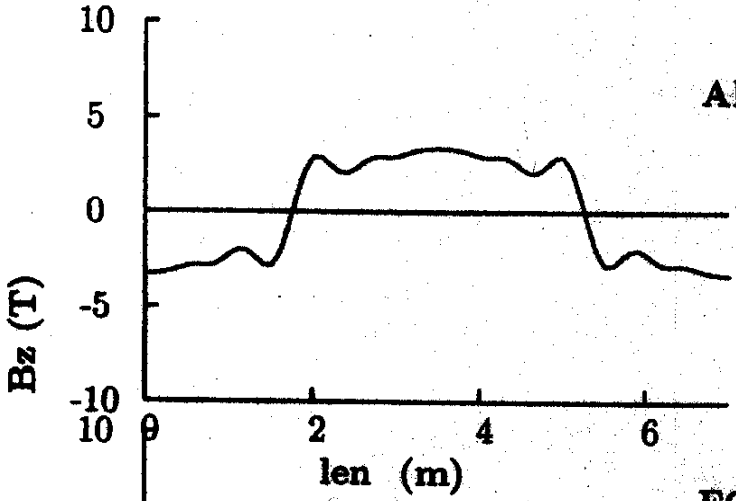
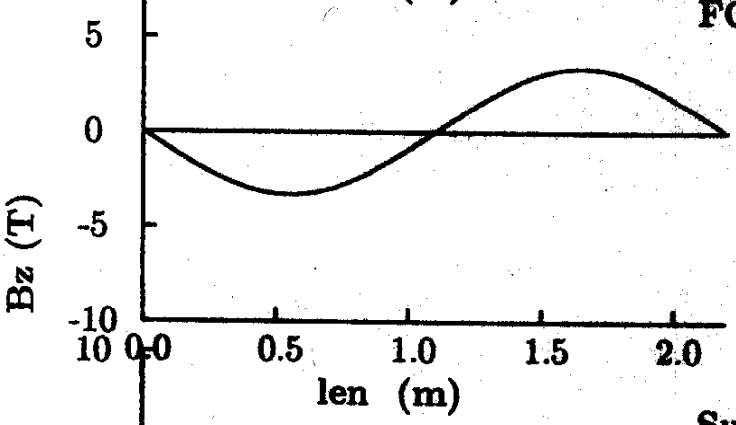
Cooling Lattices

R. Palmer



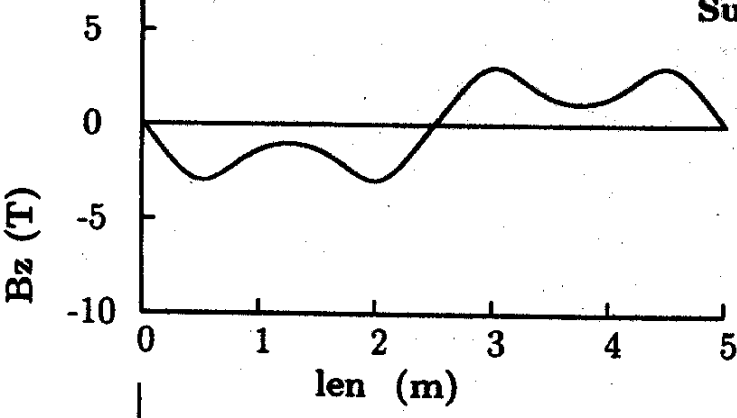
Alternating Solenoid

$B_z(max) = 3.4$ (T)
$dB_z/dz(max) = 15$ (T/m)



FOFO

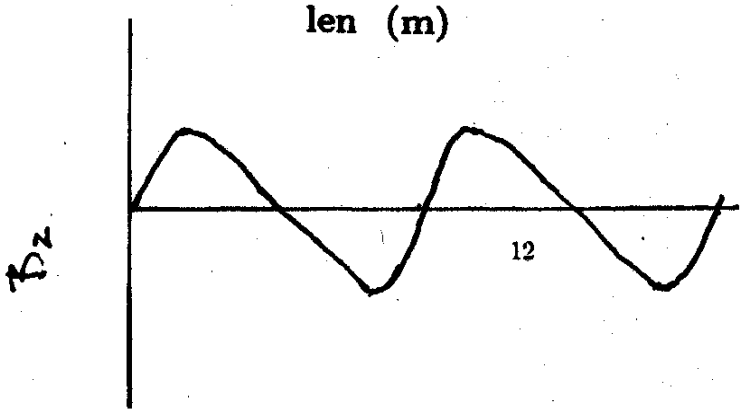
$B_z(max) = 3.4$ (T)
$dB_z/dz(max) = 9.4$ (T/m)

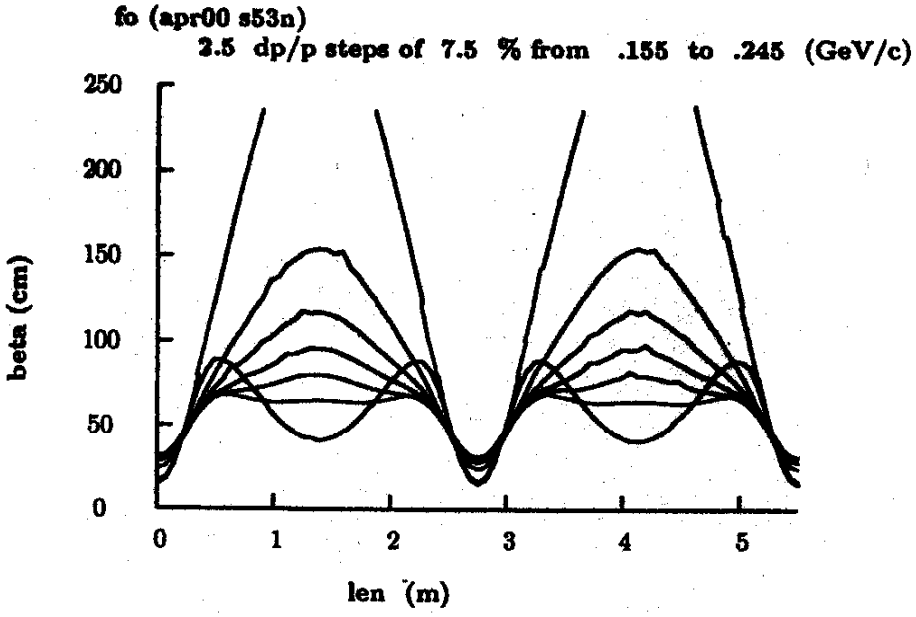


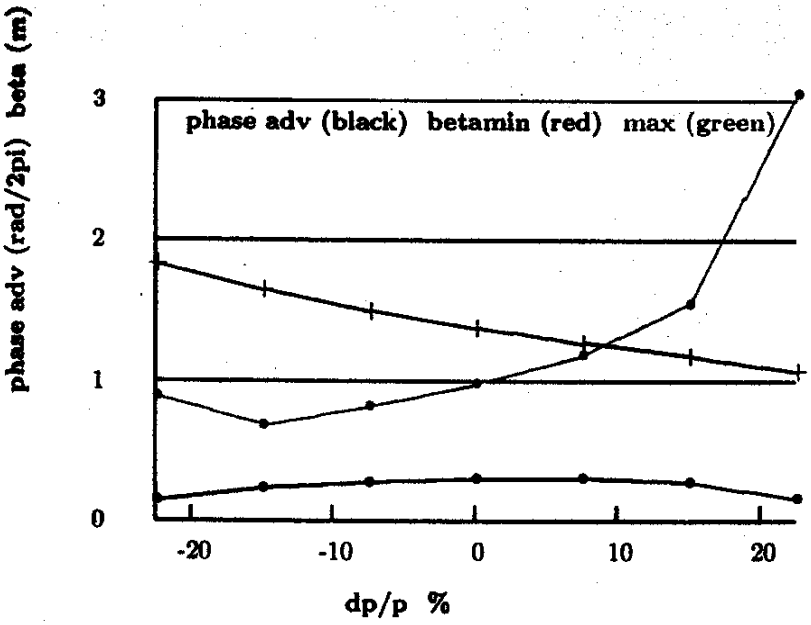
Super FOFO

$B_z(max) = 2.6$ (T)
$dB_z/dz(max) = 7$ (T/m)

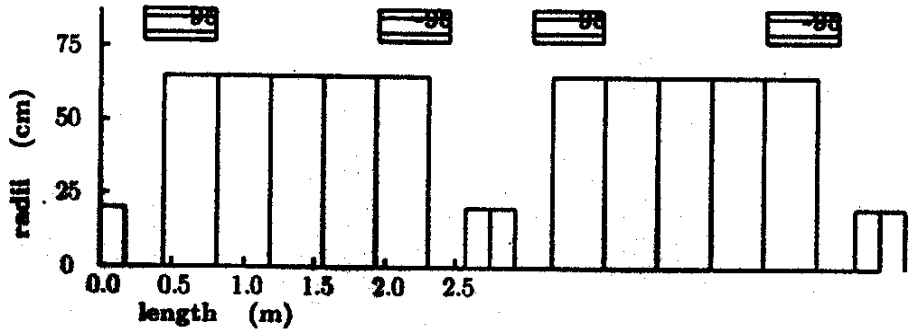
RFOFO

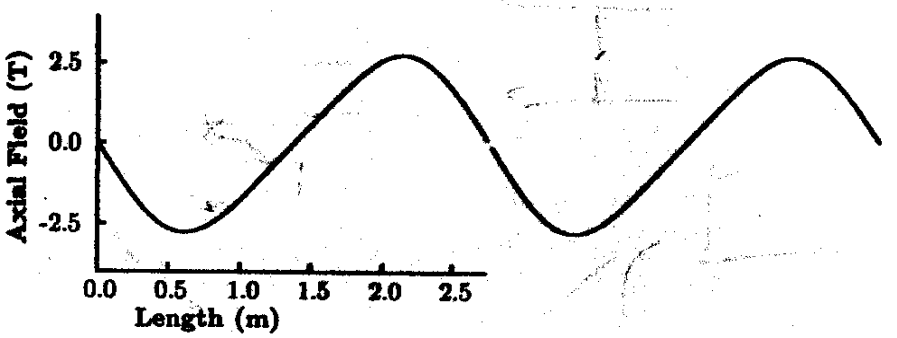


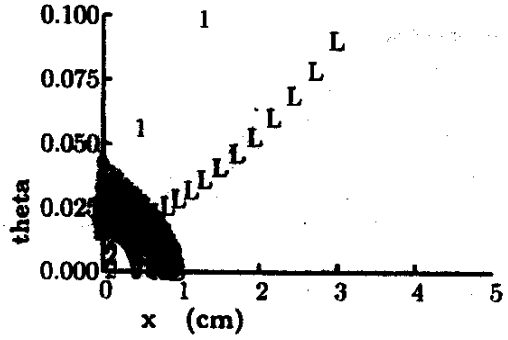

fo (apr00 s53n)
2.5 dp/p steps of 7.5 % from .155 to .245 (GeV/c)
beta (cm)
250
200
150
100
50
0
0
1
2
3
4
5
len (m)

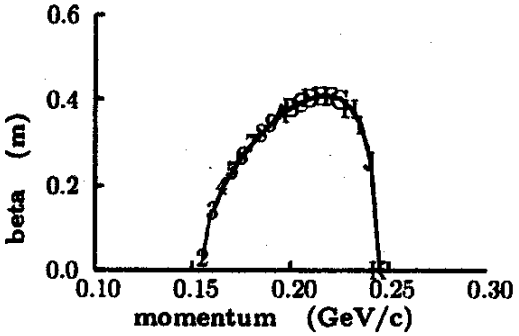

phase adv (rad/2pi) beta (m)
phase adv (black) betamin (red) max (green)
3
2
1
0
-20
-10
0
10
20
dp/p %

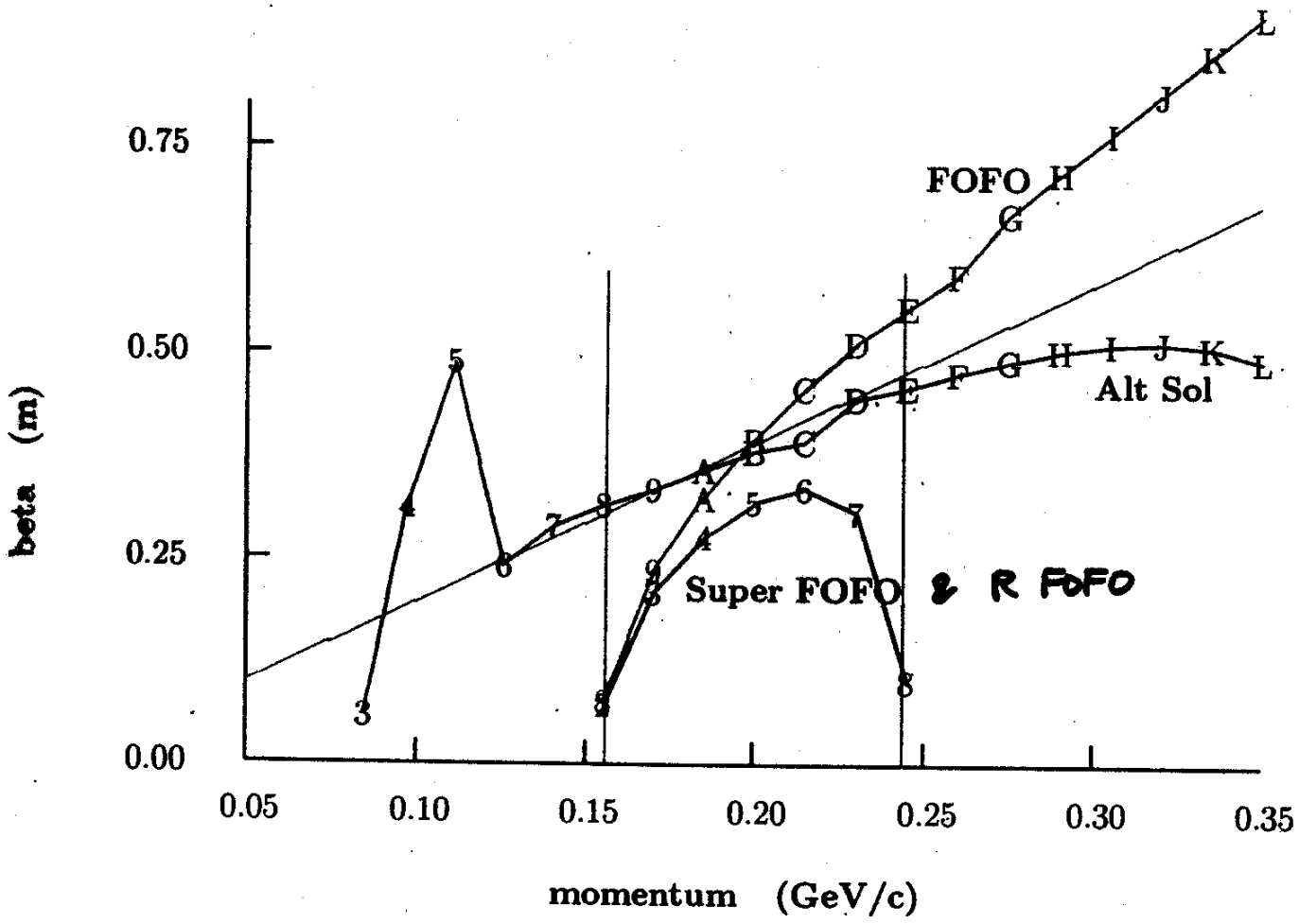
R FOFO


75
50
25
0
radii (cm)
0.0 0.5 1.0 1.5 2.0 2.5
length (m)


2.5
0.0
-2.5
Axial Field (T)
0.0 0.5 1.0 1.5 2.0 2.5
Length (m)

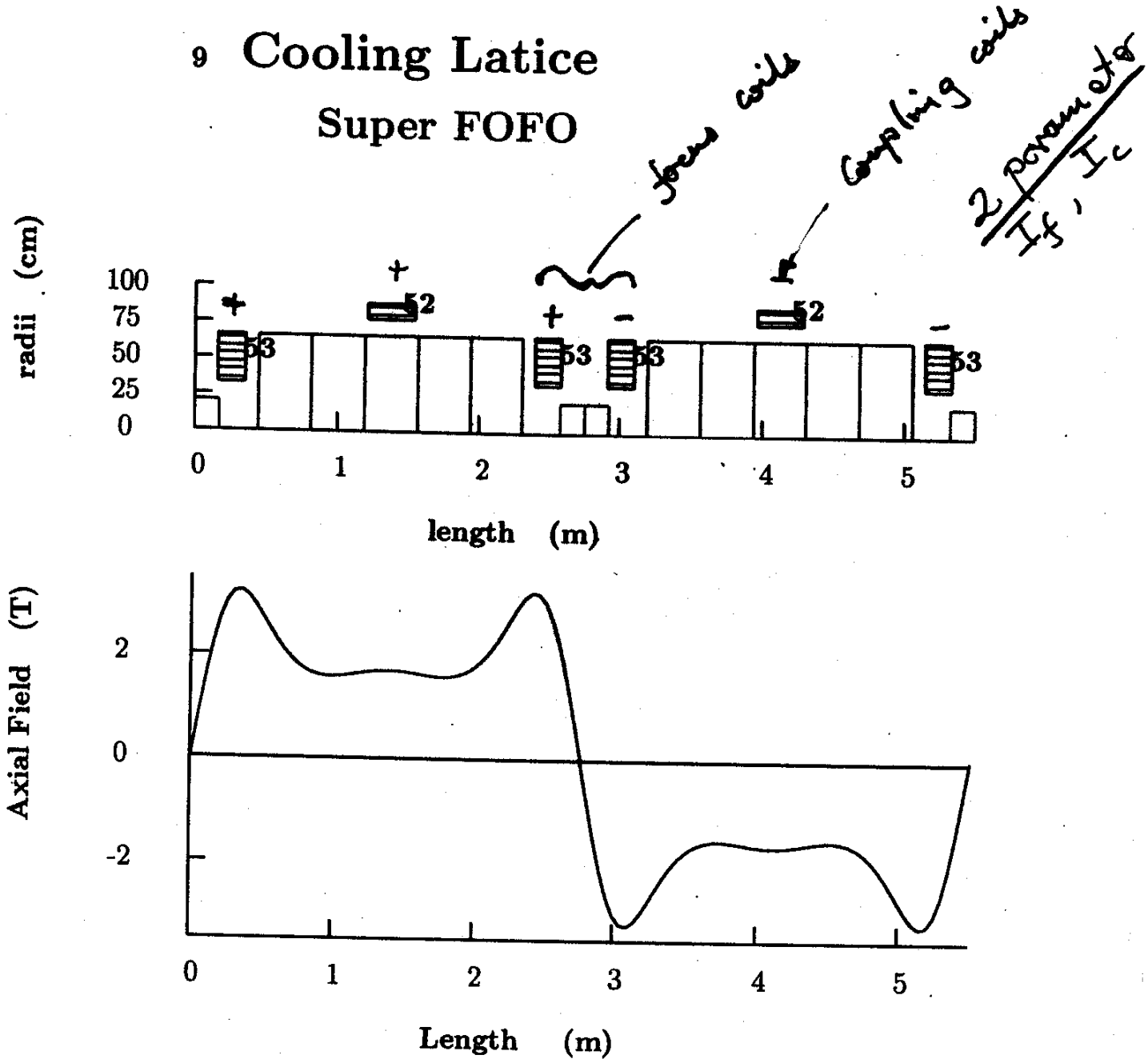

0.100
0.075
0.050
0.025
0.000
theta
0 1 2 3 4 5
x (cm)


0.6
0.4
0.2
0.0
beta (m)
0.10 0.15 0.20 0.25 0.30
momentum (GeV/c)


0.75
0.50
0.25
0.00
beta (m)
0.05
0.10
0.15
0.20
0.25
0.30
0.35
momentum (GeV/c)
FOFO
Alt Sol
Super FOFO & R FOFO

9 Cooling Latice

Super FOFO

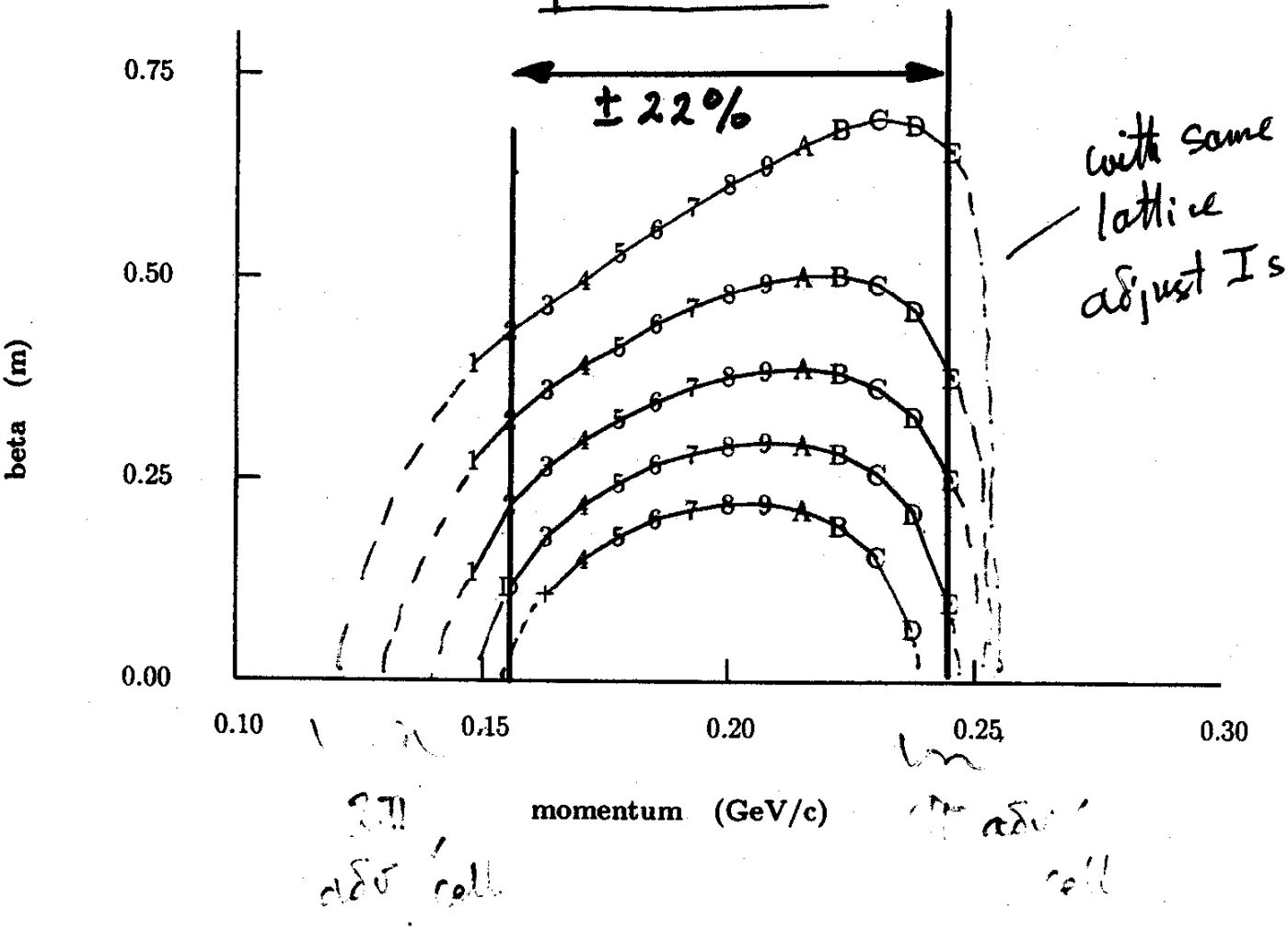


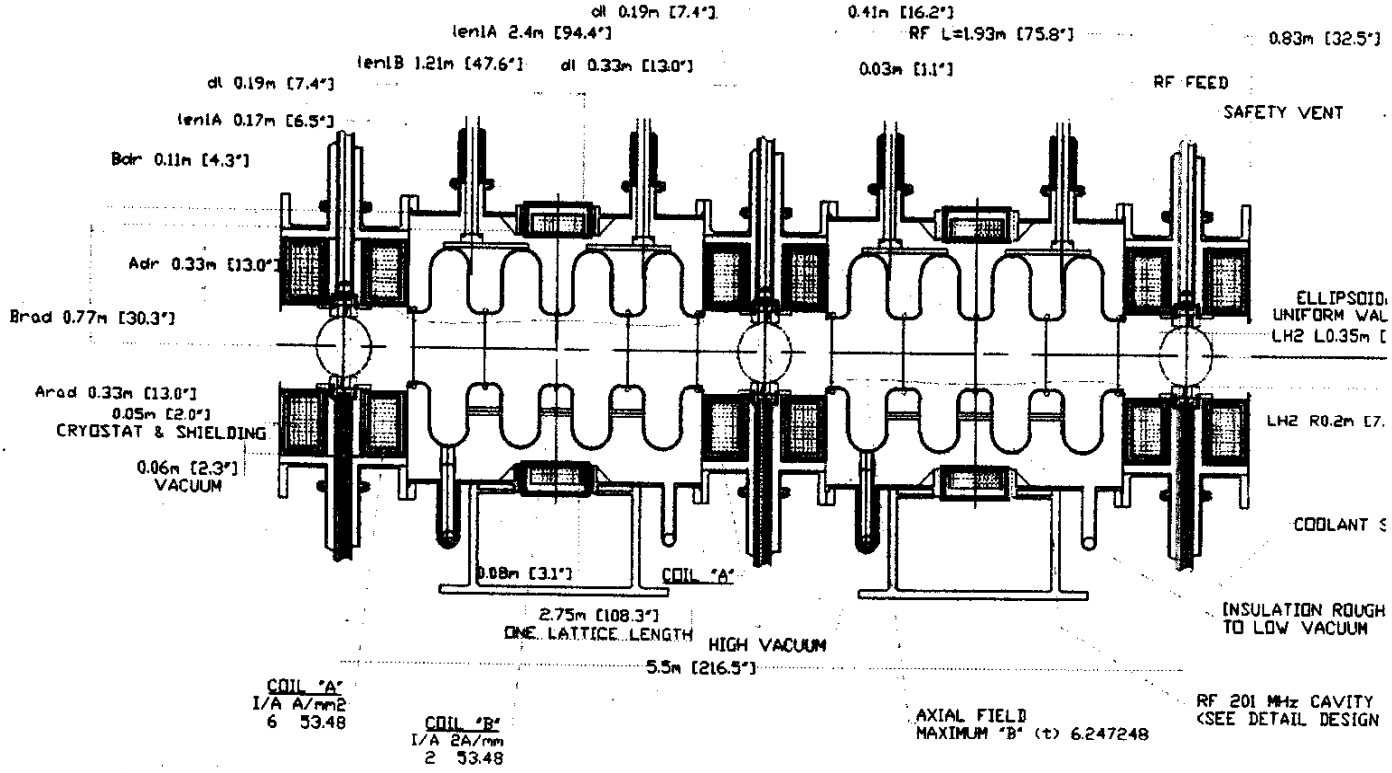
Taper β's

β too small ⇒ θ's too large – loss

β too large ⇒ θs too small – heat

more $\beta \propto \epsilon_n$

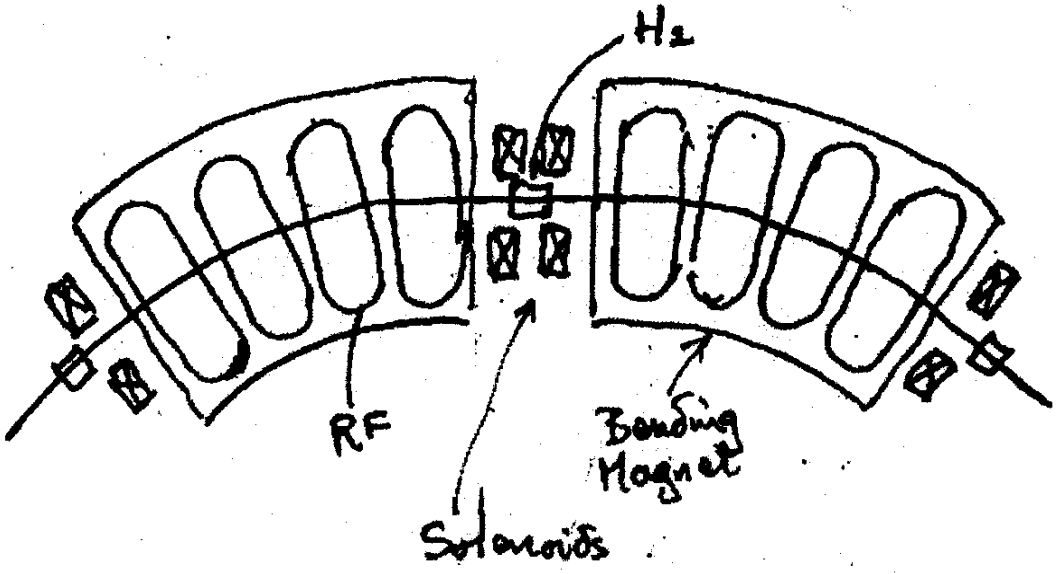




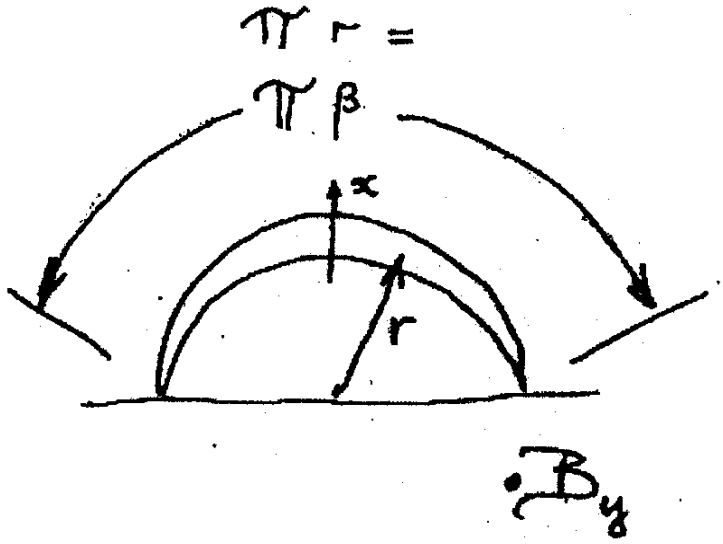
FOFOSTART201MHZrev1

Super FOFO LATTICE at start of cooling (PRELIMINAR)

E.L.Black IIT
6/22/2000
6/28/2000 REV.


H2
RF
Bending Magnet
Solenoids

0.1 Gradient focussing

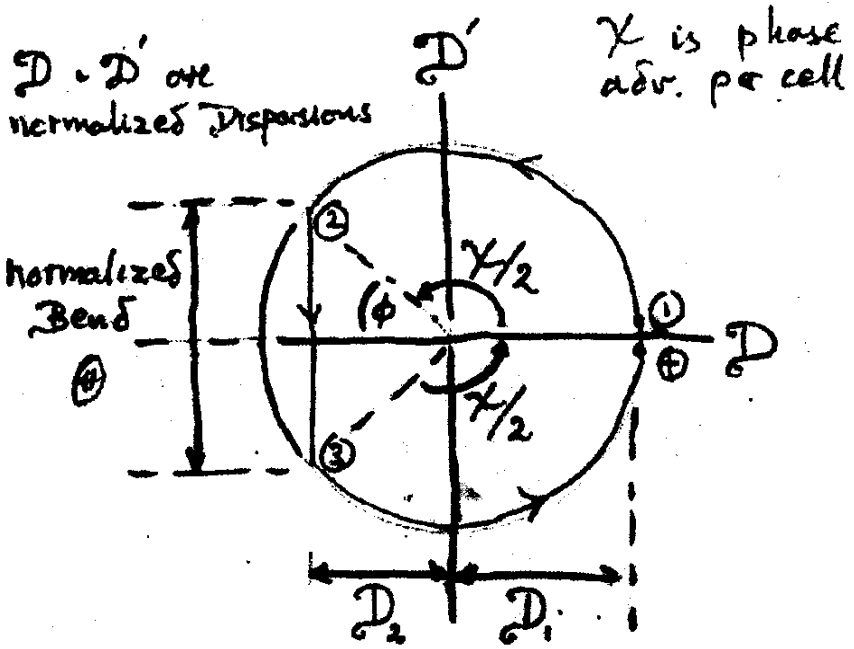


Add gradient to get:

$$B_y \propto \frac{1}{\sqrt{r}}$$

Then for same focussing:

$$B_y = B_s$$



Fig

where the un-normalized dispersions are:

$$D_i = \mathbf{D}_i \sqrt{\beta_i}$$

$$D'_i = \mathbf{D}'_i \sqrt{\frac{1}{\beta_i}}$$

A thin bend through an angle θ changes D' by θ, so we can define a normalized bend angle Θ:

$$\Delta D' = \theta = \Theta \sqrt{\beta_\theta}$$

where β_θ is the β at the bend.

Note that the normalized variables all have the dimensions of $\sqrt{length}$.

For significant emittance exchange, we require:

$$|D_1\,\delta| \;=\; k\,\sigma_1$$

where $\sigma_1 = \sqrt{\frac{\beta_1\,\epsilon_n}{\beta_v\gamma}}$, and k is of order 1 from which we obtain:

$$\theta \;=\; 2\sin(\chi) \quad \frac{k}{\delta}\;\sigma_\theta \quad \sqrt{\frac{\beta_1}{\beta_2}}$$

For good transverse cooling, we require $\sigma_\theta \approx 0.1$. Consider a fractional momentum spread $\delta = 0.07$, and $\sqrt{\frac{\beta_1}{\beta_2}} \approx 0.5$.

then:

$$\Delta\theta \;=\; \sqrt{2} \quad \frac{0.7}{.07} \qquad 0.1 \qquad .5$$

$$\approx\; 0.7 \;\text{(radians)} \quad \approx 45 \;\text{(deg)}$$

from the diagram:

$$\frac{\Theta}{2} = \mathbf{D}_1 \sin(\chi)$$

so

$$\mathbf{D}_1 = \frac{\Theta}{2 \sin(\chi)}$$

and the un-normalized dispersion:

$$\mathbf{D}_1 = \frac{\Theta}{2 \sin(\chi)} \sqrt{\beta_1 \beta_2} \; .$$

The available length for bending is about .5 m, so for 200 MeV/c, the required field is :

$$B = \frac{\Delta\theta\, p}{c\, L} = \frac{.7 \;\; .2}{.3 \;\; 0.5} \approx 1 \quad \text{(T)}$$

which is close enough to the field ($\approx$ 2 T) needed for coupling that a solution may be found.

Note: the angle of bend $\Delta\theta$ is independent of emittance.

But the required field rises inversely with the falling emittance.

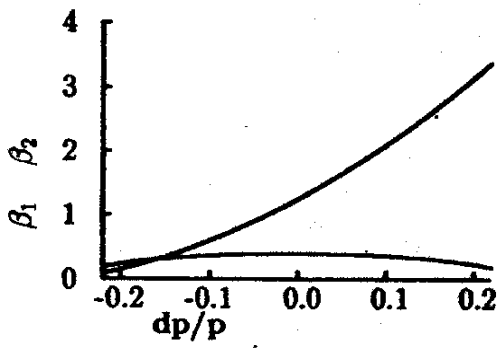
If

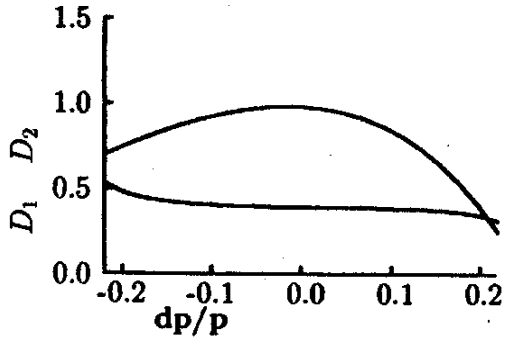
$$\theta = 45 \text{ deg.}$$

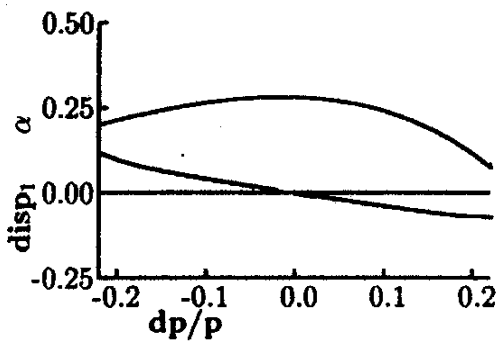
$$\chi = \pi \left(1.5 + 0.5 \frac{dp/p}{.25}\right)$$

$$\beta_1 = 0.4 \sqrt{1 - \left(\frac{dp/p}{.25}\right)^2}$$

$$\beta_2 = 2.5\,(dp/p + .25) + 10\,(dp/p + .25)^2$$







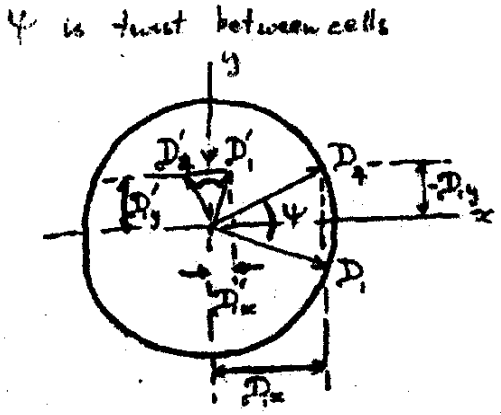
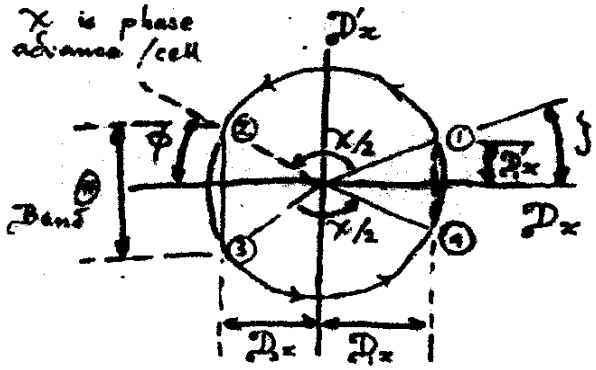


Fig 2 Dispersion & Angular Dispersion at start & end



Dispersion Phase Diagram in x
(Bend in this Plane)

Fig 3

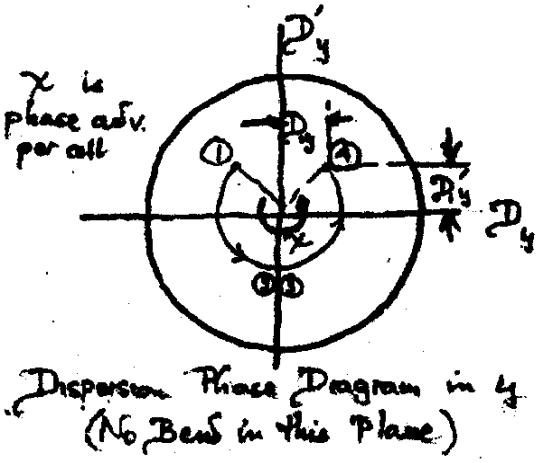
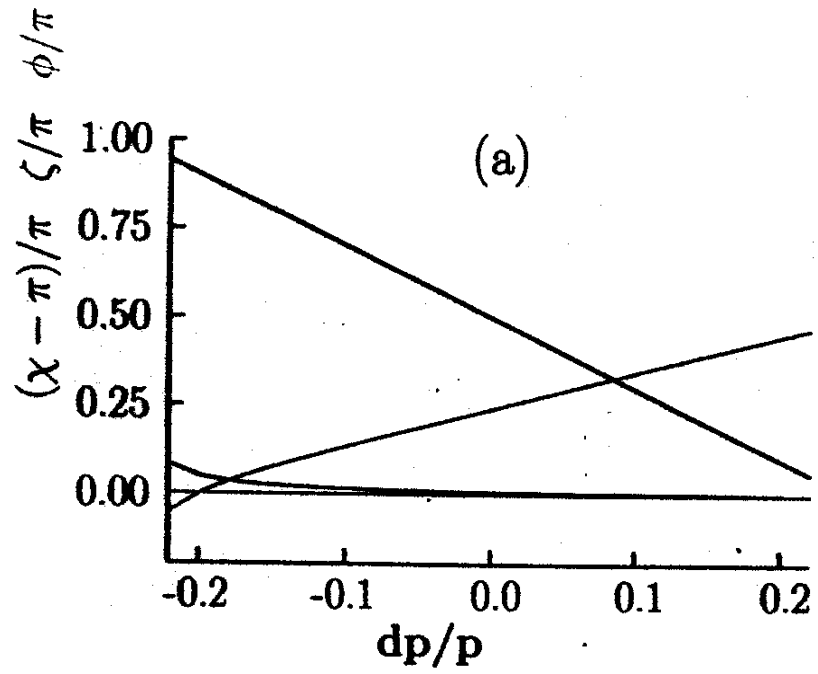
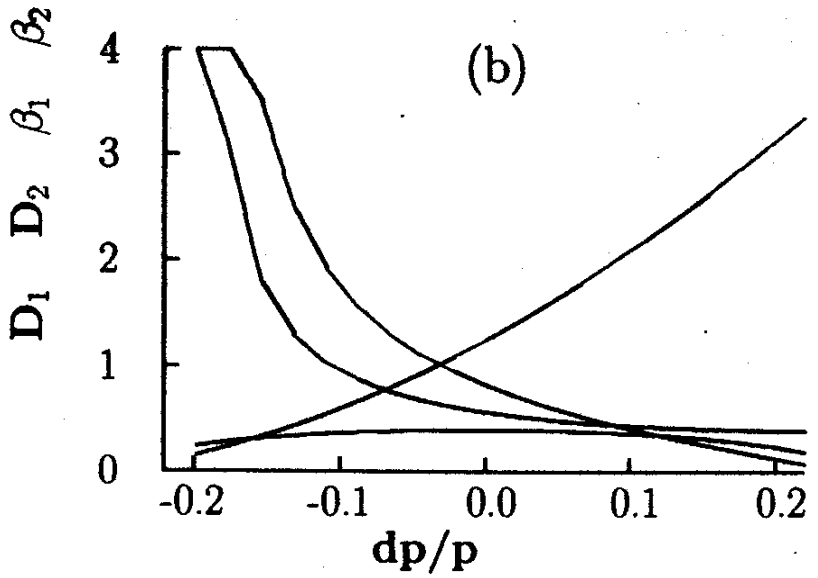

$\mathcal{D}'_y$
χ is
phase adv.
per cell
$\mathcal{D}_y$
$\mathcal{D}'_y$
$\mathcal{D}_y$
Dispersion Phase Diagram in y
(No Bend in this Plane)

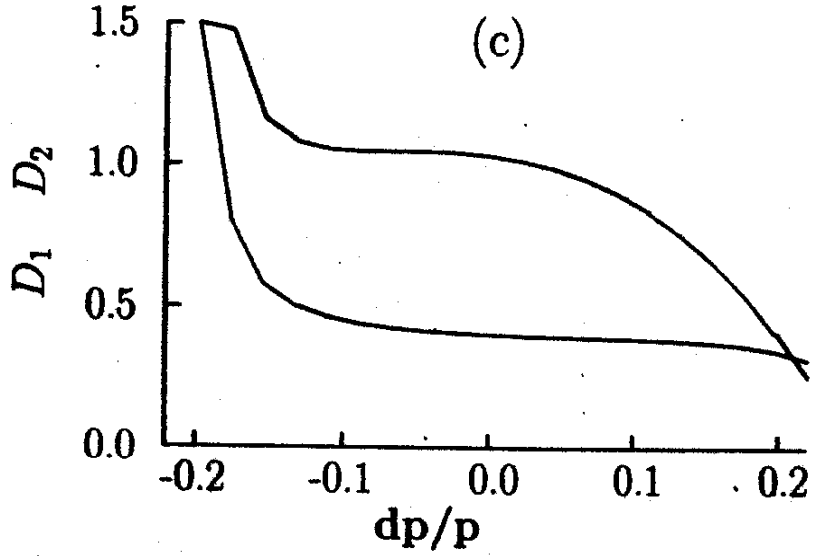
Fig 4

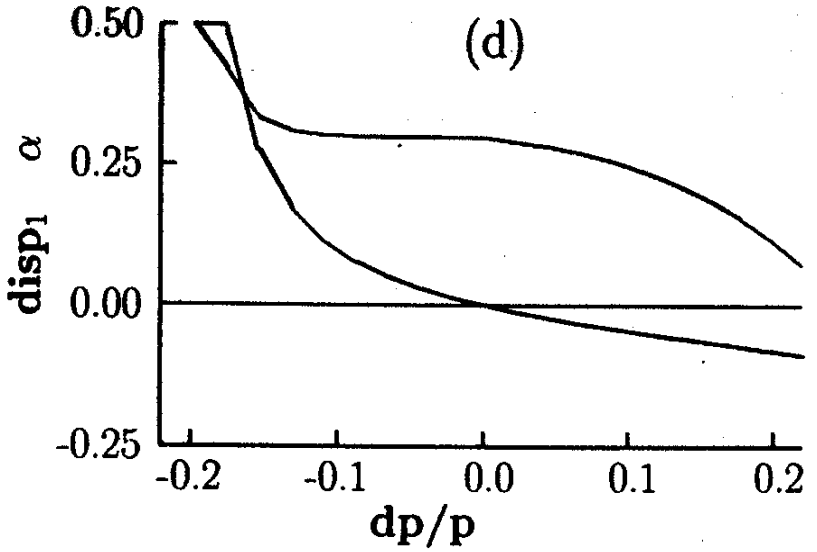
For twist angle:

$$\psi = 18 \text{ deg.}$$






(c)
1.5
1.0
0.5
0.0
D_1 D_2
-0.2
-0.1
0.0
0.1
0.2
dp/p

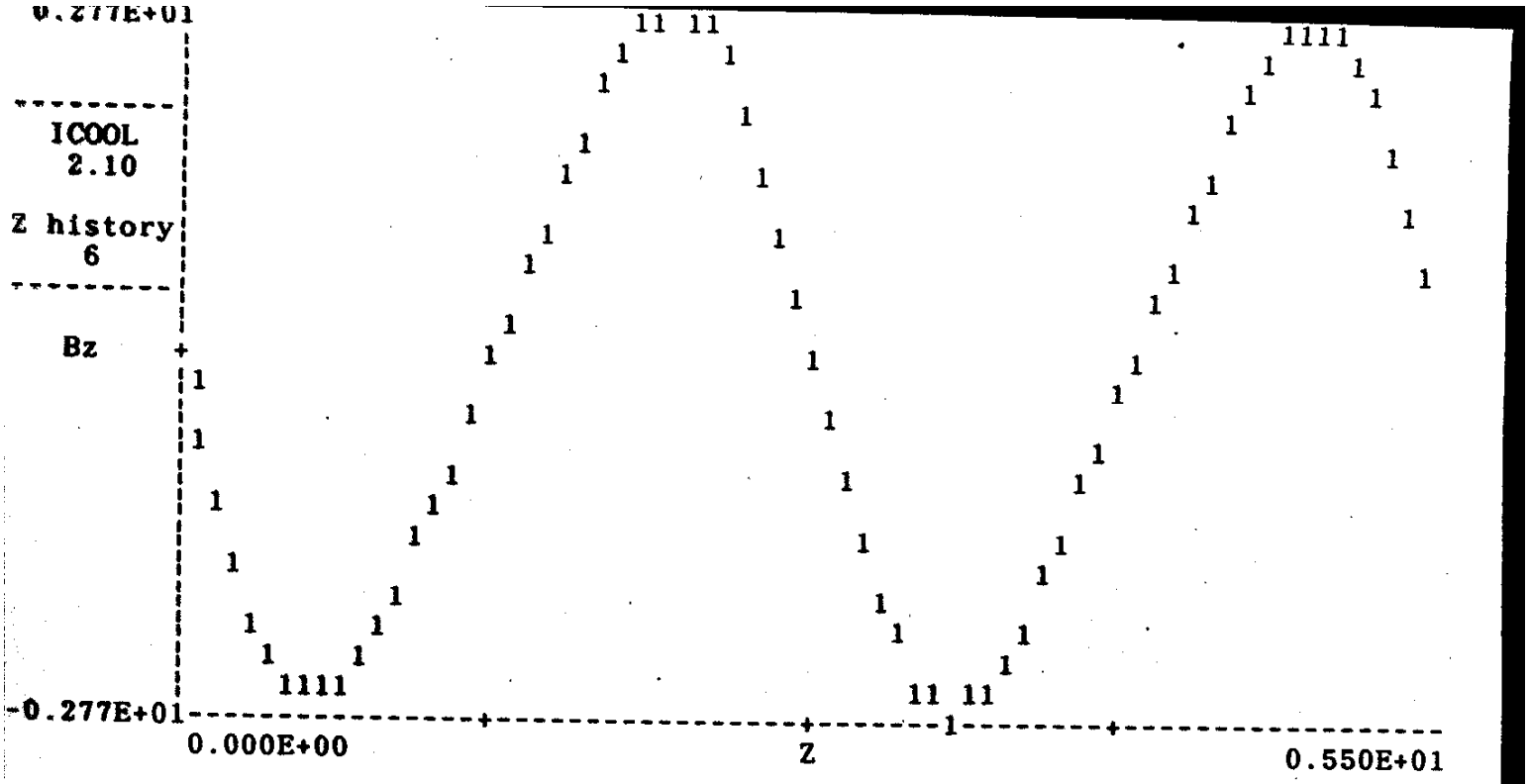

(d)
0.50
0.25
0.00
-0.25
α
disp_1
-0.2
-0.1
0.0
0.1
0.2
dp/p

```
RFOFO with no bend (2.10 aug00 bsol)
$cont npart=500  bgen=.true. varstep=.false. nprnt=-1 prlevel=3
ntuple=.false. rtuple=.false. rtuplen=1 output1=.true.
fsav=.false. fsavset=.true. izfile=140  $
 $bmt $
1 2 1. 1
0.      0.0       0.          0.         0.       0.2
0.021   0.021     0.019       0.0101     0.0101   0.009
1
1 -3.5 0. 0.
2 0.2 0.17 0.
 $ints $
 $nhs nhist=0  $
 $nsc nscat=1  $
0.065 1.3e-3 50 10 1    -0.10 1e-2 20 1 1
 $nzh nzhist=6 $
5 0.  0.01539  70  0. 0. 1
5 0.  0.01539  70  0. 0. 2
1 0. 0.07857 70 0. 0. 31
1 0. 0.07857 70 0. 0. 32
1 0. 0.07857 70 0. 0. 32
1 0. 0.07857 70 0. 0. 33
 $nrh nrhist=0 $
 $nem nemit=6 pxycorr=.true. pzcorr=.true. bzfldprd=3.5 $
 1 4 19 21 33 48
 $ncv ncovar=6 $
 1 4 19 21 33 48
SECTION
REFP
2 0.180 0. 0. 4
CELL
2
.FALSE.
BSOL
3. 42. 0.20  1. 0.   0. 0. 0. 0. 0.   0. 0. 0. 0. 0.
REPEAT
28
OUTPUT
SREGION
0.1   1   0.02        !   .1
1   0.   0.10
NONE
0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.

VAC
CBLOCK
0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
ENDR
ENDCELL
ENDSECTION
```

RFOFO with no bend

138			
0	0	0	0
.01	-.1098	0	0
.03	-.2495	0	0
.05	-.3887	0	0
.07	-.5271	0	0
.09	-.6643	0	0
.11	-.8001	0	0
.13	-.934	0	0
.15	-1.066	0	0
.17	-1.195	0	0
.19	-1.321	0	0
.21	-1.444	0	0
.23	-1.564	0	0
.25	-1.679	0	0
.27	-1.791	0	0
.29	-1.897	0	0
.31	-1.999	0	0
.33	-2.096	0	0
.35	-2.187	0	0


0.277E+01
ICOOL
2.10
Z history
6
Bz
-0.277E+01
0.000E+00
Z
0.550E+01