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Chapter 9, the first on cooling, of lectures to be given at Acc School in Vanderbilt in Jan 99, by R.B.Palmer.

Emittance Exch. & Full System

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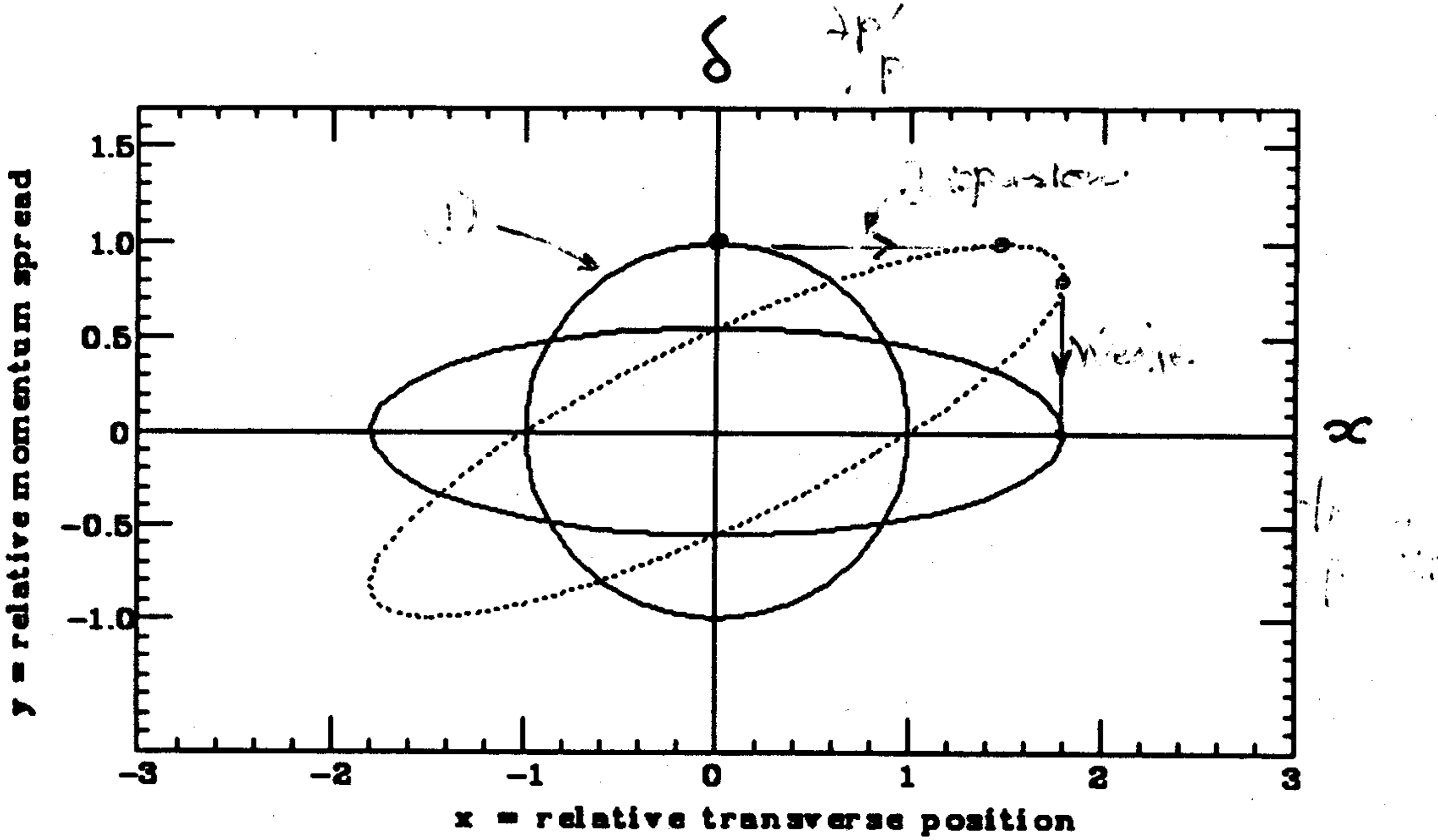
1a



δ
x
1.5
1.0
0.5
0
−0.5
−1.0
y = relative momentum spread
−3
−2
−1
0
1
2
3
x = relative transverse position

9.1 Emittance Exchange

Consider an initial upright ellipse with amplitudes a_o in y and δ_o in dp/p. If we introduce a dispersion D, then the maximum dp/p point will be displaced by

$$\Delta a = D\,\delta_o$$

and the new maximum a_1

$$a_1 \;=\; \sqrt{a_o^2 \;+\; (D\delta_o)^2} \;=\; a_o\,\sqrt{1 \;+\; \alpha_\delta^2}$$

where

$$\alpha_\delta \;=\; \frac{D\,\delta_o}{a_o}$$

This α_δ is like the Courant-Schneider alpha parameter, but in this new dp/p vs y space.

The dp/p at the point of maximum x amplitude, δ^*, is

$$\delta^* \;=\; \delta_o\,\frac{D\delta_o}{1} \;=\; \frac{\delta_o}{\sqrt{1+\alpha^2}}$$

We now pass through a wedge with central momentum loss Δp_o with a wedge angle such that

$$\Delta p \;=\; -\,(\delta p_o \;+\; \chi y)$$

As a result the ellipse is again rotated such that the new dp/p at the point of maximum amplitude, $\delta^{*\prime}$, is

$$\delta^*(new) \;=\; \delta^* \;+\; \chi\cdot a_1$$

If we choose $\delta^* \;=\; -\chi\;a_1$ then the ellipse will again be upright, but will now have a smaller dp/p:

$$\delta_2 \;=\; \delta_o\,\frac{a_o}{a_1} \;=\; \frac{\delta_o}{\sqrt{1 \;+\; \alpha_\delta^2}} \qquad (1)$$

Thus we have performed a phase rotation in dp/p y space: an emittance exchange between longitudinal and transverse emittances.

Note that in this case the final ellipse is upright and has no dispersion. If an attempt is made to remove dispersion, it will tilt the ellipse backward and generate a negative dispersion.

However, it is possible to chose an intermediate wedge that leaves an appropriate tilt to the ellipse that is removed by the application of dispersion opposite to that used initially. The situation is analogous to strong focusing in a FODO cell, where equal and opposite focusing strengths still have a net focus effect.

9.2 How to Generate Dispersion

Conventionally one would use a dipole bending magnet. This may be possible here, but is difficult in view of the very large angles present in the beam. A resonant system may be possible, but has not been studied yet. Another alternative is to use bent solenoids, whose angular acceptance is very large, but whose dynamics are relatively complicated, but it is this option which has been most studied.

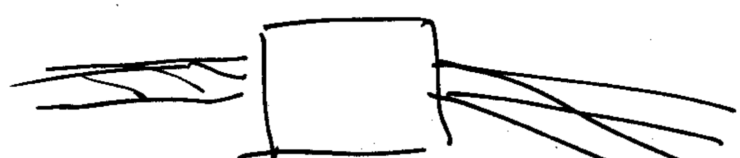
9.2.1 Dipole Magnets

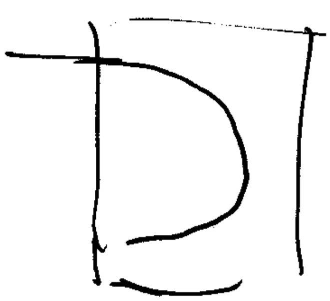
Balbakov
  Sessler

9.2.2 Alternating Resonant Dipole Magnets

9.2.3 Bent Dipoles

Bent solenoids have been used in some low energy accelerators, and are widely used in plasma confinement machines, but they are relatively unused in high energy accelerators. We will therefore digress discuss these in greater detail.



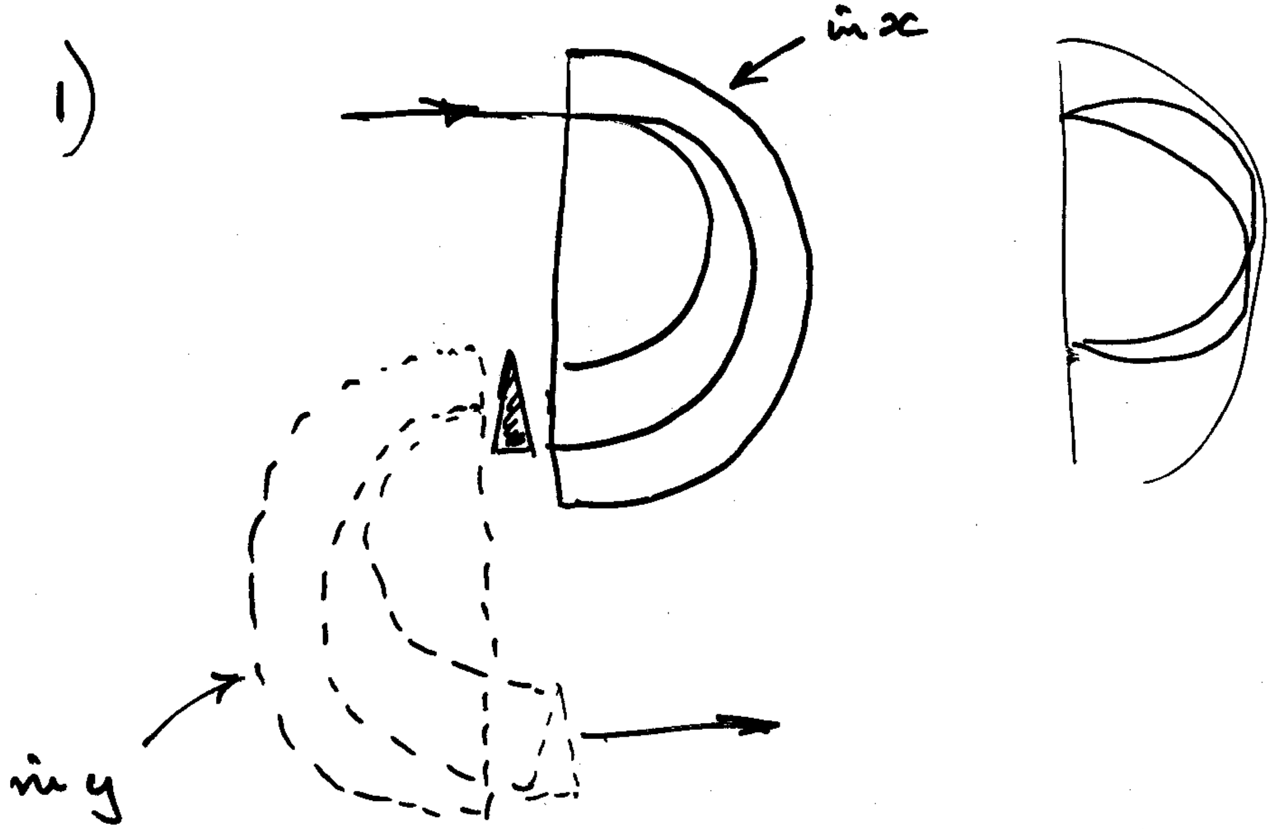


Longitudinal Cooling

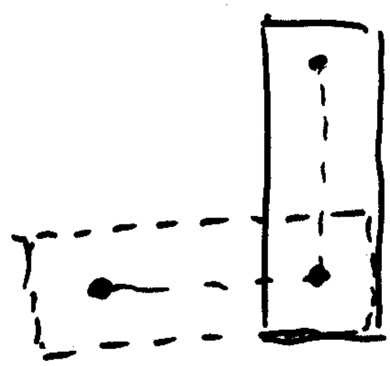
Dispersion $\approx$ 1 m $\times$ 3% $\rightarrow$ 3 cm ✓

How to generate ?

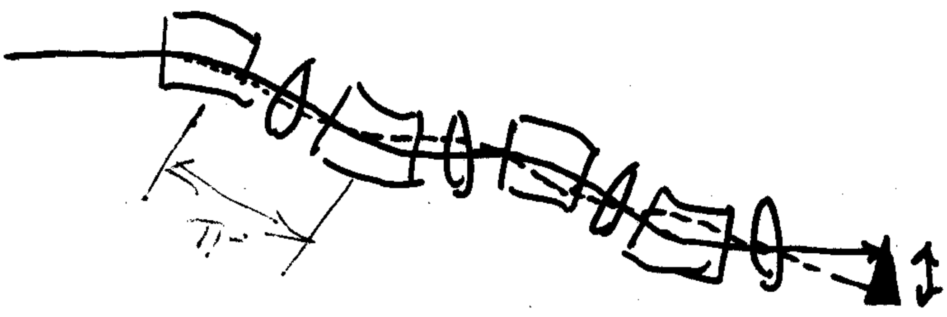
1)



ie end view



2)

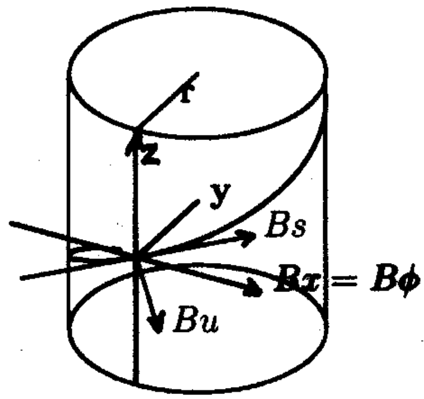


not 1) & 2) for one sign only

9.3 Bent Solenoid Dynamics

9.3.1 Momentum Drift

The dispersion occurs in a direction perpendicular to the plane of the bend, and is, in the plasma community, referred to as a momentum drift.



The field within a bent solenoid, far from its ends, is the same as that around a single vertical current I

$$B_\phi = I/r$$

Consider a particle traveling on the surface of a cylinder of radius r, If we guess the solution to be an upward drift at an angle α:

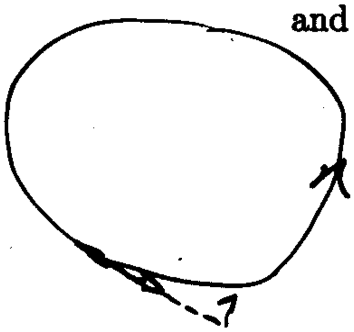
$$B_s = B_\phi \cos(\alpha)$$

$$B_u = B_\phi \sin(\alpha)$$

$$\frac{1}{\rho_s} = \frac{d^2y}{ds^2} = \frac{e}{p_s} B_u = \frac{e}{p_s} B_\phi \sin(\alpha)$$

and since $d(\phi r) = ds \cos(\alpha)$

$$\frac{1}{\rho_{cyl}} = \frac{d^2y}{d(\phi r)^2} = \frac{e}{p_s} B_\phi \frac{\sin(\alpha)}{\cos^2(\alpha)}$$



If

$$\frac{1}{\rho_{cyl}} = \frac{1}{r}$$

then the particle remains on the surface of the cylinder in a smooth upward helix, and the upward helix angle for this condition is:

$$\alpha \approx \frac{sin(\alpha)}{cos^2(\alpha)} = \frac{p_s}{e\,B_\phi\,r} \qquad (2)$$

$$\text{Dispersion} = \frac{\alpha\, s}{p} = \frac{s}{e B_\phi r}$$

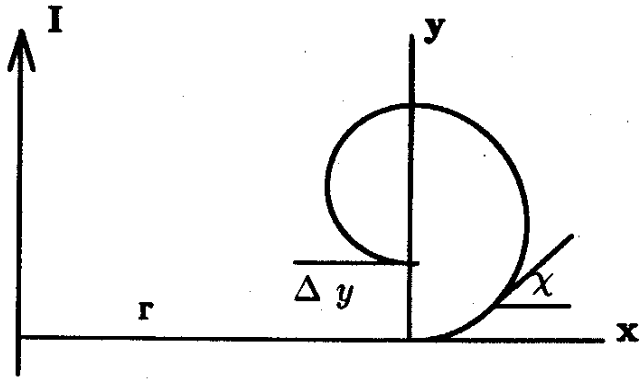
but $\frac{s}{r} = \theta_{turn}$

so $\mathcal{D} = \dfrac{\theta_{turn}}{e B_\phi}$

9.3.2 Amplitude Drift

Particles with finite amplitudes around the reference will also drift upwards at approximately the same angle α but there are some abberations. In particular, there is an additional amplitude dependent drift.

Consider now the situation with little or no momentum in the ϕ direction, but a relatively large initial radial momentum. The particle will remain perpendicular to the filed lines and will, to first order, loop in a circle. But to second order, because the B is not constant, the particle drifts upwards.



The bending radius ρ in the plane (x,y) including the current $I$is given by:

$$\rho = \frac{p}{eB_{\phi}} \propto \frac{p_t\, r}{I}$$

Defining k

$$\rho = k\, r \;=\; k\, (r_o \;+\; x)$$

Defining the track slope $\chi = tan(dy/dx)$:

$$\frac{ds}{d\chi} = \rho$$