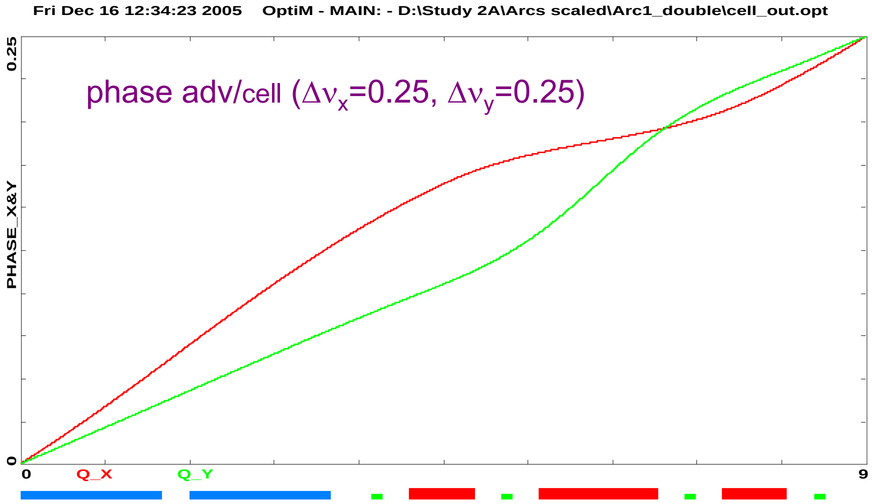
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phase adv/cell ($\Delta\nu_x$=0.25, $\Delta\nu_y$=0.25)

$P= 2014.529

$Hr=$P/$c*1e11; => 6719.745

dipoles (2 per cell)

$Lb=150; => 150 cm

$ang=(90+$ang0)/($Nin-2*$Nout); => 8.36 deg

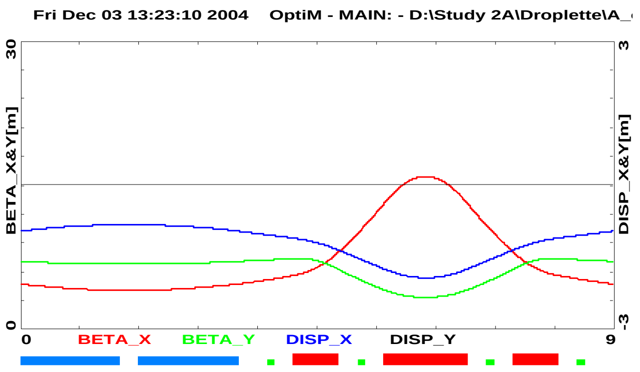
$BP=$PI*$Hr*$ang/(180*$Lb); => 6.537 kGauss

quadrupoles (triplet):

L[cm]	G[kG/cm]
68	-0.326
125	0.328
68	-0.326

Droplet Arc – Optics ‘Building Blocks’

90^0 phase advance/cell: inward and outward cells, missing dipole, empty cells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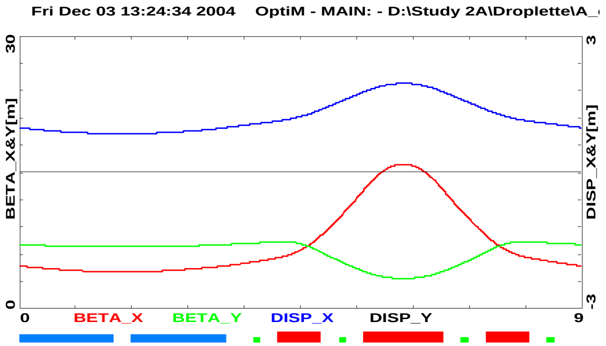
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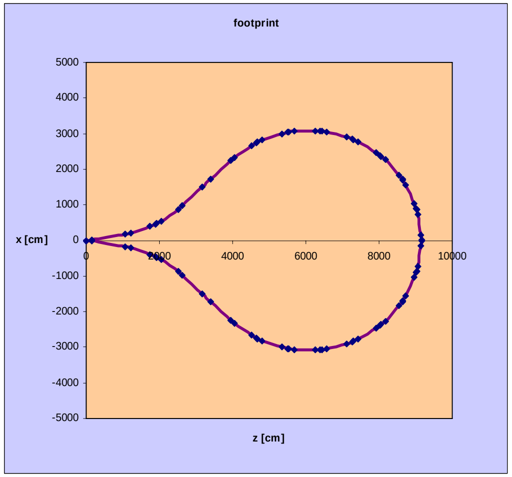

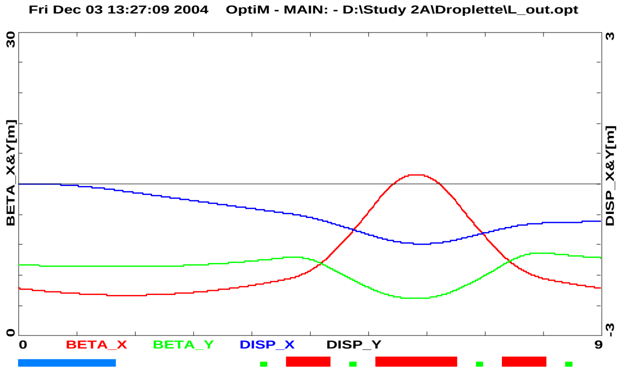
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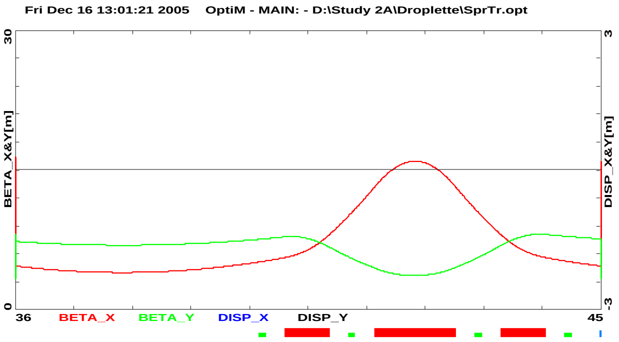
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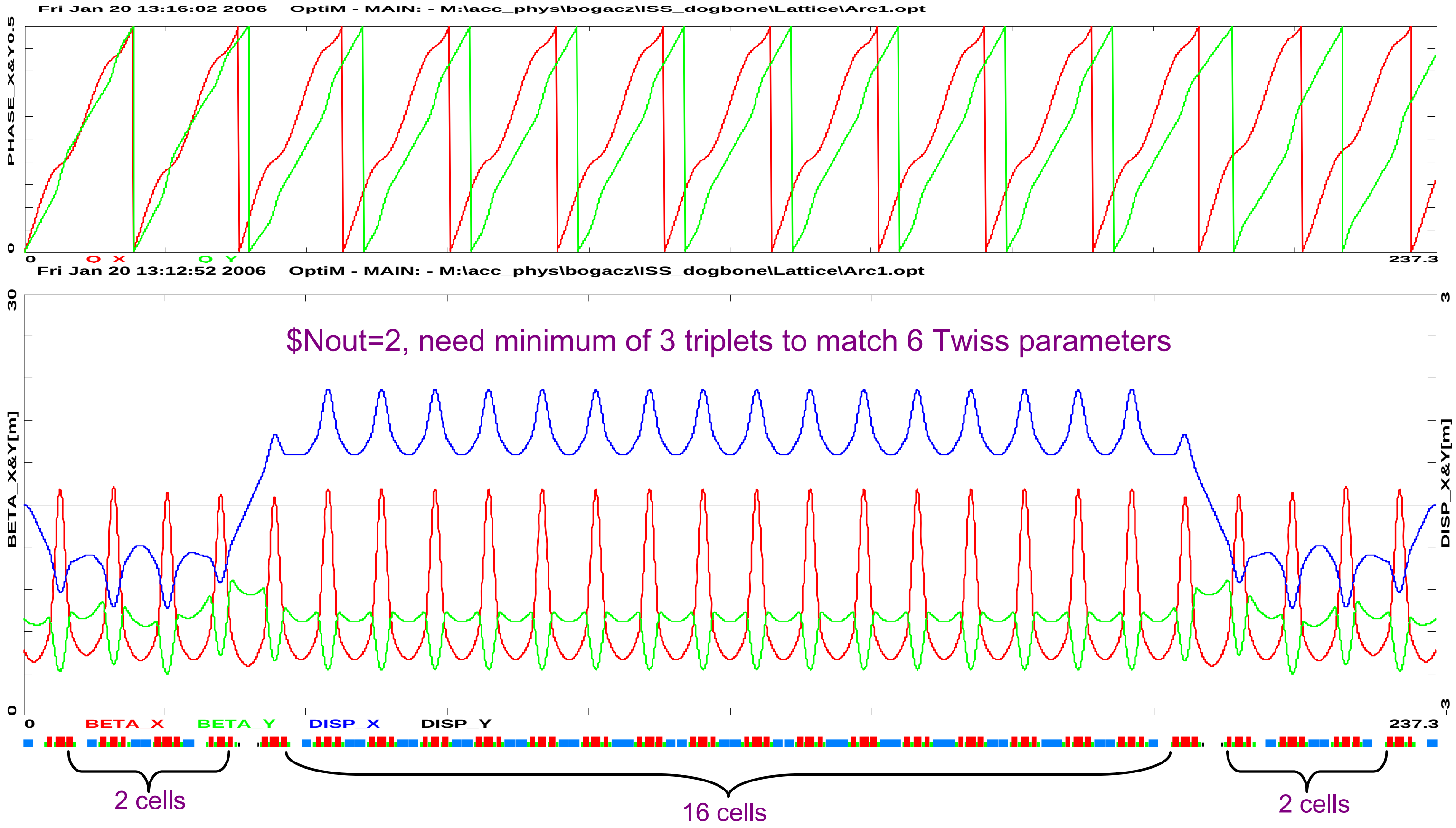
Arc 1 – Mirror-symmetric Optics

($\beta_{out} = \beta_{in}$,and $\alpha_{out} = -\alpha_{in}$, matched to the linacs)



$Nout=2, need minimum of 3 triplets to match 6 Twiss parameters

dipoles (2 per cell)

$Lb=150; => 150 cm

$ang0=10.3283 deg

$ang=(90+$ang0)/($Nin-2*$Nout); => 8.36 deg

$B=$PI*$Hr*$ang/(180*$Lb); => 6.537 kGauss

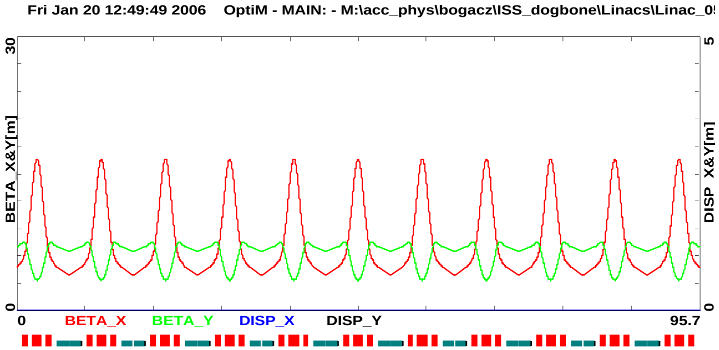
quadrupoles (triplet):

L[cm]	G[kG/cm]
68	-0.326
125	0.328
68	-0.326

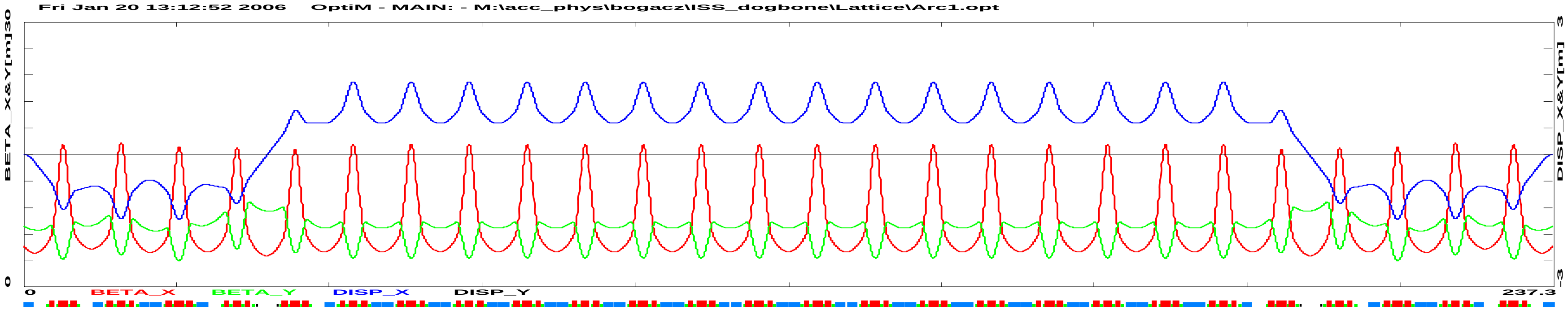
Linac-Arc1-Linac Matching

($\beta_{out} = \beta_{in}$,and $\alpha_{out} = -\alpha_{in}$, matched to the linacs)

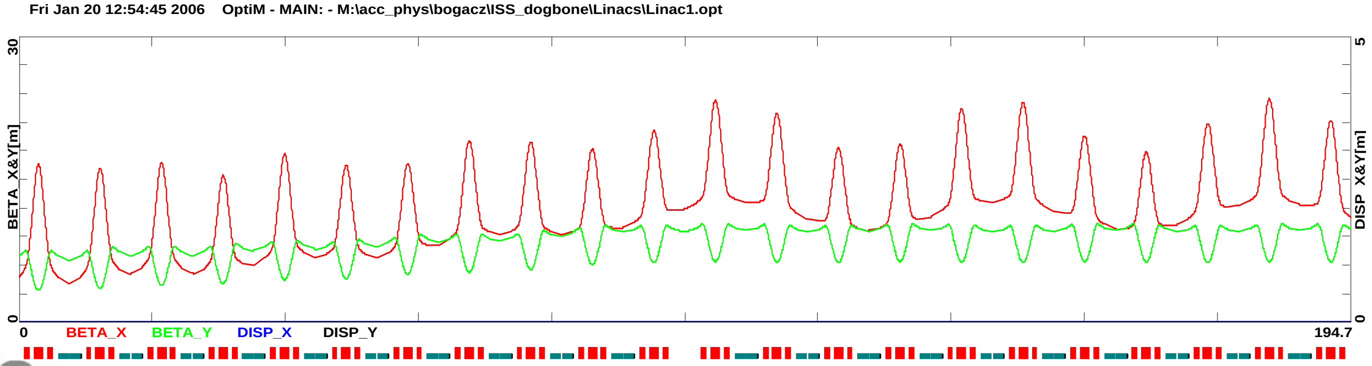
‘half pass’ (1.5-2GeV)



Arc1 (2GeV)

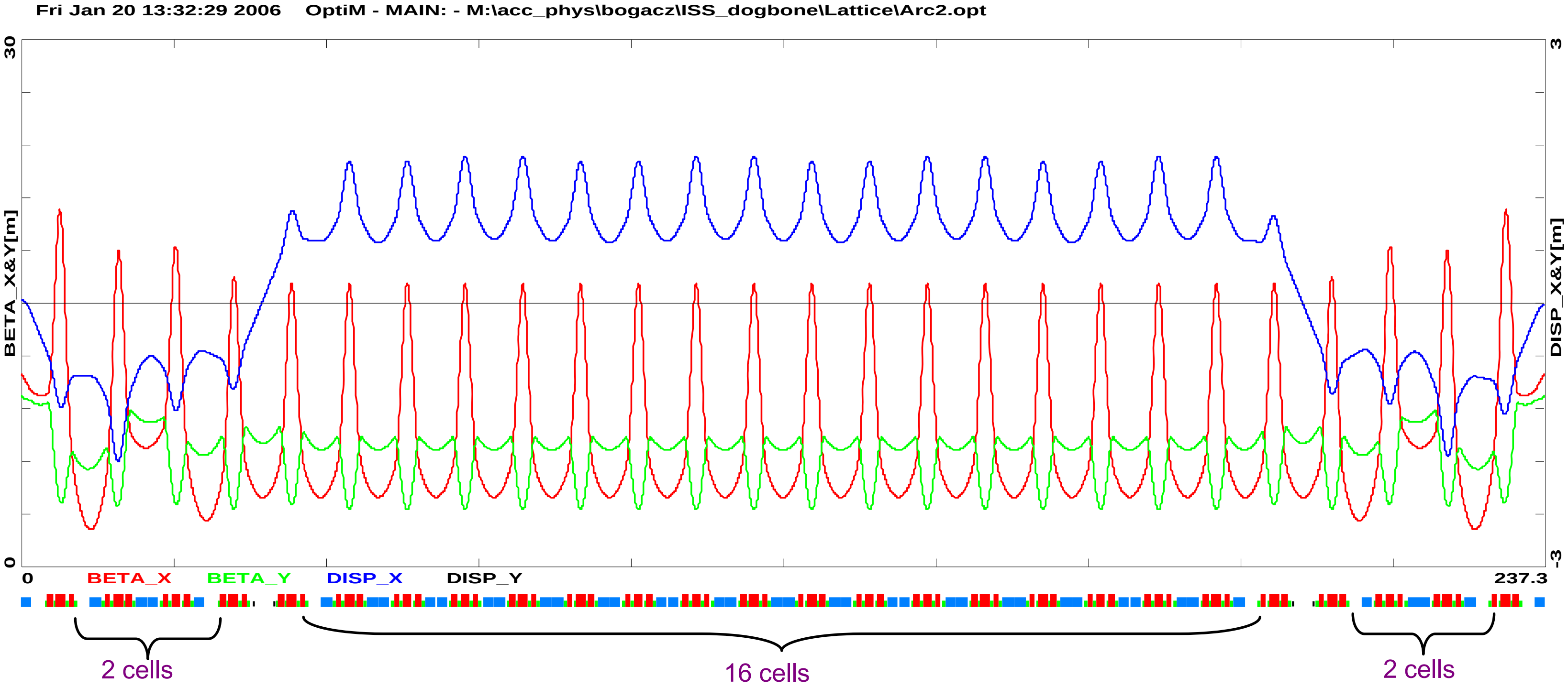


1-pass (2-3GeV)



Arc 2 – Mirror-symmetric Optics

($\beta_{out} = \beta_{in}$,and $\alpha_{out} = -\alpha_{in}$, matched to the linacs)



dipoles:

$Lb=150; => 150 cm

$E=2920.75; => 2920.75 MeV

$ang0=10.3283; => 10.33 deg.

$B0=-$PI*$Hr*$ang0/(180*$Lb); => -12.12 kGauss

$ang=8.3607 deg.

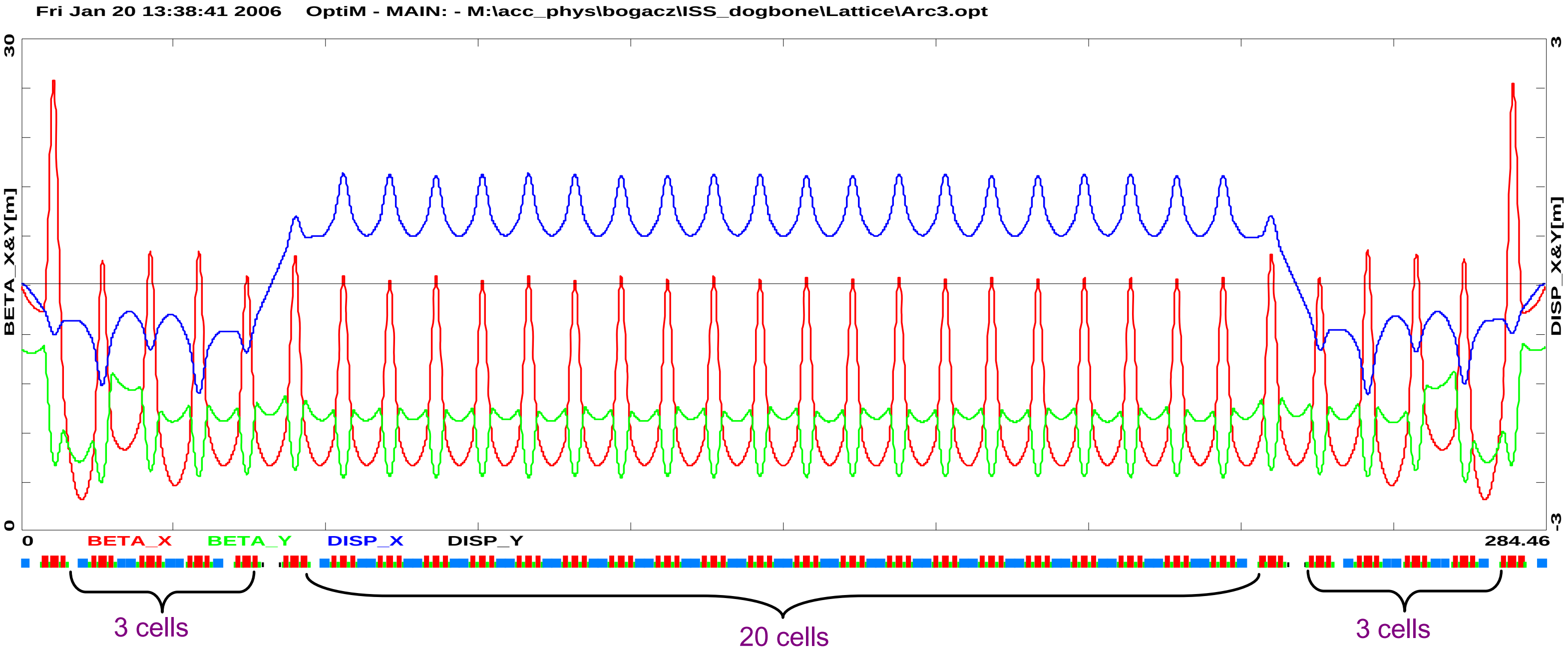
$B=$PI*$Hr*$ang/(180*$Lb); => 9.81 kGauss

quadrupoles (triplet):

L[cm]	G[kG/cm]
68	-0.490
125	0.492
68	-0.490

Arc 3 – Mirror-symmetric Optics

($\beta_{out} = \beta_{in}$,and $\alpha_{out} = -\alpha_{in}$, matched to the linacs)



dipoles:

$E=3929.86 MeV
$B0=-8.0755 kGauss
$ang0= 5.1577 deg
$BP=$PI*$Hr*$ang/(180*$Lb); => 10.64 kGauss
$ang=(90+$ang0)/($Nin-2*$Nout); => 6.797 deg
$Ang_out=$ang0+2*$Nout*$ang; => 45.94 deg
$Ang_in=2*$Nin*$ang; => 271.88 deg

quadrupoles (triplet):

L[cm]	G[kG/cm]
68	-0.6537
125	0.6565
68	-0.6537

Magnet Misalignment Errors

- Lattice sensitivity to random misalignment errors was studied via DIMAD Monte-Carlo assuming:

quadrupole misalignment errors:

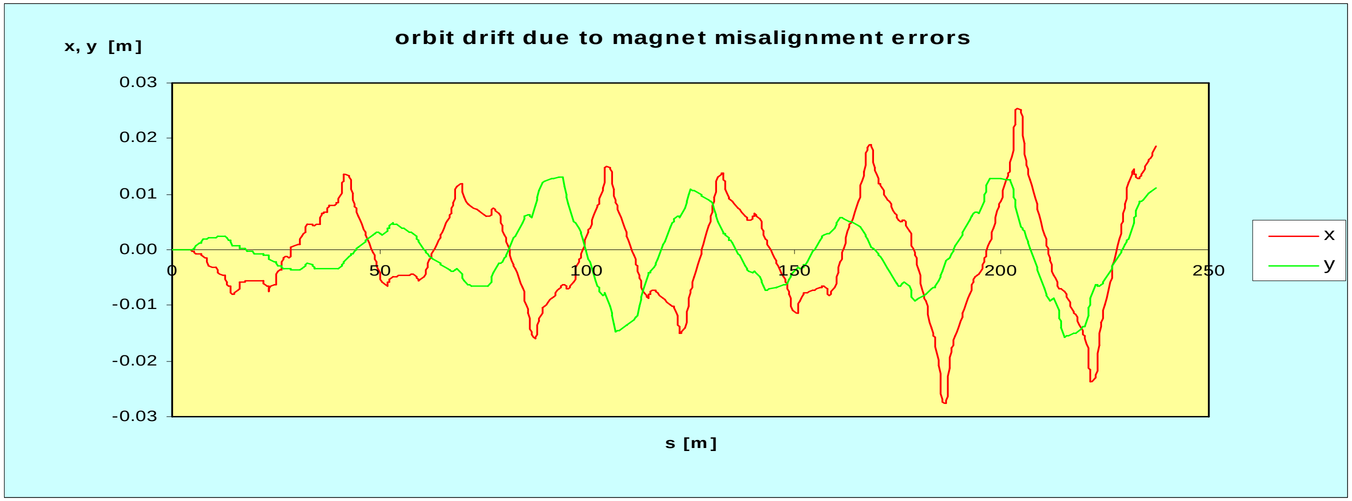
F: $\sigma_x = \sigma_y = 1$ mm

D: $\sigma_x = \sigma_y = 1$ mm

$(\sigma_{x,y'} = \sigma_{x,y}/L)$

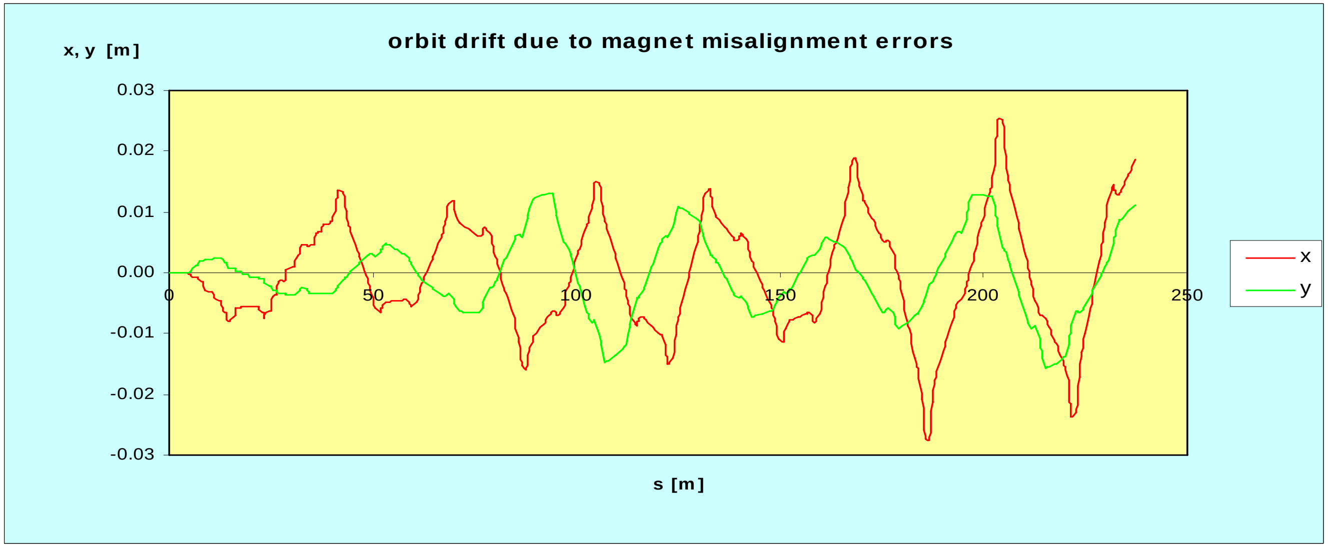
$$\left\{ \begin{array}{l} \sigma_{x'} = \sigma_{y'} = 0.8 \times 10^{-3} \\ \sigma_{x'} = \sigma_{y'} = 1.47 \times 10^{-3} \end{array} \right.$$

- Gaussian distribution was chosen for individual quad misalignments
- Resulting reference orbit distortion (uncorrected) for Arc 2 is illustrated below



- Similar level of dipole misalignment errors had virtually no effect on random steering

Arc 2 – Magnet Misalignment Errors

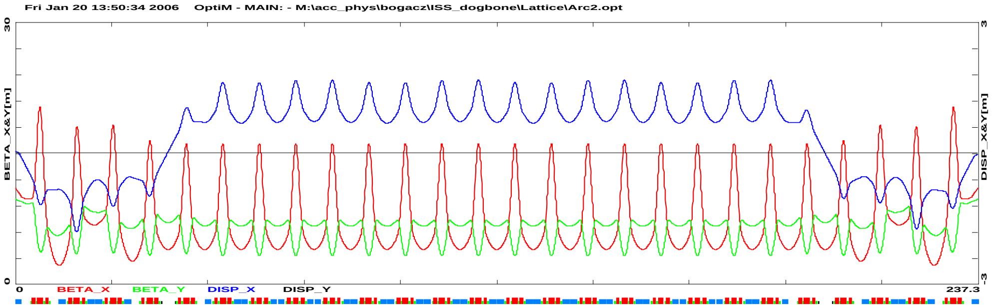


RMS Orbit Displacement [m]:

X:	0.9486e-02
y:	0.7003e-02

Extr. Orbit Displacement [m]:

X_{max}:	0.2538E-01
X_{min}:	-0.2782E-01
y_{max}:	0.1434E-01
y_{min}:	-0.1697E-01



- Same level of orbit drifts due to quad misalignments for other ‘Dogbone’ segments (Arc 1, 3 and linacs)
- Orbit drifts at the level of ~3 cm can easily be corrected by pairs of hor/vert correctors (2000 Gauss cm each) placed at every triplet girder

Focusing Error Tolerances – Quadrupole Field Spec

- Cumulative Arc-to-Arc Optics mismatch as measured by Courant-Snyder invariant change:

$$\begin{aligned}\varepsilon' &= \beta(\theta+\delta\theta)^2 + 2\alpha(\theta+\delta\theta)x + \gamma x^2 \\ &= \varepsilon\left(1 + \beta\Delta\Phi\sin(2\mu) + (\beta\Delta\Phi\cos\mu)^2\right),\end{aligned}$$

$$\begin{aligned}\varepsilon_N &= \varepsilon_0 \prod_{n=1}^{N}\left(1 + \beta_n\Delta\Phi_n\sin(2\mu_n) + (\beta_n\Delta\Phi_n\cos\mu_n)^2\right) \\ &= \varepsilon_0 \prod_{n=1}^{N}\left(1 + \frac{1}{2}(\beta_n\Delta\Phi_n)^2 + \sqrt{(\beta_n\Delta\Phi_n)^2 + \left(\frac{\beta_n\Delta\Phi_n}{2}\right)^4}\,\sin(2\mu_n+\psi_n)\right),\end{aligned}$$

- Standard deviation of Courant-Snyder invariant:

$$\boxed{\frac{\sigma_\varepsilon}{\varepsilon} = \frac{\sqrt{\overline{\Delta\varepsilon^2} - \overline{\Delta\varepsilon}^2}}{\varepsilon} \approx \sqrt{\frac{1}{2}\overline{\sum_{n=1}^{N}(\beta_n\Delta\Phi_n)^2}} = \frac{\sqrt{\overline{\Delta\Phi^2}}}{\Phi_{\max}}\sqrt{\frac{1}{2F_{\min}{}^2}\sum_{n=1}^{N}\beta_n{}^2}}$$

Focusing Error Tolerances – Quadrupole Field Spec

- By design, one can tolerate Arc-to-Arc mismatch at the level of **1%** (to be compensated by the dedicated matching quads).

$$0.01 = \frac{\sigma_\varepsilon}{\varepsilon} = \frac{\sqrt{\overline{\Delta\varepsilon^2} - \overline{\Delta\varepsilon}^2}}{\varepsilon} \approx \sqrt{\frac{1}{2}\sum_{n=1}^{N}\left(\beta_n \Delta\Phi_n\right)^2} = \frac{\sqrt{\overline{\Delta\Phi^2}}}{\Phi_{\max}} \sqrt{\frac{1}{2F_{\min}^{2}} \sum_{n=1}^{N} \beta_n^{2}}$$

- For any given Arc and the following Linac one can evaluate: $F_{\min} \approx 1\ m$ and $\sqrt{\sum_{n=1}^{N} \beta_n^{2}} \approx 50\ m$
- Thanks to well balanced, tight focusing in the Arcs and compact Spr/Rec optics the last number, 50 m, is factor of **6** smaller than the corresponding quantity for a typical CEBAF Arc-Linac segment.
- This yields the required design specification for quadrupoles of **0.2%**:

$$\frac{\sqrt{\overline{\Delta\Phi^2}}}{\Phi_{\max}} = 0.002$$

Summary

- Symmetric ‘Dogbone’ RLA (allowing to accelerate both μ^+ and μ^- species), 3.5-pass (1.5 – 5 GeV) scheme – Complete linear Optics
  - multi-pass linac optics – optimized focusing profile
  - tolerable phase ‘slippage’ in the higher pass linacs
  - mirror-symmetric Arc optics based on constant phase advance/cell (90^0) compact lattice architecture for Spr/Rec/Trans
  - geometric ‘closure’ of the ‘droplet’ Arcs
- Magnet misalignment error analysis (DIMAD Monte Carlo on the above lattice) shows quite manageable level of orbit distortion for ~1 mm level of magnet misalignment error.
- Great focusing errors tolerance for the presented lattice – 1% of Arc-to-Arc betatron mismatch limit sets the quadrupole field spec at 0.2%