FORMULATION

- Approximated Hamiltonian in accelerator frame of reference

$$H = -A_s(1+hx) - hx(1+\delta) - \frac{(1+hx)}{2(1+\delta)}\{(p_x - A_x)^2 + (p_y - A_y)^2\}\cancel{\frac{1}{2}}$$

s independent variable, $\frac{\vec{p}}{p_o}$ normalized momentum, $\vec{A}$ normalized vector potential and $\delta = \frac{p}{p_o} - 1.$

- $(x, p_x); (y, p_y)$ and (z, δ) are canonical variables.

- Metric $d\sigma^2 = dx^2 + dy^2 + (1+hx)^2 ds^2$ for horizontal bending.

- We need the vector potential. The magnetic and electric field are obtained from $\vec{B} = \mathrm{curl}\vec{A}$; $\vec{E} = c\beta_o \frac{\partial \vec{A}}{\partial z}$ (recall the metric)

⇛ Bend solenoid (horizontal as an example)

$$\vec{A}_s = -\frac{1}{2} B_o \frac{y}{(1+hx)} \vec{e_x} + \frac{B_o}{2h} \ln(1+hx) \vec{e_y}$$

it satisfy $\mathrm{div}\vec{\mathrm{A}} = 0$ and the longitudinal magnetic field is $B_s = \frac{B_o}{(1+hx)}$.

⇛ Guiding Dipole $\vec{A}_D = -\frac{1}{2}(1+hx)\vec{e_s}$ to compensate the natural drift due to the toroidal solenoid, but keeping Dispersion.

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9/14/2000

Curved rf. cavity.

General expressions straight cavity.

$$E_s = \sum_{-\infty}^{+\infty} -i\, E_n\, J_0(k_{rn} r)\, e^{i(\omega t - k_{zn} s)}$$

$$E_r = \sum_{-\infty}^{+\infty} \frac{k_{zn}}{k_{rn}} E_n\, J_1(k_{rn} r)\, e^{i(\omega t - k_{zn} s)}$$

$$B_\varphi = \sum_{-\infty}^{\infty} \frac{\omega}{c^2 k_{rn}} E_n\, J_1(k_{rn} r)\, e^{i(\omega t - k_{zn} s)}$$

$$\frac{\omega^2}{c^2} = k_{rn}^2 + k_{zn}^2$$

Write vector potential

$$A_s = \sum_{-\infty}^{+\infty} \frac{E_n}{\omega} J_0(k_{rn} r)\, e^{i(\omega t - k_{zn} s)} \approx$$

$$\sum_{-\infty}^{+\infty} \frac{E_n}{\omega}\left(1 - \frac{1}{4} k_{rn}^2 r^2\right) e^{i(\omega t - k_{zn} s)}$$

$$A_r = \sum_{-\infty}^{+\infty} \frac{i}{\omega} \frac{k_{zn}}{k_{rn}} E_n\, J_1(k_{rn} r)\, e^{i(\omega t - k_{zn} s)} \approx$$

$$\sum_{-\infty}^{+\infty} \frac{i}{\omega} k_{zn} r \frac{E_n}{2}\, e^{i(\omega t - k_{zn} s)}$$

Recall $\vec{E} = -\dfrac{\partial \vec{A}}{\partial t}$ $\quad \vec{B} = \nabla \wedge \vec{A}$

and $\nabla . \vec{A} = 0$ Coulomb gauge.

Horizontal bending, (different metric)

$$\nabla . \vec{A} = \frac{1}{1+hx} \frac{\partial}{\partial x}\left[(1+hx) A_x\right] + \frac{\partial}{\partial y} A_y + \frac{1}{1+hx} \partial_s A_s = 0$$

Find vector potential function $\vec{A}$ with Constrains

1) for $h \to 0$ we must get

$$A_x = \frac{i}{\omega} k_z \frac{E_0}{2} e^{i(\omega t - k_z s)} \quad x$$

$$A_y = \quad '' \quad y$$

$$A_s = \frac{E_0}{\omega} e^{i(\omega t - k_z s)}$$

2) $\nabla . \vec{A} = 0$

and then $\vec{E} = -\dfrac{\partial \vec{A}}{\partial t}$ $\quad \vec{B} = \nabla \wedge \vec{A}$

Assume

$$A_x = \frac{\mathcal{A}_x}{1+hx} \quad ; \quad A_y = \frac{\mathcal{A}_y}{1+hx}$$

$$A_s = \mathcal{A}_s$$

Fields

$$E_s = E_s$$

$$E_x = \frac{E_x}{1+hx}$$

$$B_x = -i k_z B_y$$

$$B_y = -i k_z B_x - \frac{h}{1+hx} A_s$$

$$B_s = 0$$