# Preliminary Report on Fitting Brookhaven Dipole Field Data Using General Surface Methods

Alex Dragt and Chad Mitchell

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This document is a preliminary report on efforts to fit the field of a planned Brookhaven dipole using general surface methods. Background for this method is provided in the attached draft Chapter 19 of the book *Lie Methods for Nonlinear Dynamics with Applications to Accelerator Physics*. All of what currently exists of the book can be downloaded from the Web site http://www.physics.umd.edu/dsat/

## 1 Description of Magnet

The attached Graphics 1 and 2 show figures provided by Brookhaven that describe the dipole. The first graphic shows a top view, and the second a perspective view. The magnet is a sector magnet (the body is bent) but the pole faces are rotated so that the magnet is parallel faced. Thus, apart from a much smaller bend angle, the geometry of the planned Brookhaven dipole is analogous to that of the dipoles in the Los Alamos Proton Storage Ring.

## 2 Brookhaven Field Data

Based on the use of Opera 3D, Brookhaven provided data on a grid with

$$x \in [-0.06, 0.06], \;\; y \in [-0.016, 0.016], \;\; z \in [-1.8, 1.8] \tag{1}$$

and spacing

$$h_x = h_y = h_z = .002 \tag{2}$$

Here all quantities are in meters. Since surface methods for general geometries require knowledge of both the field $\boldsymbol{B}$ and the scalar potential $\psi$, all of these data were provided on the grid.

Upon inspection of the Brookhaven data, the values of $\boldsymbol{B}$ and $\psi$ appeared to be inconsistent because numerical differentiation of the $\psi$ data showed that

$$\boldsymbol{B} \neq -\nabla\psi. \tag{3}$$

Here the required numerical differentiation was carried out by making a 3D spline fit to $\psi$ and then differentiating this fit. The attached Graphic 3 shows the vertical field $B_y$ as a function of $z$ along the line $(x, y) = (0, .002)$, and the $y$ component of $-\nabla\psi$ along the same line. (This line is one grid spacing $h_y$ above the $z$ axis.) Also shown is the result of renormalizing $\psi$ by a constant multiplicative factor $c$. Evidently $B_y$ and the $y$ component of the renormalized $-\nabla\psi$ can be brought into near agreement by giving $c$ the value $c \simeq (4/3)$. The same was found to be true for the two other components of $\boldsymbol{B}$ and $-\nabla\psi$. Therefore, for future use, $\psi$ was renormalized by a factor of $c$ at all mesh points, with $c$ determined by requiring an optimum agreement for the case of Graphic 3. This matter needs to be explored by further consultation with Brookhaven.

We also note, *en passant*, that $B_y$ along the $z$ axis, which is essentially what is shown in Graphic 3, changes slope (as one enters the left face of the magnet) at the points $z \simeq -1.25$ and $z \simeq -1.15$, and then takes on a nearly constant value beyond $z \simeq -1$ . Inspection of Graphic 2 show that it appears to contain a multicolored "ribbon" that begins in the midplane $y = 0$, then swoops upward to pass through the top coil, and eventually hovers over the top of the yoke. Perhaps this ribbon is meant to illustrate the magnitude of $B_y$ in the midplane. If so, note that $B_y$ changes most rapidly as one goes from outside the coil to inside the coil, as one might expect. Thus, with this interpretation, there is a correspondence between the features of Graphics 2 and 3. Finally we note that Graphic 2 also appears to depict some portion of the Opera calculational grid in the midplane.

# 3 Fit of Interior Data Using Data on the Surface of a Bent Box

A bent box with straight legs at both ends, having the general geometry depicted in Figure 19.1.1 of *Lie Methods* $\cdots$ , was selected to provide a fitting surface. Here is a description of the box and legs: The box is essentially straight, with a bending angle of 0.6 degrees. The length of the arc segment = 3.22 m, and the length of each straight leg = 0.17 m. The width of the bent box and the straight legs = 0.1 m. The height of the box and legs = 0.024 m. That is, $y = \pm.012$ m for points on the top and bottom surfaces of the bent box and the straight legs.

The Brookhaven grid field data $\boldsymbol{B}$ were interpolated onto a large number of selected points (called *integration points*) on the fitting surface and $B_n$, the normal component of $\boldsymbol{B}$, was computed and stored for each integration point. The Brookhaven grid renormalized potential data, again called $\psi$, were also interpolated to these same selected integration points and stored. From this surface data ($B_n$ and $\psi$ at the integration points) it is possible to compute the vector potential $\boldsymbol{A}$ and all its spatial derivatives at any interior point, as is needed to compute a design trajectory and the transfer map $\mathcal{M}$ about this trajectory.

How well does this fitting procedure work? One test of the procedure is to compare interior fields $\boldsymbol{B}^s$ computed from the *surface* data (using $\boldsymbol{B} = \nabla \times \boldsymbol{A}$) with the Brookhaven-provided fields $\boldsymbol{B}^{bg}$ at the interior *Brookhaven grid* points. (Observe that the computation of $\boldsymbol{B}^s$ makes no use of the interior $\boldsymbol{B}^{bg}$ data.) The attached Graphics 4 through 6 compare the components of $\boldsymbol{B}^s$ and $\boldsymbol{B}^{bg}$ as a function of $z$ along the line $(x, y) = (0, .002)$. The red points are components of the Brookhaven $\boldsymbol{B}^{bg}$ data at the grid points along the line, and the green curves are the components of $\boldsymbol{B}^s$. Evidently the agreement is generally very good. The one small discrepancy is in the peak values of $B_x$, but we note that these field values are small compared to the peak value of $|\boldsymbol{B}|$ in the middle of the magnet.

We recall, for a magnet with midplane symmetry, that the field should have the property that its $x$ and $z$ components vanish in the midplane,

$$B_x(x, 0, z) = B_z(x, 0, z) = 0. \tag{4}$$

We have verified that (4) does indeed hold for the Brookhaven data. However, consistent with the results shown in the Graphics 4 and 6, the $x$ and $z$ components need not vanish outside the midplane. Indeed, the results of Graphic 6 are consistent with what would be expected for a field that bows outward at the ends of the magnet.

The matter of the small observed discrepancy requires more study, and perhaps better data from Brookhaven. We note that the success of the comparison that we have been making depends on both the proper implementation of the surface method and the requirement that the Brookhaven data well satisfy the Maxwell equations (including, near the surface, the relation $\boldsymbol{B} = -\nabla\psi$).

The calculations for the Graphics 4 through 6, and the calculation of $\boldsymbol{A}$ and its $x$ and $y$ derivatives through order 4 as described in the next section, required approximately 24 hours using a single processor at NERSC. (We remark that the effort required to compute a design trajectory and the transfer map $\mathcal{M}$ through third order about this trajectory is essentially equivalent to the effort expended in this current computation.) Due to the small height of the box, a great number of integration points is needed to resolve the behavior of the kernels $\boldsymbol{G}^n$ and $\boldsymbol{G}^t$ on the integration surface. Considerably more computation would be required to make comparisons at grid points closer to the fitting surface, because then an even greater number of integration points would be needed to resolve the behavior of the kernels on the integration surface. However, we did carry out one such calculation (with a very large number of integration points) for the grid point $(x, y, z) = (0, 0.01, 0.072)$ m and found agreement between $B_y^s$ and $B_y^{bg}$ to within 5 parts in $10^4$. Finally, we note that the surface integration calculations can readily be done in parallel, and therefore future numerical calculations should ideally be done with a multiprocessor parallel computer.

# 4 Computation of the Vector Potential

The computed interior field $\boldsymbol{B}^s$ described in the previous section was obtained using $\boldsymbol{B}^s = \nabla \times \boldsymbol{A}$, where $\boldsymbol{A}$ was computed by integrating the surface data $B_n$ and $\psi$ against the kernels $\boldsymbol{G}^n$ and $\boldsymbol{G}^t$, respectively. The attached Graphics 7 through 9 illustrate the computed components of $\boldsymbol{A}$ as a function of $z$ along the line $(x, y) = (0, .002)$. The components $A_x$ and $A_y$ decay rather slowly in $z$. Since we would like to employ reference planes for which canonical and mechanical momenta are essentially equal, it may be necessary to integrate over a larger interval in $z$ to assure that $A_x$ and $A_y$ are sufficiently small at the endpoints. This circumstance suggests that a set of data over a larger range of $z$ values may be necessary to produce a final map computation.

In a similar way, the Taylor coefficients for each component of $\boldsymbol{A}$ can be obtained, through any desired degree, by integrating the surface data $B_n$ and $\psi$ against the known spatial derivatives of the kernels $\boldsymbol{G}^n$ and $\boldsymbol{G}^t$. For example, the attached Graphic 10 illustrates the coefficient of the monomial $x^2y^2$ appearing in the expansion of $A_z$, taken as a function of $z$ along the line $(x, y) = (0, .002)$.

## 5 Conclusion

Although much remains to be done, substantial work has been accomplished on the use of general surfaces with very promising preliminary results.