SCALING

SIMPLE ARGUMENT

Assume a focusing system

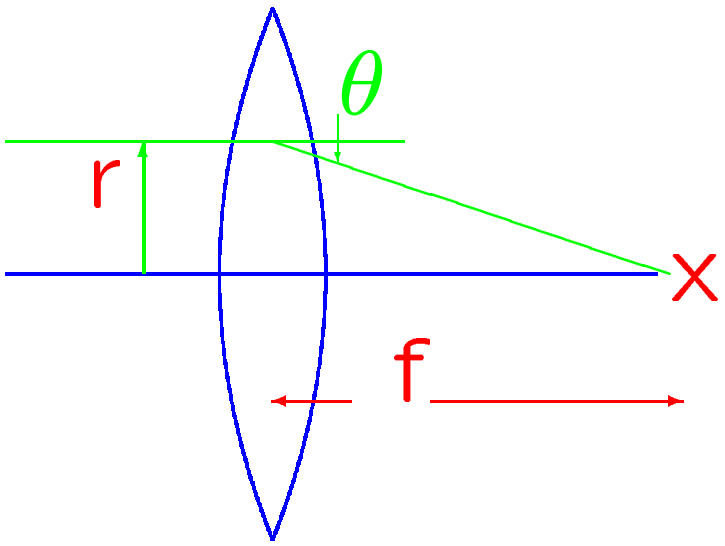
(thin-quadrupole)

$$\theta = -rkl$$

$$k = 0.3(\frac{B}{a})\frac{1}{p}[\frac{T/m}{GeV/c}]$$

$l \equiv$ eff. length of Quad

$a \equiv$ aperture Quad



Main assumption:
Length of Quad is kept in a constant ratio with the focal length

$$\boxed{l = \alpha f}$$

Substituting we get

$$\boxed{f^2 = \frac{p}{\alpha 0.3(\frac{B}{a})}}$$

New constraints: keep

apertures fixed

keep gradients fixed

LENGTHS SCALE AS $\sqrt{\gamma}$

How the emittance scale?

- $\theta = \frac{a}{f} ==> \gamma^{-1/2}$
- $\theta = \sqrt{\frac{\epsilon_N}{\gamma \beta^*}} ==>$
- $\boxed{\epsilon_N \propto \beta^*}$
- 6-D emittance

  $\epsilon_6 = \epsilon_{xN} \epsilon_{yN} \frac{\Delta p}{p} \sigma_z p$

- Fixed Energy spread ==>

- $\beta^* \propto \epsilon_N \propto \gamma^{-1/3}$

LUMINOSITY

$$\mathcal{L} = \frac{N_\mu^2 n_b n_{eff} f_{rep} \gamma}{4\pi\epsilon_N \beta^*}$$

$$\propto \gamma^{5/3}$$

keeping constant

N_μ, n_b, n_{eff} and f_{rep}

Tune Shift

Beam-beam tune shift

$$\xi = \frac{r_\mu n_\mu}{4\pi\epsilon_N} ==>$$

$$\xi \propto \gamma^{1/3}$$

Additional assumption:

Keep beam-beam tune shift

fixed

$$N_\mu \propto \gamma^{-1/3}$$

Now if current is kept constant

$$\boxed{\mathcal{L} \propto \gamma^{4/3}}$$

Summary of Scaling Relations

$Energy$	4	0.36	0.7	1.5
$Length \propto \gamma^{1/2}$	6.5	1.9	2.65	3.9
$\theta_{4\sigma} \propto \gamma^{1/2}$	4	14	10	7
$aper. \propto const.$	12	12	12	12
$\beta^* \propto \gamma^{-1/3}$	3	7	5.5	4.2
$\epsilon_N \propto \gamma^{-1/3}$	50	113	91	70
$\mathcal{L} \propto \gamma^{4/3}$	1	0.038	0.09	0.25
$N_\mu \propto \gamma^{-1/3}$	2	4.5	3.6	2.8

$Energy[TeV]$ $Length[m]$

$aperture[cm]$ $\beta^*[mm]$

$\epsilon_N[\pi \times mm - mrad]$ $N_\mu \times 10^{12}$

$\mathcal{L}[10^{35} \times cm^{-2}s^{-1}]$