$$dx = ds\ cos(\chi) \qquad dy = ds\ sin(\chi)$$

$$ds = \rho d\chi \ = \ k\ (r_o\ +\ x)\ d\chi$$

if $\rho << r_o$, then to first order :

$$y \ \approx \ \rho_o\,(1 - cos(\chi))$$

$$x \ \approx \ \rho_o\ sin(\chi)$$

where

$$\rho_o = k\ r_o$$

To second order:

$$\frac{dy}{d\chi} \ \approx k(r_o + \rho_o\ sin(\chi)\ d\chi)\ sin(\chi)$$

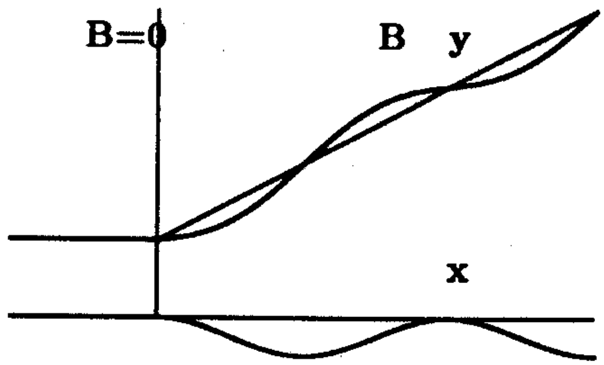
$\int_0^{2\pi} sin(\chi)\,d\chi = 0$, but $\int_0^{2\pi} sin^2(\chi)\,d\chi = \pi$, so integrating dy over one turn and substituting ρ_o/r_o for k gives a vertical "drift" of:

$$\Delta y \ \approx \ \pi\,\frac{\rho_o^2}{r_o} \qquad (3)$$

In general, a particles motion will evolve both this amplitude drift and the momentum drift discussed above.

9.3.3 First order Matching into Bent Solenoid

Jim Norem



s (along trajectory)

Inside a bent solenoid, without superimposed dipole, the equilibrium orbit is rising with an angle α. If this abruptly joins a straight solenoid, in which the equilibrium orbit is horizontal, then there will be a mismatch between them. If a dipole field is introduced over the bent portion of the solenoid, this can exactly remove the vertical drift so that the mismatch is removed.

The dipole field required is that field that will bend the reference particle with a curvature exactly equal to the bend of the solenoid:

$$B_{dipole} = \frac{p}{R\,e}$$

Guided by this field, the particle passes along the field lines of the solenoid and is not deflected by them.

There is some debate as to whether the dipole field B_d should be uniform across the solenoid, or if it should vary in proportion to the solenoidal B_s. If one considers particles inside the bend with $y' = x' = 0$ and the same momenta, but different y positions, then a graded field would seem best: the particles on the inside of the bend, if they are to remain parallel to the axis, need a greater curvature, and thus a greater B .

But a finite y but no x' or y' is not very typical: it corresponds to a large canonical angular momentum. If instead, one considers particles with random x, y, x', y' but zero canonical angular momentum, then a flat field seems to be prefered. ||

Not so more study shows that either flat or $B_\perp \propto \frac{1}{r}$ mix x & y

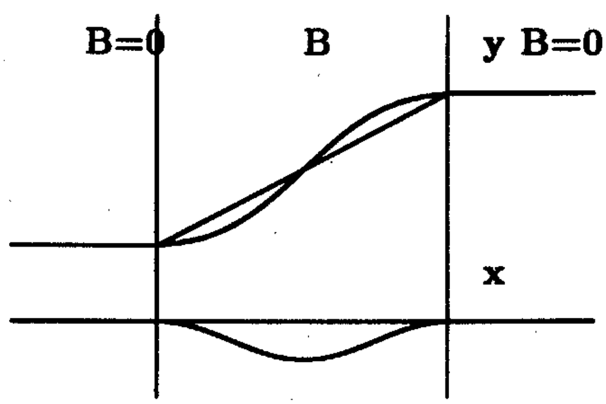
$$B_\perp \propto \frac{1}{\sqrt{r}}$$

does not mix x & y and gives equal extra focus in both directions, as in case without solenoid field.

9.3.4 Second order Matching

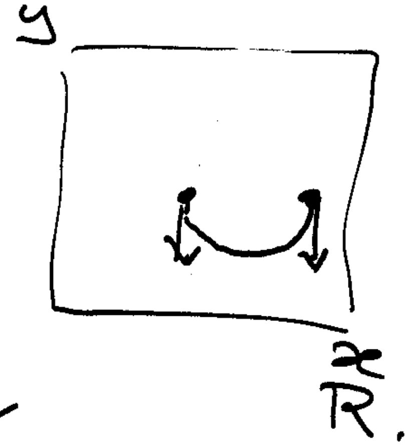
But the dipole can only remove the drift for one momentum. At other momenta there will still be an abrupt change in angle and a particle on the axis of the straight solenoid will mismatch into the bent portion and thus gain a finite betatron amplitude.

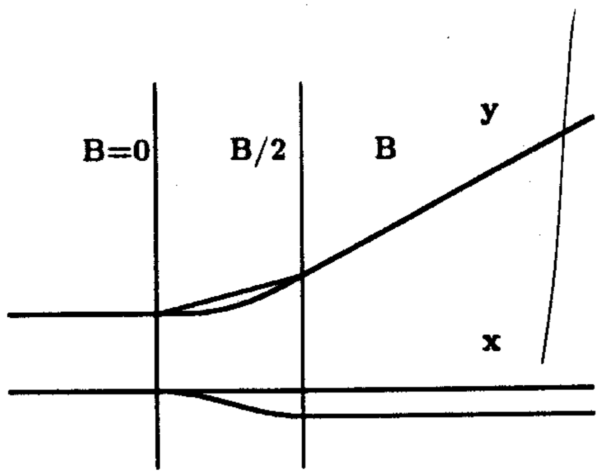
There are two ways of mitigating this problem. The first is to arrange that the length of the bend is an exact integer times the betatron (Lamor) wavelength:



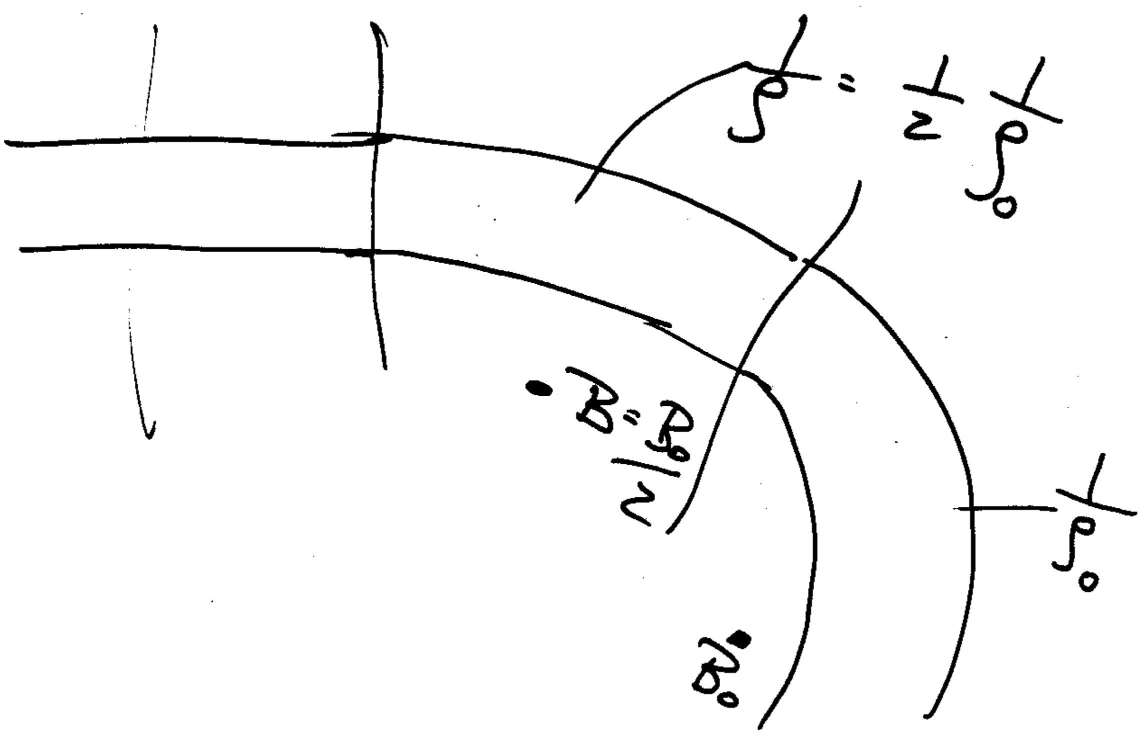
s (along trajectory)

The second method (proposed by J. Norem) is to initiate the bend in two steps. For the first half betatron wavelength the bend curvature and corresponding dipole field are set at half their final values. Only then is the full curvature and field applied. At the end of the bend, the field is similarly removed in the two stages.

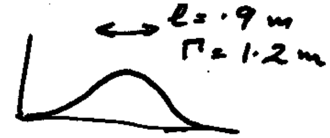




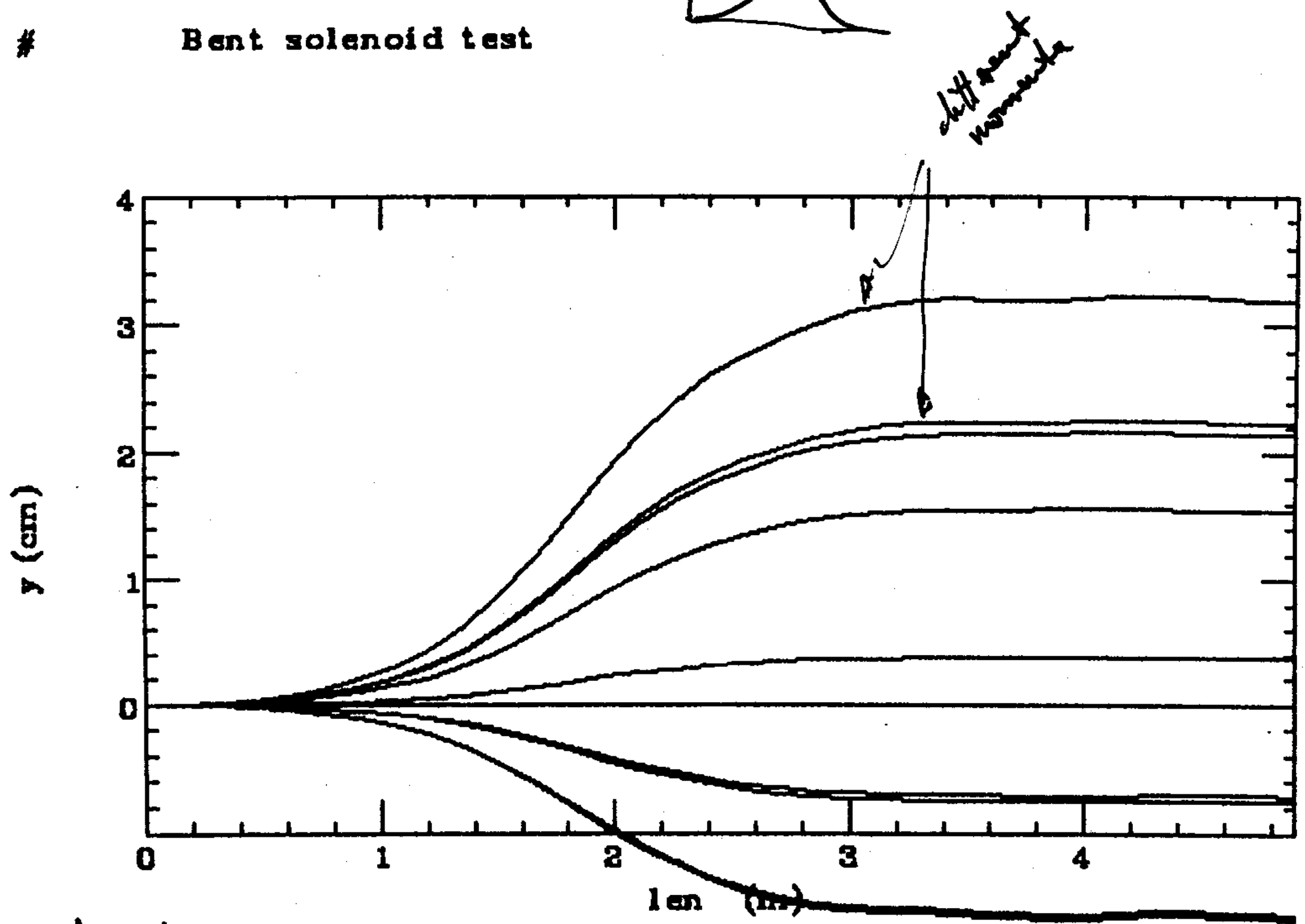
s (along trajectory)

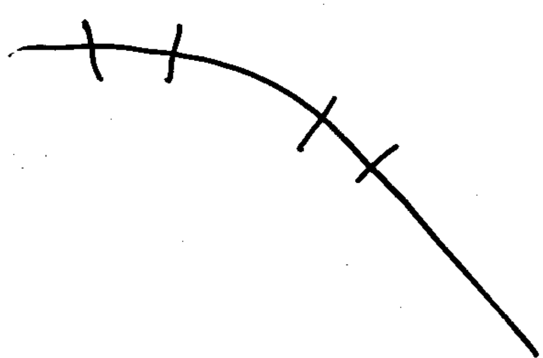


EXAMPLE with $B_\perp$ increases gently

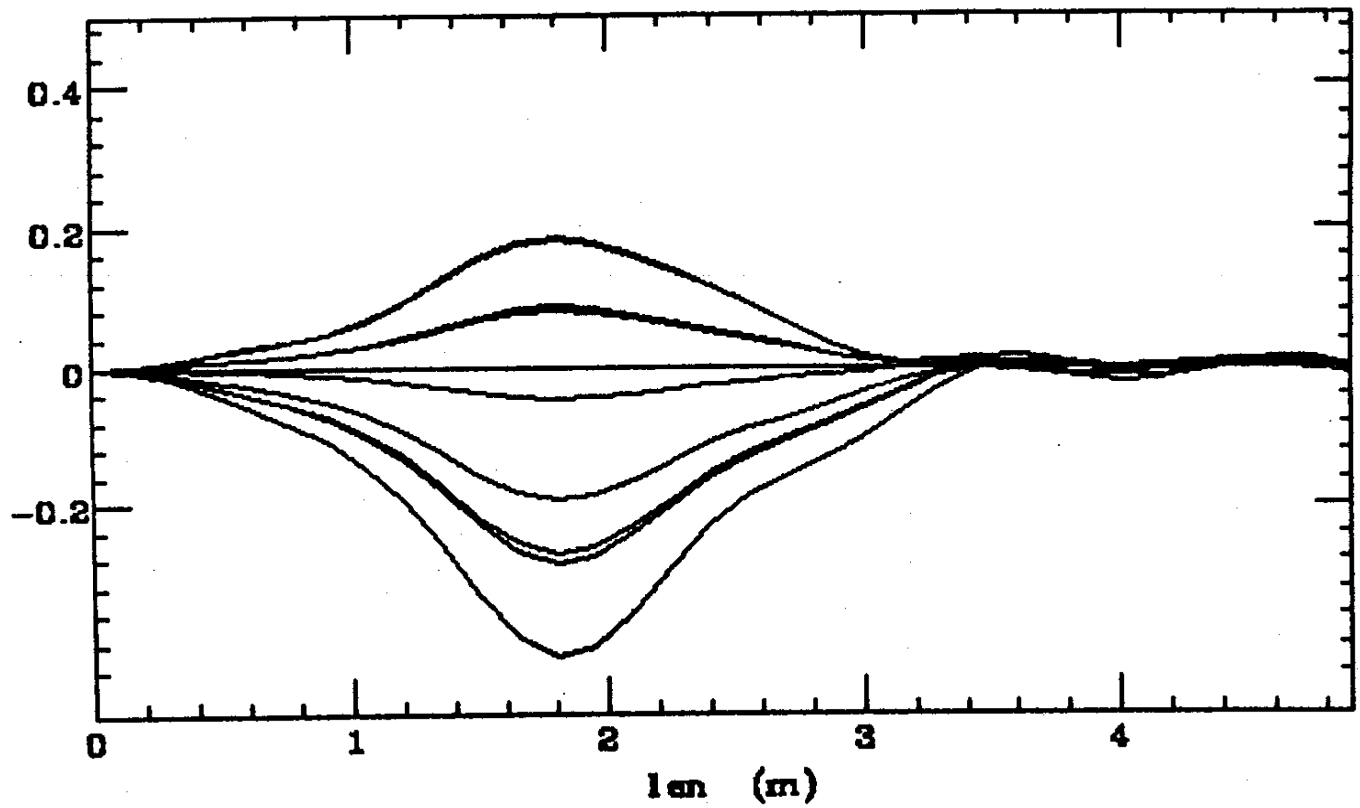


Bent solenoid test

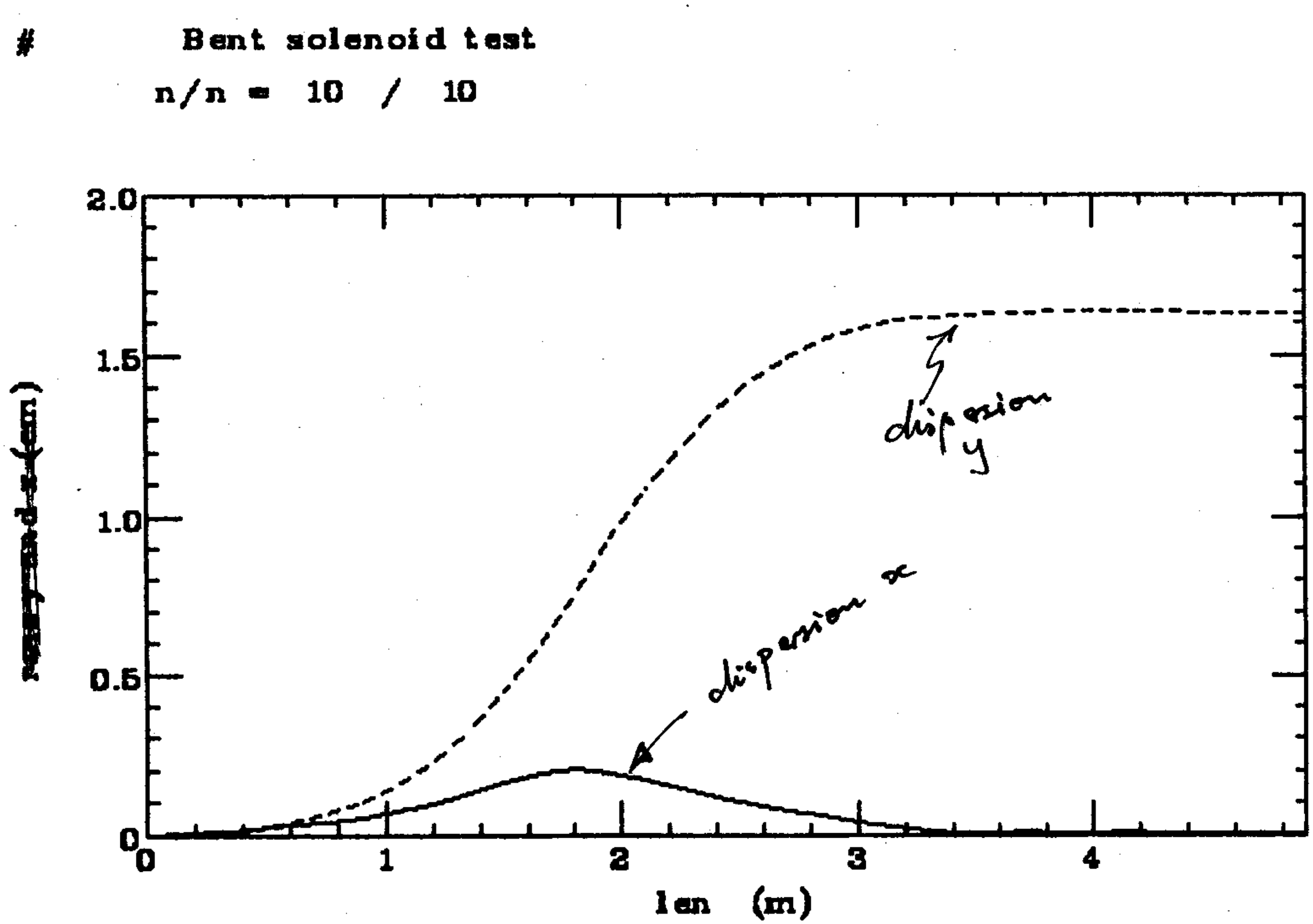




Bent solenoid test

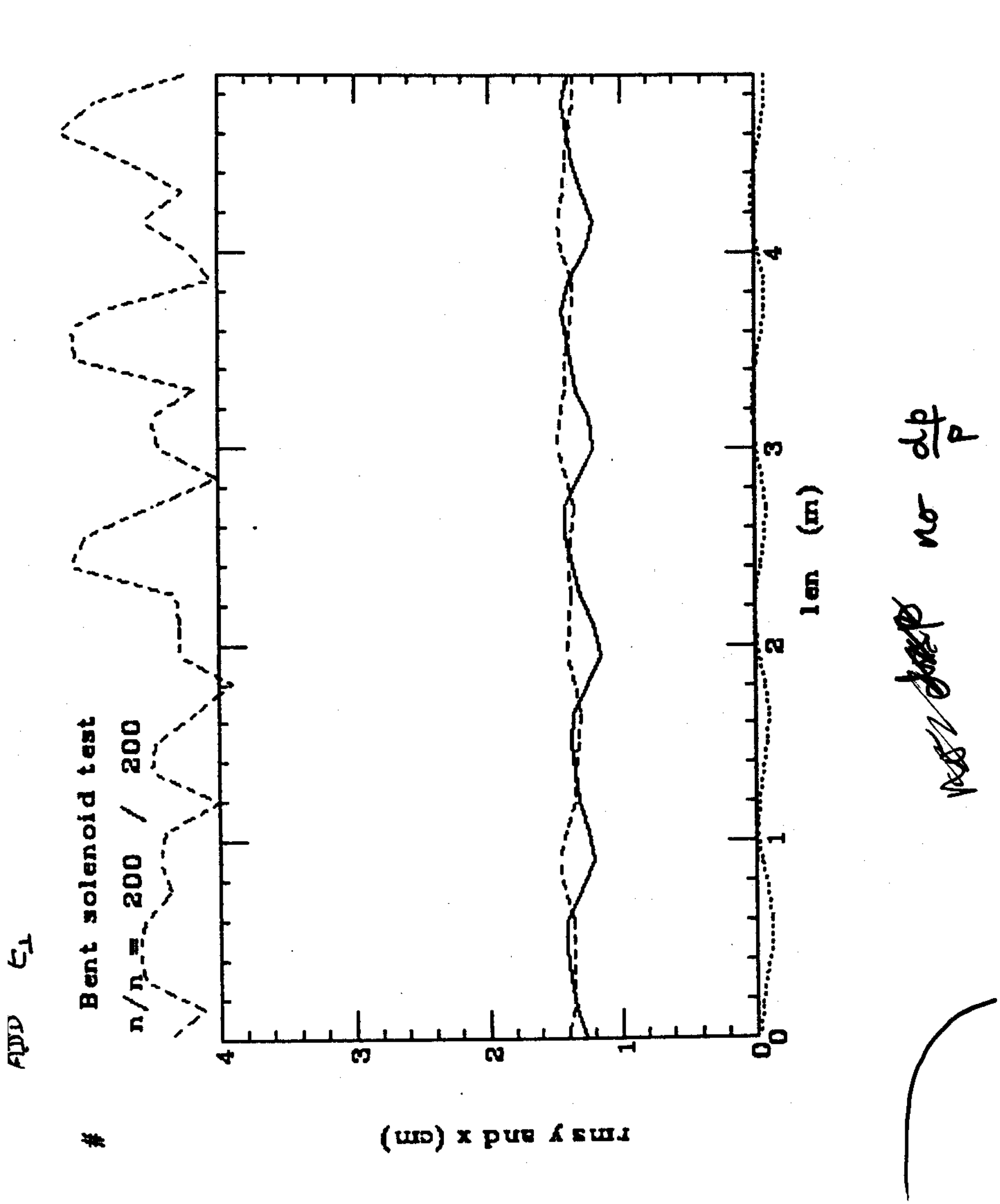


$l = .9$
$M = 1.2$



#
Bent solenoid test
n/n = 10 / 10
(cm)
2.0
1.5
1.0
0.5
0
0
1
2
3
4
len (m)
dispersion y
dispersion x

Example



ADD $\epsilon_\perp$
#
Bent solenoid test
n/n = 200 / 200
rms y and x (cm)
len (m)
0
1
2
3
4
no $\frac{dp}{p}$

9.6 Bent Solenoid Design

The bent solenoid itself is straight forward, but there are different ways to apply the dipole fields. One method is to superimpose $\cos(\theta)$ windings over the solenoid. This would give a flat field. It could be modified to give a field gradient by either adding quadrupole coils, or modifying the dipole windings.

A perhaps simpler way would be to tip the solenoid coils as in a bookshelf. This would automatically give a dipole whose strength varies as $1/R$. If the flat field is essential then quadrupole windings could be added, or the solenoid coils could be pretzeled: their tilt angle would vary with R, with less tilt on the inside and more on the outside. More study is needed.