

Helical Wiggler: no skew components

fixed point (orbit) has $B_r = 0$

either $\theta = 0$ or $\theta = \pi$

corr. to below or above resonance

$$K = \pm \frac{2\pi}{L} \quad \left\{ \begin{array}{l} + \\ - \end{array} \right\} \rightarrow \begin{array}{l} \text{RH twist} \\ \text{LH twist} \end{array}$$

$$K_s = \frac{e B_0}{p_z}$$

B_0 = uniform solenoid field

stability depends on $\left. \frac{dB_\theta}{dr} \right|_{r, \theta}$; $g \equiv -\frac{1+K^2 r^2}{p_z} \left(\frac{\partial B_\theta}{\partial r} \right)$

stability condition:

$$0 < G = [K(K_s - K) + g] \left[K(K_s - K) \frac{1-K^2 r^2}{1+K^2 r^2} + K^2 \frac{K^2 r^2}{1+K^2 r^2} - g \right]$$

$$= K^2 \frac{r}{p} \frac{dp}{dr}$$

equivalently,

$$\left| g + \frac{1}{2} \frac{K^2 r^2}{1+K^2 r^2} [2K(K_s - K) - K^2] \right| < \left| \frac{K^2 K^2 r^2 + 2K(K_s - K)}{2(1+K^2 r^2)} \right|$$

Simplified case:

neglect $K^2 r^2$, g

linear equations of motion are

$$\xi'' - (2K - K_s) \xi' + K(K_s - K) \xi = 0$$

$$\xi = \frac{\Delta r}{r}$$

$$\psi'' + (2K - K_s) \psi' + K(K_s - K) \psi = 0$$

$$\psi = \Delta \theta$$

symmetric, $\xi \rightarrow \psi, \psi \rightarrow -\xi$

$$\frac{1}{\beta^2} = K(K_s - K) + \frac{1}{4} (2K - K_s)^2 \approx \frac{K_s^2}{4} \quad \text{so } \beta = \left| \frac{2\beta_2}{r B_0} \right|$$

Gradient in B_0 changes slope of beam,

can also affect dispersion, by quadrupole field e.g.

Example:

$$L = 2.5 \text{ m}$$

$$B_0 = -3.2 \text{ T}$$

$$B_w = 0.2 \text{ T}$$

↑

$$p_z \approx 0.3 \text{ GeV/c}$$

$$R \approx 70 \text{ cm}$$

"resonant" $p_z \approx 0.38 \text{ GeV/c}$

below resonance

hard to lower resonant $Q = \frac{B_0 L}{2\pi}$:

$L \downarrow$, dispersion reduced

$B_0 \downarrow$, β gets large

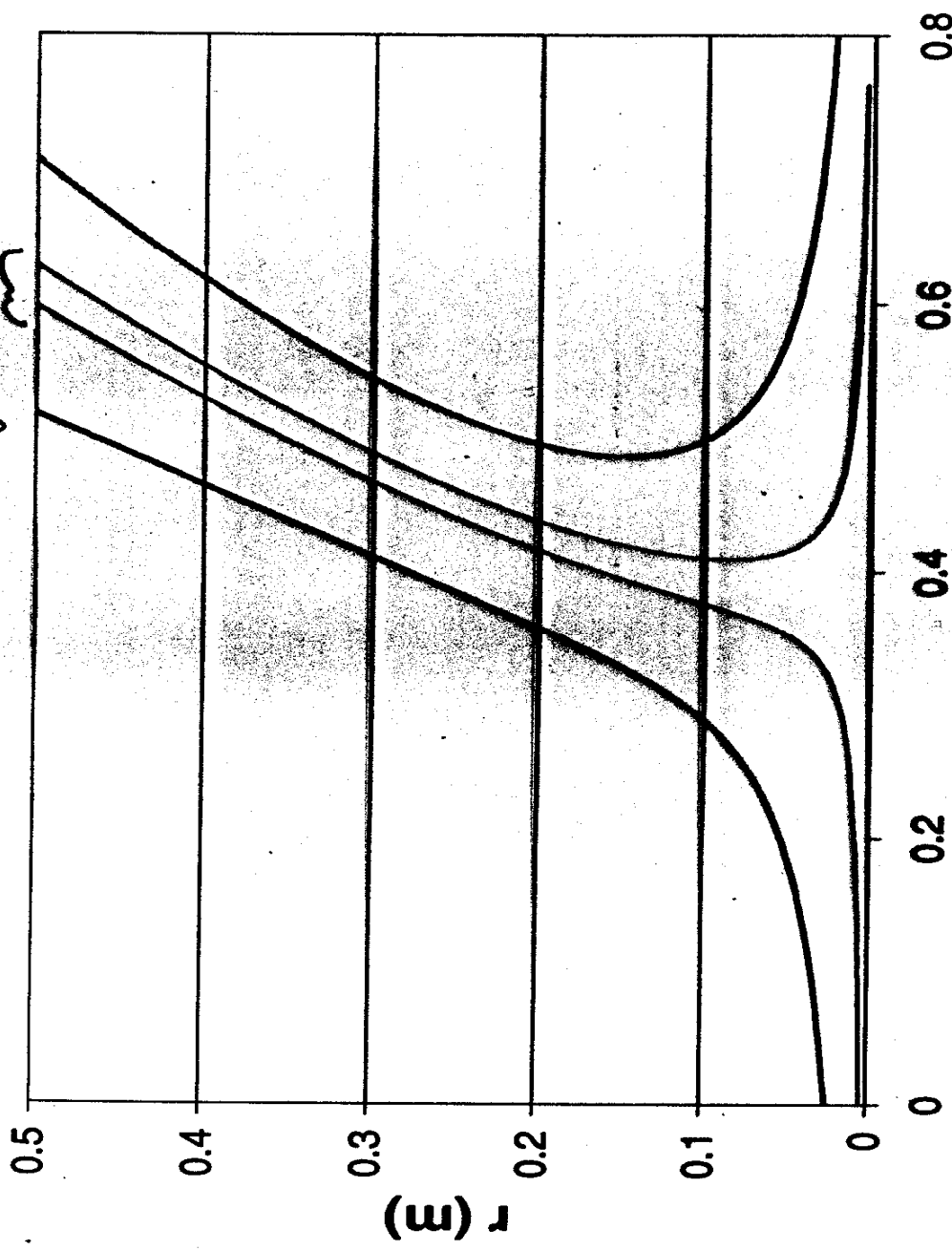
initial $\epsilon_{\perp} = 2000 \text{ } \mu\text{m mrad}$

start off with $\sigma_{p_z} = 15 \text{ MeV/c}$

$$t \equiv 0$$

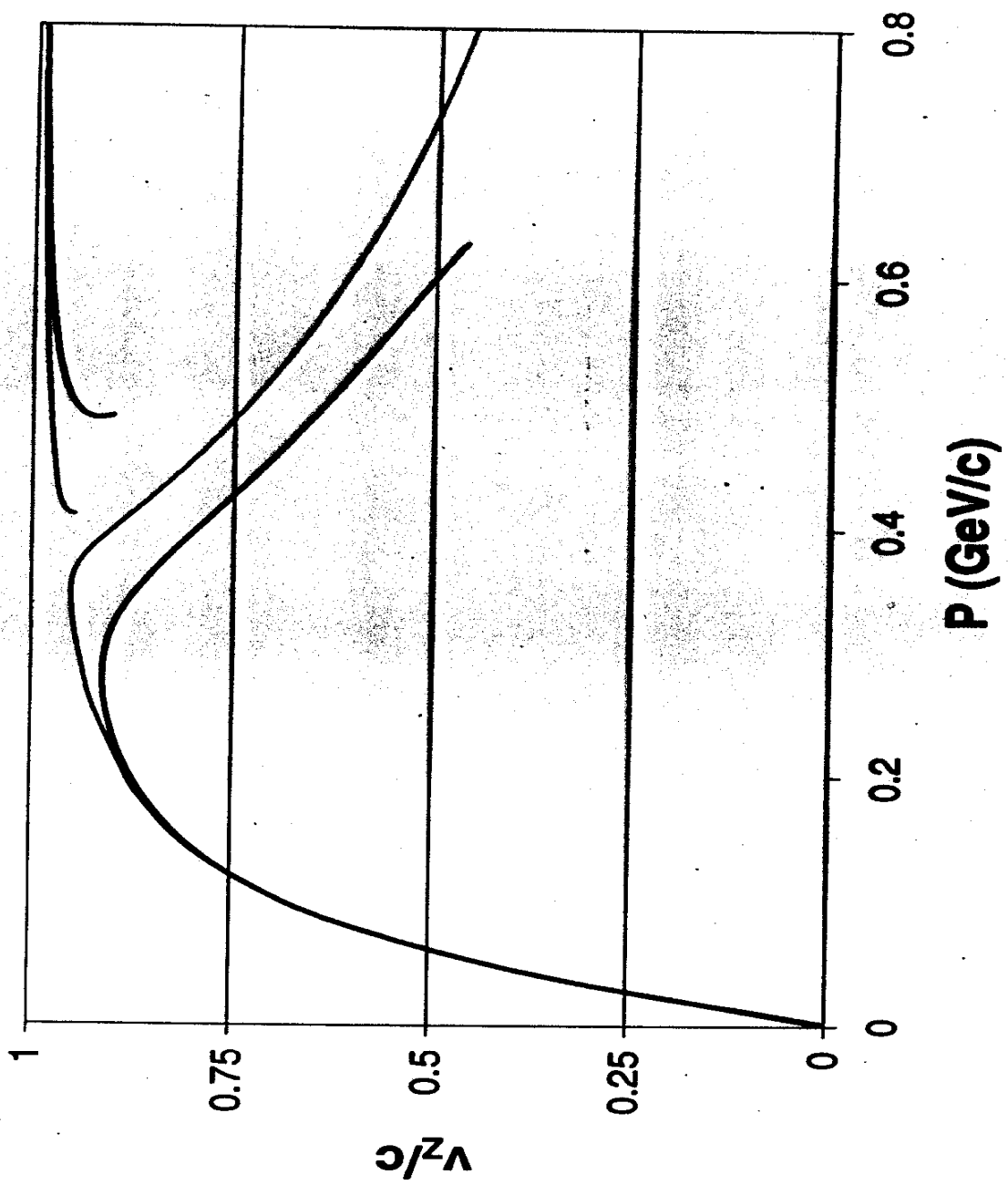
no RF, no absorber

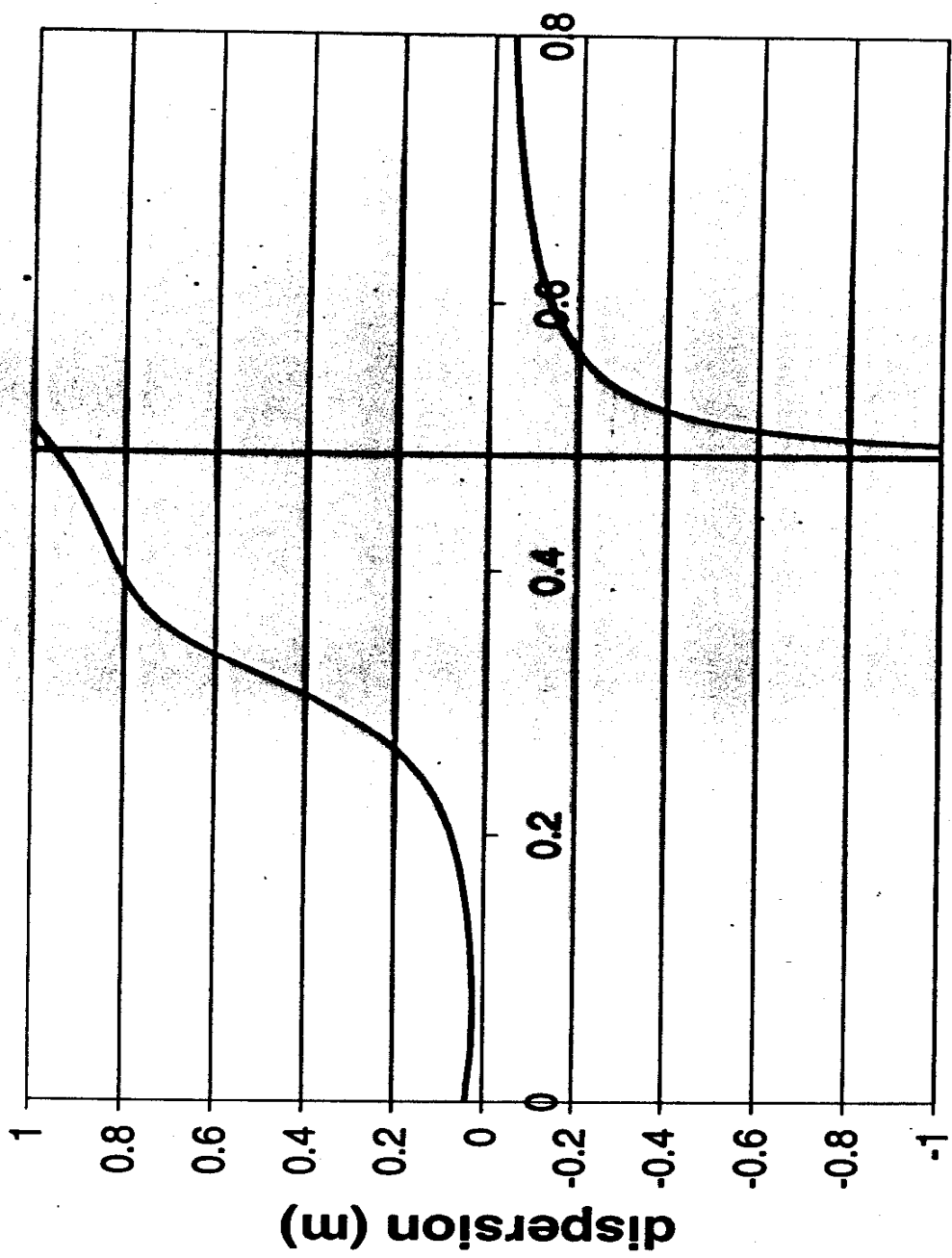
$\beta_w = 0.2T$ $\beta_w = 0.032T$



$P \text{ (GeV/c)}$

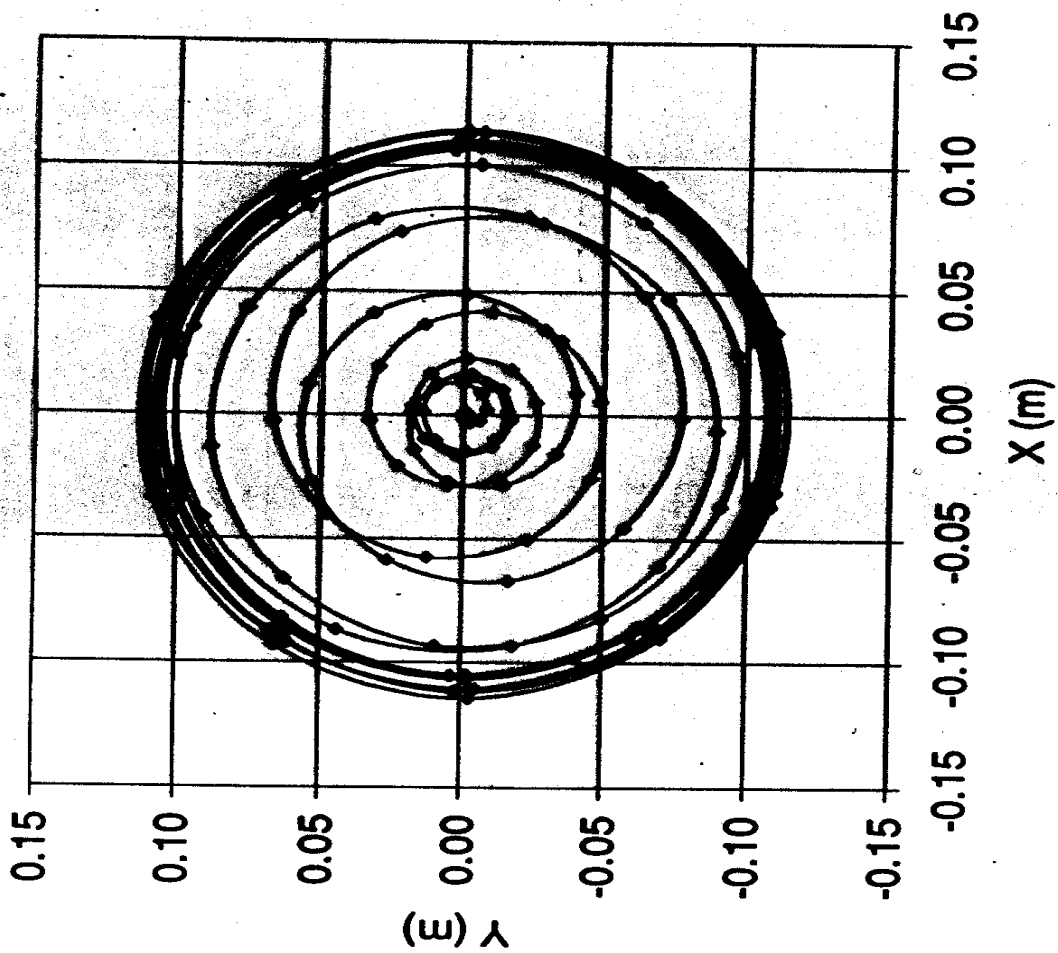
Orbit Radius



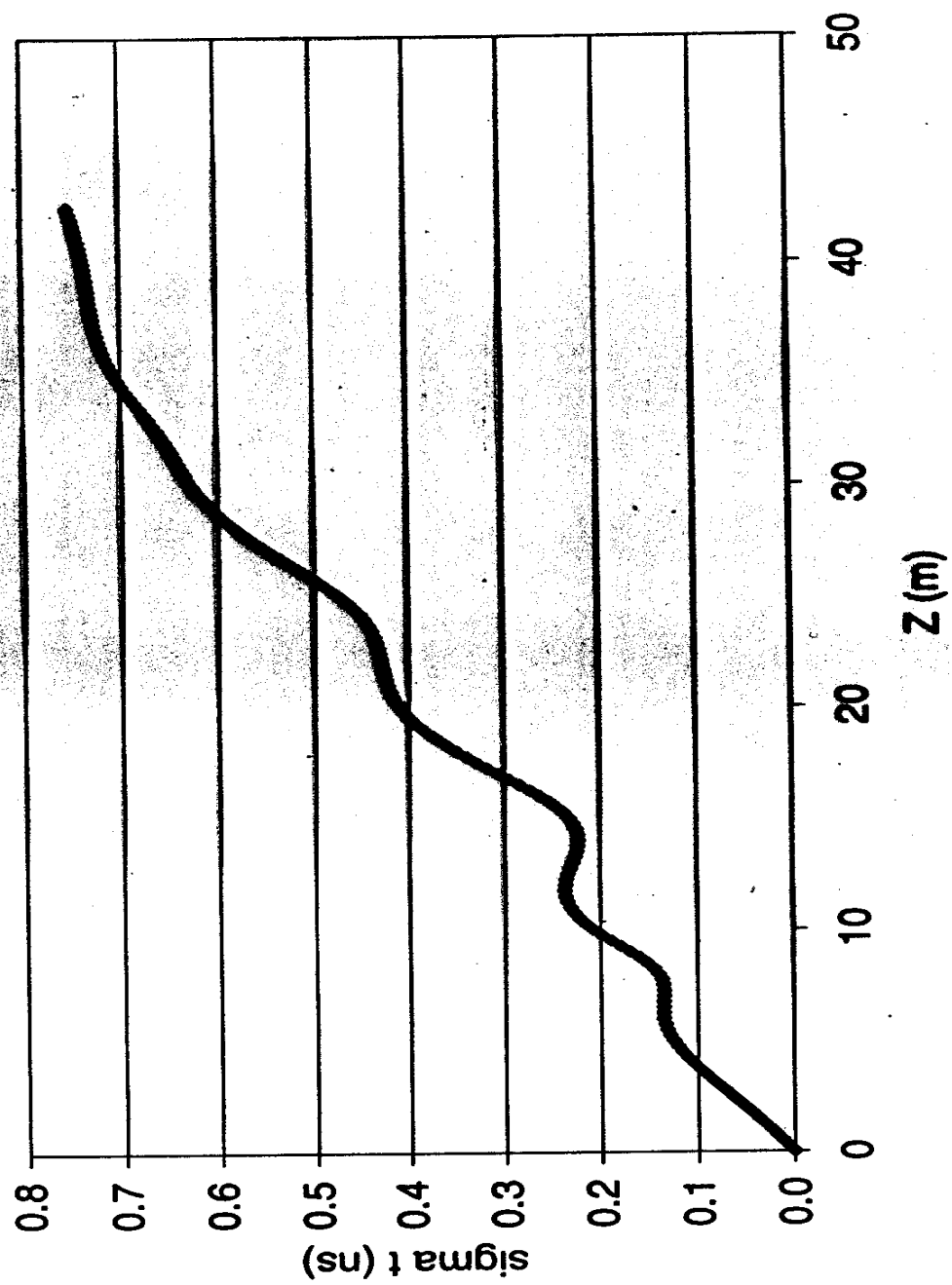


P (GeV/c)

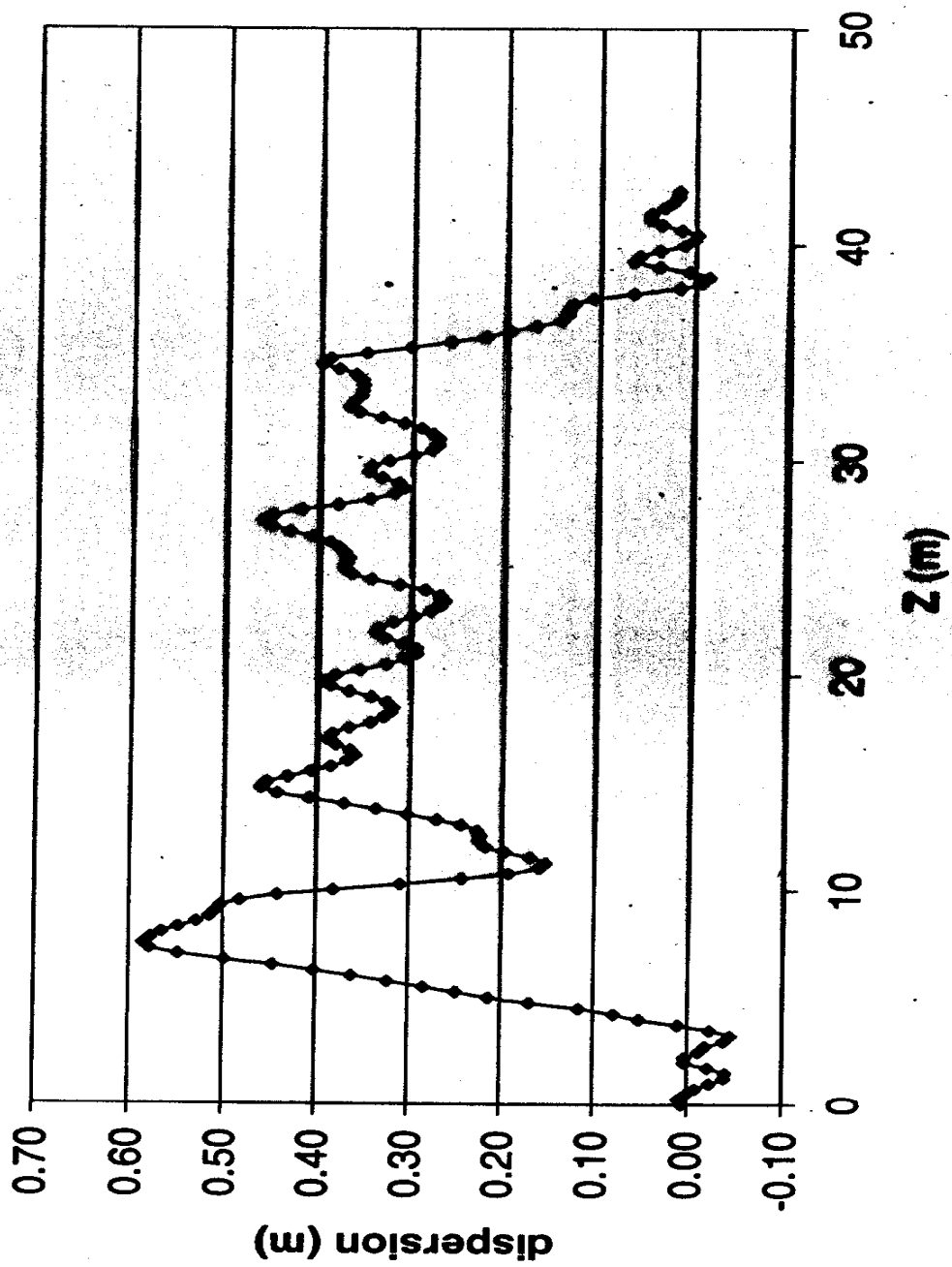
Motion of Centroid



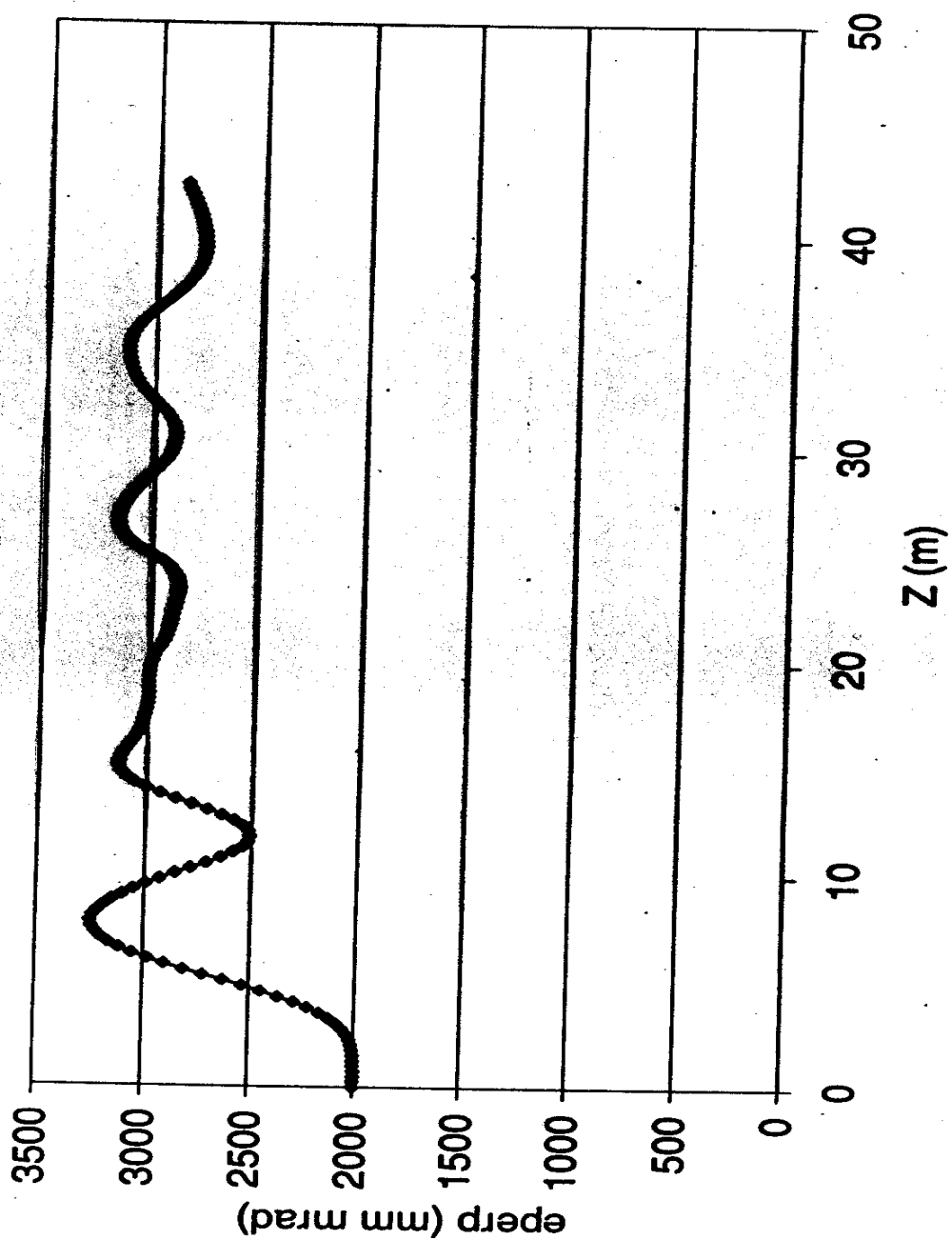
Bunch Lengthening



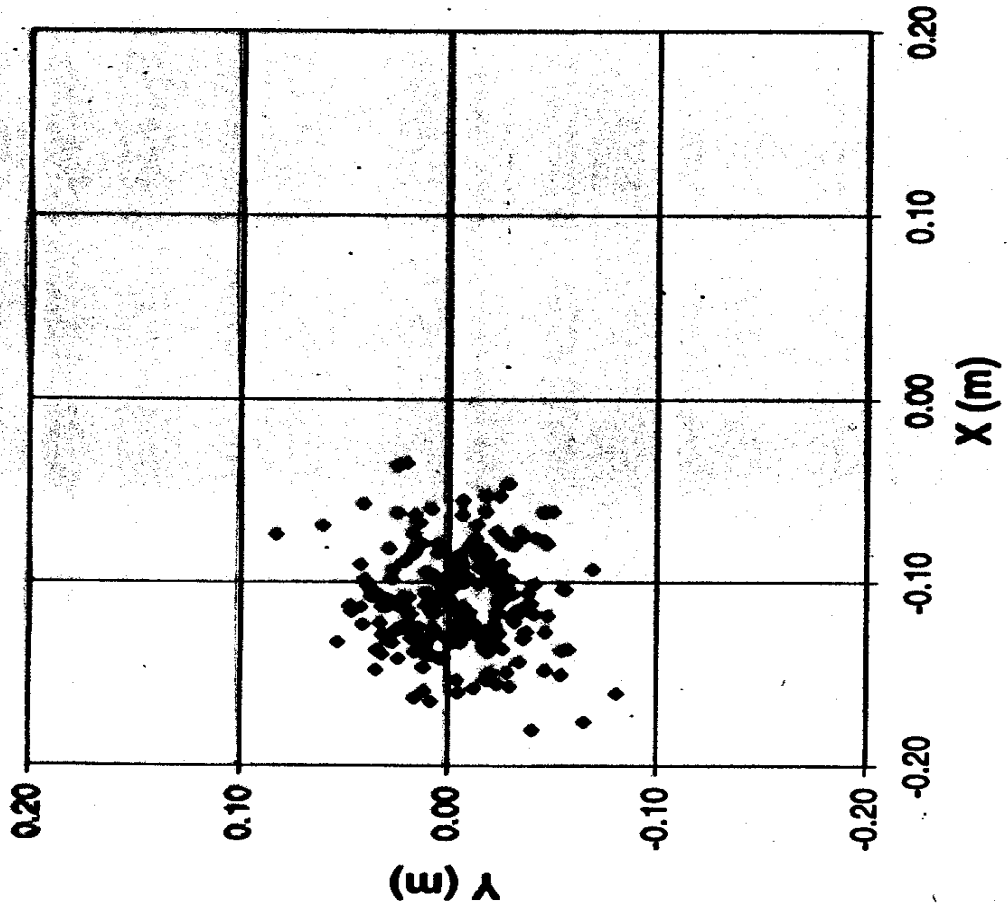
Radial Dispersion



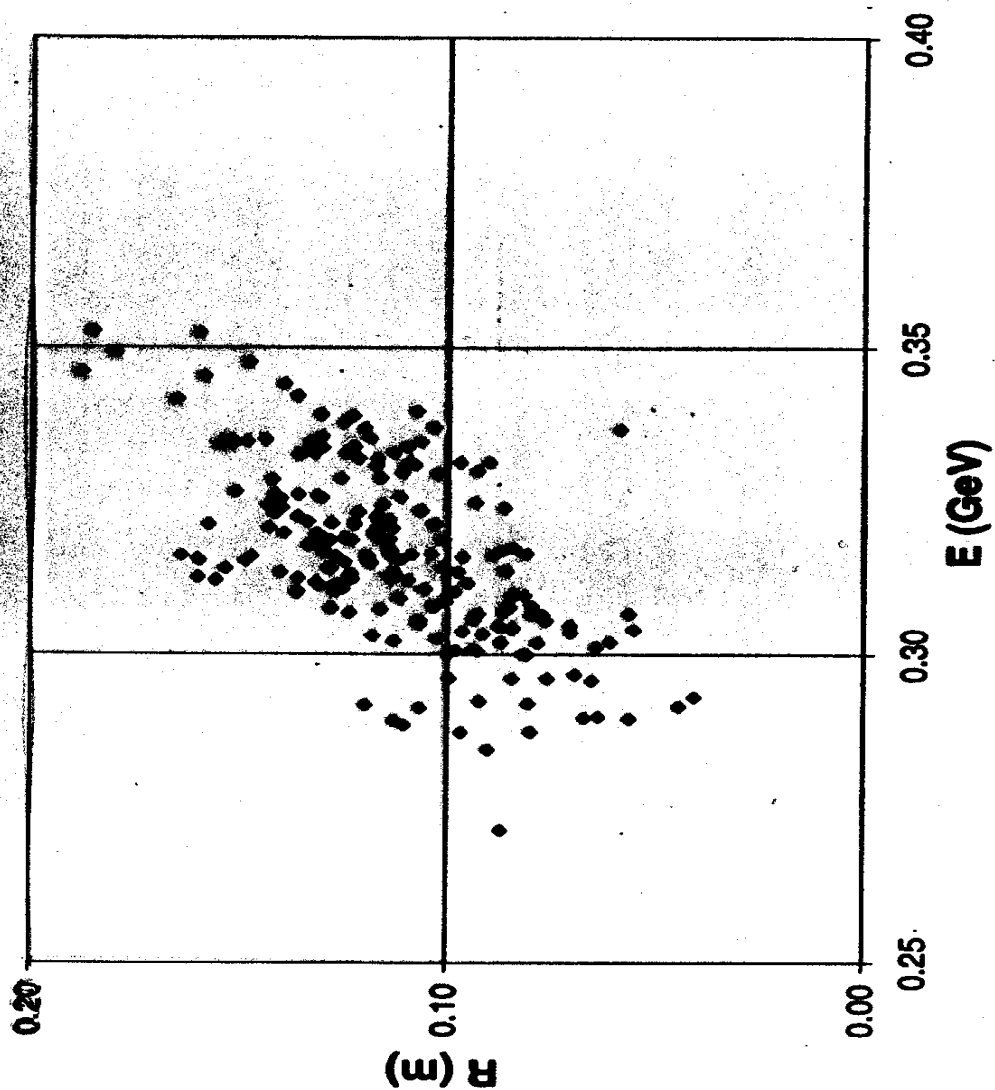
Transverse Emittance



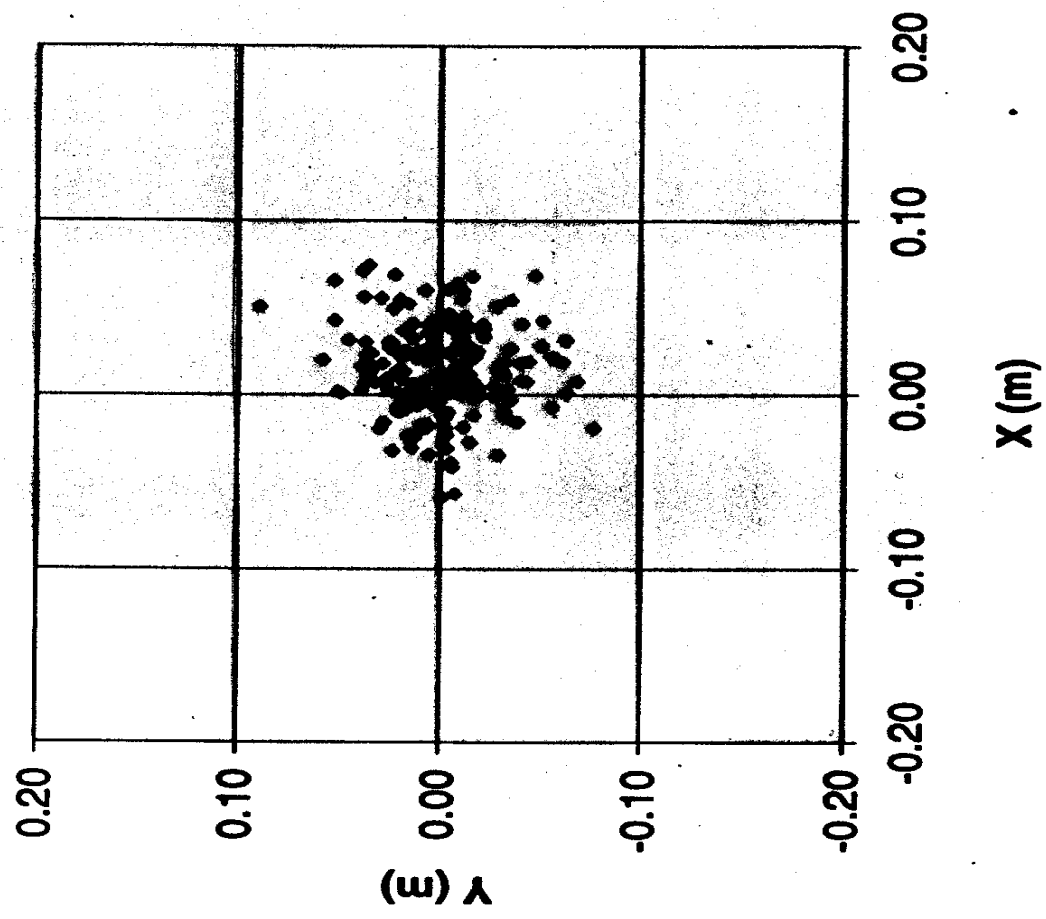
X-Section Inside Wagon



Radial Displacement



X-Section After Wiggle



Distribution after ~~154.5~~ trigger

