fixed-field-alternating-gradient (FFAG) accelerators. Positive and negative muon bunches are then injected in opposite directions into a collider storage ring and brought into collision at the interaction point. The bunches circulate and collide for many revolutions before decay has depleted the beam intensities to an uninteresting level. Useful luminosity can be delivered for about 800 revolutions for the high-energy collider and 450 revolutions for the low-energy one.

There are many interesting and challenging problems that need to be resolved before the feasibility of building a muon collider can be demonstrated. For example, (i) heating from the very intense proton bunches may require the use of of a liquid-jet target, and (ii) attaining the desired cooling factor in the ionization-cooling channel may require the development of rf cavities with thin beryllium windows operating at liquid-nitrogen temperatures in high solenoidal fields. In addition, the development of long liquid-lithium lenses may be desirable to provide stronger radial focusing for the final cooling stages.

This article describes the status of our muon-collider feasibility studies, and is organized as follows. Section **II** gives a brief summary of the physics potential of muon colliders, including physics at the accelerator complex required for a muon collider. Section **III** describes the proton-driver specifications for a muon collider, and two site-dependent examples that have been studied in some detail. Section **IV** presents pion production, capture, and the pion-decay channel, and section **V** discusses the design of the ionization-cooling channel needed to produce an intense muon beam suitable for acceleration and injection into the final collider. Sections **VI** and **VII** describe the acceleration scenario and collider ring, respectively. Section **VIII** discusses backgrounds at the collider interaction point and section **IX** deals with possible detector scenarios. A summary of the conclusions is given in section **X**.

II. THE PHYSICS POTENTIAL OF MUON COLLIDERS

A. Brief overview

The physics agenda at a muon collider falls into three categories: First Muon Collider (FMC) physics at a machine with center-of-mass energies of 100 to 500 GeV; Next Muon Collider (NMC) physics at 3–4 TeV center-of-mass energies; and front-end physics with a high-intensity muon source.

The FMC will be a unique facility for neutral Higgs boson (or techni-resonance) studies through s-channel resonance production. Measurements can also be made of the threshold cross sections for production of W^+W^-, $t\bar{t}$, Zh, and pairs of supersymmetry particles — $\chi_1^+\chi_1^-$, $\chi_2^0\chi_1^0$, $\tilde{\ell}^+\tilde{\ell}^-$ and $\tilde{\nu}\bar{\tilde{\nu}}$ — that will determine the corresponding masses to high precision. A $\mu^+\mu^- \to Z^0$ factory, utilizing the partial polarization of the muons, could allow significant improvements in $\sin^2\theta_{\rm w}$ precision and in B-mixing and CP-violating studies. In Fig. 4, we show the cross sections for standard model (SM) processes versus the CoM energy at the FMC. For the unique s-channel Higgs boson production, where $\sqrt{s}_{\mu\mu} = m_H$, results for three different beam energy resolutions are presented.

The NMC will be particularly valuable for reconstructing supersymmetric particles of high mass from their complex cascade decay chains. Also, any Z' resonances within the kinematic reach of the machine would give enormous event rates. The effects of virtual Z' states would be detectable to high mass. If no Higgs bosons exist below ~1 TeV, then the NMC would be the ideal machine for the study of strong WW scattering at TeV energies.

At the front end, a high-intensity muon source will permit searches for rare muon processes sensitive to branching ratios that are orders of magnitude below present upper limits. Also, a high-energy muon-proton collider can be constructed to probe high$-Q^2$ phenomena beyond the reach of the HERA ep collider. In addition, the decaying muons will provide high-intensity neutrino beams for precision neutrino cross-section measurements and for long-baseline experiments [57–66]. Plus, there are numerous other new physics possibilities for muon facilities [44,39] that we will not discuss in detail in this document.

B. Higgs boson physics

The expectation that there will be a light (mass below $2M_W$) SM-like Higgs boson provides a major motivation for the FMC, since such a Higgs boson can be produced with a very high rate directly in the s-channel. Theoretically, the lightest Higgs boson h^0 of the most general supersymmetric model is predicted to have mass below 150 GeV and to be very SM-like in the usual decoupling limit. Indeed, in the minimal supersymmetric model, which contains the five Higgs bosons h^0, H^0, A^0, $H^\pm$, one finds $m_{h^0} \lesssim 130$ GeV and the h^0 is SM-like if $m_{A^0} \gtrsim 130$ GeV. Experimentally, global analyses of precision electroweak data now indicate a strong preference for a light SM-like Higgs boson. The goals of the FMC for studying the SUSY Higgs sector via s-channel resonance production are: to measure the light Higgs mass, width, and branching fractions with high precision, in particular sufficient to differentiate the minimal

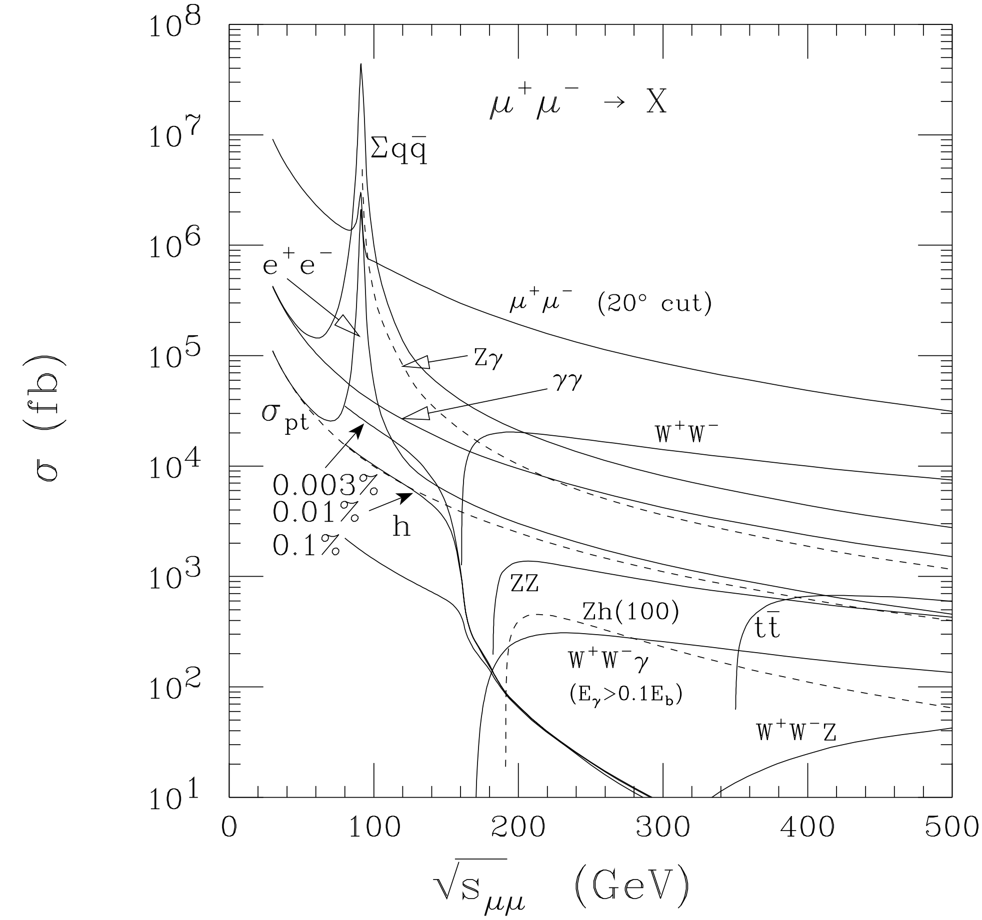

FIG. 4. Cross sections for SM processes versus the CoM energy at the FMC. $\sigma_{pt} \equiv \sigma(\mu^+\mu^- \to \gamma^* \to e^+e^-)$. For the s-channel Higgs boson production, three different beam energy resolutions of 0.003%, 0.01% and 0.1% are presented.

supersymmetric standard model (MSSM) h^0 from the SM $h_{\rm SM}$; and, to find and study the heavier neutral Higgs bosons H^0 and A^0.

The production of Higgs bosons in the s-channel with interesting rates is a unique feature of a muon collider [45,67]. The resonance cross section is

$$\sigma_h(\sqrt{s}) = \frac{4\pi ,\, (h\to\mu\bar{\mu})\, ,\, (h\to X)}{\left(s-m_h^2\right)^2 + m_h^2 \left(,\, ^h_{\rm tot}\right)^2}\,. \tag{1}$$

Gaussian beams with root-mean-square (rms) energy resolution down to $R = 0.003\%$ are realizable. The corresponding rms spread $\sigma_{\sqrt{s}}$ in CoM energy is

$$\sigma_{\sqrt{s}} = (2\ {\rm MeV}) \left(\frac{R}{0.003\%}\right) \left(\frac{\sqrt{s}}{100\ {\rm GeV}}\right)\,. \tag{2}$$

The effective s-channel Higgs cross section convolved with a Gaussian spread,

$$\bar{\sigma}_h(\sqrt{s}) = \frac{1}{\sqrt{2\pi}\,\sigma_{\sqrt{s}}} \int \sigma_h(\sqrt{\hat{s}})\ \exp\left[\frac{-\left(\sqrt{\hat{s}}-\sqrt{s}\right)^2}{2\sigma^2_{\sqrt{s}}}\right] d\sqrt{\hat{s}}, \tag{3}$$

is illustrated in Fig. 5 for $m_h = 110$ GeV, $,\,_h = 2.5$ MeV, and resolutions $R = 0.01\%$, 0.06% and 0.1%. A resolution $\sigma_{\sqrt{s}} \sim ,\,_h$ is needed to be sensitive to the Higgs width. The light Higgs width is predicted to be

$$\begin{array}{ll} ,\ \approx 2 \text{ to } 3\ {\rm MeV} & \text{if } \tan\beta \sim 1.8, \\ ,\ \approx 2 \text{ to } 800\ {\rm MeV} & \text{if } \tan\beta \sim 20, \end{array} \tag{4}$$

for 80 GeV $\lesssim m_h \lesssim$ 120 GeV, where the smaller values apply in the decoupling limit of large m_{A^0}. We note that, in the MSSM, m_{A^0} is required to be in the decoupling regime in the context of minimal super gravity (mSUGRA) boundary conditions in order that correct electroweak symmetry breaking arises after evolution of parameters from the unification scale. In particular, decoupling applies in mSUGRA at $\tan\beta \sim 1.8$, corresponding to the infrared fixed point of the top quark Yukawa coupling.

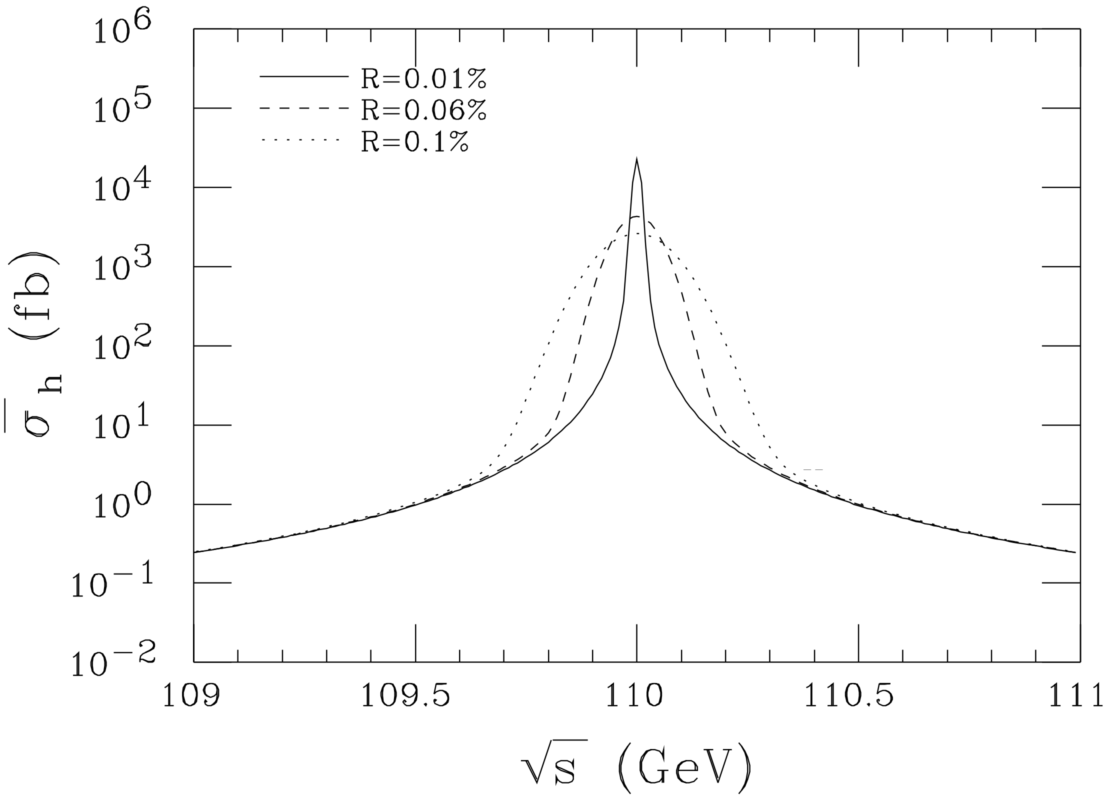

FIG. 5. Effective s-channel Higgs cross section $\bar{\sigma}_h$ obtained by convoluting the Breit-Wigner resonance formula with a Gaussian distribution for resolution R. From Ref. [45].

At $\sqrt{s} = m_h$, the effective s-channel Higgs cross section is

$$\bar{\sigma}_h \simeq \frac{4\pi}{m_h^2} \, \frac{\mathrm{BF}(h \to \mu\bar{\mu})\,\mathrm{BF}(h \to X)}{\left[1 + \frac{8}{\pi}\left(\frac{\sigma_{\sqrt{s}}}{\Gamma^h_{\rm tot}}\right)^2\right]^{1/2}} \,. \tag{5}$$

BF denotes the branching fraction for h decay; also, note that $\bar{\sigma}_h \propto 1/\sigma_{\sqrt{s}}$ for $\sigma_{\sqrt{s}} > {,}^{h}_{\rm tot}$. At $\sqrt{s} = m_h \approx 110$ GeV, the $b\bar{b}$ rates are

$$\text{signal} \approx 10^4 \text{ events} \times \mathrm{L}(\mathrm{fb}^{-1})\,, \tag{6}$$

$$\text{background} \approx 10^4 \text{ events} \times \mathrm{L}(\mathrm{fb}^{-1})\,, \tag{7}$$

assuming a b-tagging efficiency $\epsilon \sim 0.5$ and an energy resolution of 0.003%. The effective on-resonance cross sections for other m_h values and other channels (ZZ^*, WW^*) are shown in Fig. 6 for the SM Higgs. The rates for the MSSM Higgs are nearly the same as the SM rates in the decoupling regime of large m_{A^0}.

The important factors that make s-channel Higgs physics studies possible at a muon collider are energy resolutions $\sigma_{\sqrt{s}}$ of order a few MeV, little bremsstrahlung and no beamstrahlung smearing, and precise tuning of the beam energy to an accuracy $\Delta E \sim 10^{-6}E$ through continuous spin-rotation measurements [12]. As a case study, we consider a SM-like Higgs boson with $m_h \approx 110$ GeV. Prior Higgs discovery is assumed at the Tevatron (in $Wh, t\bar{t}h$ production with $h \to b\bar{b}$ decay) or at the LHC (in $gg \to h$ production with $h \to \gamma\gamma, 4\ell$ decays with a mass measurement of $\Delta m_h \sim 100$ MeV for an integrated luminosity of $L = 300$ fb^{-1}) or possibly at a NLC (in $Z^* \to Zh, h \to b\bar{b}$ giving $\Delta m_h \sim 50$ MeV for $L = 200$ fb^{-1}). A muon collider ring design would be optimized to run at energy $\sqrt{s} = m_h$. For

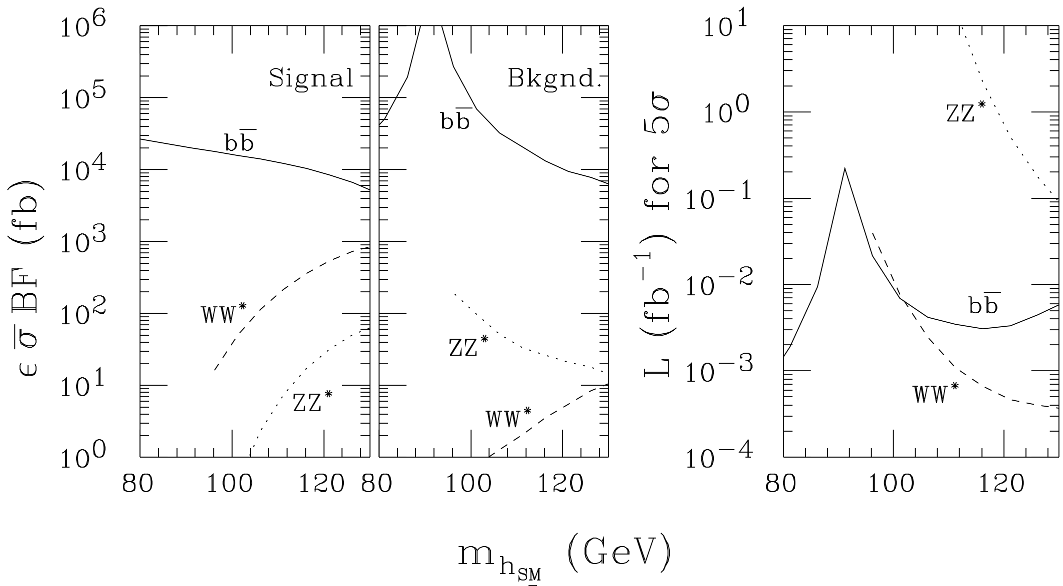

FIG. 6. The SM Higgs cross sections and backgrounds in $b\bar{b}$, WW^* and ZZ^*. Also shown is the luminosity needed for a 5 standard deviation detection in $b\bar{b}$. From Ref. [45].

an initial Higgs-mass uncertainty of $\Delta m_h \sim 100$ MeV, the maximum number of scan points required to locate the s-channel resonance peak at the muon collider is

$$n = \frac{2\Delta m_h}{\sigma_{\sqrt{s}}} \approx 100 \tag{8}$$

for a $R = 0.003\%$ resolution of $\sigma_{\sqrt{s}} \approx 2$ MeV. The necessary luminosity per scan point ($L_{\rm s.p.}$) to observe or eliminate the h-resonance at a significance level of $S/\sqrt{B} = 3$ is $L_{\rm s.p.} \sim 1.5 \times 10^{-3}\,{\rm fb}^{-1}$. (The scan luminosity requirements increase for m_h closer to M_Z; at $m_h \sim M_Z$ the $L_{\rm s.p.}$ needed is a factor of 50 higher.) The total luminosity then needed to tune to a Higgs boson with $m_h = 110$ GeV is $L_{\rm tot} = 0.15\ {\rm fb}^{-1}$. If the machine delivers $1.5 \times 10^{31}\,{\rm cm}^{-2}\,{\rm s}^{-1}$ ($0.15\ {\rm fb}^{-1}$/year), then one year of running would suffice to complete the scan and measure the Higgs mass to an accuracy $\Delta m_h \sim 1$ MeV. Figure 7 illustrates a simulation of such a scan.

Once the h-mass is determined to ~ 1 MeV, a 3-point fine scan [45] can be made across the peak with higher luminosity, distributed with L_1 at the observed peak position in $\sqrt{s}$ and $2.5L_1$ at the wings ($\sqrt{s}$ = peak $\pm\, 2\sigma_{\sqrt{s}}$). Then, with $L_{\rm tot} = 0.4\ {\rm fb}^{-1}$ the following accuracies would be achievable: 16% for , $^h_{\rm tot}$, 1% for $\sigma{\rm BF}({\rm b\bar{b}})$ and 5% for $\sigma{\rm BF}({\rm WW}^*)$. The ratio $r = {\rm BF}(WW^*)/{\rm BF}(b\bar{b})$ is sensitive to m_{A^0} for m_{A^0} values below 500 GeV. For example, $r_{\rm MSSM}/r_{\rm SM} = 0.3, 0.5, 0.8$ for $m_{A^0} = 200, 250, 400$ GeV [45]. Thus, using s-channel measurements of the h, it may be possible not only to distinguish the h^0 from the SM h_{SM} but also to infer m_{A^0}.

The study of the other neutral MSSM Higgs bosons at a muon collider via the s-channel is also of major interest. Finding the H^0 and A^0 may not be easy at other colliders. At the LHC the region $m_{A^0} > 200$ GeV is deemed to be inaccessible for $3 \lesssim \tan\beta \lesssim$ 5–10 [68]. At an NLC the $e^+e^- \to H^0A^0$ production process may be kinematically inaccessible if H^0 and A^0 are heavy (mass > 230 GeV for $\sqrt{s} = 500$ GeV). At a $\gamma\gamma$ collider, very high luminosity ($\sim$200 ${\rm fb}^{-1}$) would be needed for $\gamma\gamma \to H^0, A^0$ studies.

At a muon collider the resolution requirements for s-channel H^0 and A^0 studies are not as demanding as for the h^0 because the H^0, A^0 widths are broader; typically , ~ 30 MeV for $m_{A^0} < 2m_t$ and , ~ 3 GeV for $m_{A^0} > 2m_t$. Consequently $R \sim 0.1\%$ ($\sigma_{\sqrt{s}} \sim 70$ MeV) is adequate for a scan. This is important, since higher instantaneous luminosities (corresponding to $L \sim 2-10\ {\rm fb}^{-1}$/yr) are possible for $R \sim 0.1\%$ (as contrasted with the $L \sim 0.15\ {\rm fb}^{-1}$/yr for the much smaller $R \sim 0.003\%$ preferred for studies of the h^0). A luminosity per scan point $L_{\rm s.p.} \sim 0.1\ {\rm fb}^{-1}$ probes the parameter space with $\tan\beta > 2$. The $\sqrt{s}$-range over which the scan should be made depends on other information available to indicate the A^0 and H^0 mass range of interest. A wide scan would not be necessary if r is measured with the above-described precision to obtain an approximate value of m_{A^0}.

In the MSSM, $m_{A^0} \approx m_{H^0} \approx m_{H^\pm}$ at large m_{A^0} (as expected for mSUGRA boundary conditions), with a very close degeneracy in these masses for large $\tan\beta$. In such a circumstance, only an s-channel scan with the good resolution possible at a muon collider may allow separation of the A^0 and H^0 states; see Fig. 8.

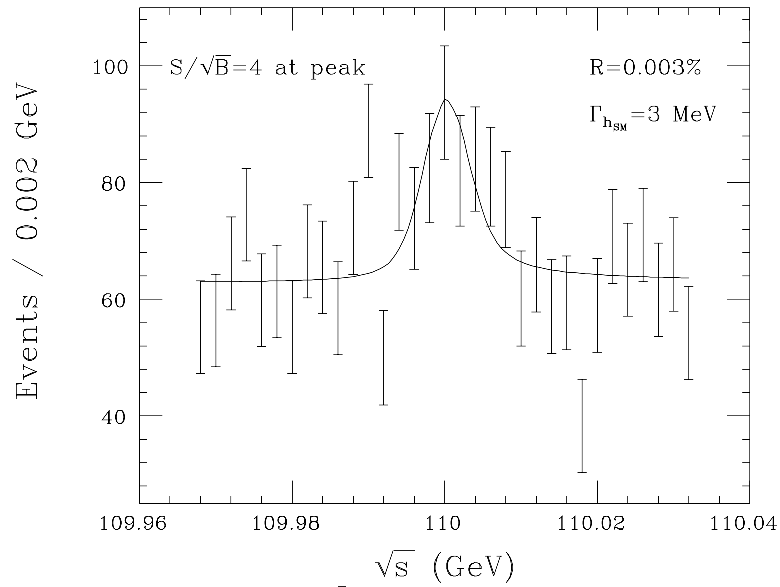

FIG. 7. Number of events and statistical errors in the $b\bar{b}$ final states as a function of $\sqrt{s}$ in the vicinity of $m_{h_{\rm SM}} = 110$ GeV, assuming $R = 0.003\%$. From Ref. [45].

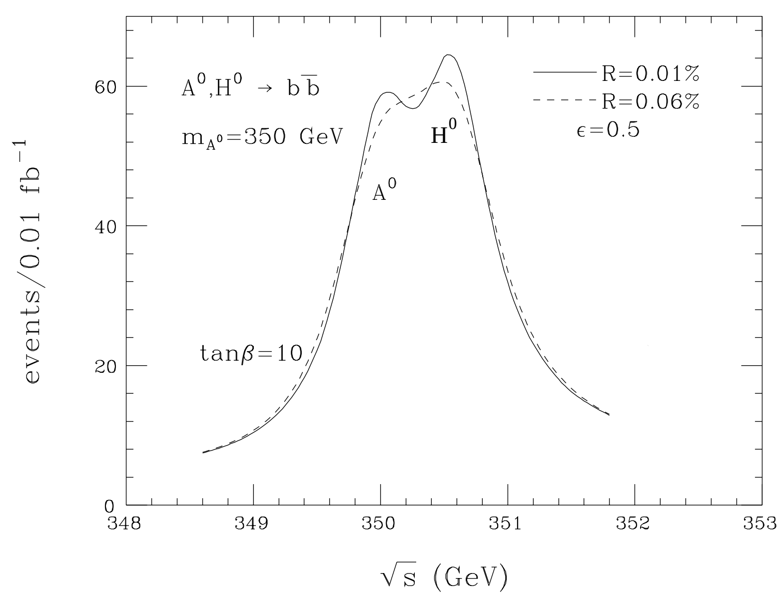

FIG. 8. Separation of A^0 and H^0 signals for $\tan\beta = 10$. From Ref. [45].

C. Light particles in technicolor models

In most technicolor models, there will be light neutral and colorless technipion resonances, π_T^0 and $\pi_T^{0\prime}$, with masses below 500 GeV. Sample models include the recent top-assisted technicolor models [69], in which the technipion masses are typically above 100 GeV, and models [70] in which the masses of the neutral colorless resonances come primarily from the one-loop effective potential and the lightest state typically has mass as low as 10 to 100 GeV. The widths of these light neutral and colorless states in the top-assisted models will be of order 0.1 to 50 GeV [71]. In the one-loop models, the width of the lightest technipion is typically in the range from 3 to 50 MeV. Neutral technirho and techniomega resonances are also a typical feature of technicolor models. In all models, these resonances are predicted to have substantial Yukawa-like couplings to muons and would be produced in the s-channel at a muon collider,

$$\mu^+\mu^- \to \pi_T^0,\, \pi_T^{0\prime},\, \rho_T^0,\, \omega_T^0, \tag{9}$$

with high event rates. The peak cross sections for these processes are estimated to be $\approx 10^4$–10^7 fb [71]. The dominant decay modes depend on eigenstate composition and other details but typically are [71]

$$\pi_T^0 \to gg,\, b\bar{b},\, \tau\bar{\tau},\, c\bar{c},\, t\bar{t}\,, \tag{10}$$
$$\pi_T^{0\prime} \to gg,\, b\bar{b},\, c\bar{c},\, t\bar{t},\, \tau^+\tau^-\,, \tag{11}$$
$$\rho_T^0 \to \pi_T\pi_T,\, W\pi_T,\, WW\,, \tag{12}$$
$$\omega_T^0 \to c\bar{c},\, b\bar{b},\, \tau\bar{\tau},\, t\bar{t},\, \gamma\pi_T^0,\, Z\pi_T^0\,. \tag{13}$$

Such resonances would be easy to find and study at a muon collider.

D. Exotic narrow resonance possibilities

There are important types of exotic physics that would be best probed in s-channel production of a narrow resonance at a muon collider. Many extended Higgs sector models contain a doubly-charged Higgs boson Δ^{--} (and its Δ^{++} partner) that couples to $\mu^-\mu^-$ via a Majorana coupling. The s-channel process $\mu^-\mu^- \to \Delta^{--}$ has been shown [72] to probe extremely small values of this Majorana coupling, in particular values naturally expected in models where such couplings are responsible for neutrino mass generation. In supersymmetry, it is possible that there is R-parity violation. If R-parity violation is of the purely leptonic type, the coupling $\lambda_{\mu\tau\mu}$ for $\mu^-\mu^+ \to \tilde{\nu}_\tau$ is very possibly the largest such coupling and could be related to neutrino mass generation. This coupling can be probed down to quite small values via s-channel $\tilde{\nu}_\tau$ production at the muon collider [73].

E. Z-factory

A muon collider operating at the Z-boson resonance energy is an interesting option for measurement of polarization asymmetries, B_s^0–$\bar{B}_s^0$ mixing, and of CP violation in the B-meson system [74]. The muon collider advantages are the partial muon beam polarization, and the long B-decay length for B-mesons produced at this $\sqrt{s}$. The left-right asymmetry A_{LR} is the most accurate measure of $\sin^2\theta_{\rm w}$, since the uncertainty is statistics dominated. The present LEP and SLD polarization measurements show deviations from the Standard Model prediction by 2.4σ in A_{LR}^0, 1.9σ in $A_{FB}^{0,b}$ and 1.7σ in $A_{FB}^{0,\tau}$ [75]. The CP angle β could be measured from $B^0 \to K_s J/\psi$ decays. To achieve significant improvements over existing measurements and those at future B-facilities, a data sample of 10^8 Z-boson events/year would be needed. This corresponds to a luminosity > 0.15 fb^{-1} /year, which is well within the domain of muon collider expectations; $R \sim 0.1\%$ would be more than adequate, given the substantial ~ 2.4 GeV width of the Z.

F. Threshold measurements at a muon collider

With 10 fb^{-1} integrated luminosity devoted to a measurement of a threshold cross-section, the following precisions on particle masses may be achievable [76]:

$$\begin{array}{ll} \mu^+\mu^- \to W^+W^- & \Delta M_W = 20 \text{ MeV}\,, \\ \mu^+\mu^- \to t\bar{t} & \Delta m_t = 0.2 \text{ GeV}\,, \\ \mu^+\mu^- \to Zh & \Delta m_h = 140 \text{ MeV} \end{array} \tag{14}$$

(if $m_h = 100$ GeV). Precision M_W and m_t measurements allow important tests of electroweak radiative corrections through the relation

$$M_W = M_Z \left[1 - \frac{\pi\alpha}{\sqrt{2}\,G_\mu\, M_W^2(1-\delta r)}\right]^{1/2}\,, \tag{15}$$

where δr represents loop corrections. In the SM, δr depends on m_t^2 and $\log m_h$. The optimal precision for tests of this relation is $\Delta M_W \approx \frac{1}{140}\Delta m_t$, so the uncertainty on M_W is the most critical. With $\Delta M_W = 20$ MeV the SM Higgs mass could be inferred to an accuracy

$$\Delta m_{h_{\rm SM}} = 30 \text{ GeV} \left(\frac{m_h}{100 \text{ GeV}}\right)\,. \tag{16}$$

Alternatively, once m_h is known from direct measurements, SUSY loop contributions can be tested.

In top-quark production at a muon collider above the threshold region, modest muon polarization would allow sensitive tests of anomalous top quark couplings [77].

One of the important physics opportunities for the First Muon Collider is the production of the lighter chargino, $\tilde{\chi}_1^+$ [78]. Fine-tuning arguments in mSUGRA suggest that it should be lighter than 200 GeV. A search at the upgraded Tevatron for the process $q\bar{q} \to \tilde{\chi}_1^+\tilde{\chi}_2^0$ with $\tilde{\chi}_1^+ \to \tilde{\chi}_1^0\ell^+\nu$ and $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\ell^+\ell^-$ decays can potentially reach masses $m_{\tilde{\chi}_1^+} \simeq m_{\tilde{\chi}_2^0} \sim 170$ GeV with 2 fb^{-1} luminosity and ~ 230 GeV with 10 fb^{-1} [79]. The mass difference $M(\tilde{\chi}_2^0) - M(\tilde{\chi}_1^0)$ can be determined from the $\ell^+\ell^-$ mass distribution.

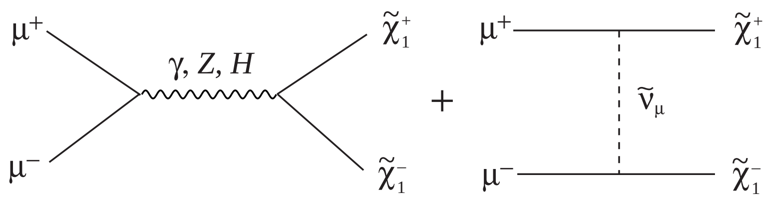

FIG. 9. Diagrams for production of the lighter chargino.

The two contributing diagrams in the chargino pair production process are shown in Fig. 9; the two amplitudes interfere destructively. The $\tilde{\chi}_1^+$ and $\tilde{\nu}_\mu$ masses can be inferred from the shape of the cross section in the threshold region [80]. The chargino decay is $\tilde{\chi}_1^+ \to f\bar{f}'\tilde{\chi}_1^0$. Selective cuts suppress the background from W^+W^- production and leave $\sim 5\%$ signal efficiency for 4 jets + $\not{E}_T$ events. Measurements at two energies in the threshold region with total luminosity $L = 50$ fb and resolution $R = 0.1\%$ can give the accuracies listed in table II on the chargino mass for the specified values of $m_{\tilde{\chi}_1^+}$ and $m_{\tilde{\nu}_\mu}$.

TABLE II. Achievable uncertainties with 50 fb^{-1} luminosity on the mass of the lighter chargino for representative $m_{\tilde{\chi}_1^+}$ and $m_{\tilde{\nu}_\mu}$ masses. From Ref. [80].

$\Delta m_{\tilde{\chi}_1^+}$ (MeV)	$m_{\tilde{\chi}_1^+}$ (GeV)	$m_{\tilde{\nu}_\mu}$ (GeV)
35	100	500
45	100	300
150	200	500
300	200	300

G. Heavy particles of supersymmetry

The requirements of gauge coupling unification can be used to predict the mean SUSY mass scale, given the value of the strong coupling constant at the Z-mass scale. Figure 10 shows the SUSY GUT predictions versus $\alpha_s(M_Z)$. For the value $\alpha_s(M_Z) = 0.1214 \pm 0.0031$ from a new global fit to precision electroweak data [75], a mean SUSY mass of order 1 TeV is expected. Thus, it is likely that some SUSY particles will have masses at the TeV scale. Large masses for the squarks of the first family are perhaps the most likely in that this would provide a simple cure for possible flavor changing neutral current difficulties.

At the LHC, mainly squarks and gluinos will be produced; these decay to lighter SUSY particles. The LHC will be a great SUSY machine, but some sparticle measurements will be very difficult or impossible there [81,82], namely: (i) the determination of the LSP mass (LHC measurements give SUSY mass differences); (ii) study of sleptons of mass $\gtrsim 200$ GeV because Drell-Yan production becomes too small at these masses; (iii) study of heavy gauginos $\tilde{\chi}_2^\pm$ and $\tilde{\chi}_{3,4}^0$, which are mainly Higgsino and have small direct production rates and small branching fractions to channels usable for detection; (iv) study of heavy Higgs bosons $H^\pm,\ H^0,\ A^0$ when the MSSM $\tan\beta$ parameter is not large and their masses are larger than $2m_t$, so that cross sections are small and decays to $t\bar{t}$ are likely to be dominant (their detection is deemed impossible if SUSY decays dominate).

Detection and study of the many scalar particles predicted in supersymmetric models could be a particularly valuable contribution of a high energy lepton collider. However, since pair production of scalar particles at a lepton collider is P-wave suppressed, energies well above threshold are needed for sufficient production rates; see Fig. 11. For scalar particle masses of order 1 TeV a collider energy of 3 to 4 TeV is needed to get past the threshold suppression. A

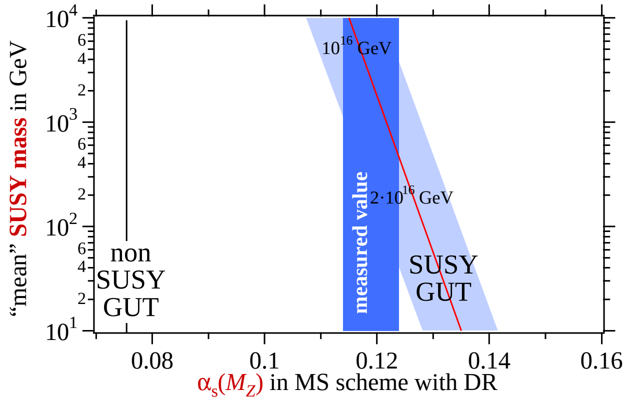

FIG. 10. α_s prediction in supersymmetric GUT with minimal particle content in the Dimensional Regularization scheme.

muon collider operating in this energy range with high luminosity ($L \sim 10^2$ to 10^3 fb^{-1}/year) would provide sufficient event rates to reconstruct heavy sparticles from their complex cascade decay chains [82,84].

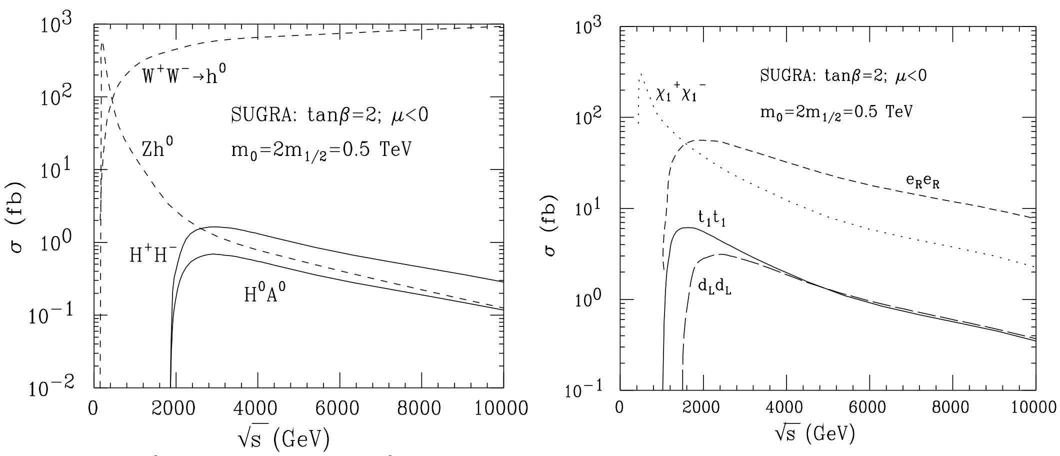

FIG. 11. Cross sections for pair production of Higgs bosons and scalar particles at a high-energy muon collider. From Ref. [83].

In string models, it is very natural to have extra Z bosons in addition to low-energy supersymmetry. The s-channel production of a Z' boson at the resonance energy would give enormous event rates at the NMC. Moreover, the s-channel contributions of Z' bosons with mass far above the kinematic reach of the collider could be revealed as contact interactions [85].

H. Strong scattering of weak bosons

The scattering of weak bosons can be studied at a high-energy muon collider through the process in Fig. 12. The amplitude for the scattering of longitudinally polarized W-bosons behaves like

$$A(W_L W_L \to W_L W_L) \sim m_H^2/v^2 \tag{17}$$

if there is a light Higgs boson, and like

$$A(W_L W_L \to W_L W_L) \sim s_{WW}/v^2 \tag{18}$$

if no light Higgs boson exists; here s_{WW} is the square of the WW CoM energy and $v = 246$ GeV. In the latter scenario, partial-wave unitarity of $W_L W_L \to W_L W_L$ requires that the scattering of weak bosons becomes strong at energy scales of order 1 to 2 TeV. Thus, subprocess energies $\sqrt{s_{WW}} \gtrsim 1.5$ TeV are needed to probe strong WW scattering effects.

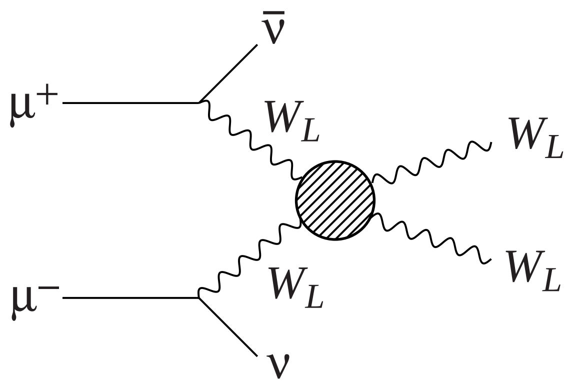

FIG. 12. Symbolic diagram for strong WW scattering.

The nature of the dynamics in the WW sector is unknown. Models for this scattering assume heavy resonant particles (isospin scalar and vector) or a non-resonant amplitude based on a unitarized extrapolation of the low-energy theorem behavior $A \sim s_{WW}/v^2$. In all models, impressive signals of strong WW scattering are obtained at the NMC, with cross sections typically of order 50 fb [86]. Event rates are such that the various weak-isospin channels ($I = 0, 1, 2$) could be studied in detail as a function of s_{WW}. After several years of operation, it would even be possible to perform such a study after projecting out the different final polarization states ($W_L W_L$, $W_L W_T$ and $W_T W_T$), thereby enabling one to verify that it is the $W_L W_L$ channel in which the strong scattering is taking place.

I. Front end physics

New physics is likely to have important lepton flavor dependence and may be most apparent for heavier flavors. The intense muon source available at the front end of the muon collider will provide many opportunities for uncovering such physics.

1. Rare muon decays

The planned muon flux of $\sim 10^{14}$ muons/sec for a muon collider dramatically eclipses the flux, $\sim 10^8$ muons/sec, of present sources. With an intense source, the rare muon processes $\mu \to e\gamma$ (for which the current branching fraction limit is 0.49×10^{-12}), $\mu N \to eN$ conversion, and the muon electric dipole moment can be probed at very interesting levels. A generic prediction of supersymmetric grand unified theories is that these lepton flavor violating or CP-violating processes should occur via loops at significant rates, e.g. BF$(\mu \to e\gamma) \sim 10^{-13}$. Lepton-flavor violation can also occur via Z' bosons, leptoquarks, and heavy neutrinos [87].

2. Neutrino flux

The decay of a muon beam leads to neutrino beams of well defined flavors. A muon collider would yield a neutrino flux 1000 times that presently available [88]. This would result in $\sim 10^6$ νN and $\bar{\nu} N$ events per year, which could be used to measure charm production ($\sim$6% of the total cross section) and measure $\sin^2\theta_{\rm w}$ (and infer the W-mass to an accuracy $\Delta M_W \simeq 30$–50 MeV in one year) [57–65].

3. Neutrino oscillations

A special purpose muon ring has been proposed [58] to store $\sim 10^{21}$ μ^+ or μ^- per year and obtain $\sim 10^{20}$ neutrinos per year from muon decays along $\sim$75-m straight sections of the ring, which would be pointed towards a distant neutrino detector. The neutrino fluxes from $\mu^- \to \nu_\mu \bar{\nu}_e e^-$ or from $\mu^+ \to \bar{\nu}_\mu \nu_e e^+$ decays can be calculated with little systematic error. Then, for example, from the decays of stored μ^-'s, the following neutrino oscillation channels could be studied by detection of the charged leptons from the interactions of neutrinos in the detector:

oscillation	detect
$\nu_\mu \to \nu_e$	e^-
$\nu_\mu \to \nu_\tau$	τ^-
$\bar{\nu}_e \to \bar{\nu}_\mu$	μ^+
$\bar{\nu}_e \to \bar{\nu}_\tau$	τ^+

The detected e^- or μ^+ have the "wrong sign" from the leptons produced by the interactions of the $\bar{\nu}_e$ and ν_μ flux. The known neutrino fluxes from muon decays could be used for long-baseline oscillation experiments at any detector on Earth. The probabilities for vacuum oscillations between two neutrino flavors are given by

$$P(\nu_a \to \nu_b) = \sin^2 2\theta \, \sin^2(1.27 \delta m^2 L/E) \tag{19}$$

with δm^2 in eV^2 and L/E in km/GeV. In a very long baseline experiment from Fermilab to the Gran Sasso laboratory or the Kamioka mine ($L = \mathcal{O}(10^4)$ km) with ν-energies $E_\nu = 20$ to 50 GeV ($L/E = 500$–200 km/GeV), neutrino charged-current interaction rates of $\sim 10^3$/year would result. In a long baseline experiment from Fermilab to the Soudan mine (L=732 km), the corresponding interaction rate is $\sim 10^4$/year. Such an experiment would have sensitivity to oscillations down to $\delta m^2 \sim 10^{-4} - 10^{-5}\,\mathrm{eV}^2$ for $\sin^2 2\theta = 1$ [58].

4. μp collider

The possibility of colliding 200-GeV muons with 1000-GeV protons at Fermilab is under study. This collider would reach a maximum $Q^2 \sim 8 \times 10^5$ GeV^2, which is $\sim$8 times the reach of the HERA *ep* collider, and deliver a luminosity $\sim 10^{33}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$, which is $\sim$300 times the HERA luminosity. The μp collider would produce $\sim 10^6$ neutral-current deep-inelastic-scattering events per year at $Q^2 > 5000$ GeV^2, which is more than a factor of 10^3 higher than at HERA. In the new physics realm, leptoquark couplings and contact interactions, if present, are likely to be larger for muons than for electrons. This μp collider would have sufficient sensitivity to probe leptoquarks up to a mass $M_{LQ} \sim 800$ GeV and contact interactions to a scale $\Lambda \sim 6$–9 TeV [89].

J. Summary of the physics potential

The First Muon Collider offers unique probes of supersymmetry (particularly s-channel Higgs boson resonances) and technicolor models (via s-channel production of techni-resonances), high-precision threshold measurements of W, t and SUSY particle masses, tests of SUSY radiative corrections that indirectly probe the existence of high-mass squarks, and a possible Z^0 factory for improved precision in polarization measurements and for B-physics studies of CP violation and mixing.

The Next Muon Collider guarantees access to heavy SUSY scalar particles and Z' states or to strong WW scattering if there are no Higgs bosons and no supersymmetry.

The Front End of a muon collider offers dramatic improvements in sensitivity for flavor-violating transitions (e.g., $\mu \to e\gamma$), access to high-Q^2 phenomena in deep-inelastic muon-proton and neutrino-proton interactions, and the ability to probe very small δm^2 via neutrino-oscillation studies in long-baseline experiments.

The muon collider would be crucial to unraveling the flavor dependence of any type of new physics that is found at the next generation of colliders.

Thus, muon colliders are robust options for probing new physics that may not be accessible at other colliders.

III. PROTON DRIVER

The overview of the required parameters is followed by a description of designs that have been studied in some detail. The section concludes with a discussion of the outstanding open issues.