Bent Solenoids

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Coupling sections, emittance growth, and drift compensation in the use of bent solenoids as beam transport elements

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Bent solenoids can transmit charged particle beams while providing momentum dispersion. While less familiar than quadrupole and dipole systems, bent solenoids can produce superficially simple transport lines and large acceptance spectrometers for use at low energies. Design issues such as drift compensation and coupling sections between straight and bent solenoids are identified, and aberrations such as shears produced by perpendicular error fields are discussed. Examples are considered which provide the basis for the design of emittance exchange elements for the cooling system of a muon collider. [S1098-4402(99)00043-9]

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I. INTRODUCTION

Solenoidal focusing systems have been used in many low energy, linear beam transport applications for many years [1]. High energy optical systems which will use solenoids with bends are also being considered for transport and cooling of muons in a muon collider [2,3], transport of muons for the study of muon decays [4,5], charge separation [6], and electron cooling of high energy proton and antiproton beams [7].

One of the basic assumptions generally made in high energy beam optics is that **B** fields are perpendicular to the direction of motion. Solenoids, which have a large parallel component of the magnetic field, $B_\parallel$, tend to be used more for low energy beams, since the focusing power K of solenoids goes like $K \sim 1/f \sim p^{-2}$, where f and p are the focal length of the solenoid section and the momentum of the particles. Straight solenoids also have the advantage that they can have large apertures, and are homogeneous and simple. Cases where the beam and the fields are roughly parallel have been worked out in detail in plasma physics examples [8].

Beam optics in straight solenoids is simple and well understood in the context of accelerator beams [1]. The optics of orbits in toroids is also well understood, since toroidal magnetic fields are used in confining plasmas. The dynamics of particle orbits in bent (toroidal) geometries, which show a variety of effects such as cross field drifts, are clearly described in Ref. [8]. Cross field drifts are due to two effects: centrifugal drift and grad B drift. Centrifugal drift is caused by the rigidity of the beam pushing particles to larger bend radii than those followed by the magnetic field lines. Grad B drifts are produced by variations in the magnetic field causing the radius of the Larmor orbit to change as the particle circles the field line. The use of external magnetic fields as well as fields from the beam itself are also common. These effects have been described in the context of plasma confinement systems such as the tokamak and the stellarator. The solutions applicable in plasma physics, however, which involve rotating the whole assembly of particles on its axis, are not generally applicable to high energy beam optics.

The use of both straight and bent elements together is not common, although these systems can do many of the things normally associated with magnetic spectrometers, such as magnetic analysis, charge separation, and transverse projections of longitudinal emittance. The magnetic fields in the transition region between the straight and bend are difficult to calculate and are dependent on physical coil geometry as well as the parameters of the bent and straight sections. Even in simple cases the magnetic field in the transition region can best be calculated numerically. When solenoids with large coils are constrained to follow bends with a small or variable radius, there are further complications and the fields and the effects they produce become even more difficult to evaluate.

This paper attempts to describe the scope of beam optics of bent solenoids, producing algorithms to evaluate aberrations and describing methods to minimize these problems. Considering the superficial simplicity of the problem, it is surprising that there are so many possible variables and effects that can be produced. Since the effects can at first seem nonintuitive, they are described systematically, with ray tracing examples using the code GPT, written by Pulsar Physics [9]. Three-dimensional numerical analysis was used because this method eliminated assumptions. While a deductive exposition starting with $d\mathbf{p}/dt = q\mathbf{v} \times \mathbf{B}$ would be valuable, there is a large range of variables and valid solutions, and this paper is intended to describe the beam effects in a more descriptive way. The approach here has been to produce accurate numerical calculations of the fields and orbits from the geometry of coils and currents, without considering approximations or analytical models.

The focus of this paper is on developing the basic principles which can be used for the design of compact high dispersion systems for longitudinal emittance exchange and momentum measurements for experimental muon

Why Bent Solenoids?

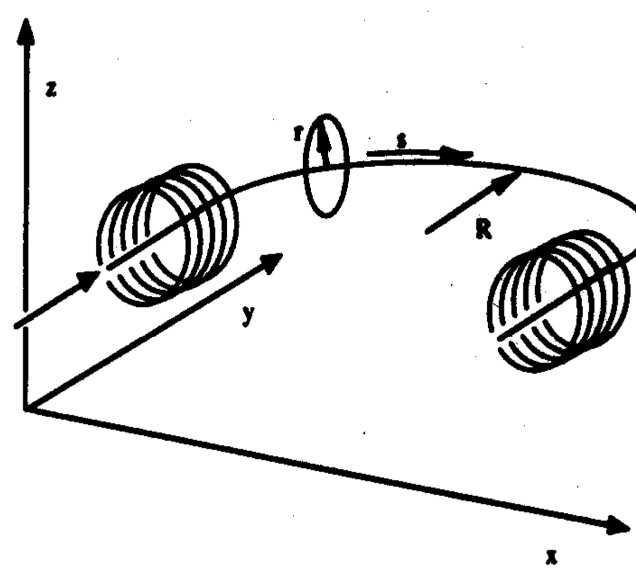
- Alternatives seemed messy
  - mixed solenoid and quads, benders
- Can be large aperture
- Seem "Simple"

Why Current Loops?

- Tried $B = B_0(1 + hx)$ etc, didn't seem to handle transition region

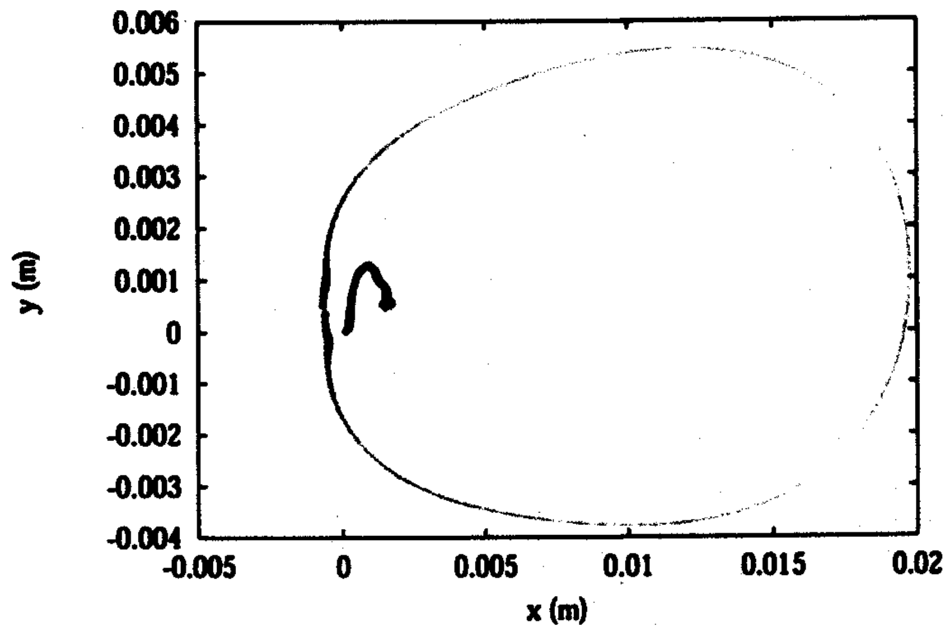
Bent Solenoids are unexpectedly complex

- Although superficially simple, the design of a bent solenoid involves many variables.
  - Bend radius
  - Coil radius
  - Bend / Straight coupling
  - Total flux through bend
  - Dipole field magnitude
  - Vertical field geometry
    - Tipped coils
    - Saddle coils
  - Coil Dimensions, shapes etc
  - . . .



- Even when all coils were optimized and all corrections were applied, (tipped coils, etc.), the orbits depended on "irrelevant" parameters like the coil radius.

- With a R=1 m bend, two coil radii were used, r=0.3 and r=0.03 m. The results are qualitatively different.



- Bent solenoid optics are complex because the optics are inherently nonlinear and non-symmetric, and dependent on a number of non obvious variables.

- Many aspects of optics in bent solenoids seemed non-intuitive, and it seemed desirable to explore some aspects of the problem.

- When starting, (in 8/97), it seemed the Hamiltonian approach did not have enough variables, and most approximations of coil geometries were not usefully precise.

- A commercial code, GPT, seemed to be reliable and relatively easy to use, and saved debugging time on my own code.

- This code was primarily used to track a cold beam through a bend to look at effects which raised the emittance.

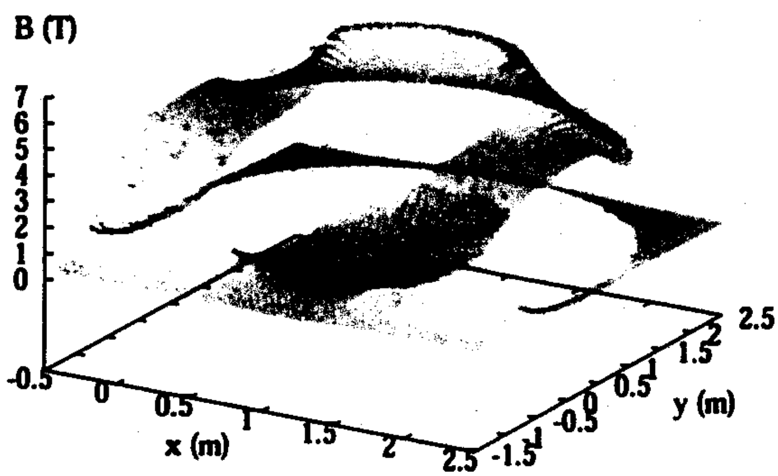
Bent Solenoid Fields

- The field in a straight solenoid is flat, and the field in a bend goes like 1/R, therefore field lines tend to move to the inside of the bend.
  - The reorganization depends on the shape of the coils
  - rectangular coils simpler to calculate, hard to build.

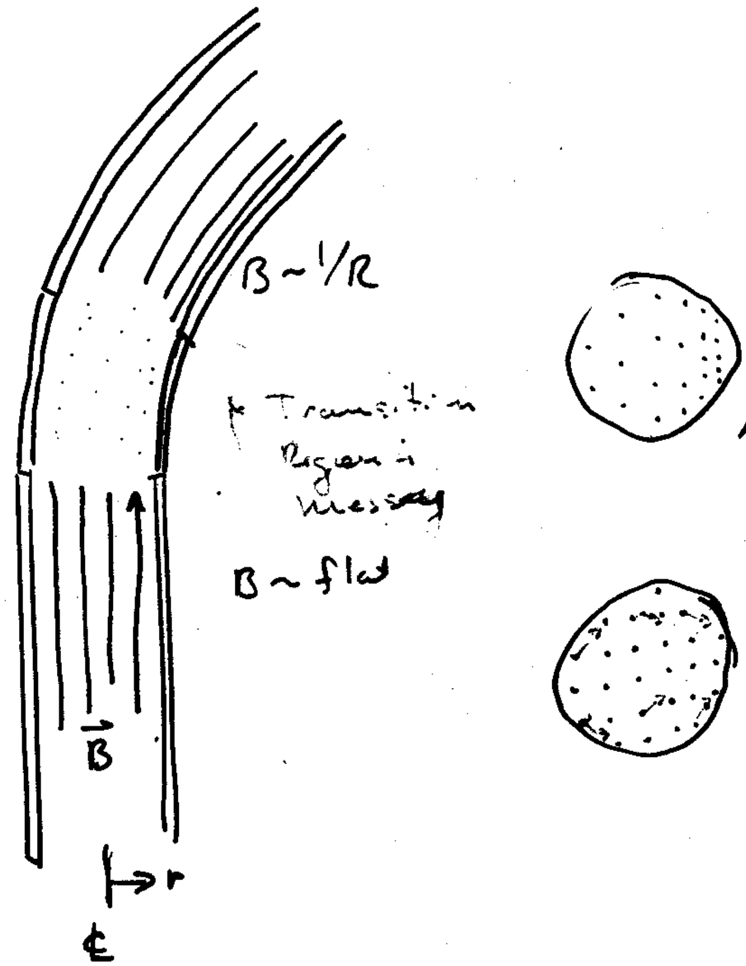
- The total flux, $\Phi=\int B dA$ and centerline field B(r=0), going through the bend are not necessarily equal to those in the straight. This effect gets worse as r/R gets large. There will be fringe fields at the ends of bends.

- Vertical fields produce fringe fields in the straights.

- All these effects remove symmetry from the problem.



The Transition between Straight & Bend



In three dimensions it is curved. Field lines move in 3 dims

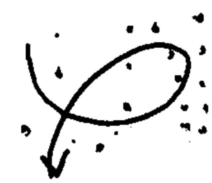
In general: $\Phi = \int_{\text{Straight}} \vec{B} \, d\vec{A} \neq \Phi \int_{\text{bend}} \vec{B} \cdot dA$

$$B_{st}(r=0) \neq B_{bend}(r=0)$$

Particle Orbits

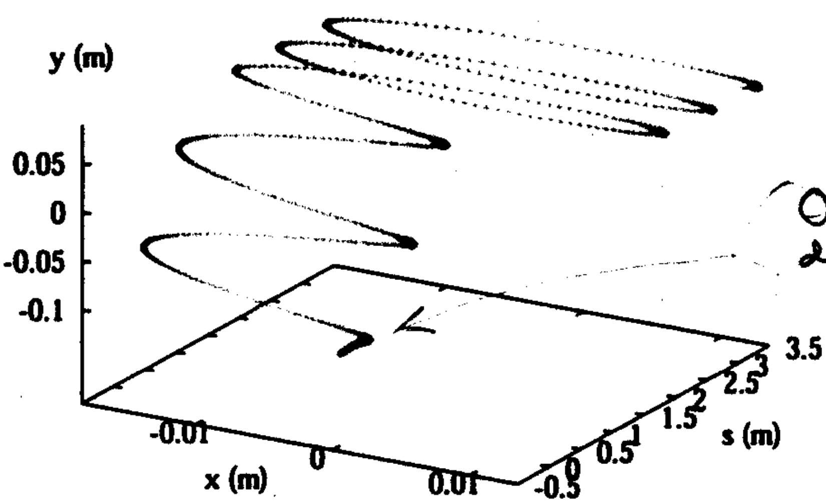
- There are two sources of vertical drift in bent solenoids so motion is nonlinear.
  - Centrifugal drift: $v_c=(\gamma m v_\parallel^2/qB^2)\ \mathbf{RxB} / R^2$
  - Grad B drift: $v_g=\pm 0.5\ v_\perp \mathbf{Rx\nabla B} / B^2$
  - Total drift = $(\gamma m/qR^2B^2)\ \mathbf{RxB}\ (v_\parallel^2 + 0.5\ v_\perp^2)$

$\vec{v} = \frac{1}{q} \frac{\vec{F} \times \vec{B}}{B^2}$



- Paraxial rays are affected by centrifugal drift, but orbits at large divergence are also subject to Grad B drift.

- Particles entering a bend follow a cusp shaped orbit without a vertical field.



"Orbits in solenoid don't look discontinuous"

F. Chen "Introduction to Plasma Physics"

- Particles want to follow field lines, however:
  - Centrifugal forces will offset the orbits from the lines.
  - The field lines themselves move.
- Coupling sections designed to minimize the effects of uncompensated centrifugal bends seem to also work to minimize the effects of moving field lines.
- While coupling sections can work well for a limited range of momenta, (Larmor lengths), it is more difficult to design a coupling section which works over a large momentum range

Coupling Sections

- Coupling sections can minimize orbit perturbations introduced by discontinuities.

  - Adiabatic couplers: Smooth Transitions in principle produce little emittance growth, but are very expensive in hardware and control of the beam

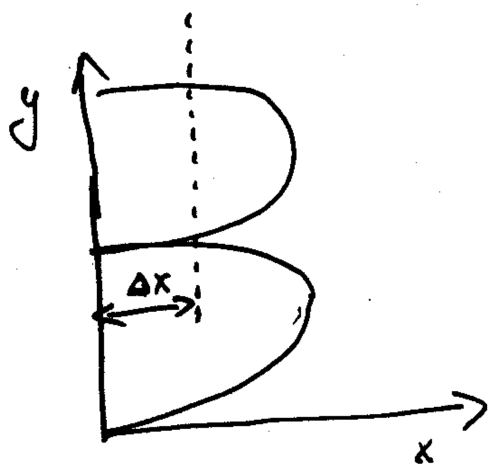
  - $L = \lambda_L/2$ Couplers: If the bend radius is twice the normal bend radius for one half a larmor period, the orbits can be made to smoothly blend with the orbits in the bent sections

  - Smooth $\lambda_L/2$ Couplers: A smooth bend profile used by Fernow also introduces minimal emittance growth in particle orbits.

- It seems that these coupling sections also damp emittance growth due to motion of field lines in the transition between straight and bend.

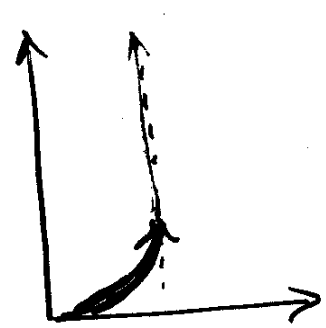
Coupling Sections

- Without a vertical field the orbit follows a cusp trajectory

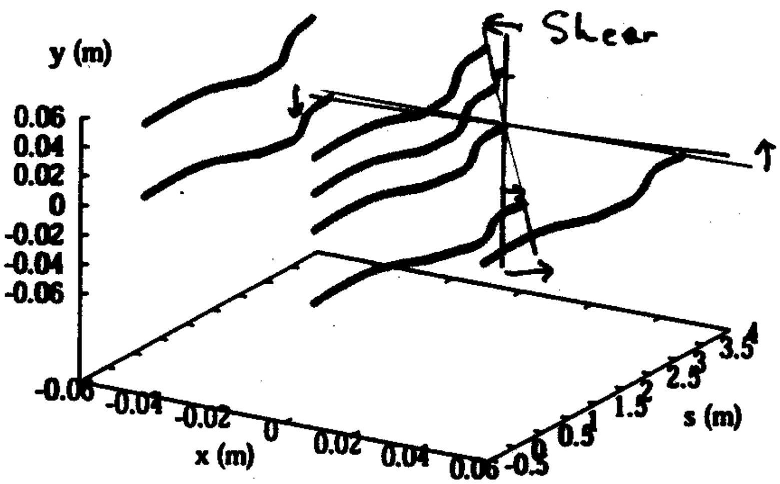


- This trajectory is composed of two drifts
  - an offset, ΔX, vertical drift
  - circular motion with $r = \Delta X$ around the offset drift

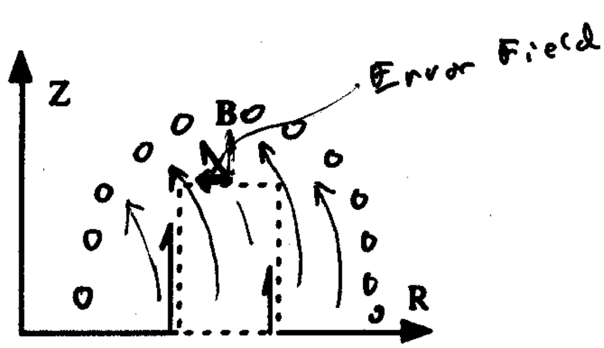
- When a short ($\lambda/2$) section is introduced so that the maximum motion in x is equal to the offset ΔX before the main bend, there are no subsequent oscillations.



- When a system is carefully designed, (with tipped coils producing a vertical field and compensating sections damping out centrifugal and magnetic field rearrangement, i.e. everything done right), one finds that the particles are still perturbed.



- These are caused by the conflicting requirements of optics, which demand that $B_v \sim F_R$, and Maxwell's equations, which demand that B_v = const, when no horizontal fields are present.
  - Maxwell wants $\int B dl = 0$.
  - $B_z \sim 1/R^n \Rightarrow \int B dl \neq 0$.



It might be interesting to combine

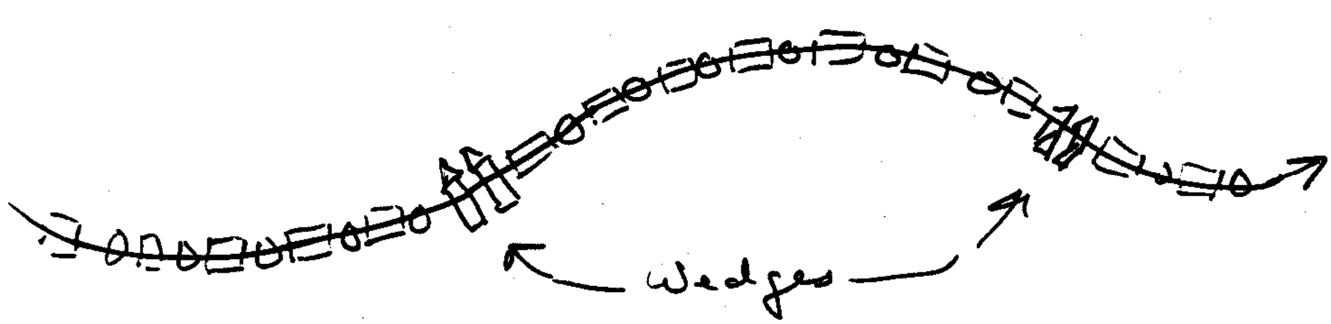
A Single Flip Channel - Damps $\varepsilon_\perp$
- Confines ε_L
- Cannot damp ε_L
- Has space for rf
- No Resonances, wide $\Delta P/P$

Plus

A "Gentle Bend" Emit Exch - Damps ε_L
- Cannot confine ε_L without rf
- Space for rf limited if R small
- Some $\varepsilon_\perp$ growth (Small R effect)

A Large Bend radius R has some advantages:
Growth in $\varepsilon_\perp$ seems due to small bend Radius. (?)
- Growth in $\varepsilon_\perp$ seems due to small bend Radius
- Dispersion is a function of bend angle, not Radius
- Low losses (?)

This system would provide "continuous" $\varepsilon_\perp$ + ε_L cooling.



Conclusions

- Bent solenoids are complex magnetic elements. Since there is less symmetry, the magnet coil parameters, fringe fields and coupling sections can be varied in a large variety of ways.

- The beam optics is also complicated. Centrifugal and grad B drifts are functions of different things. Has anyone one in HEP has ever used grad B drifts?

- Bent solenoids are not black boxes. Understanding the processes involved seems to offer the possibility of minimizing specific effects.

- Looking at single particle orbits produced from realistic coil geometries can help understand the processes involved.